Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (mm 201y), pp. 1–xx
ISSN 2093–9310
http://www.afmi.or.kr

Intuitionistic Fuzzy Soft Matrix and Its Application in Decision Making Problems

Tanushree Mitra Basu, Nirmal Kumar Mahapatra, Shyamal Kumar Mondal

Received 12 August 2011; Revised 29 September 2011; Accepted 07 October 2011

Abstract. In this paper we have introduced the concept of intuitionistic fuzzy soft matrix and defined different types of matrices along with some new operations in the parlance of intuitionistic fuzzy soft set theory. Then based on some of these new matrix operations a new efficient methodology named as $IFSM$-Algorithm has been developed to solve intuitionistic fuzzy soft set based group decision making problems and apply it in medical science to the problems of diagnosis of a disease from the myriad of symptoms as well as to evaluate the effectiveness of different habits of human being responsible for a disease.

2010 AMS Classification: 03B52, 03G24, 06D72

Keywords: Intuitionistic Fuzzy Soft Matrix, Complement, Transpose, Choice Matrix, Addition, Subtraction, Product, $IFSM$-Algorithm.

Corresponding Author: Nirmal Kumar Mahapatra (*nirmal_hridoy@yahoo.co.in*)

1. INTRODUCTION

In real life scenario, we frequently deal with information which is sometimes vague, sometimes inexact or imprecise and occasionally insufficient. Zadeh's classical concept of fuzzy sets [6] is a strong mathematical tool to deal with such type of problems. Since the initiation of fuzzy set theory, there are suggestions for non-classical and higher order fuzzy sets for different specialized purposes. Among several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) introduced by Atanassov[5] have been found to be very useful and applicable. In the year 1999, Molodtsov [1] presented soft set as a completely generic mathematical tool for modeling uncertainties. Since there is hardly any limitation in describing the objects, researchers simplify the decision making process by selecting the form of parameters they require and subsequently makes it more efficient in the absence of partial information. Maji et al.[2][3] have done further research on soft set theory. Presence of vagueness demanded Fuzzy

Soft Set (FSS) [4] to come into picture. But satisfactory evaluation of membership values is not always possible because of the insufficiency in the available information (besides the presence of vagueness) situation. Evaluation of non-membership values is also not always possible for the same reason and as a result there exists an indeterministic part upon which hesitation survives. Certainly fuzzy soft set theory is not suitable to solve such problems. In those situations Intuitionistic Fuzzy Soft Set theory (IFSS)[7] may be more applicable. In the year 2001, Maji et al.[7] have introduced the concept of **intuitionistic fuzzy soft set**. Moreover they have proposed an algorithm for solving intuitionistic fuzzy soft set based decision making problem and introduced the concept of intuitionistic fuzzy soft relation.
In 2010, Cagman et al.[10] introduced a new soft set based decision making method which selects a set of optimum elements from the alternatives. In the same year, the same authors [9] have proposed the definition of **soft matrix** which is the representation of a soft set. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer.

In this paper we have introduced the concept of **intuitionistic fuzzy soft matrix** and defined different types of matrices along with some new operations in the parlance of intuitionistic fuzzy soft set theory. Then based on some of these new matrix operations a new efficient methodology named as $IFSM$**-Algorithm** has been developed to solve intuitionistic fuzzy soft set based group decision making problems and apply it in medical science to the problems of diagnosis of a disease from the myriad of symptoms as well as to evaluate the effectiveness of different habits of human being responsible for a disease.

2. Preliminaries

Definition 2.1. [1] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U. Let $A \subseteq E$. A pair (F_A, E) is called a **soft set** over U, where F_A is a mapping given by, $F_A : E \longrightarrow P(U)$ such that $F_A(e) = \phi$ if $e \notin A$.
Here F_A is called approximate function of the soft set (F_A, E). The set $F_A(e)$ is called e-approximate value set which consists of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Example 2.2. Let U be the set of four dresses, given by, $U = \{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by,
$E = \{$ costly, cheap, comfortable, beautiful, gorgeous $\} = \{e_1, e_2, e_3, e_4, e_5\}$, where
e_1 stands for the parameter 'costly',
e_2 stands for the parameter 'cheap',
e_3 stands for the parameter 'comfortable',
e_4 stands for the parameter 'beautiful',
e_5 stands for the parameter 'gorgeous'.
Let $A \subset E$, given by, $A = \{e_1, e_2, e_3, e_4\}$ Now suppose that, F_A is a mapping, defined as "dresses(.)"and given by, $F_A(e_1) = \{d_2, d_4\}, F_A(e_2) = \{d_1, d_3\}, F_A(e_3) = \{d_2, d_3\}$,

$F_A(e_4) = \{d_4\}$. Then the soft set

$$\begin{aligned}(F_A, E) \quad = \quad & \{ \text{ costly dresses } = \{d_2, d_4\}, \text{cheap dresses} = \{d_1, d_3\}, \\ & \text{comfortable dresses} = \{d_2, d_3\}, \text{ beautiful dresses } = \{d_4\}, \\ & \text{gorgeous dresses } = \phi\}\end{aligned}$$

Definition 2.3. [4] Let U be an initial universe set and E be a set of parameters(which are fuzzy words or sentences involving fuzzy words). Let F^U denotes the set of all fuzzy subsets of U. Let $A \subset E$. Then a pair $(\tilde{F}_A, E)$ is called a **Fuzzy Soft Set** (FSS) over U, where $\tilde{F}_A$ is a mapping given by, $\tilde{F}_A : E \longrightarrow F^U$ and $\tilde{F}_A(e) = \tilde{\phi}$ if $e \notin A$ where $\tilde{\phi}$ is a null fuzzy set.

Example 2.4. Let U be the set of four ponds, say, $U = \{P_1, P_2, P_3, P_4\}$. Let E be the set of parameters where each parameter is a fuzzy word, given by,
$E = \{$ huge, average, moderate, poor $\}$.
Let $A \subset E$, given by, $A = \{$ huge, average, poor $\} = \{e_1, e_2, e_3\}$ where
e_1 stands for the parameter 'huge',
e_2 stands for the parameter 'average',
e_3 stands for the parameter 'poor'.
Now suppose that,
$\tilde{F}_A(e_1) = \{P_1/.8, P_2/.2, P_3/.5, P_4/.4\}$,
$\tilde{F}_A(e_2) = \{P_1/.3, P_2/.2, P_3/.9, P_4/.7\}$,
$\tilde{F}_A(e_3) = \{P_1/.2, P_2/.7, P_3/.8, P_4/.5\}$.

Then the fuzzy soft sets $(\tilde{F}_A, E)$ describing "the commercial benefit from the ponds" is given by,

$$\begin{aligned}(\tilde{F}_A, E) \quad = \quad & \{ \text{ huge beneficial pond } = \{P_1/.8, P_2/.2, P_3/.5, P_4/.4\}, \\ & \text{average beneficial pond} = \{P_1/.3, P_2/.2, P_3/.9, P_4/.7\}, \\ & \text{moderate beneficial pond} = \tilde{\phi}, \\ & \text{poor beneficial pond} = \{P_1/.2, P_2/.7, P_3/.8, P_4/.5\}\}\end{aligned}$$

Definition 2.5. [7] Let U be an initial universe set and E be a set of parameters which are intuitionistic fuzzy words or sentences involving intuitionistic fuzzy words. Let IF^U denotes the set of all intuitionistic fuzzy sets of U. Let $A \subset E$. A pair $(\hat{F}_A, E)$ is called an **Intuitionistic Fuzzy Soft Set**(IFSS) over U, where $\hat{F}_A$ is a mapping given by, $\hat{F}_A : E \longrightarrow IF^U$ and $\hat{F}_A(e) = \hat{\phi}$ if $e \notin A$ where $\hat{\phi}$ is null intuitionistic fuzzy set(i.e., the membership value of x, $\mu(x) = 0$; the non-membership value of x, $\nu(x) = 1$ and the indeterministic part of x, $\pi(x) = 0 \forall x \in \hat{\phi}$).

Example 2.6. Let U be the set of four cities, given by, $U = \{C_1, C_2, C_3, C_4\}$.
Let E be the set of parameters where each parameter is an intuitionistic fuzzy word, given by, $E = \{$ immensely, highly, moderately, average, less $\}$. Let $A \subset E$, given by,
$A = \{$ immensely, highly, moderately, less $\} = \{e_1, e_2, e_3, e_4\}$ where
e_1 stands for the parameter 'immensely',

e_2 stands for the parameter 'highly',
e_3 stands for the parameter 'moderately',
e_4 stands for the parameter 'less'.

Now suppose that,
$\hat{F}_A(e_1) = \{C_1/(.1, .7), C_2/(.7, .1), C_3/(.3, .6), C_4/(.4, .6)\}$,
$\hat{F}_A(e_2) = \{C_1/(.2, .7), C_2/(.8, .1), C_3/(.4, .2), C_4/(.6, .3)\}$,
$\hat{F}_A(e_3) = \{C_1/(.3, .5), C_3/(.8, .1), C_4/(.7, .2)\}$,
$\hat{F}_A(e_4) = \{C_1/(.9, .1), C_2/(.1, .8), C_3/(.5, .4), C_4/(.3, .5)\}$.

Then the intuitionistic fuzzy soft set

$$\begin{aligned}(\hat{F}_A, E) \quad = \quad \{ & \text{ immensely polluted city} = \{C_1/(.1, .7), C_2/(.7, .1), C_3/(.3, .6), C_4/(.4, .6)\},\\ & \text{highly polluted city } = \{C_1/(.2, .7), C_2/(.8, .1), C_3/(.4, .2), C_4/(.6, .3)\},\\ & \text{moderately polluted city} = \{C_1/(.3, .5), C_3/(.8, .1), C_4/(.7, .2)\},\\ & \text{average polluted city} = \hat{\phi},\\ & \text{less polluted city} = \{C_1/(.9, .1), C_2/(.1, .8), C_3/(.5, .4), C_4/(.3, .5)\}\}\end{aligned}$$

Definition 2.7. [9] Let (F_A, E) be a soft set over universal set of objects U and universal set of parameters E. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u, e) : e \in A, u \in F_A(e)\}$$

which is called a relation form of (F_A, E). Now the **characteristic function** of R_A is written by,

$$\chi_{R_A} : U \times E \longrightarrow \{0, 1\}, \ \ \chi_{R_A} \quad = \quad \begin{cases} 1, (u, e) \in R_A \\ 0, (u, e) \notin R_A \end{cases}$$

Let $U = \{u_1, u_2, ..., u_m\}$, $E = \{e_1, e_2, ..., e_n\}$, then R_A can be presented by a table as in the following form

Table-1: Tabular representation of $\hat{R}_A$

	e_1	e_2	....	e_n
u_1	$\chi_{R_A}(u_1, e_1)$	$\chi_{R_A}(u_1, e_2)$	....	$\chi_{R_A}(u_1, e_n)$
u_2	$\chi_{R_A}(u_2, e_1)$	$\chi_{R_A}(u_2, e_2)$	....	$\chi_{R_A}(u_2, e_n)$
....	....	....	....	....
u_m	$\chi_{R_A}(u_m, e_1)$	$\chi_{R_A}(u_m, e_2)$	....	$\chi_{R_A}(u_m, e_n)$

For simplicity, if $\chi_{R_A}(u_i, e_j)$ is denoted by a_{ij}, then we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ .... & .... & .... & .... \\ a_{m1} & a_{m2} & .... & a_{mn} \end{pmatrix}$$

which is called a **soft matrix** of order $m \times n$ corresponding to the soft set (F_A, E) over U. A soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. Therefore we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

Example 2.8. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of all parameters. If $A = \{e_2, e_3, e_4\}$ and $F_A(e_2) = \{u_2, u_4\}, F_A(e_3) = \phi, F_A(e_4) = U$, then we write a soft set
$(F_A, E) = \{(e_2, \{u_2, u_4\}), (e_4, U)\}$
and then the relation form of (F_A, E) is written by,
$R_A = \{(u_2, e_2), (u_4, e_2), (u_1, e_4), (u_2, e_4), (u_3, e_4), (u_4, e_4), (u_5, e_4)\}$.
Hence the soft matrix (a_{ij}) is written by,

$$(a_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Some New Concepts of Matrices in Intuitionistic Fuzzy Soft Set Theory:

Definition 3.1. Intuitionistic Fuzzy Soft Matrix: Let $(\hat{F}_A, E)$ be an intuitionistic fuzzy soft set over U. Then a subset of $U \times E$ is uniquely defined by

$$\hat{R}_A = \{(u, e) : e \in A, u \in \hat{F}_A(e)\}$$

which is called a relation form of $(\hat{F}_A, E)$. Now the relation $\hat{R}_A$ is characterized by the membership function $\mu_A : U \times E \longrightarrow [0, 1]$ and the non-membership function $\nu_A : U \times E \longrightarrow [0, 1]$

[where $\mu_A(u, e)$ is the membership value and $\nu_A(u, e)$ is the non-membership value of the object u associated with the parameter e.]
Now if the set of universe $U = \{u_1, u_2, ..., u_m\}$ and the set of parameters $E = \{e_1, e_2, ..., e_n\}$, then $\hat{R}_A$ can be presented by a tabular form as in the Table-2

Table-2: Tabular representation of $\hat{R}_A$

	e_1	e_2	....	e_n
u_1	(μ_{A11}, ν_{A11})	(μ_{A12}, ν_{A12})	....	(μ_{A1n}, ν_{A1n})
u_2	(μ_{A21}, ν_{A21})	(μ_{A22}, ν_{A22})	....	(μ_{A2n}, ν_{A2n})
....	....	....	....	....
u_m	(μ_{Am1}, ν_{Am1})	(μ_{Am2}, ν_{Am2})	....	(μ_{Amn}, ν_{Amn})

where $(\mu_{Amn}, \nu_{Amn}) = (\mu_A(u_m, e_n), \nu_A(u_m, e_n))$
For simplicity, if (μ_{Aij}, ν_{Aij}) is denoted by $\hat{a}_{ij}$, we can define a matrix

$$(\hat{a}_{ij})_{m\times n} = \begin{pmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots\cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots\cdots & \hat{a}_{2n} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ \hat{a}_{m1} & \hat{a}_{m2} & \cdots\cdots & \hat{a}_{mn} \end{pmatrix}$$

which is called an **intuitionistic fuzzy soft matrix** of order $m \times n$ corresponding to the intuitionistic fuzzy soft set $(\hat{F}_A, E)$ over U. An intuitionistic fuzzy soft set $(\hat{F}_A, E)$ is uniquely characterized by the matrix $(\hat{a}_{ij})_{m\times n}$. Therefore we shall identify any intuitionistic fuzzy soft set with its intuitionistic fuzzy soft matrix and use these two concepts as interchangeable.

Example 3.2. Let U be the set of four cities, given by, $U = \{C_1, C_2, C_3, C_4, C_5\}$.
Let E be the set of parameters (each parameter is an intuitionistic fuzzy word), given by, $E = \{$ highly, immensely, moderately, average, less $\} = \{e_1, e_2, e_3, e_4, e_5\}$(say)
Let $A \subset E$, given by, $A = \{e_1, e_2, e_3, e_5\}$ (say)
Now suppose that, $\hat{F}_A$ is a mapping, defined as "polluted cities(.)"and given by,
$\hat{F}_A(e_1) = \{C_1/(.2, .7), C_2/(.8, .1), C_3/(.4, .2), C_4/(.6, .3), C_5/(.7, .2)\}$,
$\hat{F}_A(e_2) = \{C_1/(0, 1), C_2/(.9, .1), C_3/(.3, .6), C_4/(.4, .6), C_5/(.6, .3)\}$,
$\hat{F}_A(e_3) = \{C_1/(.3, .5), C_2/(.4, .6), C_3/(.8, .1), C_4/(.1, .8), C_5/(.3, .7)\}$,
$\hat{F}_A(e_5) = \{C_1/(.9, .1), C_2/(.1, .8), C_3/(.5, .4), C_4/(.3, .5), C_5/(.1, .8)\}$
Then the intuitionistic fuzzy soft set

$$
\begin{aligned}
&(\hat{F}_A, E)\\
&= \{\ \text{highly polluted city} = \{C_1/(.2, .7), C_2/(.8, .1), C_3/(.4, .2), C_4/(.6, .3), C_5/(.7, .2)\},\\
&\quad \text{immensely polluted city} = \{C_1/(0, .9), C_2/(.9, .1), C_3/(.3, .6), C_4/(.4, .6), C_5/(.6, .3)\},\\
&\quad \text{moderately polluted city} = \{C_1/(.3, .5), C_2/(.4, .6), C_3/(.8, .1), C_4/(.1, .8), C_5/(.3, .7)\},\\
&\quad \text{less polluted city} = \{C_1/(.9, .1), C_2/(.1, .8), C_3/(.5, .4), C_4/(.3, .5), C_5/(.1, .8)\}\}
\end{aligned}
$$

Therefore the relation form of $(\hat{F}_A, E)$ is written by,
$\hat{R}_A = \{(\{C_1/(.2, .7), C_2/(.8, .1), C_3/(.4, .2), C_4/(.6, .3), C_5/(.7, .2)\}, e_1), (\{\{C_1/(0, .9),$
$C_2/(.9, .1), C_3/(.3, .6), C_4/(.4, .6), C_5/(.6, .3)\}, e_2),$
$(\{C_1/(.3, .5), C_2/(.4, .6), C_3/(.8, .1), C_4/(.1, .8), C_5/(.3, .7)\}, e_3),$
$(\{C_1/(.9, .1), C_2/(.1, .8), C_3/(.5, .4), C_4/(.3, .5), C_5/(.1, .8)\}, e_5)\}$
Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$
(\hat{a}_{ij}) = \begin{pmatrix}
(.2, .7) & (0, .9) & (.3, .5) & (0, 1) & (.9, .1)\\
(.8, .1) & (.9, .1) & (.4, .6) & (0, 1) & (.1, .8)\\
(.4, .2) & (.3, .6) & (.8, .1) & (0, 1) & (.5, .4)\\
(.6, .3) & (.4, .6) & (.1, .8) & (0, 1) & (.3, .5)\\
(.7, .2) & (.6, .3) & (.3, .7) & (0, 1) & (.1, .8)
\end{pmatrix}
$$

Definition 3.3. Row-Intuitionistic Fuzzy Soft Matrix: An intuitionistic fuzzy soft matrix of order $1\times n$ i.e., with a single row is called a **row-intuitionistic fuzzy soft matrix**. Physically, a row-intuitionistic fuzzy soft matrix formally corresponds to an intuitionistic fuzzy soft set whose universal set contains only one object.

Example 3.4. Suppose the universe set U contains only one dress d_1 and parameter set $E = \{$ costly, beautiful, cheap, comfortable $\}= \{e_1, e_2, e_3, e_4\}$.
Let $A = \{e_2, e_3, e_4\} \subset E$ and $\hat{F}_A(e_2) = \{d_1/(.8, .1)\}, \hat{F}_A(e_3) = \{d_1/(.3, .7)\}$,
$\hat{F}_A(e_4) = \{d_1/(.6, .3)\}$. Then we have an intuitionistic fuzzy soft set as
$(\hat{F}_A, E) = \{(e_2, \{d_1/(.8, .1)\}), (e_3, \{d_1/(.3, .7)\}), (e_4, \{d_1/(.6, .3)\})\}$
and then the relation form of $(\hat{F}_A, E)$ is written by,
$\hat{R}_A = \{(\{d_1/(.8, .1)\}, e_2), (\{d_1/(.3, .7)\}, e_3), (\{d_1/(.6, .3)\}, e_4)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,
$(\hat{a}_{ij}) = \begin{pmatrix} (0,1) & (.8,.1) & (.3,.7) & (.6,.3) \end{pmatrix}$ which contains a single row and so it is a row-intuitionistic fuzzy soft matrix.

Definition 3.5. Column-Intuitionistic Fuzzy Soft Matrix: An intuitionistic fuzzy soft matrix of order $m \times 1$ i.e., with a single column is called a **column-intuitionistic fuzzy soft matrix**. Physically, a column-intuitionistic fuzzy soft matrix formally corresponds to an intuitionistic fuzzy soft set whose parameter set contains only one parameter.

Example 3.6. Suppose the initial universe set U contains four dresses d_1, d_2, d_3, d_4 and the parameter set E contains only one parameter given by, $E = \{$ beautiful $\} = \{e_1\}$.

$$\hat{F} : E \longrightarrow IF^U$$

$$s.t, \qquad \hat{F}(e_1) = \{d_1/(0.7, 0.2), d_2/(0.2, 0.6), d_3/(0.8, 0.1), d_4/(0.4, 0.6)\}.$$

Then we have an intuitionistic fuzzy soft set
$(\hat{F}, E) = \{(e_1, \{d_1/(0.7, 0.2), d_2/(0.2, 0.6), d_3/(0.8, 0.1), d_4/(0.4, 0.6)\})\}$
and then the relation form of $(\hat{F}, E)$ is written by,
$\hat{R}_E = \{(\{d_1/(0.7, 0.2), d_2/(0.2, 0.6), d_3/(0.8, 0.1), d_4/(0.4, 0.6)\}, e_1)\}$
Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.7, 0.2) \\ (0.2, 0.6) \\ (0.8, 0.1) \\ (0.4, 0.6) \end{pmatrix}$$

which contains a single column and so it is an example of column-intuitionistic fuzzy soft matrix.

Definition 3.7. Square Intuitionistic Fuzzy Soft Matrix: An intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **square intuitionistic fuzzy soft matrix** if $m = n$ i.e., the number of rows and the number of columns are equal. That means a square-intuitionistic fuzzy soft matrix is formally equal to an intuitionistic fuzzy soft set having the same number of objects and parameters.

Example 3.8. Consider the Example 3.2. Here since the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ contains five rows and five columns, so it is a square-intuitionistic fuzzy soft matrix.

Definition 3.9. Complement of an Intuitionistic Fuzzy Soft Matrix: Let $(\hat{a}_{ij})$ be an $m \times n$ intuitionistic fuzzy soft matrix, where $\hat{a}_{ij} = (\mu_{ij}, \nu_{ij}) \forall i, j$. Then the **complement** of $(\hat{a}_{ij})$ is denoted by $(\hat{a}_{ij})^o$ and is defined by,
$(\hat{a}_{ij})^o = (\hat{c}_{ij})$, where $(\hat{c}_{ij})$ is also an intuitionistic fuzzy soft matrix of order $m \times n$ and $\hat{c}_{ij} = (\nu_{ij}, \mu_{ij}) \forall i, j$.

Example 3.10. Consider the Example 3.2, the complement of $(\hat{a}_{ij})$ is,

$$(\hat{a}_{ij})^o = \begin{pmatrix} (.7,.2) & (.9,0) & (.5,.3) & (1,0) & (.1,.9) \\ (.1,.8) & (.1,.9) & (.6,.4) & (1,0) & (.8,.1) \\ (.2,.4) & (.6,.3) & (.1,.8) & (1,0) & (.4,.5) \\ (.3,.6) & (.6,.4) & (.9,.1) & (1,0) & (.5,.3) \\ (.2,.7) & (.3,.6) & (.7,.3) & (1,0) & (.8,.1) \end{pmatrix}$$

Definition 3.11. Null Intuitionistic Fuzzy Soft Matrix: An intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **null intuitionistic fuzzy soft matrix or zero intuitionistic fuzzy soft matrix** if all of its elements are $(0,1)$. A null intuitionistic fuzzy soft matrix is denoted by, $\hat{\Phi}$. Now the intuitionistic fuzzy soft set associated with a null intuitionistic fuzzy soft matrix must be a null intuitionistic fuzzy soft set.

Example 3.12. Let there are four dresses in the universe set U given by, $U = \{d_1, d_2, d_3, d_4\}$ and the parameter set $E = \{$ beautiful, cheap, comfortable, gorgeous $\} = \{e_1, e_2, e_3, e_4\}$. Let $A = \{e_1, e_2, e_3\} \subset E$. Now let $\hat{F}_A : E \longrightarrow IF^U$ s.t,
$\hat{F}_A(e_1) = \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1)\} = \hat{\phi}, \hat{F}_A(e_2) = \hat{\phi}, \hat{F}_A(e_3) = \hat{\phi}$.
Then the intuitionistic fuzzy soft set
$(\hat{F}_A, E) = \{(e_1, \hat{\phi}), (e_2, \hat{\phi}), (e_3, \hat{\phi})\}$
and the relation form of $(\hat{F}_A, E)$ is written by,
$\hat{R}_A = \{(\hat{\phi}, e_1), (\hat{\phi}, e_2), (\hat{\phi}, e_3)\}$
Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} = \hat{\Phi}$$

Definition 3.13. Complete or Absolute Intuitionistic Fuzzy Soft Matrix: An intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **complete or absolute intuitionistic fuzzy soft matrix** if all of its elements are $(1,0)$. A complete or absolute intuitionistic fuzzy soft matrix is denoted by, $\hat{C}_A$. Now the intuitionistic fuzzy soft set associated with an absolute intuitionistic fuzzy soft matrix must be an absolute intuitionistic fuzzy soft set.

Example 3.14. Let there are four dresses in the universe set U given by,
$U = \{d_1, d_2, d_3, d_4\}$ and the parameter set $E = \{$beautiful, cheap, comfortable, gorgeous$\} = \{e_1, e_2, e_3, e_4\}$. Let $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ and $\hat{F}_A : E \longrightarrow IF^U$ s.t,
$\hat{F}_A(e_1) = \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}$,
$\hat{F}_A(e_2) = \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}$,
$\hat{F}_A(e_3) = \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}$,
$\hat{F}_A(e_4) = \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}$.
Then the intuitionistic fuzzy soft set
$(\hat{F}_A, E) = \{(e_1, \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\})$,
$(e_2, \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\})$,

$(e_3, \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\})$,
$(e_4, \{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\})\}$
and the relation form of $(\hat{F}_A, E)$ is written by,
$\hat{R}_A = \{(\{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}, e_1)$,
$(\{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}, e_2)$,
$(\{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}, e_3)$,
$(\{d_1/(1,0), d_2/(1,0), d_3/(1,0), d_4/(1,0)\}, e_4)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (1,0) & (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) & (1,0) \end{pmatrix} = \hat{C}_A$$

Definition 3.15. Diagonal Intuitionistic Fuzzy Soft Matrix: A square intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a **diagonal-intuitionistic fuzzy soft matrix** if all of its non-diagonal elements are $(0, 1)$.

Example 3.16. Suppose the initial universe set $U = \{d_1, d_2, d_3, d_4, d_5\}$ and the parameter set
$E = \{e_1, e_2, e_3, e_4, e_5\}$. Let $\hat{F} : E \longrightarrow IF^U$ s.t,
$\hat{F}(e_1) = \{d_1/(0.8, 0.1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}$,
$\hat{F}(e_2) = \{d_1/(0,1), d_2/(0.3, 0.7), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}$,
$\hat{F}(e_3) = \{d_1/(0,1), d_2/(0,1), d_3/(1,0), d_4/(0,1), d_5/(0,1)\}$,
$\hat{F}(e_4) = \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0.7, 0.2), d_5/(0,1)\}$,
$\hat{F}(e_5) = \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0.6, 0.3)\}$.
Then the intuitionistic fuzzy soft set
$(\hat{F}, E) = \{(e_1, \{d_1/(0.8, 0.1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}), (e_2, \{d_1/(0,1),$
$d_2/(0.3, 0.7), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}), (e_3, \{d_1/(0,1), d_2/(0,1), d_3/(1,0),$
$d_4/(0,1), d_5/(0,1)\}), (e_4, \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0.7, 0.2), d_5/(0,1)\}),$
$(e_5, \{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0.6, 0.3)\})\}$
and the relation form of $(\hat{F}, E)$ is written by,
$\hat{R}_A = \{(\{(\{d_1/(0.8, 0.1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}, e_1)$,
$(\{d_1/(0,1), d_2/(0.3, 0.7), d_3/(0,1), d_4/(0,1), d_5/(0,1)\}, e_2)$,
$(\{d_1/(0,1), d_2/(0,1), d_3/(1,0), d_4/(0,1), d_5/(0,1)\}, e_3)$,
$(\{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0.7, 0.2), d_5/(0,1)\}, e_4)$,
$(\{d_1/(0,1), d_2/(0,1), d_3/(0,1), d_4/(0,1), d_5/(0.6, 0.3)\}, e_5)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is written by,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.8, 0.1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0.3, 0.7) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0.7, 0.2) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0.6, 0.3) \end{pmatrix}$$

whose all non-diagonal elements are $(0, 1)$ and so it is a diagonal-intuitionistic fuzzy soft matrix.

Definition 3.17. Transpose of a Square Intuitionistic Fuzzy Soft Matrix: The **transpose** of a square intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ of order $m \times n$ is another square intuitionistic fuzzy soft matrix of order $n \times m$ obtained from $(\hat{a}_{ij})$ by interchanging its rows and columns. It is denoted by $(\hat{a}_{ij})^T$. Therefore the intuitionistic fuzzy soft set associated with $(\hat{a}_{ij})^T$ becomes a new intuitionistic fuzzy soft set over the same universe and over the same set of parameters.

Example 3.18. Consider the Example 3.2. Here $(\hat{F}_A, E)$ be an intuitionistic fuzzy soft set over the universe U and over the set of parameters E, given by,

$$
\begin{aligned}
(\hat{F}_A, E) \\
= \quad & \{ \text{highly polluted city} = \{C_1/(.2,.7), C_2/(.8,.1), C_3/(.4,.2), C_4/(.6,.3), C_5/(.7,.2)\}, \\
& \text{immensely polluted city} = \{C_1/(0,.9), C_2/(.9,.1), C_3/(.3,.6), C_4/(.4,.6), C_5/(.6,.3)\}, \\
& \text{moderately polluted city} = \{C_1/(.3,.5), C_2/(.4,.6), C_3/(.8,.1), C_4/(.1,.8), C_5/(.3,.7)\}, \\
& \text{less polluted city} = \{C_1/(.9,.1), C_2/(.1,.8), C_3/(.5,.4), C_4/(.3,.5), C_5/(.1,.8)\}\}
\end{aligned}
$$

whose associated intuitionistic fuzzy soft matrix is,

$$
(\hat{a}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (.3,.5) & (0,1) & (.9,.1) \\ (.8,.1) & (.9,.1) & (.4,.6) & (0,1) & (.1,.8) \\ (.4,.2) & (.3,.6) & (.8,.1) & (0,1) & (.5,.4) \\ (.6,.3) & (.4,.6) & (.1,.8) & (0,1) & (.3,.5) \\ (.7,.2) & (.6,.3) & (.3,.7) & (0,1) & (.1,.8) \end{pmatrix}
$$

Now its transpose intuitionistic fuzzy soft matrix is,

$$
(\hat{a}_{ij})^T = \begin{pmatrix} (.2,.7) & (.8,.1) & (.4,.2) & (.6,.3) & (.7,.2) \\ (0,.9) & (.9,.1) & (.3,.6) & (.4,.6) & (.6,.3) \\ (.3,.5) & (.4,.6) & (.8,.1) & (.1,.8) & (.3,.7) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (.9,.1) & (.1,.8) & (.5,.4) & (.3,.5) & (.1,.8) \end{pmatrix}
$$

Therefore the intuitionistic fuzzy soft set associated with $(\hat{a}_{ij})^T$ is,

$$
\begin{aligned}
(\hat{G}_B, E) \\
= \quad & \{ \text{highly polluted city} = \{C_1/(.2,.7), C_2/(0,.9), C_3/(.3,.5), C_4/(0,1), C_5/(.9,.1)\}, \\
& \text{immensely polluted city} = \{C_1/(.8,.1), C_2/(.9,.1), C_3/(.4,.6), C_4/(0,1), C_5/(.1,.8)\}, \\
& \text{moderately polluted city} = \{C_1/(.4,.2), C_2/(.3,.6), C_3/(.8,.1), C_4/(0,1), C_5/(.3,.7)\}, \\
& \text{average polluted city} = \{C_1/(.6,.3), C_2/(.4,.6), C_3/(.1,.8), C_4/(0,1), C_5/(.3,.5)\}, \\
& \text{less polluted city} = \{C_1/(.7,.2), C_2/(.6,.3), C_3/(.3,.7), C_4/(0,1), C_5/(.1,.8)\}\}
\end{aligned}
$$

where B= { highly, immensely, moderately, average, less }$\subseteq E$ and G_B is a mapping from B to IF^U.

Definition 3.19. Choice Matrix: It is a square matrix whose rows and columns both indicate parameters(which are intuitionistic fuzzy words or sentences involving intuitionistic fuzzy words). If $\hat{\xi}$ is a choice matrix, then its element $\hat{\xi}_{ij}$ is defined as follows:

$$\hat{\xi}_{ij} = \begin{cases} (1,0) \text{ when } i^{th} \text{ and } j^{th} \text{ parameters are both } \textbf{choice} \text{ parameters of the decision makers} \\ (0,1) \text{ otherwise, i.e. when atleast one of the } i^{th} \text{ or } j^{th} \text{ parameters be not under } \textbf{choice} \end{cases}$$

There are different types of choice matrices according to the number of decision makers. We may realize this by the following example.

Example 3.20. Suppose that U be a set of four factories, say, $U = \{f_1, f_2, f_3, f_4\}$
Let E be a set of parameters, given by,
$E = \{$ costly, excellent work culture, assured production, good location, cheap$\}$
$= \{e_1, e_2, e_3, e_4, e_5\}$(say)
Now let the intuitionistic fuzzy soft set $(\hat{F}, A)$ describing "the quality of the factories", is given by,

$$\begin{aligned}(\hat{F}, E) = \{ & \text{costly factories } = \{f_1/(0.9,0.1), f_2/(0.2,0.7), f_3/(0.4,0.5), f_4/(0.8,0.1)\}, \\ & \text{factories with excellent work culture } = \{f_1/(0.8,0.1), f_2/(0.3,0.5), f_3/(0.5,0.4), f_4/(0.4,0.5)\}, \\ & \text{factories with assured production } = \{f_1/(0.9,0.1), f_2/(0.2,0.7), f_3/(0.4,0.5), f_4/(0.8,0.1)\}, \\ & \text{factories with good location} = \{f_1/(0.7,0.3), f_2/(0.9,0.1), f_3/(0.4,0.5), f_4/(0.8,0.2)\}, \\ & \text{cheap factories} = \{f_1/(0.1,0.8), f_2/(0.7,0.1), f_3/(0.5,0.3), f_4/(0.2,0.7)\}\}\end{aligned}$$

Suppose Mr.X wants to buy a factory on the basis of his choice parameters excellent work culture, assured production and cheap which form a subset P of the parameter set E.
Therefore $P = \{e_2, e_3, e_5\}$
Now **the choice matrix of Mr.X** is,

$$(\hat{\xi}_{ij})_P = e_P \begin{pmatrix} & & e_P & & \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \end{pmatrix}$$

Now suppose Mr.X and Mr.Y together wants to buy a factory according to their choice parameters. Let the choice parameter set of Mr.Y be, $Q = \{e_1, e_2, e_3, e_4\}$
Then **the combined choice matrix of Mr.X and Mr.Y** is

$$(\hat{\xi}_{ij})_{(P,Q)} = e_P \begin{pmatrix} & & e_Q & & \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) & (0,1) \end{pmatrix}$$

[Here the entries $\hat{e}_{ij} = (1,0)$ indicates that e_i is a choice parameter of Mr.X and e_j is a choice parameter of Mr.Y. Now $\hat{e}_{ij} = (0,1)$ indicates either e_i fails to be a choice parameter of Mr.X or e_j fails to be a choice parameter of Mr.Y.]
Again the above combined choice matrix of Mr.X and Mr.Y may be also presented

in its transpose form as,

$$(\hat{\xi}_{ij})_{(Q,P)} = e_Q \begin{pmatrix} & & e_P & & \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

Now let us see the form of the combined choice matrix associated with three decision makers. Suppose that Mr.Z is willing to buy a factory together with Mr.X and Mr.Y on the basis of his choice parameters excellent work culture, assured production and good location which form a subset R of the parameter set E.
Therefore $R = \{e_2, e_3, e_4\}$
Then **the combined choice matrix of Mr.X, Mr.Y and Mr.Z** will be of three different types which are as follows,

(i) $$(\hat{\xi}_{ij})_{(R,P\wedge Q)} = e_R \begin{pmatrix} & & e_{(P\wedge Q)} & & \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

[Since the set of common choice parameters of Mr.X and Mr.Y is, $P \wedge Q = \{e_2, e_3\}$. Here the entries $\hat{e}_{ij} = (1,0)$ indicates that e_i is a choice parameter of Mr.Z and e_j is a common choice parameter of Mr.X and Mr.Y. Now $\hat{e}_{ij} = (0,1)$ indicates either e_i fails to be a choice parameter of Mr.Z or e_j fails to be a common choice parameter of Mr.X and Mr.Y.]

(ii) $$(\hat{\xi}_{ij})_{(P,Q\wedge R)} = e_P \begin{pmatrix} & & e_{(Q\wedge R)} & & \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (1,0) & (0,1) \end{pmatrix}$$ [Since $Q \wedge R = \{e_2, e_3, e_4\}$]

(iii) $$(\hat{\xi}_{ij})_{(Q,R\wedge P)} = e_Q \begin{pmatrix} & & e_{(R\wedge P)} & & \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$ [Since $R \wedge P = \{e_2, e_3\}$]

Definition 3.21. Symmetric Intuitionistic Fuzzy Soft Matrix: A square intuitionistic fuzzy soft matrix $\hat{A}$ of order $n \times n$ is said to be a **symmetric Intuitionistic fuzzy soft matrix**, if its transpose be equal to it, i.e., if $\hat{A}^T = \hat{A}$. Hence the Intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is symmetric, if $\hat{a}_{ij} = \hat{a}_{ji}, \forall i, j$.

Therefore if $(\hat{a}_{ij})$ be a symmetric Intuitionistic fuzzy soft matrix then the Intuitionistic fuzzy soft sets associated with $(\hat{a}_{ij})$ and $(\hat{a}_{ij})^T$ both be the same.

Example 3.22. Let the set of universe $U = \{u_1, u_2, u_3, u_4\}$ and the set of parameters $E = \{e_1, e_2, e_3, e_4\}$. Now suppose that, $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ and $\hat{F}_A : A \longrightarrow IF^U$ s.t, $(\hat{F}_A, E)$ forms an intuitionistic fuzzy soft set given by,
$(\hat{F}_A, E) = \{(e_1, \{u_1/(0.2, 0.8), u_2/(0.3, 0.7), u_3/(0.8, 0.1)u_4/(0.5, 0.4)\}), (e_2, \{u_1/(0.3, 0.7), u_2/(0.6, 0.2), u_3/(0.1, 0.7), u_4/(0.7, 0.2)\}), (e_3, \{u_1/(0.8, 0.1), u_2/(0.1, 0.7), u_3/(0.7, 0.2), u_4/(0.2, 0.7)\}), (e_4, \{u_1/(0.5, 0.4), u_2/(0.7, 0.2), u_3/(0.2, 0.7), u_4/(0.4, 0.6)\})\}$
The intuitionistic fuzzy soft matrix associated with this intuitionistic fuzzy soft set $(\hat{F}_A, E)$ is,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.2, 0.8) & (0.3, 0.7) & (0.8, 0.1) & (0.5, 0.4) \\ (0.3, 0.7) & (0.6, 0.2) & (0.1, 0.7) & (0.7, 0.2) \\ (0.8, 0.1) & (0.1, 0.7) & (0.7, 0.2) & (0.2, 0.7) \\ (0.5, 0.4) & (0.7, 0.2) & (0.2, 0.7) & (0.4, 0.6) \end{pmatrix}$$

Since here $\hat{a}_{ij} = \hat{a}_{ji}, \forall i, j$; $(\hat{a}_{ij})$ is a symmetric intuitionistic fuzzy soft matrix.

Definition 3.23. Addition of Intuitionistic Fuzzy Soft Matrices: Two intuitionistic fuzzy soft matrices $\hat{A}$ and $\hat{B}$ are said to be conformable for addition, if they be of the same order. The addition of two intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ of order $m \times n$ is defined by,
$(\hat{a}_{ij}) \oplus (\hat{b}_{ij}) = (\hat{c}_{ij})$, where $(\hat{c}_{ij})$ is also an $m \times n$ intuitionistic fuzzy soft matrix and $\hat{c}_{ij} = (\max\{\mu_{\hat{a}_{ij}}, \mu_{\hat{b}_{ij}}\}, \min\{\nu_{\hat{a}_{ij}}, \nu_{\hat{b}_{ij}}\}) \forall i, j$.

Example 3.24. Consider the intuitionistic fuzzy soft matrix of example 3.1,

$$(\hat{a}_{ij}) = \begin{pmatrix} (.2, .7) & (0, .9) & (.3, .5) & (0, 1) & (.9, .1) \\ (.8, .1) & (.9, .1) & (.4, .6) & (0, 1) & (.1, .8) \\ (.4, .2) & (.3, .6) & (.8, .1) & (0, 1) & (.5, .4) \\ (.6, .3) & (.4, .6) & (.1, .8) & (0, 1) & (.3, .5) \\ (.7, .2) & (.6, .3) & (.3, .7) & (0, 1) & (.1, .8) \end{pmatrix}$$

Now consider another intuitionistic fuzzy soft matrix $(\hat{b}_{ij})$ associated with the intuitionistic fuzzy soft set $(\hat{G}_B, E)$ (also describing "the pollution of the cities") over the same universe U.
Let $B = \{e_1, e_4, e_5\} \subset E$ and

$$\begin{aligned} (\hat{G}, B) \quad = \quad \{ & \text{ highly polluted city } = \{C_1/(.3, .7), C_2/(.9, .1), C_3/(.4, .5), C_4/(.7, .2), C_5/(.6, .2)\}, \\ & \text{average polluted city} = \{C_1/(.2, .7), C_2/(.3, .7), C_3/(.7, .1), C_4/(.2, .8), C_5/(.3, .6)\}, \\ & \text{less polluted city} = \{C_1/(.8, .1), C_2/(.2, .7), C_3/(.6, .4), C_4/(.3, .5), C_5/(.2, .6)\}\} \end{aligned}$$

and then the relation form of $(\hat{G}_B, E)$ is written by,
$\hat{R}_B = \{(\{C_1/(.3, .7), C_2/(.9, .1), C_3/(.4, .5), C_4/(.7, .2), C_5/(.6, .2)\}, e_1), (\{C_1/(.2, .7), C_2/(.3, .7), C_3/(.7, .1), C_4/(.2, .8), C_5/(.3, .6)\}, e_2), (\{d_1/0.3, d_2/0.7, d_3/0.6, d_4/0.2\}, e_4), (\{C_1/(.8, .1), C_2/(.2, .7), C_3/(.6, .4), C_4/(.3, .5), C_5/(.2, .6)\}, e_5)\}$

Hence the intuitionistic fuzzy soft matrix $(\hat{b}_{ij})$ is written by,

$$(\hat{b}_{ij}) = \begin{pmatrix} (.3,.7) & (0,1) & (0,1) & (.2,.7) & (.8,.1) \\ (.9,.1) & (0,1) & (0,1) & (.3,.7) & (.2,.7) \\ (.4,.5) & (0,1) & (0,1) & (.7,.1) & (.6,.4) \\ (.7,.2) & (0,1) & (0,1) & (.2,.8) & (.3,.5) \\ (.6,.2) & (0,1) & (0,1) & (.3,.6) & (.2,.6) \end{pmatrix}$$

Therefore the sum of the intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ is,

$$(\hat{a}_{ij}) \oplus (\hat{b}_{ij}) = \begin{pmatrix} (.3,.7) & (0,0.9) & (.3,.5) & (0.2,0.7) & (.9,.1) \\ (.9,.1) & (.9,0.1) & (.4,0.6) & (0.3,.7) & (.2,.7) \\ (.4,.2) & (.3,0.6) & (.8,0.1) & (0.7,.1) & (.6,.4) \\ (.7,.2) & (.4,0.6) & (.1,0.8) & (0.2,.8) & (.3,.5) \\ (.7,.2) & (.6,0.3) & (.3,0.7) & (0.3,.6) & (.2,.6) \end{pmatrix}$$

Definition 3.25. Subtraction of Intuitionistic Fuzzy Soft Matrices: Two intuitionistic fuzzy soft matrices $\hat{A}$ and $\hat{B}$ are said to be conformable for subtraction, if they be of the same order. For any two intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ of order $m \times n$, the subtraction of $(\hat{b}_{ij})$ from $(\hat{a}_{ij})$ is defined as,
$(\hat{a}_{ij}) \ominus (\hat{b}_{ij}) = (\hat{c}_{ij})$, where $(\hat{c}_{ij})$ is also an $m \times n$ intuitionistic fuzzy soft matrix and $\hat{c}_{ij} = (\min\{\mu_{\hat{a}_{ij}}, \mu_{\hat{b}^o_{ij}}\}, \max\{\nu_{\hat{a}_{ij}}, \nu_{\hat{b}^o_{ij}}\})\forall i, j$ where $(\hat{b}^o_{ij})$ is the complement of $(\hat{b}_{ij})$

Example 3.26. Consider the intuitionistic fuzzy soft matrices $(\hat{a}_{ij})$ and $(\hat{b}_{ij})$ of Example 3.24. Now,

$$(\hat{a}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (.3,.5) & (0,1) & (.9,.1) \\ (.8,.1) & (.9,.1) & (.4,.6) & (0,1) & (.1,.8) \\ (.4,.2) & (.3,.6) & (.8,.1) & (0,1) & (.5,.4) \\ (.6,.3) & (.4,.6) & (.1,.8) & (0,1) & (.3,.5) \\ (.7,.2) & (.6,.3) & (.3,.7) & (0,1) & (.1,.8) \end{pmatrix}$$

and $$(\hat{b}_{ij})^o = \begin{pmatrix} (.7,.3) & (1,0) & (1,0) & (.7,.2) & (.1,.8) \\ (.1,.9) & (1,0) & (1,0) & (.7,.3) & (.7,.2) \\ (.5,.4) & (1,0) & (1,0) & (.1,.7) & (.4,.6) \\ (.2,.7) & (1,0) & (1,0) & (.8,.2) & (.5,.3) \\ (.2,.6) & (1,0) & (1,0) & (.6,.3) & (.6,.2) \end{pmatrix}$$

Therefore the subtraction of the intuitionistic fuzzy soft matrix $(\hat{b}_{ij})$ from the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is,

$$(\hat{a}_{ij}) \ominus (\hat{b}_{ij}) = \begin{pmatrix} (.2,.7) & (0,.9) & (0.3,.5) & (0,0.2) & (.1,.8) \\ (.1,.9) & (0.9,.1) & (0.4,.6) & (0,0.3) & (.1,.8) \\ (.2,.7) & (0.3,.6) & (0.8,.1) & (0,0.7) & (.4,.6) \\ (.2,.7) & (0.4,.6) & (0.1,.8) & (0,0.2) & (.3,.5) \\ (.2,.6) & (0.6,.3) & (0.3,.7) & (0,0.3) & (.1,.8) \end{pmatrix}$$

Properties: Let $\hat{A}$ and $\hat{B}$ be two intuitionistic fuzzy soft matrices of order $m \times n$. Then
(i) $\hat{A} \oplus \hat{B} = \hat{B} \oplus \hat{A}$
(ii) $(\hat{A} \oplus \hat{B}) \oplus \hat{C} = \hat{A} \oplus (\hat{B} \oplus \hat{C})$
(iii)$\hat{A} \ominus \hat{B} \neq \hat{B} \ominus \hat{A}$

(iv) $(\hat{A} \ominus \hat{B}) \ominus \hat{C} \neq \hat{A} \ominus (\hat{B} \ominus \hat{C})$
(v) $\hat{A} \oplus \hat{A}^o \neq \hat{C}_A$
(vi) $\hat{A} \ominus \hat{A} \neq \hat{\Phi}$

Definition 3.27. Product of an Intuitionistic Fuzzy Soft Matrix with a Choice Matrix: Let U be the set of universe and E be the set of parameters. Suppose that $\hat{A}$ be any intuitionistic fuzzy soft matrix and $\hat{\beta}$ be any choice matrix of a decision maker concerned with the same universe U and E. Now if the number of columns of the intuitionistic fuzzy soft matrix $\hat{A}$ be equal to the number of rows of the choice matrix $\hat{\beta}$, then $\hat{A}$ and $\hat{\beta}$ are said to be conformable for the product $(\hat{A} \otimes \hat{\beta})$ and the product $(\hat{A} \otimes \hat{\beta})$ becomes an intuitionistic fuzzy soft matrix. We may denote the product by $\hat{A} \otimes \hat{\beta}$ or simply by $\hat{A}\hat{\beta}$.
If $\hat{A} = (\hat{a}_{ij})_{m \times n}$ and $\hat{\beta} = (\hat{\beta}_{jk})_{n \times p}$, then
$\hat{A}\hat{\beta} = (\hat{c}_{ik})$
where $\hat{c}_{ik} = (\max_{j=1}^{n} \min\{\mu_{\hat{a}_{ij}}, \mu_{\hat{\beta}_{jk}}\}, \min_{j=1}^{n} \max\{\nu_{\hat{a}_{ij}}, \nu_{\hat{\beta}_{jk}}\})$
It is to be noted that, $\hat{\beta}\hat{A}$ cannot be defined here.

Example 3.28. Let U be the set of four dresses, given by, $U = \{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by, $E = \{$ cheap, beautiful, comfortable, gorgeous $\} = \{e_1, e_2, e_3, e_4\}$(say). Suppose that the set of choice parameters of Mr.X be, $A = \{e_1, e_3\}$. Now let according to the choice parameters of Mr.X, we have the intuitionistic fuzzy soft set $(\hat{F}, A)$ which describes "the attractiveness of the dresses" and the intuitionistic fuzzy soft matrix of the intuitionistic fuzzy soft set $(\hat{F}, A)$ be,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.8, 0.1) & (0.2, 0.7) & (0.7, 0.2) & (0.3, 0.5) \\ (0.3, 0.6) & (0.7, 0.1) & (0.4, 0.6) & (0.8, 0.1) \\ (0.7, 0.2) & (0.4, 0.5) & (0.5, 0.3) & (0.6, 0.2) \\ (0.5, 0.4) & (0.1, 0.8) & (0.9, 0.1) & (0.2, 0.7) \end{pmatrix}$$

Again the choice matrix of Mr.X is,

$$(\hat{\xi}_{ij})_A = e_A \begin{pmatrix} & e_A & & \\ (1, 0) & (0, 1) & (1, 0) & (0, 1) \\ (0, 1) & (0, 1) & (0, 1) & (0, 1) \\ (1, 0) & (0, 1) & (1, 0) & (0, 1) \\ (0, 1) & (0, 1) & (0, 1) & (0, 1) \end{pmatrix}$$

Since the number of columns of the intuitionistic fuzzy soft matrix $(\hat{a}_{ij})$ is equal to the number of rows of the choice matrix $(\hat{\xi}_{ij})_A$, they are conformable for the product.

$$\text{Therefore} \begin{pmatrix} (0.8, 0.1) & (0.2, 0.7) & (0.7, 0.2) & (0.3, 0.5) \\ (0.3, 0.6) & (0.7, 0.1) & (0.4, 0.6) & (0.8, 0.1) \\ (0.7, 0.2) & (0.4, 0.5) & (0.5, 0.3) & (0.6, 0.2) \\ (0.5, 0.4) & (0.1, 0.8) & (0.9, 0.1) & (0.2, 0.7) \end{pmatrix} \otimes \begin{pmatrix} (1, 0) & (0, 1) & (1, 0) & (0, 1) \\ (0, 1) & (0, 1) & (0, 1) & (0, 1) \\ (1, 0) & (0, 1) & (1, 0) & (0, 1) \\ (0, 1) & (0, 1) & (0, 1) & (0, 1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8, 0.1) & (0, 1) & (0.8, 0.1) & (0, 1) \\ (0.4, 0.6) & (0, 1) & (0.4, 0.6) & (0, 1) \\ (0.7, 0.2) & (0, 1) & (0.7, 0.2) & (0, 1) \\ (0.9, 0.1) & (0, 1) & (0.9, 0.1) & (0, 1) \end{pmatrix}$$

4. A Generalized Intuitionistic Fuzzy Soft Set Based Group Decision Making Problem

Let N number of decision makers want to select an object jointly from the m number of objects which have n number of features i.e., parameters(E). Suppose that each decision maker has freedom to take his decision of inclusion and evaluation of parameters associated with the selected object, i.e., each decision maker has his own choice parameters belonging to the parameter set E and has his own view of evaluation. Here it is assumed that **the parameter evaluation of the objects by the decision makers must be intuitionistic fuzzy** and may be presented in linguistic form or intuitionistic fuzzy soft set format, alternatively, in the form of intuitionistic fuzzy soft matrix. **Now the problem is to find out the object out of these m objects which satisfies all the choice parameters of all decision makers jointly as much as possible.**

5. A New Approach to Solve Intuitionistic Fuzzy Soft Set Based Group Decision Making Problems

This new approach is specially based on choice matrices. These choice matrices represent the choice parameters of the decision makers and also help us to solve the intuitionistic fuzzy soft set (or intuitionistic fuzzy soft matrix) based decision making problems with least computational complexity.

The Stepwise Solving Procedure: To solve such type of intuitionistic fuzzy soft set (or intuitionistic fuzzy soft matrix) based decision making problems, we are presenting the following stepwise procedure which comprises of the newly proposed choice matrices, intuitionistic fuzzy soft matrices and the operations on them.

$IFSM$-Algorithm
Step 1: If the parameter evaluation of the objects by the decision makers are not given in intuitionistic fuzzy soft matrix form, then first construct the intuitionistic fuzzy soft matrices according to the given evaluations.
Step 2: Construct the combined choice matrix with respect to the choice parameters of the decision makers.
Step 3: Compute the product intuitionistic fuzzy soft matrices by multiplying each given intuitionistic fuzzy soft matrix with the combined choice matrix as per the rule of multiplication of intuitionistic fuzzy soft matrices.
Step 4: Compute the sum of these product intuitionistic fuzzy soft matrices to have the resultant intuitionistic fuzzy soft matrix($\hat{R}_{IFS}$).
Step 5: Then compute the weight of each object(O_i) by adding the membership values of the entries of its concerned row(i-th row) of $\hat{R}_{IFS}$ and denote it as $W(O_i)$.
Step 6: The object having the highest weight becomes the optimal choice object. If more than one object have the highest weight then go to the next step.
Step 7: Now we have to consider the "sum of the non-membership values"(Θ) of the entries of the rows associated with those equal weighted objects. The object with the minimum Θ-value will be the optimal choice object. Now if the Θ-values of those

objects also be the same, any one of them may be chosen as the optimal choice object.

To illustrate the basic idea of the $IFSM$-algorithm, now we apply it to the following intuitionistic fuzzy soft set (or intuitionistic fuzzy soft matrix) based decision making problems.

Example 5.1. Let U be the set of four dresses, given by, $U = \{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by,
$E = \{$ cheap, beautiful, comfortable, gorgeous $\} = \{e_1, e_2, e_3, e_4\}$(say).
Suppose that, three friends Mr.X, Mr.Y and Mr.Z together want to buy a dress among these four dresses for their common friend Mr.D according to their choice parameters, $A = \{e_1, e_3\}, B = \{e_2, e_3\}, C = \{e_1, e_4\}$ respectively. Now let according to the choice parameter evaluation of the dresses by Mr.X, Mr.Y and Mr.Z, we have the intuitionistic fuzzy soft sets $(\hat{F}_A, E)$, $(\hat{G}_B, E)$, $(\hat{H}_C, E)$ which describe "the attractiveness of the dresses "according to Mr.X, Mr.Y and Mr.Z respectively and given by,

$$\begin{aligned}(\hat{F}_A, E) \;&=\; \{\text{ cheap dresses } = \{d_1/(0.9, 0.1), d_2/(0.3, 0.5), d_3/(0.7, 0.1), d_4/(0.2, 0.7)\},\\ &\qquad \text{comfortable dresses } = \{d_1/(1, 0), d_2/(0.6, 0.3), d_3/(0.3, 0.5), d_4/(0.2, 0.7)\}\}\end{aligned}$$

$$\begin{aligned}(\hat{G}_B, E) \;&=\; \{\text{beautiful dresses } = \{d_1/(0.4, 0.6), d_2/(0.8, 0.1), d_3/(0.5, 0.2), d_4/(0.3, 0.5)\},\\ &\qquad \text{comfortable dresses } = \{d_1/(0.8, 0.1), d_2/(0.6, 0.2), d_3/(0.4, 0.5), d_4/(0.2, 0.8)\}\}\\ (\hat{H}_C, E) \;&=\; \{\text{ cheap dresses } = \{d_1/(0.9, 0.1), d_2/(0.4, 0.5), d_3/(0.6, 0.3), d_4/(0.3, 0.5)\},\\ &\qquad \text{gorgeous dresses } = \{d_1/(0.2, 0.7), d_2/(0.3, 0.5), d_3/(0.6, 0.2), d_4/(0.9, 0)\}\}\end{aligned}$$

The problem is to select the dress among the four dresses which satisfies the choice parameters of Mr.X, Mr.Y and Mr.Z as much as possible.

Now let us apply our newly proposed $IFSM$-algorithm to solve this problem.
(1) The intuitionistic fuzzy soft matrices of the intuitionistic fuzzy soft sets $(\hat{F}_A, E), (\hat{G}_B, E)$ and $(\hat{H}_C, E)$ are respectively,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.9, 0.1) & (0, 1) & (1, 0) & (0, 1)\\ (0.3, 0.5) & (0, 1) & (0.6, 0.3) & (0, 1)\\ (0.7, 0.1) & (0, 1) & (0.3, 0.5) & (0, 1)\\ (0.2, 0.7) & (0, 1) & (0.2, 0.7) & (0, 1) \end{pmatrix}, (\hat{b}_{ik}) = \begin{pmatrix} (0, 1) & (0.4, 0.6) & (0.8, 0.1) & (0, 1)\\ (0, 1) & (0.8, 0.1) & (0.6, 0.2) & (0, 1)\\ (0, 1) & (0.5, 0.2) & (0.4, 0.5) & (0, 1)\\ (0, 1) & (0.3, 0.5) & (0.2, 0.8) & (0, 1) \end{pmatrix},$$

$$(\hat{c}_{il}) = \begin{pmatrix} (0.9, 0.1) & (0, 1) & (0, 1) & (0.2, 0.7)\\ (0.4, 0.5) & (0, 1) & (0, 1) & (0.3, 0.5)\\ (0.6, 0.3) & (0, 1) & (0, 1) & (0.6, 0.2)\\ (0.3, 0.5) & (0, 1) & (0, 1) & (0.9, 0) \end{pmatrix}$$

(2) The combined choice matrices of Mr.X, Mr.Y, Mr.Z in different forms are,

$$e_A \begin{pmatrix} & e_{B\wedge C} & & \\ (0, 1) & (0, 1) & (0, 1) & (0, 1)\\ (0, 1) & (0, 1) & (0, 1) & (0, 1)\\ (0, 1) & (0, 1) & (0, 1) & (0, 1)\\ (0, 1) & (0, 1) & (0, 1) & (0, 1) \end{pmatrix} [\text{ Since } B \wedge C = \phi, A = \{e_1, e_3\}]$$

$$e_B \begin{pmatrix} & e_{C\wedge A} & & \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} [\text{ Since } C\wedge A = \{e_1\}, B = \{e_2, e_3\}]$$

$$e_C \begin{pmatrix} & e_{A\wedge B} & & \\ (0,1) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{pmatrix} [\text{ Since } A\wedge B = \{e_3\}, C = \{e_1, e_4\}]$$

(3) Corresponding product intuitionistic fuzzy soft matrices are,

$$U_A \begin{pmatrix} & e_A & & \\ (0.9,0.1) & (0,1) & (1,0) & (0,1) \\ (0.3,0.5) & (0,1) & (0.6,0.3) & (0,1) \\ (0.7,0.1) & (0,1) & (0.3,0.5) & (0,1) \\ (0.2,0.7) & (0,1) & (0.2,0.7) & (0,1) \end{pmatrix} \otimes\ e_A \begin{pmatrix} & e_{B\wedge C} & & \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$U_B \begin{pmatrix} & e_B & & \\ (0,1) & (0.4,0.6) & (0.8,0.1) & (0,1) \\ (0,1) & (0.8,0.1) & (0.6,0.2) & (0,1) \\ (0,1) & (0.5,0.2) & (0.4,0.5) & (0,1) \\ (0,1) & (0.3,0.5) & (0.2,0.8) & (0,1) \end{pmatrix} \otimes\ e_B \begin{pmatrix} & e_{C\wedge A} & & \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.5,0.2) & (0,1) & (0,1) & (0,1) \\ (0.3,0.5) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$U_C \begin{pmatrix} & e_C & & \\ (0.9,0.1) & (0,1) & (0,1) & (0.2,0.7) \\ (0.4,0.5) & (0,1) & (0,1) & (0.3,0.5) \\ (0.6,0.3) & (0,1) & (0,1) & (0.6,0.2) \\ (0.3,0.5) & (0,1) & (0,1) & (0.9,0) \end{pmatrix} \otimes\ e_C \begin{pmatrix} & e_{A\wedge B} & & \\ (0,1) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0,1) & (0,1) & (0.9,0.1) & (0,1) \\ (0,1) & (0,1) & (0.4,0.5) & (0,1) \\ (0,1) & (0,1) & (0.6,0.2) & (0,1) \\ (0,1) & (0,1) & (0.9,0) & (0,1) \end{pmatrix}$$

[As per the rule of multiplication of intuitionistic fuzzy soft matrices.]

(4) The sum of these product intuitionistic fuzzy soft matrices is,

$$\begin{pmatrix} (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{pmatrix} \oplus \begin{pmatrix} (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.8,0.1) & (0,1) & (0,1) & (0,1) \\ (0.5,0.2) & (0,1) & (0,1) & (0,1) \\ (0.3,0.5) & (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$\oplus \begin{pmatrix} (0,1) & (0,1) & (0.9,0.1) & (0,1) \\ (0,1) & (0,1) & (0.4,0.5) & (0,1) \\ (0,1) & (0,1) & (0.6,0.2) & (0,1) \\ (0,1) & (0,1) & (0.9,0) & (0,1) \end{pmatrix} = \begin{pmatrix} (0.8,0.1) & (0,1) & (0.9,0.1) & (0,1) \\ (0.8,0.1) & (0,1) & (0.4,0.5) & (0,1) \\ (0.5,0.2) & (0,1) & (0.6,0.2) & (0,1) \\ (0.3,0.5) & (0,1) & (0.9,0) & (0,1) \end{pmatrix} =$$
$\hat{R}_{IFS}$

(5) Now the weights of the dresses are,

(i) $W(d_1) = 0.8 + 0 + 0.9 + 0 = 1.7$
(ii) $W(d_2) = 0.8 + 0 + 0.4 + 0 = 1.2$
(iii) $W(d_3) = 0.5 + 0 + 0.6 + 0 = 1.1$
(iv) $W(d_4) = 0.3 + 0 + 0.9 + 0 = 1.2$

(6) The dress associated with the first row of the resultant intuitionistic fuzzy soft matrix($\hat{R}_{IFS}$) has the highest weight($W(d_1) = 1.7$), therefore d_1 be the optimal choice dress. **Hence Mr.X, Mr.Y and Mr.Z will buy the dress d_1 according to their choice parameters.**

6. Application in Medical Science

Generally in medical science a patient suffering from a disease may have multiple symptoms. Again it is also observed that there are certain symptoms which may be common to more than one diseases leading to diagnostic dilemma.

Example 6.1. Now we consider from medical science[8][12][13] four symptoms such as abdominal pain, fever, nausea vomiting, diarrhea which have more or less contribution in four diseases such as typhoid, peptic ulcer, food poisoning, acute viral hepatitis. Now, from medical statistics, the degree of availability of these four symptoms in these four diseases are observed as follows. The degree of belongingness of all the symptoms abdominal pain, fever, nausea vomiting and diarrhea for the diseases typhoid, peptic ulcer, food poisoning and acute viral hepatitis are $\{(0.3, 0.6), (0.8, 0.1), (0.1, 0.7), (0.2, 0.7)\}$, $\{(0.9, 0.1), (0.2, 0.6), (0.1, 0.8), (0.1, 0.7)\}$, $\{(0.6, 0.2), (0.3, 0.6), (0.6, 0.3), (0.7, 0.2)\}$ and $\{(0.2, 0.6), (0.6, 0.2), (0.5, 0.4), (0.1, 0.7)\}$ respectively.
Suppose a patient who is suffering from a disease, have the symptoms P (abdominal pain, fever and diarrhea). Now **the problem is how a doctor detects the actual disease among these four diseases for that patient**. Now we will solve this problem by applying $IFSM$-Algorithm.
Here $U = \{$typhoid, peptic ulcer, food poisoning, acute viral hepatitis$\} = \{d_1, d_2, d_3, d_4\}$, $E = \{$abdominal pain, fever, nausea vomiting, diarrhea$\} = \{e_1, e_2, e_3, e_4\}$ and $P = \{e_1, e_2, e_4\} \subset E$
(1) The intuitionistic fuzzy soft matrix obtained from the given data is,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.3,0.6) & (0.8,0.1) & (0.1,0.7) & (0.2,0.7) \\ (0.9,0.1) & (0.2,0.6) & (0.1,0.8) & (0.1,0.7) \\ (0.6,0.2) & (0.3,0.6) & (0.6,0.3) & (0.7,0.2) \\ (0.2,0.6) & (0.6,0.2) & (0.5,0.4) & (0.1,0.7) \end{pmatrix}$$

(2) The choice matrix of the patient is,

$$e_P \begin{pmatrix} & e_P & & \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) \end{pmatrix} [\text{ Since } P = \{e_1, e_2, e_4\}]$$

(3) and (4) Corresponding product intuitionistic fuzzy soft matrix is,

$$U_A \begin{pmatrix} & e_P & & \\ (0.3,0.6) & (0.8,0.1) & (0.1,0.7) & (0.2,0.7) \\ (0.9,0.1) & (0.2,0.6) & (0.1,0.8) & (0.1,0.7) \\ (0.6,0.2) & (0.3,0.6) & (0.6,0.3) & (0.7,0.2) \\ (0.2,0.6) & (0.6,0.2) & (0.5,0.4) & (0.1,0.7) \end{pmatrix} \otimes e_P \begin{pmatrix} & e_P & & \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (1,0) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0.8,0.1) & (0,1) & (0.8,0.1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) & (0.9,0.1) \\ (0.7,0.2) & (0.7,0.2) & (0,1) & (0.7,0.2) \\ (0.6,0.2) & (0.6,0.2) & (0,1) & (0.6,0.2) \end{pmatrix} = \hat{R}_{IFS}$$

[As per the rule of multiplication of fuzzy soft matrices.]

(5) Now the weights of the diseases are,

(i) $W(d_1) = 0.8 + 0.8 + 0 + 0.8 = 2.4$
(ii) $W(d_2) = 0.9 + 0.9 + 0 + 0.9 = 2.7$
(iii) $W(d_3) = 0.7 + 0.7 + 0 + 0.7 = 2.1$
(iv) $W(d_4) = 0.6 + 0.6 + 0 + 0.6 = 1.8$

(6) The disease associated with the second row of the resultant intuitionistic fuzzy soft matrix($\hat{R}_{IFS}$) has the highest weight($W(d_2) = 2.7$), therefore d_2 be the optimal choice disease. **Hence the patient is suffering from the disease peptic ulcer(d_2).**

Example 6.2. In medical science[11] there are different types of diseases and various types of reasons are responsible for them. Now suppose that according to Dr.X. personal habits are responsible for dental caries $(0.7, 0.2)$, for gum disease $(0.8, 0.1)$, for oral ulcer $(0.8, 0.2)$; food habits are responsible for dental caries $(0.8, 0.1)$, for gum disease $(0.7, 0.3)$, for oral ulcer $(0.4, 0.5)$. Again let according to Dr.Y personal habits are responsible for dental caries $(0.6, 0.3)$, for gum disease $(0.8, 0.2)$, for oral ulcer $(0.9, 0.1)$; food habits are responsible for dental caries $(0.8, 0.1)$, for gum disease $(0.7, 0.2)$, for oral ulcer $(0.5, 0.4)$ and hereditary factor is also responsible for dental caries $(0.2, 0.7)$, for gum disease $(0.4, 0.3)$, for oral ulcer $(0.6, 0.3)$. **Now the problem is to find out the disease which is mostly affected by the personal habits, food habits and hereditary factors of a human being according to both Dr.X and Dr.Y simultaneously.**

Now we will solve this problem by applying $IFSM$-Algorithm.
Here $U = \{$dental caries, gum disease, oral ulcer$\} = \{d_1, d_2, d_3\}$,
$E = \{$personal habits, food habits, hereditary factors$\} = \{e_1, e_2, e_3\}$. The choice parameter set of Dr.X is, $A = \{e_1, e_2\} \subset E$ and the choice parameter set of Dr.Y is,

$A = \{e_1, e_2, e_3\} \subseteq E$

(1) The intuitionistic fuzzy soft matrices according to Dr.X and Dr.Y are respectively,

$$(\hat{a}_{ij}) = \begin{pmatrix} (0.7,0.2) & (0.8,0.1) & (0,1) \\ (0.8,0.1) & (0.7,0.3) & (0,1) \\ (0.8,0.2) & (0.4,0.5) & (0,1) \end{pmatrix}$$

$$(\hat{b}_{ik}) = \begin{pmatrix} (0.6,0.3) & (0.8,0.1) & (0.2,0.7) \\ (0.8,0.2) & (0.7,0.2) & (0.4,0.3) \\ (0.9,0.1) & (0.5,0.4) & (0.6,0.3) \end{pmatrix}$$

(2) The combined choice matrices of Mr.X and Mr.Y in different forms are,

$$e_A \begin{pmatrix} & e_B & \\ (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) \\ (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$e_B \begin{pmatrix} & e_A & \\ (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (0,1) \end{pmatrix}$$

(3) Corresponding product intuitionistic fuzzy soft matrices are,

$$U_A \begin{pmatrix} & e_A & \\ (0.7,0.2) & (0.8,0.1) & (0,1) \\ (0.8,0.1) & (0.7,0.3) & (0,1) \\ (0.8,0.2) & (0.4,0.5) & (0,1) \end{pmatrix} \otimes \; e_A \begin{pmatrix} & e_B & \\ (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) \\ (0,1) & (0,1) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.2) & (0.8,0.2) & (0.8,0.2) \end{pmatrix}$$

$$U_B \begin{pmatrix} & e_B & \\ (0.6,0.3) & (0.8,0.1) & (0.2,0.7) \\ (0.8,0.2) & (0.7,0.2) & (0.4,0.3) \\ (0.9,0.1) & (0.5,0.4) & (0.6,0.3) \end{pmatrix} \otimes \; e_B \begin{pmatrix} & e_A & \\ (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0.8,0.1) & (0,1) \\ (0.8,0.2) & (0.8,0.2) & (0,1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) \end{pmatrix}$$

(4) The sum of these product intuitionistic fuzzy soft matrices is,

$$\begin{pmatrix} (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.2) & (0.8,0.2) & (0.8,0.2) \end{pmatrix} \oplus \begin{pmatrix} (0.8,0.1) & (0.8,0.1) & (0,1) \\ (0.8,0.2) & (0.8,0.2) & (0,1) \\ (0.9,0.1) & (0.9,0.1) & (0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.9,0.1) & (0.9,0.1) & (0.8,0.2) \end{pmatrix} = \hat{R}_{IFS}$$

(5) Now the weights of the diseases are,

(i) $W(d_1) = 0.8 + 0.8 + 0.8 = 2.4$
(ii) $W(d_2) = 0.8 + 0.8 + 0.8 = 2.4$
(iii) $W(d_3) = 0.9 + 0.9 + 0.8 = 2.6$

(6) The disease associated with the third row of the resultant intuitionistic fuzzy soft matrix($\hat{R}_{IFS}$) has the highest weight($W(d_3) = 2.6$), therefore d_3 be the optimal choice disease. **Hence oral ulcer(d_3) is mostly affected by the personal habits, food habits and hereditary factor according to the both doctors.**

7. Conclusion:

In this paper we have proposed **the concept of intuitionistic fuzzy soft matrix** and after that **different types of matrices in intuitionistic fuzzy soft set theory** have been defined. Then we have introduced here some new operations and properties on these matrices. Furthermore **an efficient solution procedure named as $IFSM$-Algorithm has been developed to solve intuitionistic fuzzy soft set(or intuitionistic fuzzy soft matrix) based group decision making problems** and it has been applied **in medical science to the problems of diagnosis of a disease from the myriad of symptoms as well as to evaluate the effectiveness of different habits of human being responsible for a disease**.

References

[1] D. Molodtsov, Soft Set Theory-First Results, Computers and Mathematics with Applications, 37, (1999), 19-31.
[2] P. K. Maji, R. Biswas and A. R. Roy, An application of soft sets in a decision making problem, Computers And Mathematics with Applications, 44, (2002), 1077-1083.
[3] P. K. Maji, R. Biswas and A. R. Roy, Soft Set Theory, Computers And Mathematics with Applications, 45, (2003), 555-562.
[4] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy Soft Sets, The Journal of Fuzzy Mathematics, 9(3), (2001), 589-602.
[5] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, (1986), 87-96.
[6] L. A. Zadeh, Fuzzy Sets, Information And Control, 8, (1965), 338-353.
[7] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic Fuzzy Soft Set, The Journal of Fuzzy Mathematics, 9(3), (2001), 677-692.
[8] L.M. Tierney, S.J. McPhee, and M.A. Papadakis, Current Medical Diagnosis and Treatment, 42nd Edition, Lange Medical Books/ McGraw-Hill Medical Publishing Division, (2003), 947, 1258.
[9] N. Cagman and S. Enginoglu, Soft Matrix Theory and It's Decision Making, Computers and Mathematics with Applications, 59 (2010), 3308-3314.
[10] N.Cagman and S.Enginoglu, Soft Set Theory and Uni-int Decision Making, European Journal of Operational Research, 207 (2010), 848-855.
[11] Carranza, Carranza's Clinical Periodontology, 10th edition, Elsevier(A Divison of Reed Elsevier Pvt. Ltd.), (2006).
[12] Hauser, L. Stephen, E. Braunwald, S.A. Fauci and D.L. Kasper, Harrison's Principles of Internal Medicine, 15th edition, I (2001), 971, 1147-1148.
[13] Hauser, L. Stephen, E. Braunwald, S.A. Fauci and D.L. Kasper, Harrison's Principles of Internal Medicine, 15th edition, II (2001), 1654, 1730.

Tanushree Mitra Basu (tanushreemitra13@gmail.com) – Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore -721 102,W.B., India

Nirmal Kumar Mahapatra (nirmal_hridoy@yahoo.co.in) – Department of Mathematics, Panskura Banamali College, Panskura RS-721152, WB, India

Shyamal Kumar Mondal (shyamal_260180@yahoo.com) – Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore -721 102,W.B., India