Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Multiple attribute group decision making based on intuitionistic fuzzy neutral geometric operators induced by interaction coefficients

Xin Mei Wang, Zi Wen Jiang, Wei Yang

Received 29 May 2017; Revised 14 June 2017; Accepted 15 July 2017

Abstract. Various intuitionistic fuzzy aggregation operators for intuitionistic fuzzy numbers (IFNs) have been proposed and applied in the multiple attribute (group) decision making in the literature. In this paper, the neutral geometric operations are provided by an interaction coefficient, and some properties of them are investigated. Based on these, the intuitionistic fuzzy weighted neutral geometric (IFWNG) operator and the intuitionistic fuzzy ordered weighted neutral geometric (IFOWNG) operators are developed. Moreover, the approach to multiple attribute group decision making based on the proposed IFWNG operator are given. Finally, an example is given to show the feasibility and validity of the new approaches.

2010 AMS Classification: 91B06, 03E72

Keywords: Intuitionistic fuzzy set, Neutral geometric operation, Intuitionistic fuzzy neutral geometric operators, Multiple attribute group decision making.

Corresponding Author: (wyoeng@126.com)

1. Introduction

Intuitionistic fuzzy set (IFS) [1] is the generalization of fuzzy set [33]. An IFS is characterized by a membership function, a non-membership function and a hesitancy function with their sum equal to 1, who form a triple called intuitionistic fuzzy number (IFN) [25, 27], and thus can depict the fuzzy character of data more comprehensively than fuzzy set only characterized by a membership function. For example, in a variety of voting events, in addition to the support and objection, there is usually the abstention which indicates the hesitation and indeterminacy of the voter to the object. Thus, IFSs are more suitable to deal with these cases, especially the multiple attribute (group) decision making problems, than fuzzy sets.

IFNs are a vital tool to express a decision maker's preference information over objects in the process of decision making. In order to obtain a decision result, an crucial step is to aggregate the given IFNs by the intuitionistic fuzzy aggregation operators which are roughly divided into two classes [4, 6, 10, 11, 16, 17, 21, 22, 23, 25, 26, 30, 31, 32, 34]. Some of the intuitionistic fuzzy aggregation operators were constructed by applying different operators to the components of the IFNs. Xu and Yager [25] introduced the intuitionistic fuzzy weighted geometric (IFWG) operator by applying the operators derived from the algebraic product triangular norm and its conorm to the components of the IFNs. Xu [26] also defined the intuitionistic fuzzy weighted averaging (IFWA) operator. Similar work has been done in [31, 22, 23] using other triangular norms and their conorm.

Other intuitionistic fuzzy aggregation operators were composed by the same operators due to that we are neutral and want to be treated fairly in some cases [11, 30]. Beliakov et al. [4] provided the IFWA operator by using the Łukasiewicz triangular norm and its conorm. Based on algebraic product triangular norm, Liao and Xu [18, 28] introduced the simple intuitionistic fuzzy weighted geometric (SIFWG) operator. Deriving the operations from algebraic product triangular norm and its conorm, Xia and Xu [30] defined the symmetric intuitionistic fuzzy weighted geometric (SIFWG) operators. Considering the interactions between membership degrees and non-membership degrees of different IFNs [10], He et al. [11, 12, 13, 14] proposed the neutral operation to define the intuitionistic fuzzy weighted neutral averaging (IFWNA) operator which are regarded as a complement to the existing works [25, 26] on IFSs, especially when one of the membership degrees of IFNs is zero.

The rest of the paper is organized as follows. Section 2 briefly reviews some basic concepts on IFSs. In Section 3, we analyze the existing neutral operation, scalar neutral operation and the IFWNA operator proposed in [11]. In Section 4, associated with the interaction coefficient, we develops a neutral geometric operation, a neutral power operation and intuitionistic fuzzy neutral geometric operators including the IFWNG operator and the IFOWNG operator. The properties of them are investigated. In Section 5, we apply the proposed IFWNG operators to multiple attritute group decision making under intuitionistic fuzzy environment and a numerical example is given to show the feasibility and validity of the new approaches. Finally, Section 6 concludes the paper.

2. Preliminaries

Atanassov [1] introduced the concept of intuitionistic fuzzy sets.

Definition 2.1 ([1])**.** Let X be a given universe. Then an intuitionistic fuzzy set (IFS) A in X is defined as follows $A = \{x, \mu_A(x), \nu_A(x) | x \in X\}$ $\mu_A(x), \nu_A(x) \in [0, 1]$ indicate the amount of guaranteed membership and non-membership of x in A, respectively and fulfill $\mu_A(x) + \nu_A(x) \leq 1$. Furthermore, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called a hesitancy degree or an intuitionistic index of x in A.

In the special case, $\pi_A(x) = 0$, i.e., $\mu_A(x) + \nu_A(x) = 1$, the IFS A reduces to a fuzzy set [33]. We recall the membership grade of x in A which is represented as a triple $(\mu_A(x), \nu_A(x), \pi_A(x))$ called an intuitionistic fuzzy number (IFN) [27], for convenience, we denote an IFN by $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$, where $\mu_\alpha, \nu_\alpha \in [0, 1]$, $\mu_\alpha + \nu_\alpha \leq 1$

and $\pi_\alpha = 1 - \mu_\alpha - \nu_\alpha$. Each IFN has a physical interpretation, for example, if $\alpha = (0.3, 0.2, 0.5)$, then it can be interpreted as "the vote for the resolution is 3 in favor, 2 against, and 5 abstentions"[9]. The following partial order $\leq$ on the set of all IFNs [7] is defined such that $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$, $\beta \leq \alpha$ if and only if $\mu_\beta \leq \mu_\alpha$ and $\nu_\alpha \leq \nu_\beta$. For an IFN α, a score function s [5] is defined as the difference of membership and non-membership functions: $s(\alpha) = \mu_\alpha - \nu_\alpha$, where $s(\alpha) \in [-1, 1]$. The larger the score $s(\alpha)$, the greater the IFN α. To make the comparison method more discriminatory, an accuracy function h [15], which is defined as follows: $h(\alpha) = \mu_\alpha + \nu_\alpha$, where $h(\alpha) \in [0, 1]$. When the scores are the same, the larger the accuracy $h(\alpha)$, the greater the IFN α. It is obvious that $h(\alpha) + \pi_\alpha = 1$. Moreover, the methods for ranking the IFNs are also a focus in the discussion of IFNs, many scholars have been present various methods to solve this problem. Up to now, there is still not a perfect way for solving it completely. This paper doesn't focus on the methods of ranking IFNs, so we only use the common method introduced by Xu and Yager [25] as follows.

Definition 2.2. Let α, β be two IFNs. Then, we have the following:
(i) If $s(\alpha) < s(\beta)$, then α is smaller than β, i.e., $\alpha \prec \beta$.
(ii) If $s(\alpha) = s(\beta)$,
(a) if $h(\alpha) < h(\beta)$, then α is smaller than β, i.e., $\alpha \prec \beta$,
(b) if $h(\alpha) = h(\beta)$, i.e., $\alpha = \beta$.

3. Analysis of the existing operations and aggregation operators for IFNs

It is pointed by He et al. [11] that different operations are needed to adapt to various decision environment, and the existing addition operation [2, 8, 25] on IFNs, which was defined as $\alpha \oplus \beta = (1 - (1 - \mu_\alpha)(1 - \mu_\beta), \nu_\alpha \nu_\beta, (1 - \mu_\alpha)(1 - \mu_\beta) - \nu_\alpha \nu_\beta)$, cannot be used in all situations. For example, let α and β be two IFNs, $\alpha = (\mu_\alpha, 0, 1 - \mu_\alpha)$, $\beta = (\mu_\beta, \nu_\beta, 1 - \mu_\beta - \nu_\beta)$ and $\nu_\beta \neq 0$, then according to the addition operation by Atanassov [2, 8, 25], we have $\nu_{\alpha \oplus \beta} = \nu_\beta \times 0 = 0$. Evidently, ν_β is not accounted for at all, which is an undesirable feature for an averaging operation. Thus, He et al. [11] introduced some new operations on the IFNs, including the neutral operation and the scalar neutral operation, taking the attitude of the decision makers and the interactions between different IFNs into consideration.

Let α and β be two IFNs. Then the neutral operation and scalar neutral operation [11] are equivalently defined as follows:

$$\alpha \boxplus \beta = \left(\frac{(1-\pi_\alpha\pi_\beta)(\mu_\alpha+\mu_\beta)}{\mu_\alpha+\nu_\alpha+\mu_\beta+\nu_\beta}, \frac{(1-\pi_\alpha\pi_\beta)(\nu_\alpha+\nu_\beta)}{\mu_\alpha+\nu_\alpha+\mu_\beta+\nu_\beta}, \pi_\alpha\pi_\beta \right), \tag{3.1}$$

$$\alpha_{\boxplus}^{\lambda} = \left(\frac{(1-\pi_\alpha^\lambda)\mu_\alpha}{\mu_\alpha+\nu_\alpha}, \frac{(1-\pi_\alpha^\lambda)\nu_\alpha}{\mu_\alpha+\nu_\alpha}, \pi_\alpha^\lambda \right). \tag{3.2}$$

The following properties are essential to define the aggregation operators under multiple attribute decision making environment on the operation $\boxplus$.

Proposition 3.1 ([11]). *Let α and β be two IFNs and $\lambda, \lambda_1, \lambda_2 > 0$. Then*
(1) $\alpha \boxplus \beta = \beta \boxplus \alpha$,
(2) $\alpha_{\boxplus}^{\lambda} \boxplus \beta_{\boxplus}^{\lambda} = (\alpha \boxplus \beta)_{\boxplus}^{\lambda}$,

(3) $\alpha_{\boxplus}^{\lambda_1} \boxplus \alpha_{\boxplus}^{\lambda_2} = \alpha_{\boxplus}^{\lambda_1+\lambda_2}$.

Based on the operation $\boxplus$, an intuitionistic fuzzy weighted neutral averaging (IFWNA) operator was defined as fellows.

Definition 3.2 ([11])**.** Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be a collection of IFNs, where $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i}, \pi_{\alpha_i})$, $\mu_{\alpha_i}, \nu_{\alpha_i} \in [0,1]$, $\mu_{\alpha_i} + \nu_{\alpha_i} + \pi_{\alpha_i} = 1$ and $1 \le i \le n$. If

$$\begin{aligned} & IFWN_{\omega}^{\boxplus}(\alpha_1, \alpha_2, \cdots, \alpha_n) \\ = & \left(\frac{(1-\prod_{i=1}^{n} \pi_{\alpha_i}^{\omega_i}) \sum_{i=1}^{n} \omega_i \mu_{\alpha_i}}{\sum_{i=1}^{n} \omega_i \mu_{\alpha_i} + \sum_{i=1}^{n} \omega_i \nu_{\alpha_i}}, \frac{(1-\prod_{i=1}^{n} \pi_{\alpha_i}^{\omega_i}) \sum_{i=1}^{n} \omega_i \nu_{\alpha_i}}{\sum_{i=1}^{n} \omega_i \mu_{\alpha_i} + \sum_{i=1}^{n} \omega_i \nu_{\alpha_i}}, \prod_{i=1}^{n} \pi_{\alpha_i}^{\omega_i} \right), \end{aligned} \tag{3.3}$$

then $IFWN_{\omega}^{\boxplus}$ is called an intuitionistic fuzzy weighted neutral averaging (IFWNA) operator w.r.t. the operation $\boxplus$, where ω_i is the weight of α_i, $\omega_i \in [0,1]$, $1 \le i \le n$ and $\sum_{i=1}^{n} \omega_i = 1$.

Next, we analyse the neutral operation and the IFWNA operator from the following two aspects.

(1) We notice that the drawback of the addition $\oplus$ in Ref. [2, 8, 25] leads that if only one non-membership degree of an IFN equals to zero, the non-membership degree of the aggregation result of n IFNs is zero even if the other non-membership degrees of $n-1$ IFNs are not zero. Although He et al.'s operation overcomes the drawback, a similar circumstance happens for the hesitancy degrees of the aggregation result of n IFNs by Eq. (3.3), that is, if only one hesitancy degree of an IFN is zero, the hesitancy degree of the aggregation result of n IFNs is zero even if the other hesitancy degrees of $n-1$ IFNs are not zero. In fact, the IFWA operator determined by the Łukasiewicz triangular norm and its conorm [4], that is,

$$IFWA_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \left(\sum_{i=1}^{n} \omega_i \mu_{\alpha_i}, \sum_{i=1}^{n} \omega_i \nu_{\alpha_i}, \sum_{i=1}^{n} \omega_i \pi_{\alpha_i} \right) \tag{3.4}$$

can overcome the above drawbacks no matter there is only one non-membership degree or only one hesitancy degree of an IFN is zero.

(2) We find that for the given IFNs α and β, the identity $\alpha^{\lambda} \boxplus \beta^{\lambda} = (\alpha \boxplus \beta)^{\lambda}$ ($\lambda > 0$) does not hold in general which leads the representation of the IFWNA operator is not unique and can yield a contradictory ranking result. These are illustrated by the following examples:

Example 3.3. Let $\alpha = (0.7, 0.2, 0.1)$ and $\beta = (0.3, 0.4, 0.3)$ be two IFNs and $\lambda = 0.3$. Then by computing Eqs. (3.1) and (3.2), we have $\alpha^{\lambda} \boxplus \beta^{\lambda} = (0.42, 0.23, 0.35)$ and $(\alpha \boxplus \beta)^{\lambda} = (0.41, 0.24, 0.35)$. Thus $\alpha^{\lambda} \boxplus \beta^{\lambda} \neq (\alpha \boxplus \beta)^{\lambda}$.

Example 3.4. Let $\alpha_1 = (0.1, 0.1, 0.8)$, $\alpha_2 = (0.1, 0.4, 0.5)$, $\beta_1 = (0.1, 0.8, 0.1)$ and $\beta_2 = (0.6, 0.2, 0.2)$ be four IFNs and $\omega = (0.5, 0.5)$ be the weighting vector. Then

$$\begin{aligned}
IFWNA_{\omega}^{\boxplus}(\alpha_1, \alpha_2) &= {\alpha_1}_{\boxplus}^{0.5} \boxplus {\alpha_2}_{\boxplus}^{0.5} = (0.0514, 0.1324, 0.8162),\\
IFWNA_{\omega}^{\boxplus}(\alpha_1, \alpha_2) &= (\alpha_1 \boxplus \alpha_2)_{\boxplus}^{0.5} = (0.0525, 0.1313, 0.8162),\\
IFWNA_{\omega}^{\boxplus}(\beta_1, \beta_2) &= {\beta_1}_{\boxplus}^{0.5} \boxplus {\beta_2}_{\boxplus}^{0.5} = (0.1703, 0.2590, 0.5707),\\
IFWNA_{\omega}^{\boxplus}(\beta_1, \beta_2) &= (\beta_1 \boxplus \beta_2)_{\boxplus}^{0.5} = (0.1768, 0.2525, 0.5707).
\end{aligned}$$

Thus $s\left({\alpha_1}_{\boxplus}^{0.5} \boxplus {\alpha_2}_{\boxplus}^{0.5}\right) = -0.081 > -0.0887 = s\left({\beta_1}_{\boxplus}^{0.5} \boxplus {\beta_2}_{\boxplus}^{0.5}\right)$, $s\left((\alpha_1 \boxplus \alpha_2)_{\boxplus}^{0.5}\right) = -0.0788 < -0.0757 = s\left((\beta_1 \boxplus \beta_2)_{\boxplus}^{0.5}\right)$ and $IFWNA_{\omega}^{\boxplus}(\alpha_1, \alpha_2) \succ IFWNA_{\omega}^{\boxplus}(\beta_1, \beta_2)$, $IFWNA_{\omega}^{\boxplus}(\alpha_1, \alpha_2) \prec IFWNA_{\omega}^{\boxplus}(\beta_1, \beta_2)$.

4. Intuitionistic fuzzy neutral geometric operator

Motivated by the neutral operations and the IFWNA operator in [11], the IFWG operators and IFOWG operators in [25], we propose an intuitionistic fuzzy weighted neutral geometric operator and an intuitionistic fuzzy ordered weighted geometric neutral operator w.r.t. a new neutral geometric operation.

4.1. **Neutral geometric operation for IFNs.**
Here, we introduce a new neutral geometric operation for the IFNs.

Let α and β be two IFNs. We define a neutral geometric operation between the IFNs α and β as follows:

$$\alpha \circledast \beta = \left(\frac{(1-\pi_\alpha\pi_\beta)\mu_\alpha\mu_\beta}{\mu_\alpha\mu_\beta + \nu_\alpha\nu_\beta}, \frac{(1-\pi_\alpha\pi_\beta)\nu_\alpha\nu_\beta}{\mu_\alpha\mu_\beta + \nu_\alpha\nu_\beta}, \pi_\alpha\pi_\beta\right). \tag{4.1}$$

The operation $\circledast$ can be illustrated by Figure 1, where $\gamma = \alpha \circledast \beta$. From Figure 1 we can find that the aggregation result of the IFNs α and β can be obtained by the following steps:

Step 1 Aggregate the membership degrees, the non-membership degrees and the hesitancy degrees of the IFNs α and β by the algebraic product triangular norm $T_P(x, y) = xy$, then we have $\mu_\alpha\mu_\beta$ (the red part), $\nu_\alpha\nu_\beta$ (the green part) and $\pi_\alpha\pi_\beta$ (the yellow part) in Figure 1 (1). Obviously, their sum is less than 1, so they can not form an IFN,

Step 2 In order to guarantee the aggregation result to be an IFN, we assign a common interaction coefficient $\frac{1-\pi_\alpha\pi_\beta}{\mu_\alpha\mu_\beta+\nu_\alpha\nu_\beta}$ to $\mu_\alpha\mu_\beta$ and $\nu_\alpha\nu_\beta$, i.e., $\frac{1-\pi_\alpha\pi_\beta}{\mu_\alpha\mu_\beta+\nu_\alpha\nu_\beta}\mu_\alpha\mu_\beta$ and $\frac{1-\pi_\alpha\pi_\beta}{\mu_\alpha\mu_\beta+\nu_\alpha\nu_\beta}\nu_\alpha\nu_\beta$, which are regarded as the membership degree and the non-membership degree of the aggregation result. At this time, the hesitancy degree of the aggregation result is still $\pi_\alpha\pi_\beta$.

The following example shows us how the operation $\circledast$ perform on the IFNs.

Example 4.1. Let $\alpha = (0.2, 0.4, 0.4)$ and $\beta = (0.4, 0.3, 0.3)$ be two IFNs. Then $\alpha \circledast \beta = \left(\begin{array}{c}\frac{(1-0.4\times0.3)0.2\times0.4}{0.2\times0.4+0.4\times0.3}, \frac{(1-0.4\times0.3)0.4\times0.3}{0.2\times0.4+0.4\times0.3}, \\ 0.4\times0.3\end{array}\right) = (0.352, 0.528, 0.12)$.

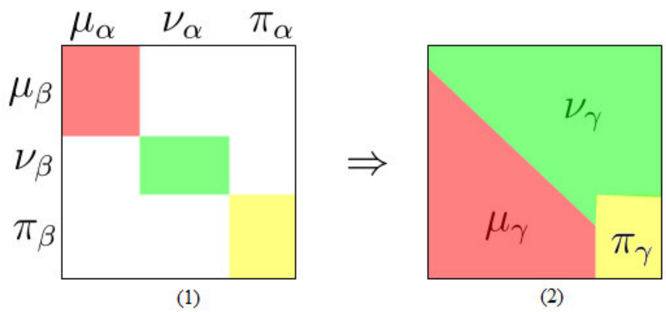

FIGURE 1. Geometric illustration of the proposed operation

Using Eq. (4.1) and the mathematical induction, we get $\alpha_{\circledast}^{n}=(\frac{(1-\pi_\alpha^n)\mu_\alpha^n}{\mu_\alpha^n+\nu_\alpha^n},\frac{(1-\pi_\alpha^n)\nu_\alpha^n}{\mu_\alpha^n+\nu_\alpha^n},\pi_\alpha^n)$. Therefore, we propose the neutral power operation $\alpha_{\circledast}^{\lambda}$ as follows:

$$\alpha_{\circledast}^{\lambda}=\left(\ \frac{(1-\pi_\alpha^\lambda)\mu_\alpha^\lambda}{\mu_\alpha^\lambda+\nu_\alpha^\lambda},\frac{(1-\pi_\alpha^\lambda)\nu_\alpha^\lambda}{\mu_\alpha^\lambda+\nu_\alpha^\lambda},\pi_\alpha^\lambda\ \right). \tag{4.2}$$

Obviously, the aggregation results of the neutral geometric operation $\circledast$ for the IFNs α and β and the associated power operation for the IFN α are also IFNs, and they possess the following properties.

Proposition 4.2. *Let α,β be two IFNs and $\lambda,\lambda_1,\lambda_2>0$. Then*
(1) $\alpha\circledast\beta=\beta\circledast\alpha$,
(2) $\alpha_{\circledast}^{\lambda}\circledast\beta_{\circledast}^{\lambda}=(\alpha\circledast\beta)_{\circledast}^{\lambda}$,
(3) $\alpha_{\circledast}^{\lambda_1}\circledast\alpha_{\circledast}^{\lambda_2}=\alpha_{\circledast}^{\lambda_1+\lambda_2}$.

Proof. (1) It is obvious.
(2) Using Eqs. (4.1) and (4.2), we have

$$\alpha_{\circledast}^{\lambda}\circledast\beta_{\circledast}^{\lambda}=\left(\ \frac{(1-\pi_\alpha^\lambda)\mu_\alpha^\lambda}{\mu_\alpha^\lambda+\nu_\alpha^\lambda},\frac{(1-\pi_\alpha^\lambda)\nu_\alpha^\lambda}{\mu_\alpha^\lambda+\nu_\alpha^\lambda},\pi_\alpha^\lambda\ \right)\circledast\left(\ \frac{(1-\pi_\beta^\lambda)\mu_\beta^\lambda}{\mu_\beta^\lambda+\nu_\beta^\lambda},\frac{(1-\pi_\beta^\lambda)\nu_\beta^\lambda}{\mu_\beta^\lambda+\nu_\beta^\lambda},\pi_\beta^\lambda\ \right).$$

For convenience, we assume that $\frac{1-\pi_\alpha^\lambda}{\mu_\alpha^\lambda+\nu_\alpha^\lambda}=A$ and $\frac{1-\pi_\beta^\lambda}{\mu_\beta^\lambda+\nu_\beta^\lambda}=B$, then the above identity can be represented as:

$$\begin{aligned}&=\left(\ A\mu_\alpha^\lambda,A\nu_\alpha^\lambda,\pi_\alpha^\lambda\ \right)\circledast\left(\ B\mu_\beta^\lambda,B\nu_\beta^\lambda,\pi_\beta^\lambda\ \right)\\&=\left(\begin{array}{c}\frac{(1-\pi_\alpha^\lambda\pi_\beta^\lambda)AB\mu_\alpha^\lambda\mu_\beta^\lambda}{AB\mu_\alpha^\lambda\mu_\beta^\lambda+AB\nu_\alpha^\lambda\nu_\beta^\lambda},\\\frac{(1-\pi_\alpha^\lambda\pi_\beta^\lambda)AB\nu_\alpha^\lambda\nu_\beta^\lambda}{AB\mu_\alpha^\lambda\mu_\beta^\lambda+AB\nu_\alpha^\lambda\nu_\beta^\lambda},\pi_\alpha^\lambda\pi_\beta^\lambda\end{array}\right)=\left(\begin{array}{c}\frac{1-\pi_\alpha^\lambda\pi_\beta^\lambda}{\mu_\alpha^\lambda\mu_\beta^\lambda+\nu_\alpha^\lambda\nu_\beta^\lambda}\mu_\alpha^\lambda\mu_\beta^\lambda,\\\frac{1-\pi_\alpha^\lambda\pi_\beta^\lambda}{\mu_\alpha^\lambda\mu_\beta^\lambda+\nu_\alpha^\lambda\nu_\beta^\lambda}\nu_\alpha^\lambda\nu_\beta^\lambda,\pi_\alpha^\lambda\pi_\beta^\lambda\end{array}\right)=(\alpha\circledast\beta)_{\circledast}^{\lambda}.\end{aligned}$$

Thus the identity holds.
(3) By Eqs. (4.1) and (4.2), we get

$$\alpha_{\circledast}^{\lambda_1}\circledast\alpha_{\circledast}^{\lambda_2}=\left(\ \frac{(1-\pi_\alpha^{\lambda_1})\mu_\alpha^{\lambda_1}}{\mu_\alpha^{\lambda_1}+\nu_\alpha^{\lambda_1}},\frac{(1-\pi_\alpha^{\lambda_1})\mu_\alpha^{\lambda_1}}{\mu_\alpha^{\lambda_1}+\nu_\alpha^{\lambda_1}},\pi_\alpha^{\lambda_1}\ \right)\circledast\left(\ \frac{(1-\pi_\alpha^{\lambda_2})\mu_\alpha^{\lambda_2}}{\mu_\alpha^{\lambda_2}+\nu_\alpha^{\lambda_2}},\frac{(1-\pi_\alpha^{\lambda_2})\mu_\alpha^{\lambda_2}}{\mu_\alpha^{\lambda_2}+\nu_\alpha^{\lambda_2}},\pi_\alpha^{\lambda_2}\ \right).$$

For convenience, we let $\frac{1-\pi_\alpha^{\lambda_1}}{\mu_\alpha^{\lambda_1}+\nu_\alpha^{\lambda_1}}=A$ and $\frac{1-\pi_\alpha^{\lambda_2}}{\mu_\alpha^{\lambda_2}+\nu_\alpha^{\lambda_2}}=B$, then the above identity can be represented as:

$$=\left(\ A\mu_\alpha^{\lambda_1}, A\mu_\alpha^{\lambda_1}, \pi_\alpha^{\lambda_1}\ \right)\circledast\left(\ B\mu_\alpha^{\lambda_2}, B\mu_\alpha^{\lambda_2}, \pi_\alpha^{\lambda_2}\ \right)$$

$$=\left(\begin{array}{c}\frac{(1-\pi_\alpha^{\lambda_1+\lambda_2})AB\mu_\alpha^{\lambda_1+\lambda_2}}{AB\mu_\alpha^{\lambda_1+\lambda_2}+AB\nu_\alpha^{\lambda_1+\lambda_2}},\\ \frac{(1-\pi_\alpha^{\lambda_1+\lambda_2})AB\nu_\alpha^{\lambda_1+\lambda_2}}{AB\mu_\alpha^{\lambda_1+\lambda_2}+AB\nu_\alpha^{\lambda_1+\lambda_2}},\pi_\alpha^{\lambda_1+\lambda_2}\end{array}\right)=\left(\begin{array}{c}\frac{1-\pi_\alpha^{\lambda_1+\lambda_2}}{\mu_\alpha^{\lambda_1+\lambda_2}+\nu_\alpha^{\lambda_1+\lambda_2}}\mu_\alpha^{\lambda_1+\lambda_2},\\ \frac{1-\pi_\alpha^{\lambda_1+\lambda_2}}{\mu_\alpha^{\lambda_1+\lambda_2}+\nu_\alpha^{\lambda_1+\lambda_2}}\nu_\alpha^{\lambda_1+\lambda_2},\pi_\alpha^{\lambda_1+\lambda_2}\end{array}\right)=\alpha_\circledast^{\lambda_1+\lambda_2}.$$

Thus the identity holds. □

4.2. **Intuitionistic fuzzy neutral geometric operator w.r.t. the proposed neutral operation.** Associated with the IFWG operators in Ref. [25] and the IFWNA operators in Ref. [11], we introduce the intuitionistic fuzzy neutral geometric operators w.r.t. the proposed neutral operations as follows.

Definition 4.3. Let $\alpha_1,\alpha_2,\cdots,\alpha_n$ be a collection of IFNs, where $\alpha_i=(\mu_{\alpha_i},\nu_{\alpha_i},\pi_{\alpha_i})$, $\mu_{\alpha_i},\nu_{\alpha_i}\in[0,1]$, $\mu_{\alpha_i}+\nu_{\alpha_i}+\pi_{\alpha_i}=1$, and $1\le i\le n$. If

$$IFWNG_\omega^\circledast(\alpha_1,\alpha_2,\cdots,\alpha_n)=\underset{i=1}{\overset{n}{\circledast}}\alpha_i{}_\circledast^{\omega_i}, \tag{4.3}$$

then $IFWNG_\omega^\circledast$ is called an intuitionistic fuzzy weighted neutral geometric (IFWNG) operator w.r.t. the operation $\circledast$, where ω_i is the weight of α_i, $\omega_i\in[0,1]$, $1\le i\le n$ and $\sum_{i=1}^n\omega_i=1$.

Proposition 4.4. *Let $\alpha_1,\alpha_2,\cdots,\alpha_n$ be a collection of IFNs. Then the aggregated result by using the IFWNG operator is also an IFN and*

$$\begin{aligned}&IFWNG_\omega^\circledast(\alpha_1,\alpha_2,\cdots,\alpha_n)\\&=\left(\frac{(1-\prod_{i=1}^n\pi_{\alpha_i}^{\omega_i})\prod_{i=1}^n\mu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^n\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^n\nu_{\alpha_i}^{\omega_i}},\frac{(1-\prod_{i=1}^n\pi_{\alpha_i}^{\omega_i})\prod_{i=1}^n\nu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^n\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^n\nu_{\alpha_i}^{\omega_i}},\prod_{i=1}^n\pi_{\alpha_i}^{\omega_i}\right)\end{aligned} \tag{4.4}$$

Proof. The first result follows immediately from Eqs. (4.1) and (4.2). Next, we prove Eq. (4.4) by the mathematical induction on n.

We firstly prove that it holds for $n=2$. By Eqs. (4.1) and (4.2), it holds that

$$\alpha_1{}_\circledast^{\omega_1}\circledast\alpha_2{}_\circledast^{\omega_2}=\left(\ \frac{(1-\pi_{\alpha_1}^{\omega_1})\mu_{\alpha_1}^{\omega_1}}{\mu_{\alpha_1}^{\omega_1}+\nu_{\alpha_1}^{\omega_1}},\frac{(1-\pi_{\alpha_1}^{\omega_1})\nu_{\alpha_1}^{\omega_1}}{\mu_{\alpha_1}^{\omega_1}+\nu_{\alpha_1}^{\omega_1}},\pi_{\alpha_1}^{\omega_1}\ \right)\circledast\left(\ \frac{(1-\pi_{\alpha_2}^{\omega_2})\mu_{\alpha_2}^{\omega_2}}{\mu_{\alpha_2}^{\omega_2}+\nu_{\alpha_2}^{\omega_2}},\frac{(1-\pi_{\alpha_2}^{\omega_2})\nu_{\alpha_2}^{\omega_2}}{\mu_{\alpha_2}^{\omega_2}+\nu_{\alpha_2}^{\omega_2}},\pi_{\alpha_2}^{\omega_2}\ \right).$$

For convenience, we assume that $\frac{1-\pi_{\alpha_1}^{\omega_1}}{\mu_{\alpha_1}^{\omega_1}+\nu_{\alpha_1}^{\omega_1}}=A_1$ and $\frac{1-\pi_{\alpha_2}^{\omega_2}}{\mu_{\alpha_2}^{\omega_2}+\nu_{\alpha_2}^{\omega_2}}=A_2$, then the above identity can be represented as:

$$=\left(\ A_1\mu_{\alpha_1}^{\omega_1}, A_1\nu_{\alpha_1}^{\omega_1}, \pi_{\alpha_1}^{\omega_1}\ \right)\circledast\left(\ A_2\mu_{\alpha_2}^{\omega_2}, A_2\nu_{\alpha_2}^{\omega_2}, \pi_{\alpha_2}^{\omega_2}\ \right)$$

$$=\left(\begin{array}{c}\frac{(1-\pi_{\alpha_1}^{\omega_1}\pi_{\alpha_2}^{\omega_2})A_1A_2\mu_{\alpha_1}^{\omega_1}\mu_{\alpha_2}^{\omega_2}}{A_1A_2\mu_{\alpha_1}^{\omega_1}\mu_{\alpha_2}^{\omega_2}+A_1A_2\nu_{\alpha_1}^{\omega_1}\nu_{\alpha_2}^{\omega_2}},\\ \frac{(1-\pi_{\alpha_1}^{\omega_1}\pi_{\alpha_2}^{\omega_2})A_1A_2\nu_{\alpha_1}^{\omega_1}\nu_{\alpha_2}^{\omega_2}}{A_1A_2\mu_{\alpha_1}^{\omega_1}\mu_{\alpha_2}^{\omega_2}+A_1A_2\nu_{\alpha_1}^{\omega_1}\nu_{\alpha_2}^{\omega_2}},\pi_{\alpha_1}^{\omega_1}\pi_{\alpha_2}^{\omega_2}\end{array}\right)=\left(\begin{array}{c}\frac{(1-\pi_{\alpha_1}^{\omega_1}\pi_{\alpha_2}^{\omega_2})\mu_{\alpha_1}^{\omega_1}\mu_{\alpha_2}^{\omega_2}}{\mu_{\alpha_1}^{\omega_1}\mu_{\alpha_2}^{\omega_2}+\nu_{\alpha_1}^{\omega_1}\nu_{\alpha_2}^{\omega_2}},\\ \frac{(1-\pi_{\alpha_1}^{\omega_1}\pi_{\alpha_2}^{\omega_2})\nu_{\alpha_1}^{\omega_1}\nu_{\alpha_2}^{\omega_2}}{\mu_{\alpha_1}^{\omega_1}\mu_{\alpha_2}^{\omega_2}+\nu_{\alpha_1}^{\omega_1}\nu_{\alpha_2}^{\omega_2}},\pi_{\alpha_1}^{\omega_1}\pi_{\alpha_2}^{\omega_2}\end{array}\right).$$

If Eq. (4.4) holds for $n=k$, that is,

$$IFWG_{\omega}^{\circledast}(\alpha_1,\alpha_2,\cdots,\alpha_{k+1}) = {\alpha_1}_{\circledast}^{\omega_1}\circledast {\alpha_2}_{\circledast}^{\omega_2}\circledast\cdots\circledast {\alpha_k}_{\circledast}^{\omega_k}\circledast {\alpha_{k+1}}_{\circledast}^{\omega_{k+1}}$$
$$=\left(\frac{(1-\prod_{i=1}^{k}\pi_{\alpha_i}^{\omega_i})\prod_{i=1}^{k}\mu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{k}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{k}\nu_{\alpha_i}^{\omega_i}},\frac{(1-\prod_{i=1}^{k}\pi_{\alpha_i}^{\omega_i})\prod_{i=1}^{k}\nu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{k}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{k}\nu_{\alpha_i}^{\omega_i}},\prod_{i=1}^{k}\pi_{\alpha_i}^{\omega_i}\right)$$
$$\circledast\left(\frac{(1-\pi_{\alpha_{k+1}}^{\omega_{k+1}})\mu_{\alpha_{k+1}}^{\omega_{k+1}}}{\mu_{\alpha_{k+1}}^{\omega_{k+1}}+\nu_{\alpha_{k+1}}^{\omega_{k+1}}},\frac{(1-\pi_{\alpha_{k+1}}^{\omega_{k+1}})\nu_{\alpha_{k+1}}^{\omega_{k+1}}}{\mu_{\alpha_{k+1}}^{\omega_{k+1}}+\nu_{\alpha_{k+1}}^{\omega_{k+1}}},\pi_{\alpha_{k+1}}^{\omega_{k+1}}\right).$$

For convenience, we assume that $\frac{1-\prod_{i=1}^{k}\pi_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{k}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{k}\nu_{\alpha_i}^{\omega_i}}=A_k$ and $\frac{1-\pi_{\alpha_{k+1}}^{\omega_{k+1}}}{\mu_{\alpha_{k+1}}^{\omega_{k+1}}+\nu_{\alpha_{k+1}}^{\omega_{k+1}}}=A_{k+1}$. Then, when $n=k+1$, using Eqs. (4.1) and (4.2), the above identity can be represented as:

$$=\left(A_k\prod_{i=1}^{k}\mu_{\alpha_i}^{\omega_i},A_k\prod_{i=1}^{k}\nu_{\alpha_i}^{\omega_i},\prod_{i=1}^{k}\pi_{\alpha_i}^{\omega_i}\right)\circledast\left(A_{k+1}\mu_{\alpha_{k+1}}^{\omega_{k+1}},A_{k+1}\nu_{\alpha_{k+1}}^{\omega_{k+1}},\pi_{\alpha_{k+1}}^{\omega_{k+1}}\right)$$
$$=\left(\frac{(1-\prod_{i=1}^{k+1}\pi_{\alpha_i}^{\omega_i})A_kA_{k+1}\prod_{i=1}^{k+1}\mu_{\alpha_i}^{\omega_i}}{A_kA_{k+1}\prod_{i=1}^{k+1}\mu_{\alpha_i}^{\omega_i}+A_kA_{k+1}\prod_{i=1}^{k+1}\nu_{\alpha_i}^{\omega_i}},\frac{(1-\prod_{i=1}^{k+1}\pi_{\alpha_i}^{\omega_i})A_kA_{k+1}\prod_{i=1}^{k+1}\nu_{\alpha_i}^{\omega_i}}{A_kA_{k+1}\prod_{i=1}^{k+1}\mu_{\alpha_i}^{\omega_i}+A_kA_{k+1}\prod_{i=1}^{k+1}\nu_{\alpha_i}^{\omega_i}},\prod_{i=1}^{k+1}\pi_{\alpha_i}^{\omega_i}\right)$$
$$=\left(\frac{(1-\prod_{i=1}^{k+1}\pi_{\alpha_i}^{\omega_i})\prod_{i=1}^{k+1}\mu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{k+1}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{k+1}\nu_{\alpha_i}^{\omega_i}},\frac{(1-\prod_{i=1}^{k+1}\pi_{\alpha_i}^{\omega_i})\prod_{i=1}^{k+1}\nu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{k+1}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{k+1}\nu_{\alpha_i}^{\omega_i}},\prod_{i=1}^{k+1}\pi_{\alpha_i}^{\omega_i}\right).$$

i.e., Eq. (4.4) holds for $n=k+1$. Therefore, Eq. (4.4) holds for all n. □

Particularly, when $\pi_{\alpha_i}=0$ $(i=1,2,\cdots,n)$, i.e., $\nu_{\alpha_i}=1-\mu_{\alpha_i}$, we have

$$IFWNG_{\omega}^{\circledast}(\alpha_1,\alpha_2,\cdots,\alpha_n)$$
$$=\left(\frac{\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\omega_i}},1-\frac{\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\omega_i}},0\right),$$

that is, the IFWNG operator w.r.t. the operation $\circledast$ reduces to the symmetric sum operator in fuzzy environment [3].

The following example indicates that how the IFWNG operator performs on the IFNs.

Example 4.5. Let $\alpha_1=(0.7,0.2,0.1),\alpha_2=(0.5,0.3,0.2),\alpha_3=(0.8,0.1,0.1)$ be three IFNs with the weighting vector $\omega=(0.4,0.3,0.3)$. Then

$$IFWNG_{\omega}^{\circledast}(\alpha_1,\alpha_2,\alpha_3)$$
$$=\left(\frac{(1-0.1^{0.4}\times0.2^{0.3}\times0.1^{0.3})0.7^{0.4}\times0.5^{0.3}\times0.8^{0.3}}{0.7^{0.4}\times0.5^{0.3}\times0.8^{0.3}+0.2^{0.4}\times0.3^{0.3}\times0.1^{0.3}},\frac{(1-0.1^{0.4}\times0.2^{0.3}\times0.1^{0.3})0.2^{0.4}\times0.3^{0.3}\times0.1^{0.3}}{0.7^{0.4}\times0.5^{0.3}\times0.8^{0.3}+0.2^{0.4}\times0.3^{0.3}\times0.1^{0.3}},0.1^{0.4}\times0.2^{0.3}\times0.1^{0.3}\right)$$
$$=(0.686,0.191,0.123).$$

Here, some properties of the IFWNG operator are obtained as follows.

Proposition 4.6. *(Idempotency) Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be a collection of IFNs. Then $IFWG^{\circledast}_{\omega}(\alpha, \alpha, \cdots, \alpha) = \alpha$.*

Proof. Using Eq. (4.4), we get

$$IFWNG^{\circledast}_{\omega}(\alpha, \alpha, \cdots, \alpha) = \left(\frac{(1-\prod_{i=1}^{n}\pi_{\alpha}^{\omega_i})\prod_{i=1}^{n}\mu_{\alpha}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\alpha}^{\omega_i}+\prod_{i=1}^{n}\nu_{\alpha}^{\omega_i}}, \frac{(1-\prod_{i=1}^{n}\pi_{\alpha}^{\omega_i})\prod_{i=1}^{n}\nu_{\alpha}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\alpha}^{\omega_i}+\prod_{i=1}^{n}\nu_{\alpha}^{\omega_i}}, \prod_{i=1}^{n}\pi_{\alpha}^{\omega_i} \right)$$

$$= \left(\frac{(1-\pi_{\alpha}^{\sum_{i=1}^{n}\omega_i})\mu_{\alpha}^{\sum_{i=1}^{n}\omega_i}}{\mu_{\alpha}^{\sum_{i=1}^{n}\omega_i}+\nu_{\alpha}^{\sum_{i=1}^{n}\omega_i}}, \frac{(1-\pi_{\alpha}^{\sum_{i=1}^{n}\omega_i})\nu_{\alpha}^{\sum_{i=1}^{n}\omega_i}}{\mu_{\alpha}^{\sum_{i=1}^{n}\omega_i}+\nu_{\alpha}^{\sum_{i=1}^{n}\omega_i}}, \pi_{\alpha}^{\sum_{i=1}^{n}\omega_i} \right) = \left(\frac{1-\pi_{\alpha}}{\mu_{\alpha}+\nu_{\alpha}}\mu_{\alpha}, \frac{1-\pi_{\alpha}}{\mu_{\alpha}+\nu_{\alpha}}\nu_{\alpha}, \pi_{\alpha} \right)$$

$$=(\mu_{\alpha}, \nu_{\alpha}, \pi_{\alpha}).$$

Thus $IFWG^{\circledast}_{\omega}(\alpha, \alpha, \cdots, \alpha) = \alpha$. □

For given IFNs α and β, we define a partial order as follows:

$$\alpha \preccurlyeq \beta \quad \text{iff} \quad \mu_{\alpha_i} \leq \mu_{\beta_i}, \nu_{\alpha_i} \geq \nu_{\beta_i} \text{ and } \pi_{\alpha_i} = \pi_{\beta_i}.$$

Proposition 4.7. *(Monotonicity) Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ and $\beta_1, \beta_2, \cdots, \beta_n$ be two collections of IFNs such that $\alpha_i \preccurlyeq \beta_i$ $(i = 1, 2, \cdots, n)$. Then $IFWNG^{\circledast}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) \preccurlyeq IFWNG^{\circledast}_{\omega}(\beta_1, \beta_2, \cdots, \beta_n)$.*

Proof. Since $\alpha_i \preccurlyeq \beta_i$ $(i = 1, 2, \cdots, n)$, i.e., $\mu_{\alpha_i} \leq \mu_{\beta_i}, \nu_{\alpha_i} \geq \nu_{\beta_i}$ and $\pi_{\alpha_i} = \pi_{\beta_i}$, we have

$$IFWNG^{\circledast}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \left(\frac{1-\prod_{i=1}^{n}\pi_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{n}\nu_{\alpha_i}^{\omega_i}}\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}, \frac{1-\prod_{i=1}^{n}\pi_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\alpha_i}^{\omega_i}+\prod_{i=1}^{n}\nu_{\alpha_i}^{\omega_i}}\prod_{i=1}^{n}\nu_{\alpha_i}^{\omega_i}, \prod_{i=1}^{n}\pi_{\alpha_i}^{\omega_i} \right)$$

$$= \left(\frac{1-\prod_{i=1}^{n}\pi_{\alpha_i}^{\omega_i}}{1+\prod_{i=1}^{n}\left(\frac{\nu_{\alpha_i}}{\mu_{\alpha_i}}\right)^{\omega_i}}, \frac{1-\prod_{i=1}^{n}\pi_{\alpha_i}^{\omega_i}}{1+\prod_{i=1}^{n}\left(\frac{\mu_{\alpha_i}}{\nu_{\alpha_i}}\right)^{\omega_i}}, \prod_{i=1}^{n}\pi_{\alpha_i}^{\omega_i} \right) \preccurlyeq \left(\frac{1-\prod_{i=1}^{n}\pi_{\beta_i}^{\omega_i}}{1+\prod_{i=1}^{n}\left(\frac{\nu_{\beta_i}}{\mu_{\beta_i}}\right)^{\omega_i}}, \frac{1-\prod_{i=1}^{n}\pi_{\beta_i}^{\omega_i}}{1+\prod_{i=1}^{n}\left(\frac{\mu_{\beta_i}}{\nu_{\beta_i}}\right)^{\omega_i}}, \prod_{i=1}^{n}\pi_{\beta_i}^{\omega_i} \right)$$

$$= \left(\frac{1-\prod_{i=1}^{n}\pi_{\beta_i}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\beta_i}^{\omega_i}+\prod_{i=1}^{n}\nu_{\beta_i}^{\omega_i}}\prod_{i=1}^{n}\mu_{\beta_i}^{\omega_i}, \frac{1-\prod_{i=1}^{n}\pi_{\beta_i}^{\omega_i}}{\prod_{i=1}^{n}\mu_{\beta_i}^{\omega_i}+\prod_{i=1}^{n}\nu_{\beta_i}^{\omega_i}}\prod_{i=1}^{n}\nu_{\beta_i}^{\omega_i}, \prod_{i=1}^{n}\pi_{\beta_i}^{\omega_i} \right) = IFWNG^{\circledast}_{\omega}(\beta_1, \beta_2, \cdots, \beta_n).$$

Thus $IFWNG^{\circledast}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) \preccurlyeq IFWNG^{\circledast}_{\omega}(\beta_1, \beta_2, \cdots, \beta_n)$. □

4.3. **Intuitionistic fuzzy ordered weighted neutral geometric operator w.r.t. the proposed neutral operation.** Inspired by the IFOWG operator and the IFOWA operator in Ref. [25, 26] and the IFOWNA operator in Ref. [11], we propose intuitionistic fuzzy ordered weighted neutral geometric operator w.r.t. the proposed neutral geometric operation.

Definition 4.8. Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be a collection of IFNs, where $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i}, \pi_{\alpha_i})$, $\mu_{\alpha_i}, \nu_{\alpha_i} \in [0, 1]$, $\mu_{\alpha_i} + \nu_{\alpha_i} + \pi_{\alpha_i} = 1$ and $1 \leq i \leq n$. If

$$IFOWNG^{\circledast}_{\omega}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \underset{i=1}{\overset{n}{\circledast}} \alpha_{(i)}{}^{\omega_i}_{\circledast}, \tag{4.5}$$

then $IFOWNG_{\omega}^{\circledast}$ is called an intuitionistic fuzzy ordered weighted neutral geometric (IFOWNG) operator w.r.t. the operation $\circledast$, where $\alpha_{(i)}$ is the ith largest value of α_i(i=1,2,$\cdots$,n), $((1),(2),\cdots,(n))$ is a permutation of $(1,2,\cdots,n)$, ω is the associated weighting vector, $\omega_i \in [0,1]$, $1 \le i \le n$ and $\sum\limits_{i=1}^{n} \omega_i = 1$.

Proposition 4.9. *Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be a collection of IFNs. Then*

$$IFOWNG_{\omega}^{\circledast}(\alpha_1,\alpha_2,\cdots,\alpha_n) = \left(\frac{(1-\prod\limits_{i=1}^{n}\pi_{\alpha_{(i)}}^{\omega_i})\prod\limits_{i=1}^{n}\mu_{\alpha_{(i)}}^{\omega_i}}{\prod\limits_{i=1}^{n}\mu_{\alpha_{(i)}}^{\omega_i}+\prod\limits_{i=1}^{n}\nu_{\alpha_{(i)}}^{\omega_i}}, \frac{(1-\prod\limits_{i=1}^{n}\pi_{\alpha_{(i)}}^{\omega_i})\prod\limits_{i=1}^{n}\nu_{\alpha_{(i)}}^{\omega_i}}{\prod\limits_{i=1}^{n}\mu_{\alpha_{(i)}}^{\omega_i}+\prod\limits_{i=1}^{n}\nu_{\alpha_{(i)}}^{\omega_i}}, \prod_{i=1}^{n}\pi_{\alpha_{(i)}}^{\omega_i} \right) \tag{4.6}$$

Proof. The proof is similar to Proposition 4.4, so we omit it here. □

Proposition 4.10. *(Idempotency) Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be a collection of IFNs. Then $IFWG_{\omega}^{\circledast}(\alpha,\alpha,\cdots,\alpha) = \alpha$.*

Proof. The proof is similar to Proposition 4.6, so we omit it. □

Proposition 4.11. *(Monotonicity) Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ and $\beta_1, \beta_2, \cdots, \beta_n$ be two collections of IFNs such that $\alpha_i \preccurlyeq \beta_i$ $(i = 1,2,\cdots,n)$. Then $IFWNG_{\omega}^{\circledast}(\alpha_1,\alpha_2,\cdots,\alpha_n) \preccurlyeq IFWNG_{\omega}^{\circledast}(\beta_1,\beta_2,\cdots,\beta_n)$.*

Proof. The proof is similar to Proposition 4.7, so we omit it here. □

5. Approaches to multiple attribute group decision making with the proposed operators and an illustrative example

In this section, we utilize the IFWA operator and the proposed aggregation operator to multiple attribute group decision making with Atanassov's intuitionistic fuzzy information:

For a multiple attribute group decision making problem with Atanassov's intuitionistic fuzzy information, let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of n alternatives, $G = \{G_1, G_2, \cdots, G_m\}$ be a set of m attributes, whose weight vector is $\omega = \{\omega_1, \omega_2, \cdots, \omega_m\}$, with $\omega_j \in [0,1]$, $j = 1,2,\cdots,m$, and $\sum\limits_{j=1}^{m} \omega_j = 1$, and let $E = \{e_1, e_2, \cdots, e_s\}$ be a set of s decision makers, whose weight vector is $w = \{w_1, w_2, \cdots, w_s\}$, with $w_k \in [0,1]$, $k = 1,2,\cdots,s$, and $\sum\limits_{k=1}^{s} w_k = 1$. Let $A_k = \left(\alpha_{ij}^{(k)}\right)_{n\times m}$ be an intuitionistic fuzzy decision matrix, where $\alpha_{ij}^{(k)} = \left(\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)}\right)$ is an attribute value provided by the decision maker e_k , denoted by an IFN, where $\mu_{ij}^{(k)}$ indicates the degree that the alternative x_i satisfies the attribute G_j, while $\nu_{ij}^{(k)}$ indicates the degree that the alternative x_i does not satisfy the attribute G_j, and $\pi_{ij}^{(k)}$ indicates the uncertainty degree of the alternative x_i to the attribute G_j, such that $\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)} \in [0,1], \mu_{ij}^{(k)} + \nu_{ij}^{(k)} + \pi_{ij}^{(k)} = 1, i = 1,2,\cdots,n; j = 1,2,\cdots,m$. If all the attributes $G_j (j = 1,2,\cdots,m)$ are of the same type, then the attribute values do not need normalization. Whereas, there are generally benefit attributes (i.e., the bigger the attribute values the better) and cost attributes (i.e., the smaller the attribute values the better) in multiple attribute decision making. In such cases, we may transform the attribute values of cost type into the attribute values of benefit type, then $A_k = \left(\alpha_{ij}^{(k)}\right)_{n\times m}$ can be transformed into the intuitionistic fuzzy decision matrix $R_k = \left(r_{ij}^{(k)}\right)_{n\times m}$, where

$$r_{ij}^{(k)} = \left(\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)}\right) = \begin{cases} \alpha_{ij}^{(k)}, & \text{for benefit attribute } G_j, \\ \overline{\alpha}_{ij}^{(k)}, & \text{for cost attribute } G_j. \end{cases} \tag{5.1}$$

and $\overline{\alpha}_{ij}^{(k)}$ is the complement of $\alpha_{ij}^{(k)}$, such that $\overline{\alpha}_{ij}^{(k)} = \left(\nu_{ij}^{(k)}, \mu_{ij}^{(k)}, \pi_{ij}^{(k)}\right)$, clearly, $\pi_{ij}^{(k)} = 1 - \mu_{ij}^{(k)} - \nu_{ij}^{(k)} = 1 - \nu_{ij}^{(k)} - \mu_{ij}^{(k)}$.

Then, we utilize the IFWNG operator to develop an approach to multiple attribute group decision making with intuitionistic fuzzy information, which involves the following steps.

Step 1 Obtain the normalized intuitionistic fuzzy matrices $R_k = \left(r_{ij}^{(k)}\right)_{n\times m}$ $(k = 1, 2, \cdots, s)$ by Eq. (5.1).

Step 2 Aggregate all the individual intuitionistic fuzzy decision matrices R_k into the collective intuitionistic fuzzy decision matrix $R = (r_{ij})_{n\times m}$ by the IFWA operator (3.4):

$$r_{ij} = IFWA_w\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \cdots, r_{ij}^{(s)}\right) = \left(\sum_{k=1}^{s} w_k\mu_{ij}^{(k)}, \sum_{k=1}^{s} w_k\nu_{ij}^{(k)}, \sum_{k=1}^{s} w_k\pi_{ij}^{(k)}\right), \tag{5.2}$$

or the IFWNG operator (4.4):

$$\begin{aligned} r_{ij} =& IFWNG_w^{\circledast}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \cdots, r_{ij}^{(s)}\right) \\ =& \left(\frac{(1-\prod_{k=1}^{s} {\pi_{ij}^{(k)}}^{w_k}) \prod_{k=1}^{s} {\mu_{ij}^{(k)}}^{w_k}}{\prod_{k=1}^{s} {\mu_{ij}^{(k)}}^{w_k} + \prod_{k=1}^{s} {\nu_{ij}^{(k)}}^{w_k}}, \frac{(1-\prod_{k=1}^{s} {\pi_{ij}^{(k)}}^{w_k}) \prod_{k=1}^{s} {\nu_{ij}^{(k)}}^{w_k}}{\prod_{k=1}^{s} {\mu_{ij}^{(k)}}^{w_k} + \prod_{k=1}^{s} {\nu_{ij}^{(k)}}^{w_k}}, \prod_{k=1}^{s} {\pi_{ij}^{(k)}}^{w_k} \right). \end{aligned} \tag{5.3}$$

Step 3 Aggregate all the preference IFNs r_{ij} $(j = 1, 2, \cdots, m)$ into the collective IFNs r_i $(i = 1, 2, ..., n)$ by the IFWA operator (3.4):

$$r_i = IFWNG_\omega^{\circledast}(r_{i1}, r_{i2}, \cdots, r_{im}) = \left(\sum_{j=1}^{m} \omega_j\mu_{ij}, \sum_{j=1}^{m} \omega_j\nu_{ij}, \sum_{j=1}^{m} \omega_j\pi_{ij} \right), \tag{5.4}$$

or the IFWNG operator (4.4):

$$\begin{aligned} r_i =& IFWNG_\omega^{\circledast}(r_{i1}, r_{i2}, \cdots, r_{im}) \\ =& \left(\frac{(1-\prod_{j=1}^{m} \pi_{ij}^{\omega_j}) \prod_{j=1}^{m} \mu_{ij}^{\omega_j}}{\prod_{j=1}^{m} \mu_{ij}^{\omega_j} + \prod_{j=1}^{m} \nu_{ij}^{\omega_j}}, \frac{(1-\prod_{j=1}^{m} \pi_{ij}^{\omega_i}) \prod_{j=1}^{m} \nu_{ij}^{\omega_j}}{\prod_{j=1}^{m} \mu_{ij}^{\omega_j} + \prod_{j=1}^{m} \nu_{ij}^{\omega_j}}, \prod_{j=1}^{m} \pi_{ij}^{\omega_j} \right). \end{aligned} \tag{5.5}$$

Step 4 Calculate the scores and accuracy degrees of r_i $(i = 1, 2, \cdots, n)$ respectively and rank r_i $(i = 1, 2, \cdots, n)$ in descending order by Definition 2.2.

Step 5 Select the best alternative according to the ranking of r_i $(i = 1, 2, \cdots, n)$.

Example 5.1. [27] Now we consider a software selection problem in which alternatives are the software packages to be selected, and criteria are those attributes under consideration (adapted from Wang and Lee [24]). A computer center in a university desires to select a new information system in order to improve work productivity. After preliminary screening, four alternatives x_i $(i = 1, 2, 3, 4)$ have remained in the candidate list. Three experts e_k $(k = 1, 2, 3)$, from a committee to act as decision makers, whose weight vector is $w = (0.4, 0.3, 0.3)^T$. Four attributes need to be considered: (1) costs of hardware/software investment (G_1); (2) contribution to organization performance (G_2); (3) effort to transform from current systems (G_3); and (4) outsourcing software developer reliability (G_4). The weight vector of the attributes G_j $(j = 1, 2, 3, 4)$ is $\omega = (0.30, 0.25, 0.25, 0.20)^T$. The experts e_k $(k = 1, 2, 3)$ evaluate the software packages x_i $(i = 1, 2, 3, 4)$ with respect to the attributes G_j $(j = 1, 2, 3, 4)$, and construct the following three intuitionistic fuzzy decision matrices $A_k = \left(\alpha_{ij}^{(k)}\right)_{4\times4}$ $(k = 1, 2, 3)$ (see Tables 1-3). We select the software packages

Table 1. Intuitionistic Fuzzy Decision Matrix A_1

	G_1	G_2	G_3	G_4
x_1	(0.5,0.4,0.1)	(0.5,0.5,0.0)	(0.7,0.3,0.0)	(0.3,0.6,0.1)
x_2	(0.4,0.5,0.1)	(0.6,0.4,0.0)	(0.2,0.5,0.3)	(0.5,0.3,0.2)
x_3	(0.8,0.2,0.0)	(0.7,0.3,0.0)	(0.4,0.6,0.0)	(0.5,0.2,0.3)
x_4	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.5,0.1,0.4)	(0.8,0.1,0.1)

Table 2. Intuitionistic Fuzzy Decision Matrix A_2

	G_1	G_2	G_3	G_4
x_1	(0.6,0.3,0.1)	(0.3,0.5,0.2)	(0.5,0.2,0.3)	(0.4,0.5,0.1)
x_2	(0.3,0.4,0.3)	(0.5,0.3,0.2)	(0.2,0.6,0.2)	(0.6,0.4,0.0)
x_3	(0.9,0.1,0.0)	(0.5,0.2,0.3)	(0.4,0.4,0.2)	(0.4,0.3,0.3)
x_4	(0.6,0.2,0.2)	(0.7,0.3,0.0)	(0.4,0.2,0.4)	(0.7,0.1,0.2)

Table 3. Intuitionistic Fuzzy Decision Matrix A_3

	G_1	G_2	G_3	G_4
x_1	(0.4,0.5,0.1)	(0.5,0.4,0.1)	(0.6,0.2,0.2)	(0.3,0.5,0.2)
x_2	(0.4,0.6,0.0)	(0.7,0.3,0.0)	(0.3,0.5,0.2)	(0.5,0.5,0.0)
x_3	(0.7,0.3,0.0)	(0.6,0.4,0.0)	(0.3,0.5,0.2)	(0.6,0.2,0.2)
x_4	(0.7,0.2,0.1)	(0.5,0.3,0.2)	(0.9,0.1,0.0)	(0.6,0.4,0.0)

by the following steps:

Step 1 Among the considered attributes, G_1 is of cost type, and G_j $(j = 2, 3, 4)$ are of benefit type, i.e., the attributes have two different types, and thus we need to transform the attribute values of cost type into the attribute values of benefit type by using Eq. (5.1), then $A_k = \left(\alpha_{ij}^{(k)}\right)_{4\times4}$ $(k = 1, 2, 3)$ are transformed into $R_k = \left(r_{ij}^{(k)}\right)_{4\times4}$ $(k = 1, 2, 3)$ (see Tables 4-6):

Table 4. Intuitionistic Fuzzy Decision Matrix R_1

	G_1	G_2	G_3	G_4
x_1	(0.4,0.5,0.1)	(0.5,0.5,0.0)	(0.7,0.3,0.0)	(0.3,0.6,0.1)
x_2	(0.5,0.4,0.1)	(0.6,0.4,0.0)	(0.2,0.5,0.3)	(0.5,0.3,0.2)
x_3	(0.2,0.8,0.0)	(0.7,0.3,0.0)	(0.4,0.6,0.0)	(0.5,0.2,0.3)
x_4	(0.3,0.5,0.2)	(0.6,0.2,0.2)	(0.5,0.1,0.4)	(0.8,0.1,0.1)

Table 5. Intuitionistic Fuzzy Decision Matrix R_2

	G_1	G_2	G_3	G_4
x_1	(0.3,0.6,0.1)	(0.3,0.5,0.2)	(0.5,0.2,0.3)	(0.4,0.5,0.1)
x_2	(0.4,0.3,0.3)	(0.5,0.3,0.2)	(0.2,0.6,0.2)	(0.6,0.4,0.0)
x_3	(0.1,0.9,0.0)	(0.5,0.2,0.3)	(0.4,0.4,0.2)	(0.4,0.3,0.3)
x_4	(0.2,0.6,0.2)	(0.7,0.3,0.0)	(0.4,0.2,0.4)	(0.7,0.1,0.2)

Table 6. Intuitionistic Fuzzy Decision Matrix R_3

	G_1	G_2	G_3	G_4
x_1	(0.5,0.4,0.1)	(0.5,0.4,0.1)	(0.6,0.2,0.2)	(0.3,0.5,0.2)
x_2	(0.6,0.4,0.0)	(0.7,0.3,0.0)	(0.3,0.5,0.2)	(0.5,0.5,0.0)
x_3	(0.3,0.7,0.0)	(0.6,0.4,0.0)	(0.3,0.5,0.2)	(0.6,0.2,0.2)
x_4	(0.2,0.7,0.1)	(0.5,0.3,0.2)	(0.9,0.1,0.0)	(0.6,0.4,0.0)

Step 2 Utilize the IFWA operator (3.4) to aggregate all the individual intuitionistic fuzzy decision matrices $R_k = \left(r_{ij}^{(k)}\right)_{4\times 4}$ $(k = 1, 2, 3)$ into the collective intuitionistic fuzzy decision matrix $R = (r_{ij})_{4\times 4}$ (see Table 7):

Table 7. Intuitionistic Fuzzy Decision Matrix R

	G_1	G_2	G_3	G_4
x_1	(0.40, 0.50, 0.10)	(0.44, 0.47, 0.09)	(0.61, 0.24, 0.15)	(0.33, 0.54, 0.13)
x_2	(0.50, 0.37, 0.13)	(0.60, 0.34, 0.06)	(0.23, 0.53, 0.24)	(0.53, 0.39, 0.08)
x_3	(0.20, 0.80, 0.00)	(0.61, 0.30, 0.09)	(0.37, 0.51, 0.12)	(0.50, 0.23, 0.27)
x_4	(0.24, 0.59, 0.17)	(0.60, 0.26, 0.14)	(0.59, 0.13,0.28)	(0.71, 0.19, 0.10)

Step 3 Aggregate all the preference values r_{ij} $(i = 1, 2, 3, 4)$ in the ith row of R by using the IFWNG operator (4.4), and get the overall preference value r_i corresponding to the alternative x_i:

$$r_1 = (0.4546, 0.4318, 0.1136)\,, r_2 = (0.4622, 0.4245, 0.1133)\,,$$
$$r_3 = (0.4592, 0.5408, 0.0000)\,, r_4 = (0.5355, 0.2995, 0.1650)\,.$$

Step 4 We calculate the scores of r_i $(i = 1, 2, 3, 4)$, respectively:

$$s(r_1) = 0.4546 - 0.4318 = 0.0228, s(r_2) = 0.4622 - 0.4245 = 0.0377,$$
$$s(r_3) = 0.4592 - 0.5408 = -0.0816, s(r_4) = 0.5355 - 0.2995 = 0.2360.$$

Step 5 Since $s(r_4) > s(r_2) > s(r_1) > s(r_3)$, we have $r_4 \succ r_2 \succ r_1 \succ r_3$, that is, x_4 is the best software package.

Table 8. Ranking order obtained by different combination of the IFWA operator and the IFWNG operator

Operator used in Step 2.	Operator used in Step 3.	Ranking order
IFWA [4]	IFWNG	$r_4 \succ r_2 \succ r_1 \succ r_3$
IFWA [4]	IFWA [4]	$r_4 \succ r_2 \succ r_1 \succ r_3$
IFWNG	IFWA [4]	$r_4 \succ r_2 \succ r_1 \succ r_3$
IFWNG	IFWNG	$r_4 \succ r_2 \succ r_1 \succ r_3$
SIFWG [30]	SIFWG [30]	$r_4 \succ r_2 \succ r_1 \succ r_3$
SIFWG [18]	SIFWG [18]	$r_4 \succ r_2 \succ r_1 \succ r_3$

We find that the obtained result is identical with that of Xu [27]. Furthermore, in order to make a comparative study to the aggregated results, we also give the ranking order by different combinations of the IFWA operator and the IFWNG operator, the SIFWG operator [30] and the SIFWG operator [18] in Steps 2 and 3 (see Table 8). These operators are all neutral to to the membership degrees and the non-membership degrees of the aggregated IFNs and can obviously be divided into two cases: some of them are immune to the appearance of zero in the components of the IFN such as the IFWA operator [4], other possess shortcoming when zero appears in the components of IFN as has been stated before such as the IFWNG operator, SIFWG [30] and SIFWG [18]. All the methods yield the same ranking order, that is, $r_4 \succ r_2 \succ r_1 \succ r_3$. But the IFWNG operator can consider the interactions between membership degrees and non-membership degrees of different IFNs with an interaction coefficient. Therefore, the method proposed by this paper is effective and valid.

6. Conclusions

By analysing the existing neutral operations and the IFWNA operator on the IFNs proposed in Ref. [11], we have presented a neutral geometric operation on the IFNs based on the interaction coefficient determined by the membership degrees, the non-membership degrees and the hesitancy degrees of the IFNs under multi-attribute decision making environment. Moreover, we have developed the IFWNG operator and the IFOWNG operator and investigated their properties. Compared to the existing neutral operators in Ref. [4, 18, 30], the principal advantages of the proposed operators consider not only the neutral attitude to the intuitionistic fuzzy information of the decision makers, but also the interactions of the components of the IFNs. Finally, an approach to multiple attribute group decision making based on the proposed operators have been given under intuitionistic fuzzy environment, and an example has been provided to show the feasibility and validity of the new approach to the application of group decision making. In the future, Associated with the proposed operations and operators, we will investigate the compatibility measures and consensus models for group decision making with intuitionistic fuzzy preference relations as that in Ref. [18, 19, 20, 28, 29].

Acknowledgements. This research was supported by the Innovation and Entrepreneurship Project of Linyi University (No. 201610452167).

References

[1] K. Atanassov, Intuitionistic fuzzy set, Fuzzy Set. Syst. 20 (1986) 87–96.

[2] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, Fuzzy Set. Syst. 61 (1994) 137–142.

[3] G. Beliakov, A. Pradera and T. Calvo, Aggregation Functions: A Guide for Practitioners, Springer, Heidelberg, Berlin, New York 2007.

[4] G. Beliakov, H. Bustince, D. P. Goswami, U. K. Mukherjee and N. R. Pal, On averaging operators for Atanassov's intuitionistic fuzzy sets, Inform. Sci. 181 (2011) 1116–1124.

[5] S. M. Chen and J. M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Set. Syst. 67 (1994) 163–172.

[6] S. M. Chen and C. H. Chang, Fuzzy multiattribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators, Inform. Sci. 352 (2016) 133–149.

[7] G. Deschrijver and E. Kerre, On the relationship between some extensions of fuzzy set theory, Fuzzy Set. Syst. 133 (2003) 227–235.

[8] S. K. De, R. Biswas and A. R. Roy, Some operations on intuitionistic fuzzy sets, Fuzzy Set. Syst. 114 (2000) 477–484.

[9] W. L. Gau and D. J. Buehrer, Vague sets, IEEE T Syst. Man Cy. B 23 (1993) 610–614.

[10] Y. He, H. Chen, Z. He and L. Zhou, Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making, Inform. Sci. , 259 (2014) 142–159.

[11] Y. He, H. Chen, Z. He and L. Zhou, Multi-attribute decision making based on neutral averaging operators for intuitionistic fuzzy information, Appl. Soft Comput. 27 (2015) 64–76.

[12] Y. D. He, Z. He and H. Y. Chen, Intuitionistic Fuzzy Interaction Bonferroni Means and Its Application to Multiple Attribute Decision Making, IEEE. T. Syst. Man Cy. B. 45 (2015) 116–128.

[13] Y. D. He and Z. He, Extensions of Atanassov's intuitionistic fuzzy interaction Bonferroni means and Their Application to Multiple attribute decision making, IEEE T. Fuzzy Syst. 24 (2016) 558–573.

[14] Y. D. He, Z. He, D. H. Lee, K. J. Kim, L. Zhang and X. Yang, Robust Fuzzy Programming Method for MRO Problems Considering Location Effect, Dispersion Effect and Model Uncertainty, Comput. Ind. Eng. 105 (2017) 76–83.

[15] D. H. Hong and C. H. Choi, Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Set. Syst. 114 (2000) 103–113.

[16] D. F. Li, Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets, Expert. Syst. Appl. 37 (2010) 8673–8678.

[17] H. C. Liao and Z. S. Xu, Intuitionistic fuzzy hybrid weighted aggregation operators, Int. J. Intell. Syst. 29 (2014) 971–993.

[18] H. C. Liao and Z. S. Xu, Consistency of the fused intuitionistic fuzzy preference relation in group intuitionistic fuzzy analytic hierarchy process, Appl. Soft Comput. 35 (2015) 812–826.

[19] H. C. Liao, Z. S. Xu, X. J. Zeng and D. L. Xu, An enhanced consensus reaching process in group decision making with intuitionistic fuzzy preference relations, Inform. Sci. 329 (2016) 274–286.

[20] H. C. Liao , Z. Li , X. J Zeng and W. Liu, A comparison of distinct consensus measures for group decision making with intuitionistic fuzzy preference relations, Int. J. Comput. Int. Syst. 10 (2017) 456–469.

[21] W. Liu and H. C. Liao, A bibliometric analysis of fuzzy decision research during 1970-2015, Int. J. Fuzzy Syst. 19 (2017) 1–14.

[22] W. Wang and X. Liu, Intuitionistic fuzzy geometric aggregation operators based on Einstein operations, Int. J. Intell. Syst. 26 (2011) 1049–1075.

[23] W. Wang and X. Liu, Intuitionistic fuzzy information aggregation using Einstein operations, IEEE T. Fuzzy Syst. 20 (2012) 923–938.

[24] T. C. Wang and H. D. Lee, Developing a fuzzy TOPSIS approach based on subjective weights and objective weights, Expert. Syst. Appl. 36 (2009) 8980–8985.
[25] Z. S. Xu and R. R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, Int. J. Gen. Syst. 35 (2006) 417–433.
[26] Z. S. Xu, Intuitionistic fuzzy aggregation operators, IEEE T. Fuzzy Syst. 15 (2007) 1179–1187.
[27] Z. S. Xu, Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators, Knowl.-based Syst. 24 (2011) 749–760.
[28] Z. S. Xu and H. C. Liao, Intuitionistic fuzzy analytic hierarchy process, IEEE T. Fuzzy Syst. 22 (2014) 749–761.
[29] Z. S. Xu and H. C. Liao, A survey of approaches to decision making with intuitionistic fuzzy preference relations, Knowl.-based Syst. 80 (2015) 131–142.
[30] M. M. Xia and Z. S. Xu, Entropy/cross entropy-based group decision making under intuitionistic fuzzy environment, Inform. Fusion 13 (2012) 31–47.
[31] M. M. Xia and Z. S. Xu, Bin Zhu, Some issues on intuitionistic fuzzy aggregation operators based on Archimedean t-conorm and t-norm, Knowl.-based Syst. 31 (2012) 78–88.
[32] W. Yang and Z. P. Chen, The quasi-arithmetic intuitionistic fuzzy OWA operators, Knowl.-based Syst. 27 (2012) 219–233.
[33] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.
[34] H. Zhao, Z. S. Xu, M. F. Ni and S. S. Liu, Generalized aggregation operators for intuitionistic fuzzy sets, Int. J. Intell. Syst. 25 (2010)1–30.

Xin Mei Wang (2313170092@qq com)
School of Mathematics and Statistics, Linyi University, Linyi 276005, China

Zi Wen Jiang (jiangziwensss@163.com)
School of Mathematics and Statistics, Linyi University, Linyi 276005, China

Wei Yang (wyoeng@126.com)
School of Mathematics and Statistics, Linyi University, Linyi 276005, China