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Some properties of soft topological vector spaces

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Abstract. In this paper studies on soft topological vector spaces, as introduced by us, have been continued. Separation properties of soft topological vector spaces have been dealt with. Also, some properties related to continuity and boundedness of soft linear mapping on soft topological vector spaces have been investigated.

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1. Introduction

Several attempts have been made for developing frameworks which enable us to handle problems with vagueness and uncertainties: examples include the theory of fuzzy sets by Zadeh [22] and the theory of Rough sets by Pawlak [18]. The starting point of this work is Molodtsov's soft set theory [16]. Molodtsov's motivation for introducing this mathematical tool is to overcome some of the difficulties involving the parametrization process in handling uncertainties. Although somewhat different from the initial motivation, the investigation of mathematical aspects of soft sets is of continuous interest of many authors [1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 19, 21] . Very recently Shi et al. in [20] has commented that the soft topology in the sense of Shabir & Naz [19] can be interpreted as a crisp topology. In [3], we defined soft topology in a different perspective using elementary union and elementary intersection which is different from that of Naz's topology and study some basic properties of this new type of soft topological space. The importance of this study lies on the fact that the operations of elementary union and elementary intersection are not distributive. Also if we take a soft set and its elementary complement then the law of excluded-middle is not valid in general. In [4], we introduced the notion of a soft topological vector space (soft tvs) with the soft topology of [3] and studied some basic properties of such spaces. The problem of normability of soft tvs was also addressed to. To develop further results of soft tvs, in this paper, we continue our investigation by studying the neighborhood properties of soft null vector and use it to introduce the separation axioms in soft tvs. The soft continuity of soft linear mapping is also studied and some characterizing properties of soft continuity in soft tvs is investigated.

2. Preliminaries

Definition 2.1. Let X be a universal set, A be a set of parameters, $P(X)$ denote the power set of X and $B \subseteq A$. A pair (F, B) is called a soft set [16] over X, where F is a mapping given by $F : B \to P(X)$.

In [15], the soft sets are redefined as follows: Let B be the set of parameters and $B \subseteq A$. Then for each soft set (F, B) over X a soft set (H, A) is constructed over X, where $\forall \lambda \in A$, $H(\lambda) = \begin{cases} F(\lambda) & if\ \lambda \in B \\ \phi & if\ \lambda \in A \setminus B. \end{cases}$

Thus the soft sets (F, B) and (H, A) are equivalent to each other and the usual set operations of the soft sets $(F_i, B_i), i \in \Delta$ is the same as those of the soft sets $(H_i, A), i \in \Delta$. For this reason, in this paper, we have considered our soft sets over the same parameter set A.

Let $(F, A), (G, A)$ be two soft sets over X. Then

(i) an absolute soft set [14], denoted by $(\tilde{X}, A)$, is defined by for all $\lambda \in A$, $F(\lambda) = X$,

(ii) a null soft set [14], denoted by $(\tilde{\Phi}, A)$, is defined by for all $\lambda \in A$, $F(\lambda) = \phi$,

(iii) the complement or relative complement of a soft set (F, A) [14], denoted by $(F, A)^C$, is defined by $(F, A)^C = (F^C, A)$, where $F^C(\lambda) = X \setminus F(\lambda), \forall \lambda \in A$,

(iv) (F, A) is said to be a soft subset [16] of (G, A) if $F(\lambda)$ is a subset of $G(\lambda)$, $\forall \lambda \in A$,

(v) union [14] of (F, A) and (G, A) is denoted by $(F, A)\tilde{\cup}(G, A)$ and defined by $[(F, A)\tilde{\cup}(G, A)](\lambda) = F(\lambda) \cup G(\lambda), \forall \lambda \in A$,

(vi) intersection [14] of (F, A) and (G, A) is denoted by $(F, A)\tilde{\cap}(G, A)$ and defined by $[(F, A)\tilde{\cap}(G, A)](\lambda) = F(\lambda) \cap G(\lambda), \forall \lambda \in A$.

In [19], soft topology is defined as a collection of soft sets over X containing $(\tilde{X}, A)$ and $(\tilde{\Phi}, A)$ which is closed under arbitrary union and finite intersection.

Definition 2.2 ([5])**.** Let X be a non-empty set and A be a non-empty parameter set. A function $\tilde{x} : A \to X$ is said to be a soft element of X. A soft element $\tilde{x}$ of X is said to belong to a soft set (F, A) of X , which is denoted by $\tilde{x}\tilde{\in}(F, A)$, if $\tilde{x}(\lambda) \in F(\lambda)$, for all $\lambda \in A$.

Definition 2.3 ([5, 6])**.** A soft element of $\mathbb{R}$ ($\mathbb{C}$) is called a soft real (complex) number. The set of all soft real (complex) numbers is denoted by $\mathbb{R}(A)(\mathbb{C}(A))$ and set of all non-negative soft real numbers by $\mathbb{R}(A)^*$. To avoid ambiguity of notations we use $\tilde{r}$,$\tilde{s}$,$\tilde{t}$ to denote soft real (complex) numbers whereas $\bar{r}$,$\bar{s}$,$\bar{t}$ will denote a particular type of soft real (complex) numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc. For two soft real numbers $\tilde{r}, \tilde{s}$ define:

(i) $\tilde{r}\tilde{\leq}\tilde{s}$ if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, for all $\lambda \in A$,
(ii) $\tilde{r}\tilde{\geq}\tilde{s}$ if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$, for all $\lambda \in A$,
(iii) $\tilde{r}\tilde{<}\tilde{s}$ if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$, for all $\lambda \in A$,

(iv) $\tilde{r}\tilde{>}\tilde{s}$ if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$, for all $\lambda \in A$.

Definition 2.4 ([4])**.** The inverse of any soft real or soft complex number $\tilde{r}$, denoted by $\tilde{r}^{-1}$, defined by $\tilde{r}^{-1}(\lambda) = (\tilde{r}(\lambda))^{-1}, \tilde{r}(\lambda) \neq 0$ for each $\lambda \in A$.

Remark 2.5 ([7])**.** Let X be any non-empty set. By $S(\tilde{X})$ we denote the collection of all soft sets (F, A) over X for which $F(\lambda) \neq \phi$, for all $\lambda \in A$ together with the null soft set $(\tilde{\Phi}, A)$. For any soft set $(F, A) \in S(\tilde{X})$, the collection of all soft elements of (F, A) is denoted by $SE(F, A)$. For a collection $\mathscr{B}$ of soft elements of $(\tilde{X}, A)$, the soft set generated by $\mathscr{B}$ is denoted by $SS(\mathscr{B})$.

Proposition 2.6 ([7])**.** *For any soft sets* $(F, A), (G, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$, $(F, A)\tilde{\subseteq}(G, A)$ *iff every soft element of* (F, A) *is also a soft element of* (G, A).

Definition 2.7 ([7])**.** For any two soft sets $(F, A), (G, A) \in S(\tilde{X})$, elementary union '$\Cup$', elementary intersection '$\Cap$' and elementary complement '$(F, A)^{\mathbb{C}}$' of (F, A) are defined by:

(i) $(F, A) \Cup (G, A) = SS(SE(F, A) \cup SE(G, A))$,
(ii) $(F, A) \Cap (G, A) = SS(SE(F, A) \cap SE(G, A))$,
(iii) $(F, A)^{\mathbb{C}} = SS(SE(F, A)^{C})$.

Remark 2.8. (1) [3] Elementary union and intersection are not distributive.

(2) [7] In general, $(F, A)\Cup(F, A)^{\mathbb{C}}\tilde{\subseteq}(\tilde{X}, A)$, $(F, A)\Cup(F, A)^{\mathbb{C}} \neq (\tilde{X}, A)$ and $\left((F, A)^{\mathbb{C}}\right)^{\mathbb{C}} \neq (F, A)$. If $(F, A)^{\mathbb{C}} \neq (\tilde{\Phi}, A)$, then $(F, A) \Cup (F, A)^{\mathbb{C}} = (\tilde{X}, A)$ and $\left((F, A)^{\mathbb{C}}\right)^{\mathbb{C}} = (F, A)$.

Proposition 2.9 ([7])**.** *For any two soft sets* (F, A), $(G, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$,

(1) $(F, A) \Cup (G, A) = (F, A)\tilde{\cup}(G, A)$,
(1) $(F, A) \Cap (G, A) = (F, A)\tilde{\cap}(G, A)$, *if* $(F, A) \Cap (G, A) \neq (\tilde{\Phi}, A)$.

Definition 2.10 ([6])**.** Let (F, A), (G, A) be two soft real (complex) sets. Then their sum, difference, product and division are defined parameterwise. For example,

(i) $(F + G)(\lambda) = F(\lambda) + G(\lambda)$, for each $\lambda \in A$,
(ii) $(F/G)(\lambda) = F(\lambda)/G(\lambda)$, provided $G(\lambda) \neq 0$, for each $\lambda \in A$, etc.

Definition 2.11 ([7])**.** A mapping $d : SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(A)^{*}$, is said to be a soft metric on the soft set $(\tilde{X}, A)$, if d satisfies the following conditions: $(M1)$. $d(\tilde{x}, \tilde{y})\tilde{\geq} \bar{0}$, for all $\tilde{x}, \tilde{y}\tilde{\in}\tilde{X}$,

$(M2)$. $d(\tilde{x}, \tilde{y}) = \bar{0}$ if and only if $\tilde{x} = \tilde{y}$,
$(M3)$. $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$, for all $\tilde{x}, \tilde{y}\tilde{\in}(\tilde{X}, A)$,
$(M4)$. For all $\tilde{x}, \tilde{y}, \tilde{z}\tilde{\in}(\tilde{X}, A)$, $d(\tilde{x}, \tilde{z})\tilde{\leq}d(\tilde{x}, \tilde{y}) + d(\tilde{y}, \tilde{z})$.
A soft metric space is denoted by $(\tilde{X}, d, A)$.

Theorem 2.12 ([7])**.** *If a soft metric d satisfies the condition:*

$(M5)$. *for* $(\xi, \eta) \in X \times X$, *and* $\lambda \in A$,

$$\{d(\tilde{x}, \tilde{y})(\lambda) : \tilde{x}(\lambda) = \xi, \tilde{y}(\lambda) = \eta\}$$

is a singleton set
and

if for $\lambda \in A$, $d_{\lambda} : X \times X \to \mathbb{R}^{+}$ *is defined by* $d_{\lambda}(\tilde{x}(\lambda), \tilde{y}(\lambda)) = d(\tilde{x}, \tilde{y})(\lambda)$, $\tilde{x}, \tilde{y}\tilde{\in}\tilde{X}$, *then* d_{λ} *is a metric on* X.

Remark 2.13. [7] Every crisp metric d on a crisp set X can be extended to a soft metric as $d(\tilde{x},\tilde{y})(\lambda) = d(\tilde{x}(\lambda),\tilde{y}(\lambda)), \lambda \in A$ on the soft set $(\tilde{X}, A)$. Such a soft metric is said to be the soft metric generated by the crisp metric d.

Definition 2.14. [7] In a soft metric space $(\tilde{X}, d, A)$, for $\tilde{r}\tilde{>}\bar{0}$ and $\tilde{a}\tilde{\in}(\tilde{X}, A)$, $B(\tilde{a},\tilde{r})$ $\left(= \{\tilde{x}\tilde{\in}\tilde{X} : d(\tilde{x},\tilde{a})\tilde{\leq}\tilde{r}\}\right)$ and $SS(B(\tilde{a},\tilde{r}))$ are respectively called open ball and soft open ball centered at $\tilde{a}$ and radius $\tilde{r}$.

Proposition 2.15 ([7])**.** *Let $(\tilde{X}, d, A)$ be a soft metric space satisfying $(M5)$. Then for every open ball $B(\tilde{a},\tilde{r})$ in $(\tilde{X}, d)$, $(SS(B(\tilde{a},\tilde{r})))(\lambda) = B(\tilde{a}(\lambda),\tilde{r}(\lambda))$, is an open ball in (X, d_λ), for each $\lambda \in A$.*

Definition 2.16 ([7])**.** In a soft metric space $(\tilde{X}, d, A)$, $\mathscr{B} \subset SE(\tilde{X}, A)$ is said to be open if $\tilde{a} \in \mathscr{B}$ implies $B(\tilde{a},\tilde{r}) \subseteq \mathscr{B}$ for some $\tilde{r}\tilde{>}\bar{0}$. $SS(\mathscr{B})$ is called soft open.

Definition 2.17 ([7])**.** Let $(\tilde{X}, d, A)$ be a soft metric space satisfying $(M5)$. Then the collection τ of all soft open sets form a topology on $(\tilde{X}, A)$ with respect to elementary union and elementary intersection of soft sets. This topology will be called "soft metric topology" on $(\tilde{X}, A)$.

Definition 2.18 ([8])**.** Let X be a vector space over a field K and A be the parameter set. For $(F_i, A), (F, A) \in (\tilde{X}, A)$, $\tilde{x} \in SE(\tilde{X}, A)$, $i = 1, 2, ..n$ and $\alpha \in K$,

$$\left(\sum_{i=1}^{n}(F_i, A)\right)(\lambda) = \sum_{i=1}^{n} F_i(\lambda),$$
$$(\alpha(F, A))(\lambda) = \alpha F(\lambda)$$

and

$$(\tilde{x} + (F, A))(\lambda) = \tilde{x}(\lambda) + F(\lambda),\ \forall\lambda \in A.$$

Definition 2.19 ([8])**.** Let X be a vector space over a field K and A be the parameter set. Let (G, A) be a soft set over X.

(i) (G, A) is said to be a soft vector space or a soft linear space of X over K, if $G(\lambda)$ is a vector subspace of X, for each $\lambda \in A$.

(ii) (G, A) is said to be respectively the null and the absolute soft vector space, if $F(\lambda) = \{\theta\}$ and $F(\lambda) = X$, for each $\lambda \in A$, where θ is the null vector of X.

(iii) A soft element of a soft vector space (G, A) is called soft vector.

(iv) A soft element of $(\tilde{K}, A)$ is called soft scalar.

(v) The null soft vector, denoted by Θ, is defined by $\Theta(\lambda) = \theta$, for each $\lambda \in A$.

(vi) For $\tilde{x},\tilde{y}\tilde{\in}(G, A)$ and a soft scalar $\tilde{k}$, $\tilde{x} + \tilde{y}$ and $\tilde{k}.\tilde{x}$ are defined by:

$$(\tilde{x} + \tilde{y})(\lambda) = \tilde{x}(\lambda) + \tilde{y}(\lambda)$$

and

$$(\tilde{k}.\tilde{x})(\lambda) = \tilde{k}(\lambda).\tilde{x}(\lambda),$$

for each $\lambda \in A$.

Definition 2.20 ([3])**.** $\tau\left(\subset S(\tilde{X})\right)$ containing $(\tilde{\Phi}, A)$ and $(\tilde{X}, A)$ is said to be a soft topology on $(\tilde{X}, A)$, if it is closed under arbitrary elementary union and finite elementary intersection. The triplet $(\tilde{X}, \tau, A)$ is called a soft topological space and members of τ are called soft open sets.

Definition 2.21 ([3])**.** In a soft topological space $(\tilde{X}, \tau, A)$, a soft set $(F, A) \in S(\tilde{X})$ is said to be soft closed set, if its relative complement $(F, A)^C \in S(\tilde{X})$ and $(F, A)^{\mathbb{C}} \in \tau$.

Definition 2.22 ([3])**.** In a soft topological space$(\tilde{X}, \tau, A)$, $\mathscr{B} \subset \tau$, containing $(\tilde{\Phi}, A)$, is said to be an open base of τ, if for any $\tilde{x}\tilde{\in}(\tilde{X}, A)$ and for any $(F, A) \in \tau$ with $\tilde{x}\tilde{\in}(F, A)$, there exists $(G, A) \in \mathscr{B}$ such that $\tilde{x}\tilde{\in}(G, A)\tilde{\subseteq}(F, A)$.

The members of $\mathscr{B}$ are called soft basic open sets in $(\tilde{X}, \tau, A)$.

In $(\tilde{X}, \tau, A)$, $(F, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$ is a soft neighborhood (soft nbd) of a soft element $\tilde{x}$, if there exists a soft set $(G, A) \in \tau$ such that $\tilde{x}\tilde{\in}(G, A)\tilde{\subseteq}(F, A)$.

Definition 2.23 ([3])**.** Let X and Y be two non-empty sets and $\{f_\lambda : X \to Y, \lambda \in A\}$ be a collection of functions. Then a function $f : SE(\tilde{X}) \to SE(\tilde{Y})$, associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$, defined by $[f(\tilde{x})](\lambda) = f_\lambda(\tilde{x}(\lambda))$, for each $\lambda \in A$, is called a soft function.

Definition 2.24 ([3])**.** Let $(\tilde{X}, \tau, A)$ and $(\tilde{Y}, \nu, A)$ be two soft topological spaces and $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$. Then we denote this soft function as $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$.

(i) $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ is said to be soft continuous at $\tilde{x}_0\tilde{\in}(\tilde{X}, A)$, if for every $(V, A) \in \nu$ such that $f(\tilde{x}_0)\tilde{\in}(V, A)$, there exists $(U, A) \in \tau$ such that $\tilde{x}_0\tilde{\in}(U, A)$ and $f(U, A)\tilde{\subseteq}(V, A)$.

(ii)f is said to be soft continuous in $(\tilde{X}, \tau, A)$, if it is soft continuous at each $\tilde{x}\tilde{\in}(\tilde{X}, A)$.

(iii) A bijective soft function $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ is said to be soft homeomorphism, if f and f^{-1} are soft continuous.

Proposition 2.25 ([3])**.** *For a soft function $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$, the following relation holds:*

$$(1) \Leftrightarrow (2),\ (2) \Leftrightarrow (3) \text{ and } (2) \Rightarrow (4),$$

where

(1) *f is soft continuous on $(\tilde{X}, \tau, A)$,*
(2) *for all $(V, A) \in \nu$, $f^{-1}(V, A) \in \tau$,*
(3) *there exists a subbase $\wp$ for ν such that $f^{-1}(V, A) \in \tau$, for all $(V, A) \in \wp$,*
(4) *for any soft closed set (F, A) in $(\tilde{Y}, \nu, A)$, $f^{-1}(F, A)$ is soft closed in $(\tilde{X}, \tau, A)$.*

Definition 2.26 ([3])**.** A soft topological space is called: $(\tilde{X}, \tau, A)$,

(i) a soft T_0 space, if for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda), \forall \lambda \in A$, there exists $(F, A) \in \tau$ such that $\forall \lambda \in A$,

$$[\tilde{x}(\lambda) \in (F, A)(\lambda) \text{ and } \tilde{y}(\lambda) \notin (F, A)(\lambda)]$$

or

$$[\tilde{y}(\lambda) \in (F, A)(\lambda) \text{ and } \tilde{x}(\lambda) \notin (F, A)(\lambda)],$$

(ii) a soft T_1 space, if for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda), \forall \lambda \in A$, there exist $(F, A), (G, A) \in \tau$ such that $\forall \lambda \in A$,

$$[\tilde{x}(\lambda) \in (F, A)(\lambda),\ \tilde{y}(\lambda) \notin (F, A)(\lambda)]$$

and

$$[\tilde{y}(\lambda) \in (G, A)(\lambda),\ \tilde{x} \notin (G, A)(\lambda)],$$

(iii) a soft T_2 space, if for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda), \forall \lambda \in A$, there exist $(F, A), (G, A) \in \tau$ such that $\tilde{x}\tilde{\in}(F, A)$, $\tilde{y}\tilde{\in}(G, A)$ and $(F, A)\tilde{\cap}(G, A) = (\tilde{\Phi}, A)$.

Definition 2.27 ([3])**.** A soft set of $S(\tilde{X})$ containing exactly one soft element $\tilde{x}$ will be denoted by $(\tilde{x}, A)$, i.e., $(\tilde{x}, A)(\lambda) = \{\tilde{x}(\lambda)\},\ \forall \lambda \in A$.

Proposition 2.28 ([3])**.** *In a soft T_1 topological space $(\tilde{X}, \tau, A)$, $(\tilde{x}, A)$ is soft closed, for all $\tilde{x}\tilde{\in}(\tilde{X}, A)$.*

Definition 2.29 ([3])**.** $(\tilde{X}, \tau, A)$ is called soft regular space, if for any soft closed set (F, A) and any soft element $\tilde{x}$ such that $\tilde{x}(\lambda) \notin (F, A)(\lambda)$, $\forall \lambda \in A$, there exist $(G, A), (H, A) \in \tau$ such that $(F, A)\tilde{\subseteq}(G, A), \tilde{x}\tilde{\in}(H, A)$ and $(F, A) \Cap (G, A) = (\tilde{\Phi}, A)$.
If in addition, $(\tilde{X}, \tau, A)$ is soft T_1, then it is called soft T_3 space.

Definition 2.30. Let $(\tilde{X}, A)$ be an absolute soft vector space. A soft set $(F, A) \in S(\tilde{X})$ is said to be:
(i) [8] convex, if for all $\tilde{x}, \tilde{y}\tilde{\in}(F, A)$ and for all soft real numbers $\tilde{t}$ with $\tilde{t}(\lambda) \in [0, 1]$, for all $\lambda \in A$, $\tilde{t}\tilde{x} + (\bar{1} - \tilde{t})\tilde{y}\tilde{\in}(F, A)$,
(ii) [4] balanced, if for all $\tilde{x}\tilde{\in}(F, A)$ and $\tilde{\alpha}\tilde{\in}(\widetilde{K}, A)$ with $\mid \tilde{\alpha} \mid \tilde{\leq}\bar{1}$, $\tilde{\alpha}(\lambda) \neq 0$, *for all* $\lambda \in A$ or $\mid \tilde{\alpha} \mid = \bar{0}$, $\tilde{\alpha}\tilde{x}\tilde{\in}(F, A)$,
(iii) [4] absorbing, if for all $\tilde{x}\tilde{\in}(F, A)$, there exists a soft real number $\tilde{t}$ with $\tilde{t}(\lambda) > 0$, for each $\lambda \in A$, such that $\tilde{t}^{-1}\tilde{x}\tilde{\in}(F, A)$,
(iv)[4] symmetrical, if $(-\bar{1})(F, A) = (F, A)$.

Definition 2.31 ([4])**.** Let $(\tilde{X}, A)$ be the absolute soft vector space of X over $\mathbb{K}$, the field of real or complex numbers. Let τ be a soft topology on $(\tilde{X}, A)$ as in Definition 2.20 and ν be the soft topology on $(\widetilde{K}, A)$ induced by the usual soft metric. Then $(\tilde{X}, \tau, A)$ is called a soft topological vector space (in short soft tvs), if the mappings

$$f : (\tilde{x}, \tilde{y}) \quad \to \quad \tilde{x} + \tilde{y}$$

and

$$g : (\tilde{\alpha}, \tilde{x}) \quad \to \quad \tilde{\alpha}\tilde{x}$$

are continuous in the sense that for any soft nbd (W, A) of $\tilde{x}+\tilde{y}$, there exist soft nbds $(V_1, A), (V_2, A)$ of $\tilde{x}$ and $\tilde{y}$ respectively such that $(V_1, A) + (V_2, A)\tilde{\subseteq}(W, A)$ and for any soft nbd (U, A) of $\tilde{\alpha}\tilde{x}$, there exist soft nbds (U_1, A) of $\tilde{\alpha}$ in $(\widetilde{K}, \nu, A)$and (U_2, A) of $\tilde{x}$ in $(\tilde{X}, \tau, A)$ such that $(U_1, A) \cdot (U_2, A)\tilde{\subseteq}(U, A)$.

Proposition 2.32 ([4])**.** *In a soft topological vector space $(\tilde{X}, \tau, A)$, for any $\tilde{x}\tilde{\in}(\tilde{X}, A)$ and $\tilde{k}\tilde{\in}(\widetilde{K}, A)$ with $\tilde{k}(\lambda) \neq 0$, for each $\lambda \in A$, soft translation operator $T_{\tilde{x}}$ and soft multiplication operator $M_{\tilde{k}}$ are soft homeomorphism from $(\tilde{X}, \tau, A)$ to $(\tilde{X}, \tau, A)$.*

Proposition 2.33 ([4])**.** *In a soft topological vector space $(\tilde{X}, \tau, A)$,*
(1) *every soft nbd of Θ contains a balanced soft nbd of Θ,*
(2) *every convex soft nbd of Θ contains a balanced convex soft nbd of Θ.*

Definition 2.34 ([4])**.** In a soft tvs $(\tilde{X}, \tau, A)$, a soft set $(F, A) \in S(\tilde{X})$ is said to be bounded, if for any soft nbd (V, A) of Θ, there exists $\tilde{\alpha}\tilde{>}\bar{0}$ such that $(F, A)\tilde{\subseteq}\tilde{\beta}(V, A)$, for each $\tilde{\beta}\tilde{\geq}\tilde{\alpha}$.

3. Soft neighborhood system and separation properties of soft topological vector spaces

Unless otherwise stated, throughout the rest of the paper $(\tilde{X}, \tau, A), (\tilde{Y}, \nu, A)$ will denote soft topological vector spaces.

Definition 3.1. A soft nbd base of Θ in $(\tilde{X}, \tau, A)$ is a collection $\mathcal{B}$ of soft nbds of Θ such that for any soft nbd (V, A) of Θ, there exists $(U, A) \in \mathcal{B}$ such that $(U, A)\tilde{\subset}(V, A)$.

Example 3.2. Consider the set $\mathbb{R}^2$ with the usual Euclidean metric "d_u". Let "d" be the soft metric generated by "d_u" as in Remark 2.13. Then $\mathbb{R}^2$ is a soft topological vector space with the soft metric topology. Now if $\mathcal{B}$ is the collection of all soft open balls centered at Θ, then $\mathcal{B}$ is a soft nbd base at Θ.

Proposition 3.3. *Let $\mathcal{B}$ be a soft nbd base of Θ in $(\tilde{X}, \tau, A)$. Then for all $(V, A) \in \mathcal{B}$, $(-V, A)\,(= -\bar{1}(V, A))$ is also a soft nbd of Θ.*

Proposition 3.4. *In $(\tilde{X}, \tau, A)$, there exists a symmetric soft nbd base of Θ.*

Proposition 3.5. *For each soft nbd (W, A) of Θ in $(\tilde{X}, \tau, A)$ and for finite number of soft elements $\bar{k}_i$, where $\bar{k}_i = \bar{1}$ or $-\bar{1}$, $i = 1, 2, ..n$, there exists a symmetric soft nbd (U, A) of Θ such that $\bar{k}_1(U, A) + \bar{k}_2(U, A) + ... + \bar{k}_n(U, A)\tilde{\subset}(W, A)$.*

Proposition 3.6. (1) *Let $\mathcal{B}$ be a soft nbd base of the soft element Θ in $(\tilde{X}, \tau, A)$. Then $\mathcal{B}' = \{\tilde{x} + (U, A) : (U, A) \in \mathcal{B}\}$ is a soft nbd base of $\tilde{x}$.*

(2) *Let $\mathcal{B}$ be a soft nbd base of the soft element $\tilde{x}$ in $(\tilde{X}, \tau, A)$. Then $\mathcal{B}' = \{\tilde{t}(U, A) : (U, A) \in \mathcal{B}\}$ is a soft nbd base of $\tilde{t}\tilde{x}$, where $\tilde{t}$ is a non-zero soft scalar.*

Definition 3.7. Let $(\tilde{X}, \tau, A)$ be a soft topological space and $(F, A) \in S(\tilde{X})$. Then

(i) a collection $\mathbb{B}$ of members of $S(\tilde{X})$ is said to be a soft cover of (F, A), if $(F, A)\tilde{\subset} \Cup \{(U, A) : (U, A) \in \mathbb{B}\}$,

(ii) a soft cover $\mathbb{B}$ of (F, A) is said to be a soft open cover of (F, A), if the members of $\mathbb{B}$ are soft open,

(iii) a sub-collection of $\mathbb{B}$ is said to be a subcover of (F, A), if it is also a cover of (F, A).

Definition 3.8. In a soft topological space $(\tilde{X}, \tau, A)$, $(F, A) \in S(\tilde{X})$ is said to be soft compact, if every soft open cover of (F, A) has a finite subcover.

Proposition 3.9. *Let (F, A) be soft compact and (C, A) be soft closed in $(\tilde{X}, \tau, A)$ with $(F, A)\tilde{\cap}(C, A) = (\tilde{\Phi}, A)$. Then there exists a symmetric soft nbd (V, A) of Θ such that $[(F, A) + (V, A)]\tilde{\cap}[(C, A) + (V, A)] = (\tilde{\Phi}, A)$.*

Proof. By the continuity of the mapping $f(\tilde{x}, \tilde{y}) = \tilde{x} + \tilde{y}$ at (Θ, Θ), for any soft nbd (U, A) of Θ, there exist nbds (V_1, A) and (V_2, A) of Θ such that $(V_1, A) + (V_2, A)\tilde{\subseteq}(U, A)$. Let $(V', A) = [(V_1, A) \Cap (-V_1, A)] \Cap [(V_2, A) \Cap (-V_2, A)]$. Then (V', A) is a symmetric soft nbd of Θ and $(V', A)\tilde{\subseteq}\ (V_1, A), (V_2, A)$. Thus

$$(V', A) + (V', A)\tilde{\subseteq}(V_1, A) + (V_2, A)\tilde{\subseteq}(U, A).$$

Repeating the process, we can say that there exists a symmetric soft nbd (V'', A) of Θ such that $(V'', A) + (V'', A) + (V'', A) + (V'', A)\tilde{\subseteq}(U, A)$. Since (C, A) is soft closed, $(C,A)^{\mathbb{C}}$ is soft open and $(C,A)^{\mathbb{C}} = (C,A)^C$. So $(F,A)\tilde{\subseteq}\ (C,A)^C$. Hence to each $\tilde{x}\tilde{\in}(F,A)$ there exists a symmetric soft nbd $(V_{\tilde{x}},A)$ of Θ such that

$$\tilde{x} + (V_{\tilde{x}},A) + (V_{\tilde{x}},A) + (V_{\tilde{x}},A) + (V_{\tilde{x}},A)\tilde{\subset}(C,A)^C.$$

Since (F,A) is soft compact, there exist a finite number of symmetric soft nbds, say $(V_{\tilde{x}_1},A)$, $(V_{\tilde{x}_2},A)$, ...$(V_{\tilde{x}_n},A)$ such that $(F,A)\ \tilde{\subset}\ \underset{i=1}{\overset{n}{\Cup}}(\tilde{x}_i + (V_{\tilde{x}_i},A))$, where $\tilde{x}_1,\tilde{x}_2,...,\tilde{x}_n\tilde{\in}(F,A)$.

Let $(V,A) = \underset{i=1}{\overset{n}{\Cap}}(V_{\tilde{x}_i},A)$. Then (V,A) is a symmetric nbd of Θ. Thus

$$(F,A) + (V,A)\tilde{\subset}\underset{i=1}{\overset{n}{\Cup}}[\tilde{x}_i + (V_{\tilde{x}_i},A)] + (V,A)\tilde{\subset}\underset{i=1}{\overset{n}{\Cup}}[\{\tilde{x}_i + (V_{\tilde{x}_i},A)\} + (V_{\tilde{x}_i},A))].$$

So $(\tilde{x}_i + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A))\,\tilde{\cap}(C,A) = (\tilde{\Phi},A)$
$\Rightarrow [\tilde{x}_i + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A)]\tilde{\cap}[(C,A) - (V_{\tilde{x}_i},A)] = (\tilde{\Phi},A)$
$\Rightarrow [\tilde{x}_i + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A)]\,\tilde{\cap}[(C,A) + (V_{\tilde{x}_i},A)] = (\tilde{\Phi},A)$
$\Rightarrow \tilde{x}_i + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A)\tilde{\cap}[(C,A) + (V,A)] = (\tilde{\Phi},A),\ \forall\ i = 1,\ 2,...,n.$
Hence $\underset{i=1}{\overset{n}{\Cup}}[\tilde{x}_i + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A) + (V_{\tilde{x}_i},A)]\,\tilde{\cap}[(C,A) + (V,A)] = (\tilde{\Phi},A)$.
Therefore $[(F,A) + (V,A)]\,\tilde{\cap}\,[(C,A) + (V,A)] = (\tilde{\Phi},A)$. □

Corollary 3.10. *Every soft T_1 soft tvs is soft T_2 and soft regular.*

Corollary 3.11. *Let $(\tilde{X},\tau,A)$ be a soft tvs and $(V,A)\in\tau$. Then for any soft set $(F,A)\in S(\tilde{X})$, $(F,A)+(V,A)$ is soft open.*

4. Soft linear mappings on soft topological vector spaces

Definition 4.1. Let X and Y be two vector spaces and $\{T_\lambda : X\to Y,\lambda\in A\}$ be a collection of linear operators. Then a soft function $T : SE(\tilde{X})\to SE(\tilde{Y})$, associated with the family of functions $\{T_\lambda : X\to Y,\lambda\in A\}$, defined by $[T(\tilde{x})](\lambda) = T_\lambda(\tilde{x}(\lambda))$, for each $\lambda\in A$, is called a soft linear operator or soft linear mapping.

A soft linear mapping $T : SE(\tilde{X})\to SE(\tilde{K})$, where $(\tilde{K},A) = \mathbb{R}(A)$ *or* $\mathbb{C}(A)$, is said to be a soft linear functional.

Remark 4.2. A soft linear mapping as in Definition 4.1 is same as a linear operator in [9] satisfying $(L3)$.

Proposition 4.3. *Let $(\tilde{X},A)$ and $(\tilde{Y},A)$ be two soft vector spaces and $T : SE(\tilde{X})\to SE(\tilde{Y})$ be a soft linear mapping. Then for any convex (balanced) soft set (V,A), $T(V)$ is convex (balanced) soft set.*

Proposition 4.4. *A soft linear mapping $T : (\tilde{X},\tau,A)\to(\tilde{Y},\tau,A)$ is soft continuous if and only if T is soft continuous at Θ.*

Definition 4.5. Let $(\tilde{X},\tau,A)$ be a soft tvs and $T : SE(\tilde{X})\to SE(\tilde{K})$ be a non-null soft linear functional. Then T is said to be bounded on some soft set $(F,A)\in S(\tilde{X})$, if there exists $\tilde{M}\tilde{>}\bar{0}$, such that $|T(\tilde{x})|\tilde{\leq}\tilde{M}$, for all $\tilde{x}\tilde{\in}(F,A)$.

Definition 4.6. In a soft topological space $(\tilde{X},\tau,A)$, $(F,A)\in S(\tilde{X})$ is called dense in $(\tilde{X},A)$, if $(\tilde{X},A) = \overline{(F,A)}$ = Elementary intersection of soft closed sets containing (F,A).

Example 4.7. Let $X = \{a, b, c\}, A = \{\alpha, \beta\}$ and $\tau = \{(\tilde{\Phi}, A), (\tilde{X}, A), (F, A), (G, A)\}$, where $F(\alpha) = \{a, b\},\ F(\beta) = \{a, c\}$ and $G(\alpha) = \{c\}, G(\beta) = \{b, c\}$. Consider a soft set (U, A) such that $U(\alpha) = \{b, c\}, U(\beta) = \{a, b\}$. Then $\overline{(U, A)} = (\tilde{X}, A)$, i.e. (U, A) is dense in $(\tilde{X}, A)$.

Proposition 4.8. *Let $(\tilde{X}, \tau, A)$ be a soft T_1 soft tvs and $T : SE(\tilde{X}) \to SE(\tilde{K})$ be a non-null soft linear functional. Then the followings are equivalent:*

(1) *T is soft continuous.*
(2) *$N(T) = SS\{\tilde{x}\tilde{\in}(\tilde{X}, A) : T(\tilde{x}) = \bar{0}\}$ is soft closed.*
(3) *$N(T)$ is not dense in $(\tilde{X}, A)$.*
(4) *T is bounded on some soft nbd of the null soft vector Θ_X in $(\tilde{X}, A)$.*

Proof. $(1) \Rightarrow (2)$: $N(T) = SS\{\tilde{x}\tilde{\in}(\tilde{X}, A) : T(\tilde{x}) = \bar{0}\} = T^{-1}(\bar{0}, A)$. By (1), T is soft continuous. Also, $(\bar{0}, A)$ is soft closed. Then $T^{-1}(\bar{0}, A)$ is soft closed in $(\tilde{X}, \tau, A)$. Thus (2) holds.

$(2) \Rightarrow (3)$: By (2), $N(T)$ is soft closed. Then $N(T) = \overline{N(T)}$ and $N(T) \neq X$, since T is non-null. Thus $N(T)$ is not dense in $(\tilde{X}, A)$. So (3) holds.

$(3) \Rightarrow (4)$: Suppose (3)holds. Then $\overline{N(T)}\tilde{\subsetneq}(\tilde{X}, A)$. Thus $(\tilde{X}, A) \setminus \overline{N(T)} \neq (\tilde{\Phi}, A)$. So, there exists $\tilde{x}\tilde{\in}\left((\tilde{X}, A) \setminus \overline{N(T)}\right)$. Hence there exists a soft nbd (U, A) of Θ_X such that $(\tilde{x} + (U, A))\tilde{\cap}N(T) = (\tilde{\Phi}, A)$, i.e., $(\tilde{x} + (U, A))(\lambda) \cap N(T)(\lambda) = \phi$, for all $\lambda \in A$. Again, there exists a balanced soft nbd (V, A) of Θ_X such that $(V, A)\tilde{\subset}(U, A)$. Therefore, either T is bounded on (V, A), in which case (4) holds or T is not bounded on (V, A).

Let $T(\tilde{x}) = \tilde{\alpha}\tilde{\in}(\tilde{K}, A)$. Since T is not bounded on (V, A), there exists $\lambda \in A$ and $\tilde{y}\tilde{\in}(V, A)$ such that $|\tilde{\alpha}(\lambda)| < |T(\tilde{y})(\lambda)|$. On the other hand,

$$\tilde{\alpha}(\lambda) = [T(\tilde{y})](\lambda). \left([T(\tilde{y})](\lambda)\right)^{-1}.\tilde{\alpha}(\lambda)$$

and

$$|\left([T(\tilde{y})](\lambda)\right)^{-1}.\tilde{\alpha}(\lambda)| = \frac{|\tilde{\alpha}(\lambda)|}{|T(\tilde{y})(\lambda)|} < 1.$$

Then $\tilde{\alpha}(\lambda) \in k[T(V, A)](\lambda)$, where $|k| < 1$. Since $T(V, A)$ is balanced soft set, $[T(V, A)](\lambda)$ is balanced for all $\lambda \in A$. Thus $k\left([T(V, A)](\lambda)\right) \subset [T(V, A)](\lambda)$. So $\tilde{\alpha}(\lambda) \in [T(V, A)](\lambda)$. Hence there exists $\tilde{z}\tilde{\in}(V, A)$ such that

$$\tilde{\alpha}(\lambda) = [T(\tilde{x})](\lambda) = -[T(\tilde{z})](\lambda),$$

since $[T(V, A)](\lambda)$ is balanced, for all $\lambda \in A$. Therefore

$$[T(\tilde{x} + \tilde{z})](\lambda) = [T(\tilde{x}) + T(\tilde{z})](\lambda) = 0, \text{ i.e.. } (\tilde{x} + \tilde{z})(\lambda) \in [N(T)](\lambda),$$

as $[N(T)](\lambda) = N(T_\lambda) = \{x \in X : T_\lambda(x) = 0\}$, and so $(\tilde{x} + (V, A))(\lambda) \cap N(T)(\lambda) \neq \phi$, which is impossible as $(\tilde{x} + (U, A))(\lambda) \cap N(T)(\lambda) = \phi$, for all $\lambda \in A$. Hence, T is bounded in (V, A).

$(4) \Rightarrow (1)$: Suppose T is bounded on some soft nbd (V, A) of Θ_X. Then there exists $\tilde{M}\tilde{>}\bar{0}$ such that $|T(\tilde{x})| \tilde{\leq} \tilde{M}, \forall\, \tilde{x} \tilde{\in} (V, A)$. For any $\tilde{\epsilon}\tilde{>}\bar{0}$, consider $\frac{\tilde{\epsilon}}{\bar{2}}\tilde{M}(V, A) = (W, A)$. Thus (W, A) is a soft nbd of Θ_X and $|T(\xi)| \tilde{\leq} \tilde{\epsilon}/\bar{2} \tilde{<} \tilde{\epsilon}, \forall\, \xi \tilde{\in} (W, A)$. So T is soft continuous at Θ_X and thus T is soft continuous on $(\tilde{X}, A)$. Hence (1) holds. □

Definition 4.9. A sequence $\{\tilde{x}_n\}$ of soft elements in a soft metric space $(\tilde{X}, d, A)$ is said to be soft convergent, if there is $\tilde{x}\tilde{\in}(\tilde{X}, A)$ such that $d(\tilde{x}_n, \tilde{x}) \to \bar{0}$ as $n \to \infty$. This means that for every $\tilde{\epsilon}\tilde{>}\bar{0}$, chosen arbitrarily, there exists a soft natural number $\tilde{N}$, such that $0 < d(\tilde{x}_n, \tilde{x})(\lambda) < \tilde{\epsilon}(\lambda)$, whenever $n > \tilde{N}(\lambda)$, $\forall \lambda \in A$. We denote this by $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$ and $\tilde{x}$ is said to be the limit of the sequence $\{\tilde{x}_n\}$.

Proposition 4.10. *Limit of a sequence in a soft metric space, if exists is unique.*

Proposition 4.11. *A sequence $\{\tilde{x}_n\}$ of soft elements in a soft metric space $(\tilde{X}, d, A)$, where d satisfies $(M5)$, is soft convergent to $\tilde{x}$ iff $\{\tilde{x}_n(\lambda)\}$ is convergent to $\tilde{x}(\lambda)$ in (X, d_λ), $\forall \lambda \in A$, where d_λ defined as in Theorem 2.12.*

Definition 4.12. In a soft topological space $(\tilde{X}, \tau, A)$, a sequence $\{\tilde{x}_n\}$ is said to converge to a soft element $\tilde{x}$, if for any soft nbd (V, A) of $\tilde{x}$ there exists a soft natural number $\tilde{N}$ such that $\tilde{x}_n(\lambda) \in V(\lambda)$, $\forall n \geq \tilde{N}(\lambda)$, $\forall \lambda \in A$.

Definition 4.13. A soft tvs $(\tilde{X}, \tau, A)$ is said to be soft metrizable, if there exists a translation invariant soft metric d satisfying $(M5)$ on $(\tilde{X}, A)$ such that $\tau_d = \tau$, where τ_d is the soft metric topology of d.

Proposition 4.14. *Let $(\tilde{X}, A)$ be the absolute soft vector space and d be a translation invariant soft metric on $(\tilde{X}, A)$. Then*

(1) *$d(\bar{n}\tilde{x}, \Theta)\tilde{\leq}\bar{n}d(\tilde{x}, \Theta)$, where $\bar{n}$ is a soft natural number,*

(2) *if further $(\tilde{X}, \tau, A)$ is a soft tvs which is metrizable by a translation invariant metric $'d'$, then $\tilde{x}_n \to \tilde{\Theta}$ implies there exists a sequence of soft reals $\{\tilde{\gamma}_n\}$ such that for each $\lambda \in A$, $\tilde{\gamma}_n(\lambda) \to \infty$ in (X, d_λ) but $\tilde{\gamma}_n\tilde{x}_n \to \Theta$ in $(\tilde{X}, d, A)$.*

Proof. (1) $d(\bar{n}\tilde{x}, \Theta)\tilde{\leq}d(\bar{n}\tilde{x}, \overline{(n-1)}\tilde{x}) + d(\overline{(n-1)}\tilde{x}, \overline{(n-2)}\tilde{x}) + ... + d(\bar{2}\tilde{x}, \tilde{x}) + d(\tilde{x}, \Theta)$
$= d(\tilde{x}, \Theta) + d(\tilde{x}, \Theta) + ...n \text{ times}$
$= \bar{n}d(\tilde{x}, \Theta)$.

(2) Since $\tilde{x}_n \to \Theta$, we have $d(\tilde{x}_n, \Theta)(\lambda) \to 0$ as $n \to \infty$, for all $\lambda \in A$. Choose $\lambda \in A$. Then, for a positive integer k, there exists an index n_k (with $n_k < n_{k+1}$) such that $d(\tilde{x}_n, \Theta)(\lambda) = d_\lambda(\tilde{x}_n(\lambda), \theta) \leq \frac{1}{k^2}$, for all $n \geq n_k$.

Let $\gamma_n^\lambda = k$, if $n_k \leq n < n_{k+1}$. Then $\gamma_n^\lambda \to \infty$ as $n \to \infty$ and $d_\lambda(\gamma_n^\lambda \tilde{x}_n(\lambda), \theta) \leq \gamma_n^\lambda d_\lambda(\tilde{x}_n(\lambda), \theta) < k.\frac{1}{k^2}$, for all $n \geq n_k$. Thus, $d_\lambda(\gamma_n^\lambda \tilde{x}_n(\lambda), \theta) \to 0$ as $n \to \infty$.

Let $\tilde{\gamma}_n(\lambda) = \gamma_n^\lambda$, $\forall \lambda \in A$. Then

$$d(\tilde{\gamma}_n(\lambda)\tilde{x}_n(\lambda), \theta) \to 0 \text{ as } n \to \infty, \text{ i.e., } d(\tilde{\gamma}_n\tilde{x}_n, \Theta)(\lambda) \to 0 \text{ as } n \to \infty.$$

This is true, for all $\lambda \in A$. So, $\tilde{\gamma}_n\tilde{x}_n \to \Theta$. □

Proposition 4.15. *Suppose (V, A) is a soft nbd of Θ in $(\tilde{X}, \tau, A)$. If $\{\tilde{r}_n\}$ is a sequence of soft real numbers such that $\bar{0}\ \tilde{<}\tilde{r}_1\tilde{<}\tilde{r}_2\tilde{<}...\tilde{<}\tilde{r}_n\tilde{<}...$ with $\tilde{r}_n(\lambda) \to \infty$ as $n \to \infty$, for each $\lambda \in A$, then $(\tilde{X}, A) = \underset{n=1}{\Cup} \tilde{r}_n(V, A)$.*

Proof. Let $\tilde{x}\tilde{\in}(\tilde{X}, A)$. Using the continuity of the mapping g of Definition 2.31 at $(\bar{0}, \tilde{x})$, we get that for any soft nbd (V, A) of Θ, there exists $\tilde{\delta}\tilde{>}\bar{0}$ such that $\tilde{t}\tilde{x}\ \tilde{\in}(V, A)$, for all $\tilde{t}$ with $|\tilde{t}|\tilde{<}\tilde{\delta}$. Choose $\lambda \in A$. As $\tilde{r}_n(\lambda) \to \infty$, there exists $n_0 \in N$ such that $\frac{1}{\tilde{r}_n(\lambda)} < \tilde{\delta}(\lambda)$, $\forall n \geq n_0$. Then

$$\frac{1}{\tilde{r}_n(\lambda)}\tilde{x}(\lambda) \in (V, A)(\lambda),\ \forall n \geq n_0, \text{ i.e., } \tilde{x}(\lambda) \in \tilde{r}_n(\lambda)(V, A)(\lambda),\ \forall n \geq n_0.$$

Thus $X = \bigcup_{n=1}^{\infty}(\tilde{r}_n(\lambda)(V, A)(\lambda))$. This is true, for all $\lambda \in A$. So,

$$(\tilde{X},A)(\lambda)=\tilde{\bigcup}_{n=1}^{\infty}(\tilde{r}_n(\lambda)(V,A)(\lambda))=\tilde{\bigcup}_{n=1}^{\infty}\left[\tilde{r}_n(V,A)\right](\lambda), \text{ for all } \lambda\in A.$$

Hence, $(\tilde{X},A)=\tilde{\bigcup}_{n=1}^{\infty}\left[\tilde{r}_n(V,A)\right]=\Cup_{n=1}^{\infty}\left[\tilde{r}_n(V,A)\right].$ □

Remark 4.16. In a soft tvs $(\tilde{X},\tau,A)$, every soft nbd of Θ is absorbing.

Lemma 4.17. *For a sequence $\{\tilde{x}_n\}$ in a soft tvs $(\tilde{X},\tau,A)$ converging to Θ, the soft set $(\tilde{x}_n,A)(=SS\{\tilde{x}_1,\ \tilde{x}_2,...,\ \tilde{x}_n,....\})$ is bounded in $(\tilde{X},\tau,A)$.*

Proof. Since $\{\tilde{x}_n\}$ is a sequence of soft vectors converging to Θ, for any balanced soft nbd (V,A) of Θ, there exists a soft natural number $\tilde{N}$ such that $\tilde{x}_n(\lambda)\in V(\lambda)$, $\forall n\geq\tilde{N}(\lambda),\ \forall\lambda\in A$. Choose $\lambda\in A$ and let $\tilde{x}_n(\lambda)\in V(\lambda),\ \forall n\geq n_0=\tilde{N}(\lambda)$. Then $\tilde{x}_n(\lambda)\notin V(\lambda),\ i=1,2,..n_0$. Since every soft nbd of Θ is absorbing, there exist $t_1,\ t_2,...,t_{n_0}>0$ such that $\tilde{x}_i(\lambda)\in t_iV(\lambda),\ i=1,2,..,n_0$. Let $t_\lambda=max\{t_i,1\},\ i=1,2,..,n_0$. Since (V,A) is balanced, $\tilde{x}_n(\lambda)\in t_\lambda V(\lambda),\forall n$. Similarly, for each $\lambda\in A$, we have some t_λ such that $\tilde{x}_n(\lambda)\in t_\lambda V(\lambda),\ \forall n$. Let $t_\lambda=\tilde{t}(\lambda)$. Then, there exists $\tilde{t}\tilde{>}\bar{0}$ such that $\tilde{x}_n\tilde{\in}\tilde{s}(V,A),\ \forall\tilde{s}\tilde{>}\tilde{t},\ \forall n$. Thus $(\tilde{x}_n,A)(=SS\{\tilde{x}_1,\ \tilde{x}_2,...,\ \tilde{x}_n,....\})\tilde{\subset}\tilde{s}(V,A)$. So, the soft set $(\tilde{x}_n,A)$ is bounded in $(\tilde{X},\tau,A)$. □

Proposition 4.18. *In $(\tilde{X},\tau,A)$ the following statements are equivalent:*
(1) *(E,A) is a soft bounded set in $(\tilde{X},\tau,A)$,*
(2) *for any sequence $\{\tilde{x}_n\}$ of soft elements in (E,A) and for any sequence of soft real numbers $\tilde{\alpha}_n$ converging to $\bar{0}$, $\tilde{\alpha}_n\tilde{x}_n$ converges to Θ.*

Proof. (1) $\Rightarrow$ (2) : Let (W,A) be any symmetric soft nbd of Θ. Since (W,A) is bounded there exists $\tilde{t}\tilde{\geq}\bar{0}$ such that $(E,A)\tilde{\subset}\tilde{s}(W,A)$, for all $\tilde{s}\tilde{\geq}\tilde{t}$. Since $\tilde{\alpha}_n\to\bar{0}$, there exists $\tilde{N}$ such that $\frac{\bar{1}}{|\tilde{\alpha}_n|}\tilde{\geq}\tilde{t}$, $\forall n\geq\tilde{N}(\lambda)$. Then there exists

$$(E,A)(\lambda)\subset\frac{1}{|\tilde{\alpha}_n(\lambda)|}(W,A)(\lambda)\ ,\forall n\geq\tilde{N}(\lambda)$$

or, $\quad|\tilde{\alpha}_n(\lambda)|(E,A)(\lambda)\subset(W,A)(\lambda),\ \forall n\geq\tilde{N}(\lambda)$

or, $\quad\tilde{\alpha}_n(\lambda)(E,A)(\lambda)\subset(W,A)(\lambda),\ \forall n\geq\tilde{N}(\lambda)$

or, $\quad\tilde{\alpha}_n(\lambda)\tilde{x}_n(\lambda)\subset(W,A)(\lambda),\ \forall n\geq\tilde{N}(\lambda).$

Thus $\tilde{\alpha}_n\tilde{x}_n\to\Theta$.

(2) $\Rightarrow$ (1) : Suppose (2) holds and assume that (1) do not hold. Then there exists a soft nbd (V,A) of Θ such that for any positive integer n, there exists $\tilde{x}_n\tilde{\in}(E,A)$, such that $\tilde{x}_n\tilde{\notin}\bar{n}(V,A)$. Thus $\tilde{x}_n$ is a sequence in (E,A) and $\frac{\bar{1}}{n}$ converges to $\bar{0}$. But $\frac{\bar{1}}{n}\tilde{x}_n\tilde{\notin}(V,A)$, for all $n=1,2,...,$ So $\frac{\bar{1}}{n}\tilde{x}_n\nrightarrow\Theta$, which is contradictory to (2). Hence The result holds. □

Definition 4.19. A soft linear mapping $f:(\tilde{X},\tau,A)\to(\tilde{Y},\nu,A)$ is said to be soft bounded, if $f(E,A)$ is bounded whenever (E,A) is bounded.

Proposition 4.20. *Let $f:(\tilde{X},\tau,A)\to(\tilde{Y},\nu,A)$ be a soft linear mapping. Then among the following statements (1) $\Rightarrow$ (2) $\Rightarrow$ (3) holds.*
If further $(\tilde{X},\tau,A)$ is metrizable, then (3) $\Rightarrow$ (4) holds.
(1) *f is soft continuous.*
(2) *f is soft bounded.*

(3) *If* $\tilde{x}_n \to \Theta$*, then* $(f(\tilde{x}_n), A)$ *is a bounded soft set.*
(4) *If* $\tilde{x}_n \to \Theta_X$*, then* $\{f(\tilde{x}_n)\} \to \Theta_Y$*.*

Proof. (1) $\Rightarrow$ (2) : Let (E, A) be any soft bounded set in $(\tilde{X}, \tau, A)$ and (W, A) be a nbd of Θ_Y in $(\tilde{Y}, \nu, A)$. From (1), there exists a nbd (V, A) of Θ_X such that $f(V, A)\tilde{\subset}(W, A)$. Since (E, A) is bounded in $(\tilde{X}, \tau, A)$, there exists $\tilde{t}\tilde{>}\bar{0}$ such that $(E, A)\tilde{\subset}\tilde{s}(V, A)$, for all $\tilde{s}\tilde{>}\tilde{t}$. Then, $f(E, A)\tilde{\subset}f(\tilde{s}(V, A)) = \tilde{s}f(V, A)$ [Since f is soft linear] $\tilde{\subset}\tilde{s}(W, A)$, for all $\tilde{s}\tilde{>}\tilde{t}$. Thus $f(E, A)$ is bounded soft set, whenever (E, A) is bounded. So (2) holds.

(2) $\Rightarrow$ (3): The proof is obvious.

(3) $\Rightarrow$ (4) : Suppose $(\tilde{X}, \tau, A)$ is metrizable. Then by Proposition 4.14, $\tilde{x}_n \to \tilde{\Theta}$ implies there exists a sequence of soft reals $\{\tilde{\gamma}_n\}$ such that $\tilde{\gamma}_n(\lambda) \to \infty$, for each $\lambda \in A$ but $\tilde{\gamma}_n\tilde{x}_n \to \Theta$. Thus by (3), $(f(\tilde{\gamma}_n\tilde{x}_n), A)$ is bounded in $(\tilde{Y}, \nu, A)$. Now $\frac{1}{\tilde{\gamma}_n(\lambda)} \to 0, \forall\lambda \in A$. Thus $\frac{\bar{1}}{\tilde{\gamma}_n} \to \bar{0}$. So, by the previous theorem, $\frac{\bar{1}}{\tilde{\gamma}_n}(f(\tilde{\gamma}_n\tilde{x}_n)) \to \Theta_Y$, i.e., $f(\tilde{x}_n) \to \Theta_Y$ [as f is soft linear]. Hence the result holds. □

5. Conclusion

There is a wide scope of work on soft tvs such as finite dimensionality of soft tvs, extension of Banach Steinhaus theorem, weak and weak* topology etc. Metrizability in soft tvs is a problem of another direction.

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