Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 11, No. 2, (February 2016), pp. 4–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

The category of intuitionistic sets

J. Kim, P. K. Lim, J. G. Lee, K. Hur

Received 20 February 2017; Revised 4 August 2017; Accepted 13 September

Abstract. First, we introduce the concept of an intuitionistic set as the generalzation of an ordinary set and the specialization of an intuitionistic fuzzy set and thus neutrosophic crisp set, and list its some properties. Second, we introduce the category **ISet** consisting of intuitionistic sets and morphisms between them and study the category **ISet** in the viewpoint of topological universe. Finally, we find relationships between two categories **ISet** and **NCSet**. In particular, we prove that Two categories **ISet** and $^*\mathbf{NCSet}_*$ are isomorphism.

2010 AMS Classification: 03E72, 18B05

Keywords: Neutrosophic set, Cartesian closed category, Topological universe, Neutrosophic crisp set.

Corresponding Author: J. Kim (junhikim@wku.ac.kr)

1. Introduction

In 1983, Atanassove [1] proposed the notion of intuitionistic fuzzy set as the generalization of fuzzy sets by introduced by Zadeh [30] considering the degree of membership and non-membership (See [2, 3, 4, 5, 6], in order to refer to the details of intuitionistic fuzzy sets). In 1998, Smarandache [28] introduced the concept of a neutrosophic set as the generalization of fuzzy sets introduced by Atanassove considering the degree of membership, the degree of indeterminacy and the degree of non-membership.

After Zadeh [30] introdced the concept of a fuzzy set as as the generalization of an ordinary set, many researchers [8, 11, 12, 13, 16, 17, 21, 23, 24, 25] have investigated fuzzy sets in the sense of category theory, for instance, $\mathbf{Set}(\mathbf{H})$, $\mathbf{Set_f}(\mathbf{H})$, $\mathbf{Set_g}(\mathbf{H})$, $\mathbf{Fuz}(\mathbf{H})$. Among them, the category $\mathbf{Set}(\mathbf{H})$ is the most useful one as the "standard" category, because $\mathbf{Set}(\mathbf{H})$ is very suitable for describing fuzzy sets and mappings between them. In particular, Carrega [8], Dubuc [11], Eytan [12], Goguen [13], Pittes [23], Ponasse [24, 25] had studied $\mathbf{Set}(\mathbf{H})$ in topos view-point. However Hur et al. investigated $\mathbf{Set}(\mathbf{H})$ in the view of topological universe. Also Hur et al.

[17] introduced the category **ISet**(**H**) consisting of intuitionistic H-fuzzy sets and morphisms between them, and studied **ISet**(**H**) in the sense of topological universe. Recently, Lim et al. [21] introduced the new category **VSet**(**H**) and investigated it in the sense of topological universe. Furthermore, Hur et al. [18] define the category **NCSet** consisting of neutrosophc crisp spaces and morphisms between them and study its some properties.

On the other hand, Gunduz and Davvaz [14] defined intuitionistic fuzzy submodules and investigated their inverse system. Yang and Li [29] defined the concept of generalized intuitionistic fuzzy sets, formed a category consisting of the classes of generalized intuitionistic fuzzy sets and morphisms between them and studied its some properties.

In 1996, Coker [9] introduced the concept of an intuitionistic set (called an intuitionistic crisp set by Salama et al.[27]) as the generalzation of an ordinary set and the specialization of an intuitionistic fuzzy set and also neutrosophic crisp set (See [26, 28]). After that time, he [10] applied the notion to topology, and Bayhan and Coker [7] dealt with pairwise separation axioms in intuitionistic topological spaces and some relationships between categories **Dbl-Top** and **Bitop**. Furthermore, Lee and Chu [20] introduced the category **ITop** and investigated some relationships between **ITop** and **Top**.

The concept of a topological universe was introduced by Nel [22], which implies a Cartesian closed category and a concrete quasitopos. Furthermore the concept has already been up to effective use for several areas of mathematics.

In this paper, first, we list some definitions and results related to category theory and also some operations on intuitionistic sets and their some results. Second, we introduce the category **ISet** consisting of intuitionistic sets and morphisms between them and study the category **ISet** in the view-point of topological universe. Finally, we find relationships between two categories **ISet** and **NCSet**. In particular, we prove that Two categories **ISet** and ***NCSet**$_*$ are isomorphism (See Theorem 5.7).

2. Preliminaries

In this section, we list some basic definitions and well-known results from [15, 19, 22] which are needed in the next sections.

Definition 2.1 ([19]). Let **A** be a concrete category and $((Y_j, \xi_j))_J$ a family of objects in **A** indexed by a class J. For any set X, let $(f_j : X \to Y_j)_J$ be a source of mappings indexed by J. Then an **A**-structure ξ on X is said to be initial with respect to (in short, w.r.t.) $(X, (f_j), (Y_j, \xi_j))_J$, if it satisfies the following conditions:

(i) for each $j \in J$, $f_j : (X, \xi) \to (Y_j, \xi_j)$ is an **A**-morphism,

(ii) if (Z, ρ) is an **A**-object and $g : Z \to X$ is a mapping such that for each $j \in J$, the mapping $f_j \circ g : (Z, \rho) \to (Y_j, \xi_j)$ is an **A**-morphism, then $g : (Z, \rho) \to (X, \xi)$ is an **A**-morphism.

In this case, **A** is called a topological category and $(f_j : (X, \xi) \to (Y_j, \xi_j))_J$ is called an initial source in **A**.

Dual notion: cotopological category.

Result 2.2 ([19], Theorem 1.5). *A concrete category* **A** *is topological if and only if it is cotopological.*

Result 2.3 ([19], Theorem 1.6)**.** *Let* **A** *be a topological category over Set, then it is complete and cocomplete.*

Definition 2.4 ([19])**.** Let **A** be a concrete category.

(i) The **A**-fibre of a set X is the class of all **A**-structures on X.

(ii) **A** is said to be properly fibred over **Set**, it satisfies the followings:

(a) (Fibre-smallness) for each set X, the **A**-fibre of X is a set,

(b) (Terminal separator property) for each singleton set X, the **A**-fibre of X has precisely one element,

(c) if ξ and η are **A**-structures on a set X such that $id : (X, \xi) \to (X, \eta)$ and $id : (X, \eta) \to (X, \xi)$ are **A**-morphisms, then $\xi = \eta$.

Definition 2.5 ([15])**.** A category **A** is said to be Cartesian closed, if it satisfies the following conditions:

(i) for each **A**-object A and B, there exists a product $A \times B$ in **A**,

(ii) exponential objects exist in A, i.e., for each **A**-object A, the functor $A \times - : A \to A$ has a right adjoint, i.e., for any **A**-object B, there exist an **A**-object B^A and a **A**-morphism $e_{A,B} : A \times B^A \to B$ (called the evaluation) such that for any **A**-object C and any **A**-morphism $f : A \times C \to B$, there exists a unique **A**-morphism $\bar{f} : C \to B^A$ such that the diagram commutes:

Definition 2.6 ([15])**.** A category **A** is called a topological universe over **Set**, if it satisfies the following conditions:

(i) **A** is well-structured, i.e. (a) **A** is concrete category; (b) **A** satisfies the fibre-smallness condition; (c) **A** has the terminal separator property,

(ii) **A** is cotopological over **Set**,

(iii) final episinks in **A** are preserved by pullbacks, i.e., for any episink $(g_j : X_j \to Y)_J$ and any **A**-morphism $f : W \to Y$, the family $(e_j : U_j \to W)_J$, obtained by taking the pullback f and g_j, for each $j \in J$, is again a final episink.

Definition 2.7 ([26])**.** Let X be a non-empty set. Then A is called a neutrosophic crisp set (in short, NCS) in X if A has the form $A = (A_1, A_2, A_3)$, where A_1, A_2, and A_3 are subsets of X,

The neutrosophic crisp empty [resp., whole] set, denoted by ϕ_N [resp., X_N] is an NCS in X defined by $\phi_N = (\phi, \phi, X)$ [resp., $X_N = (X, X, \phi)$]. We will denote the set of all NCSs in X as $NCS(X)$.

Definition 2.8 ([26])**.** Let $A = (A_1, A_2, A_3), B = (B_1, B_2, B_3) \in NCS(X)$. Then

(i) A is said to be contained in B, denoted by $A \subset B$, if

$$A_1 \subset B_1,\ A_2 \subset B_2 \text{ and } A_3 \supset B_3,$$

(ii) A is said to equal to B, denoted by $A = B$, if

$$A \subset B \text{ and } B \subset A,$$

(iii) the complement of A, denoted by A^c, is an NCS in X defined as:

$$A^c = (A_3, A_2^c, A_1),$$

(iv) the intersection of A and B, denoted by $A \cap B$, is an NCS in X defined as:

$$A \cap B = (A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3),$$

(v) the union of A and B, denoted by $A \cup B$, is an NCS in X defined as:

$$A \cup B = (A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3).$$

Let $(A_j)_{j\in J} \subset NCS(X)$, where $A_j = (A_{j,1}, A_{j,2}, A_{j,3})$. Then

(vi) the intersection of $(A_j)_{j\in J}$, denoted by $\bigcap_{j\in J} A_j$ (simply, $\bigcap A_j$), is an NCS in X defined as:

$$\bigcap A_j = (\bigcap A_{j,1}, \bigcap A_{j,2}, \bigcup A_{j,3}),$$

(vii) the the union of $(A_j)_{j\in J}$, denoted by $\bigcup_{j\in J} A_j$ (simply, $\bigcup A_j$), is an NCS in X defined as:

$$\bigcup A_j = (\bigcup A_{j,1}, \bigcup A_{j,2}, \bigcap A_{j,3}).$$

3. Intuitionistic sets

Definition 3.1 ([9, 27])**.** Let X be a non-empty set. Then A is called an intuitionistic set (in short, IS) of X, if it is an object having the form

$$A = (A_T, A_F),$$

such that $A_T \cap A_F = \phi$, where A_T [resp. A_F] is called the set of members [resp. nonmembers] of A.

In fact, A_T [resp. A_F] is a subset of X agreeing or approving [resp. refusing or opposing] for a certain opinion, view, suggestion or policy.

The intuitionistic crisp empty set [resp. the intuitionistic crisp whole set] of X, denoted by ϕ_I [resp. X_I], is defined by $\phi_I = (\phi, X)$ [resp. $X_I = (X, \phi)$].

In general, $A_T \cup A_F \neq X$.

We will denote the set of all ISs of X as $IS(X)$.

For each ordinary subset A of X, we can identify A as the pair (A, ϕ). Then we can consider an IS of X as the generalization of an ordinary subset of X. Furthermore, it is clear that $A = (\chi_{A_T}, \chi_{A_F})$ [resp. $A = (A_T, A_T, A_F)$] is an intuitionistic fuzzy set [resp. an neutrosophic crisp set] in X, for each $A \in IS(X)$. Thus we can consider an intuitionistic fuzzy set [resp. a neutrosophic crisp set] in X as the generalization of an IS of X.

Example 3.2. Let $X = \{a, b, c, d, e, f\}$, $A_T = \{a, c, f\}$ and $A_F = \{b, d\}$. Then $A = (A_T, A_F)$ is an IS of X.

Definition 3.3 ([9])**.** Let $A, B \in IS(X)$ and let $(A_j)_{j\in J} \subset IS(X)$.

(i) We say that A is contained in B, denoted by $A \subset B$, if $A_T \subset B_T$ and $A_F \supset B_F$.

(ii) We say that A equals to B, denoted by $A = B$, if $A \subset B$ and $B \subset A$.

(iii) The complement of A denoted by A^c, is an IS of X defined as:

$$A^c = (A_F, A_T).$$

(iv) The union of A and B, denoted by $A \cup B$, is an IS of X defined as:

$$(A \cup B)_T = A_T \cup B_T, (A \cup B)_F = A_F \cap B_F.$$

(v) The union of $(A_j)_{j\in J}$, denoted by $\bigcup_{j\in J} A_j$ (in short, $\bigcup A_j$), is an IS of X defined as:

$$(\bigcup_{j\in J} A_j)_T = \bigcup_{j\in J} (A_j)_T, (\bigcup_{j\in J} A_j)_F = \bigcap_{j\in J} (A_j)_F.$$

(vi) The intersection of A and B, denoted by $A \cap B$, is an IS of X defined as:

$$(A \cap B)_T = A_T \cap B_T, (A \cap B)_F = A_F \cup B_F.$$

(vii) The intersection of $(A_j)_{j\in J}$, denoted by $\bigcap_{j\in J} A_j$ (in short, $\bigcap A_j$), is an IS of X defined as:

$$(\bigcap_{j\in J} A_j)_T = \bigcap_{j\in J}(A_j)_T, (\bigcap_{j\in J} A_j)_F = \bigcup_{j\in J}(A_j)_F.$$

(viii) $A - B = A \cap B^c$.
(xi) $[\,]A = (A_T, A_T^c)$, $\quad <> A = (A_F^c, A_F)$.

Example 3.4. Let A be the IS of X Example 3.2. Then $A^c = (A_F, A_T)$. Thus

$$(A \cup A^c)_T = A_T \cup A_T^c = \{a, c, f\} \cup \{b, d\} = \{a, b, c, d, f\} \neq X$$

and

$$(A \cup A^c)_F = A_F \cap A_F^c = \{b, d\} \cap \{a, c, f\} = \phi.$$

On the other hand,

$$(A \cap A^c)_T = A_T \cap A_T^c = \{a, c, f\} \cap \{b, d\} = \phi$$

and

$$(A \cap A^c)_F = A_F \cup A_F^c = \{b, d\} \cup \{a, c, f\} = \{a, b, c, d, f\} \neq X.$$

So $A \cup A^c \neq X_I$ and $A \cap A^c \neq \phi_I$.

The followings are the immediate results of Definition 3.1.

Proposition 3.5. *Let $A, B, C \in IS(X)$. Then*
(1) $\phi_I \subset A \subset X_I$,
(2)([9], Corollary 2.7) *if $A \subset B$ and $B \subset C$, then $A \subset C$,*
(3) *$A \cap B \subset A$ and $A \cap B \subset B$,*
(4) *$A \subset A \cup B$ and $B \subset A \cup B$,*
(5) *$A \subset B$ if and only if $A \cap B = A$,*
(6) *$A \subset B$ if and only if $A \cup B = B$.*

Also the followings are the immediate results of Definition 3.1 and Example 3.4.

Proposition 3.6. (See [9], Corollary 2.7) *Let $A, B, C \in IS(X)$. Then*
(1) (Idempotent laws): $A \cup A = A$, $A \cap A = A$,
(2) (Commutative laws): $A \cup B = B \cup A$, $A \cap B = B \cap A$,
(3) (Associative laws): $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$,
(4) (Distributive laws): $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,
(5) (Absorption laws): $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$,
(6) (DeMorgan's laws): $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$,
(7) $(A^c)^c = A$,
(8) (8a) $A \cup \phi_I = A$, $A \cap \phi_I = \phi_I$,
(8b) $A \cup X_I = X_I$, $A \cap X_I = A$,
(8c) $X_I^c = \phi_I$, $\phi_I^c = X_I$,
(8d) *in general,* $A \cup A^c \neq X_I$, $A \cap A^c \neq \phi_I$.

The followings are the immediate results of Definition 3.1.

Proposition 3.7. *Let $A \in IS(X)$ and let $(A_j)_{j\in J} \subset IS(X)$. Then*
(1)([9], Corollary 2.7) $(\bigcap A_j)^c = \bigcup A_j^c$, $(\bigcup A_j)^c = \bigcap A_j^c$,
(2) $A \cap (\bigcup A_j) = \bigcup(A \cap A_j)$, $A \cup (\bigcap A_j) = \bigcap(A \cup A_j)$.

Definition 3.8 ([9]). Let $f : X \to Y$ be a mapping, and let $A \in IS(X)$ and $B \in IS(Y)$. Then

(i) the image of A under f, denoted by $f(A)$, is an IS in Y defined as:

$$f(A) = (f(A)_T, f(A)_F),$$

where $f(A)_T = f(A_T)$ and $f(A)_F = (f(A^c_F))^c$.

(ii) the preimage of B, denoted by $f^{-1}(B)$, is an IS in X defined as:

$$f^{-1}(B) = (f^{-1}(B)_T, f^{-1}(B)_T),$$

where $f^{-1}(B)_T = f^{-1}(B_T)$ and $f^{-1}(B)_F = f^{-1}(B_F)$.

Result 3.9 ([9]). *Let $f : X \to Y$ be a mapping and let $A, B, C \in IS(X)$, $(A_j)_{j \in J} \subset IS(X)$ and $D, E, F \in IS(Y)$, $(D_k)_{k \in K} \subset IS(Y)$. Then the followings hold:*

(1) *if $B \subset C$, then $f(B) \subset f(C)$ and if $E \subset F$, then $f^{-1}(E) \subset f^{-1}(F)$.*

(2) *$A \subset f^{-1}f(A))$ and if f is injective, then $A = f^{-1}f(A))$,*

(3) *$f(f^{-1}(D)) \subset D$ and if f is surjective, then $f(f^{-1}(D)) = D$,*

(4) *$f^{-1}(\bigcup D_k) = \bigcup f^{-1}(D_k)$, $f^{-1}(\bigcap D_k) = \bigcap f^{-1}(D_k)$,*

(5) *$f(\bigcup A_j) = \bigcup f(A_j)$, $f(\bigcap A_j) \subset \bigcap f(A_j)$,*

(6) *$f(A) = \phi_I$ if and only if $A = \phi_I$ and hence $f(\phi_I) = \phi_I$, in particular if f is surjective, then $f(X_I) = Y_I$,*

(7) *$f^{-1}(Y_I) = Y_I$, $f^{-1}(\phi_I) = \phi_I$.*

Definition 3.10 ([9]). Let X be a non-empty set and let $p \in X$ be fixed. Then

(i) the intuitionistic set $(\{p\}, \{p\}^c)$ is called an intuitionistic point (in short IP) in X and denoted by p_I,

(ii) the intuitionistic set $(\phi, \{p\}^c)$ is called an intuitionistic vanishing point (in short IVP) in X and denoted by p_{IV}.

We will denoet the set of all IPs [resp. IVPs] in X as $IP(X)$ [resp. $IVP(X)$].

Definition 3.11 ([9]). Let X be a non-empty set, let $p \in X$ and let $A \in IS(X)$. Then

(i) p_I is said to be contained in X, denoted by $p_I \in A$, if $p \in A_T$,

(ii) p_{IV} is said to be contained in X, denoted by $p_{IV} \in A$, if $p \notin A_T$.

Result 3.12 ([9], Proposition 3.5). *Let $A, B \in IS(X)$. Then*

(1) *$A \subset B$ if and only if $p_I \in B$, for each $p_I \in A$ if and only if $p_{IV} \in B$, for each $p_{IV} \in A$,*

(2) *$A = B$ if and only if $p_I \in A \Longleftrightarrow p_I \in B$, for each $p_I \in IP(X)$*
$p_{IV} \in A \Longleftrightarrow p_{IV} \in B$, for each $p_{IV} \in IVP(X)$.

Result 3.13 ([9], Proposition 3.4). *Let $(A_j)_{j \in J} \subset IS(X)$ and let $p \in X$.*

(1) *$p_I \in \bigcap A_j$ if and only if $p_I \in A_j$, for each $j \in J$.*

(1$'$) *$p_{IV} \in \bigcap A_j$ if and only if $p_{IV} \in A_j$, for each $j \in J$.*

(2) *$p_I \in \bigcup A_j$ if and only if there exists $j \in J$ such that $p_I \in A_j$.*

(2$'$) *$p_{IV} \in \bigcup A_j$ if and only if there exists $j \in J$ such that $p_{IV} \in A_j$.*

From Definitions 3.3 and 3.10, it follows that $[\ ]A = \bigcup_{p_I \in A} p_I$ but in general, $A \neq \bigcup_{p_I \in A} p_I$, for each $A \in IS(X)$.

Example 3.14. In Example 3.2, consider the IS $A = (\{a, c, f\}, \{b, d\})$ in X. Then clearly, $a_I, c_I, f_I \in A$. Thus

$$a_I \cup c_I \cup f_I = (\{a\} \cup \{c\} \cup \{f\}, \{b, c, d, e, f\} \cap \{a, b, d, e, f\} \cap \{a, b, c, d, e\}) = (\{a, c, f\}, \{b, c, d, e\}).$$

So $[\ ]A = \bigcup_{p_I \in A} p_I$ but $A \neq \bigcup_{p_I \in A} p_I$.

Result 3.15 ([9], Proposition 3.6)**.** *Let $A \in IS(X)$. Then $A = A_I \cup A_{IV}$, where $A_I = \bigcup_{p_I \in A} p_I$ and $A_{IV} = \bigcup_{p_{IV} \in A} p_{IV}$.*

In fact, $A_I = [\ ]A$ and $A_{IV} = (\phi, A_F)$.

Definition 3.16 ([9])**.** Let $f : X \to Y$ be a mapping and let $p \in X$. Then

(i) the image of p_I under f, denoted by $f(p_I)$, is an IP in Y defined as follows:

$$f(p_I) = (\{q\}, \{q\}^c) = q_I,$$

where $q = f(p)$,

(ii) the image of p_{IV} under f, denoted by $f(p_{IV})$, is an IVP in Y defined as follows:

$$f(p_{IV}) = (\phi, \{q\}^c) = q_{IV},$$

where $q = f(p)$.

Definition 3.17 ([7])**.** Let X, Y be non-empty sets and let $A \in IS(X)$, $B \in IS(Y)$. Then the Cartesian product of A and B, denoted by $A \times B$, is an IS in $X \times Y$ defined as:

$$A \times B = ((A \times B)_T, (A \times B)_F),$$

where $(A \times B)_T = A_T \times B_T$ and $(A \times B)_F = (A_F^c \times B_F^c)^c$.

Definition 3.18 ([7])**.** Let X, Y be non-empty sets, let $(p, q) \in X \times Y$ and let $A \in IS(X)$, $B \in IS(Y)$. Then

(i) $(p, q)_I \in A \times B$, if $(p, q) \in (A \times B)_T = A_T \times B_T$,

(ii) $(p, q)_{IV} \in A \times B$, if $(p, q) \notin (A \times B)_F = (A_F^c \times B_F^c)^c$, i.e., $(p, q) \in A_F^c \times B_F^c$.

4. Properties of **ISet**

Definition 4.1. A pair (X, A) is called an intuitionistic space (in short, ISp), if $A \in IS(X)$.

Definition 4.2. Let (X, A_X), (Y, A_Y) be two ISps and let $f : X \to Y$ be a mapping. Then $f : (X, A_X) \to (Y, A_Y)$ is called a morphism, if $A_X \subset f^{-1}(A_Y)$, equivalently,

$$p_I \in f^{-1}(A_Y), \forall p_I \in A_X, \text{ i.e.,}$$

$$p \in f^{-1}(A_{Y,T}) = f^{-1}(A_Y)_T, \forall p \in A_{X,T}.$$

In particular, $f : (X, A_X) \to (Y, A_Y)$ is called an epimorphism [resp., a monomorphism and an isomorphism], if it is surjective [resp., injective and bijective].

The following is an immediate result of Definitions 4.2.

Proposition 4.3. *For each ISp (X, A_X), the identity mapping $id : (X, A_X) \to (X, A_X)$ is a morphism.*

Proposition 4.4. *Let (X, A_X), (Y, A_Y), (Z, A_Z) be ISps and let $f : X \to Y$, $g : Y \to Z$ be mappings. If $f : (X, A_X) \to (Y, A_Y)$ and $f : (Y, A_Y) \to (Z, A_Z)$ are morphisms, then $g \circ f : (X, A_X) \to (Z, A_Z)$ is a morphism.*

Proof. By the hypotheses, $A_X \subset f^{-1}(A_Y)$ and $A_Y \subset g^{-1}(A_Z)$. Then by Result 3.9 (1), $f^{-1}(A_Y) \subset f^{-1}(g^{-1}(A_Z)) = (g \circ f)^{-1}(A_{Z,T})$. Thus $A_{X,T} \subset (g \circ f)^{-1}(A_{Z,T})$. So $g \circ f$ is a morphism. □

From Propositions 4.3 and 4.4, we can form the concrete category **ISet** consisting of ISs and morphisms between them. Every **ISet**-morphism will be called a **ISet**-mapping.

Theorem 4.5. *The category* **ISet** *is topological over* **Set**.

Proof. Let X be any set and let $((X_j, A_j))_{j \in J}$ be any families of ISps indexed by a class J. Suppose $(f_j : X \to (X_j, A_j))_J$ is a source of ordinary mappings. We define the IS A_X of X by

$$A_X = \bigcap f_j^{-1}(A_j).$$

Then clearly, $A_{X,T} = \bigcap f_j^{-1}(A_{j,T})$, $A_{X,F} = \bigcup f_j^{-1}(A_{j,F})$ and $A_{X,T} \cap A_{X,F} = \phi$. Thus (X, A_X) is an ISp. Furthermore, $A_X \subset f_j^{-1}(A_j)$, for each $j \in J$. So each $f_j : (X, A_X) \to (X_j, A_j)$ is an **ISet**-mapping.

Now let (Y, A_Y) be any ISp and suppose $g : Y \to X$ is an ordinary mapping, for which $f_j \circ g : (Y, A_Y) \to (X_j, A_j)$ is a **ISet**-mapping for each $j \in J$. Then by Definition 4.2, $A_Y \subset (f_j \circ g)^{-1}(A_j) = g^{-1}(f_j^{-1}(A_j))$, for each $j \in J$. Thus by Result 3.9 (4),

$$A_Y \subset g^{-1}(\bigcap f_j^{-1}(A_j)) = g^{-1}(A_X).$$

So $A_Y \subset g^{-1}(A_X)$. Hence $g : (Y, A_Y) \to (X, A_X)$ is an **ISet**-mapping. Therefore $(f_j : (X, A_X) \to (X_j, A_j)_J$ is an initial source in **ISet**. This completes the proof. □

Example 4.6. (1) Let X be a set, let (Y, A_Y) be an ISp and let $f : X \to Y$ be an ordinary mapping. Then clearly, there exists a unique IS A_X of X for which $f : (X, A_X) \to (Y, A_Y)$ is an **ISet**-mapping. In fact, $A_X = f^{-1}(A_Y)$.

In this case, A_X is called the inverse image under f of the ICS structure A_Y.

(2) Let $((X_j, A_j))_{j \in J}$ be any family of ISps and let $X = \Pi_{j \in J} X_j$. For each $j \in J$, let $pr_j : X \to X_j$ be the ordinary projection. Then there exists a unique IS A_X in X for which $pr_j : (X, A_X \to (X_j, A_j)$ is an **ISet**-mapping for each $j \in J$.

In this case, A_X is called the product of $(A_j)_{j \in J}$, denoted by $A_X = \Pi A_j$ and $(\Pi X_j, \Pi A_j)$ is called the product ISp of $((X_j, A_j))_{j \in J}$.

In fact, $A_X = \bigcap_{j \in J} pr_j^{-1}(A_j)$.

In particular, if $J = \{1, 2\}$, then $A_1 \times A_2 = (A_{1,T} \times A_{2,T}, A_{1,F} \times A_{2,F})$, where $A_1 = (A_{1,T}, A_{1,F}) \in ICS(X_1)$ and $A_2 = (A_{2,T}, A_{2,F}) \in ICS(X_2)$.

The following is obvious from Result 2.2. But we show directly it.

Corollary 4.7. *The category* **ISet** *is cotopological over* **Set**.

Proof. Let X be any set and let $((X_j, A_j))_J$ be any family of ISps indexed by a class J. Suppose $(f_j : X_j \to X)_J$ is a sink of ordinary mappings. We define A_X as

$$A_X = \bigcup f_j(A_j).$$

Then clearly, $A_{X,T} = \bigcup f_j(A_{j,T})$, $A_{X,F} = \bigcap f_j(A_{j,F})$ and $A_{X,T} \bigcap A_{X,F} = \phi$. Thus $A_X \in IS(X)$ and each $f_j : (X_j, A_j) \to (X, A_X)$ is an **ISet**-mapping.

Now for each ISp (Y, A_Y), let $g : X \to Y$ be an ordinary mapping for which each $g \circ f_j : (X_j, A_j) \to (Y, A_Y)$ is an **ISet**-mapping. Then clearly for each $j \in J$,

$$A_j \subset (g \circ f_j)^{-1}(A_Y) = f_j^{-1}(g^{-1}(A_Y)).$$

Thus $\bigcup A_j \subset \bigcup f_j^{-1}(g^{-1}(A_Y))$. So $f_j(\bigcup A_j) \subset f_j(\bigcup f_j^{-1}(g^{-1}(A_Y)))$. By Result 3.9 (5) and the definition of A_X,

$$f_j(\bigcup A_j) = \bigcup f_j(A_j) = A_X$$

and

$$f_j(\bigcup f_j^{-1}(g^{-1}(A_Y))) = \bigcup (f_j \circ f_j^{-1})(g^{-1}(A_Y)) = g^{-1}(A_Y).$$

Hence $A_X \subset g^{-1}(A_Y)$. Therefore $g : (X, A_X) \to (Y, A_Y)$ is an **ISet**-mapping.
This completes the proof. □

Theorem 4.8. *Final episinks in* **ISet** *are preserved by pullbacks.*

Proof. Let $(g_j : (X_j, A_j) \to (Y, A_Y))_J$ be any final episink in **ISet** and let $f : (W, A_W) \to (Y, A_Y)$ be any **ISet**-mapping. For each $j \in J$, let

$$U_j = \{(w, x_j) \in W \times X_j : f(w) = g_j(x_j)\}.$$

For each $j \in J$, we define the IS $A_{U_j} = (A_{U_{j,T}}, A_{U_{j,F}})$ of U_j by:

$$A_{U_{j,T}} = A_{W,T} \times A_{j,T},\ A_{U_{j,F}} = (A^c_{W,F} \times A^c_{j,F})^c.$$

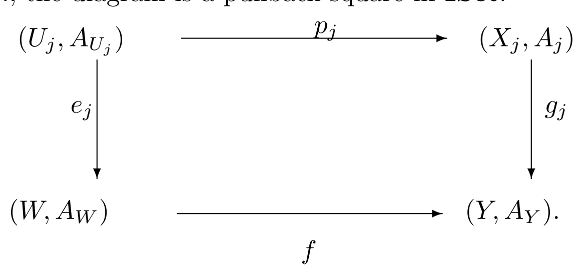
For each $j \in J$, let $e_j : U_j \to W$ and $p_j : U_j \to X_j$ be ordinary projections of U_j. Then clearly,

$$A_{U_{j,T}} \subset e_j^{-1}(A_{W,T}),\ A_{U_{j,F}} \supset e_j^{-1}(A_{W,F})$$

and

$$A_{U_{j,T}} \subset p_j^{-1}(A_{j,T}),\ A_{U_{j,F}} \supset p_j^{-1}(A_{j,F}).$$

Thus $A_{U_j} \subset e_j^{-1}(A_W)$ and $A_{U_j} \subset p_j^{-1}(A_j)$. So $e_j : (U_j, A_{U_j}) \to (W, A_W)$ and $p_j : (U_j, A_{U_j}) \to (X_j, A_j)$ are **ISet**-mappings. Moreover, $g_h \circ p_h = f \circ e_j$, for each $j \in J$, i.e., the diagram is a pullback square in **ISet**:



Now in order to prove that $(e_j)_J$ is an episink in **ISet**, i.e., each e_j is surjective, let $w_I \in W$. Since $(g_j)_J$ is an episink, there exists $j \in J$ such that $g_j(x_j) = f(w)$ for some $x_{j,I} \in X_j$. Thus $(w, x_j)_I \in U_j$ and $w_I = e_j(w, x_j)_I$. So $(e_j)_J$ is an episink in **ISet**.

Finally, let us show that $(e_j)_J$ is final in **ISet**. Let A^*_W be the final structure in W w.r.t. $(e_j)_J$ and let $w_I \in A_W$. Since $f : (W, A_W) \to (Y, A_Y)$ is an **ISet**-mapping, $w \in A_{W,T} \cap f^{-1}(A_{Y,T})$. Then $w \in A_{W,T}$ and $f(w) \in A_{Y,T}$. Since $(g_j)_J$ is final,

$$w \in A_{W,T},\ x_j \in \bigcup_J \bigcup_{x'_j \in g_j^{-1}(f(w))} A_{j,T}.$$

So $(w, x_j) \in A_{U_{j,T}}$. Since A^*_W is the final structure in W w.r.t. $(e_j)_J$, $w_I \in A^*_W$, i.e., $A_W \subset A^*_W$. On the other hand, since $(e_j : (U_j, A_{U_j}) \to (W, A_W))_J$ is final, $1_W : (W, A^*_W) \to (W, A_W)$ is an **ISet**-mapping and thus $A^*_W \subset A_W$. Hence $A^*_W = A_W$. Therefore $(e_j)_J$ is final. This completes the proof. □

For any singleton set $\{a\}$, $(\{a\}, \phi) = \{a\}_I$ and $(\phi, \{a\}) = \phi_I$ are ISs of $\{a\}$. Then IS of $\{a\}$ is not unique. Thus by Definition 2.6, Corollary 4.7 and Theorem 4.8, we have the following result.

Theorem 4.9. *The category* **ISet** *satisfies all the conditions of a topological universe over* **Set** *except the terminal separator property.*

Theorem 4.10. *The category* **ISet** *is Cartesian closed over* **Set**.

Proof. It is clear that **ISet** has products by Theorem 4.5. Then it is sufficient to see that **ISet** has exponential objects.

For any ISps $\mathbf{X} = (X, A_X)$ and $\mathbf{Y} = (Y, A_Y)$, let Y^X be the set of all ordinary mappings from X to Y. We define the IS $A_{Y^X} = (A_{Y^X,T}, A_{Y^X,F})$ in Y^X by: for each $f \in Y^X$ and each $x \in X$,

$$f_I \in A_{Y^X} \text{ if and only if } f(x_I) \in A_Y, \text{ i.e., } f \in A_{Y^X,T} \text{ if and only if } f(x) \in A_{Y,T}.$$

In fact,

$$A_{Y^X,T} = \{f \in Y^X : f(x) \in A_{Y,T} \text{ for each } x \in X\}.$$

Furthermore, $A_{Y^X,T} \cap A_{Y^X,F} = \phi$. Then clearly, (Y^X, A_{Y^X}) is an ISp.

Let $\mathbf{Y^X} = (Y^X, A_{Y^X})$ and let $f_I \in IP(Y^X)$. Then by the definition of A_{Y^X},

$$A_{Y^X,T} \subset f^{-1}(A_{Y,T}) \text{ and } A_{Y^X,F} \supset f^{-1}(A_{Y,F}).$$

We define $e_{X,Y} : X \times Y^X \to Y$ by $e_{X,Y}(x, f) = f(x)$, for each $(x, f) \in X \times Y^X$. Let $(x, f)_I \in A_X \times A_{Y^X}$. Then by Definition 3.18 and the definition of $e_{X,Y}$,

$$(x, f) \in A_{X,T} \times A_{Y^X,T} \text{ and } e_{X,Y}(x, f)_I = f(x_I).$$

Thus by the definition of A_{Y^X},

$$(x, f) \in f^{-1}(A_{Y,T}) \times f^{-1}(A_{Y,T}).$$

So $(x, f) \in e^{-1}_{X,Y}(A_{Y,T})$. Hence $A_X \times A_{Y^X} \subset e^{-1}_{X,Y}(A_Y)$. Therefore $e_{X,Y} : \mathbf{X} \times \mathbf{Y^X} \to \mathbf{Y}$ is an **ISet**-mapping.

For any $\mathbf{Z} = (Z, A_Z) \in$ **ISet**, let $h : \mathbf{X} \times \mathbf{Z} \to \mathbf{Y}$ be an **ISet**-mapping. We define $\bar{h} : Z \to Y^X$ by for each $z \in Z$ and each $x \in X$,

$$[\bar{h}(z)](x) = h(x, z).$$

Let $(x, z)_I \in A_X \times A_Z$. Since $h : \mathbf{X} \times \mathbf{Z} \to \mathbf{Y}$ is an **ISet**-mapping,

$$A_X \times A_Z \subset h^{-1}(A_Y).$$

Then by Definitions 3.18 and 4.2, $(x, z) \in h^{-1}(A_{Y,T})$. Thus $h((x, z)) \in A_{Y,T}$. By the definition of $\bar{h}$, $[\bar{h}(z)](x) \in A_{Y,T}$. So by the definition of A_{Y^X},

$$[\bar{h}(z_1)](A_{Z,T}) \subset A_{Y^X,T}.$$

Hence $A_Z \subset \bar{h}^{-1}(A_{Y^X})$. Therefore $\bar{h} : \mathbf{Z} \to \mathbf{Y}^{\mathbf{X}}$ is an **ISet**-mapping. Furthermore, $\bar{h}$ is the unique **ISet**-mapping such that $e_{X,Y} \circ (1_X \times \bar{h}) = h$. This completes the proof. □

5. The relationships between **ISet** and **NCSet**

Definition 5.1 ([18]). The concrete category **NCSet** is defined as:

(i) an object is (X, A) called a neutrosophic crisp space, where X is any non-empty set and $A \in NCS(X)$,

(ii) a morphism $f : (X, A_X) \to (Y, A_Y)$ is a mapping satisfying $A_X \subset f^{-1}(A_Y)$, equivalently,

$$A_{X,1} \subset f^{-1}(A_{Y,1}),\ A_{X,2} \subset f^{-1}(A_{Y,2}) \text{ and } A_{X,3} \supset f^{-1}(A_{Y,3}),$$

where $A_X = (A_{X,1}, A_{X,2}, A_{X,3})$ and $A_Y = (A_{Y,1}, A_{Y,2}, A_{Y,3})$.

In this case, the morphism f is called a **NCSet**-mapping.

We will define newly a neutrosophic crisp space as following.

Definition 5.2. (X, A) is called a *neutrosophic crisp space, if $A \in NCS(X)$ such that

$$A_1 \subset A_3^c \subset A_2.$$

In this case, we will denote the set of all *neutrosophic crisp spaces as $^*NCS(X)$.

Remark 5.3. We can easily see that the class $^*NCS(X)$ and **NCSet**-mappings forms a concrete category (will be denoted by ***NCSet**). Furthermore, we can easily prove that the category ***NCSet** satisfies the all properties corresponding to **NCSet** (See Section 4 in [18]). In this case, a morphism in ***NCSet** will be called a ***NCSet**-mapping.

Lemma 5.4. *Define* $G : {}^*\mathbf{NCSet} \to \mathbf{ISet}$ *as follows:*

$$G(X, (A_1, A_2, A_3)) = (X, (A_1, A_3)) \text{ and } G(f) = f.$$

for each $(X, (A_1, A_2, A_3)) \in {}^*\mathbf{NCSet}$. *Then* G *is a functor.*

Proof. Since $(X, (A_1, A_2, A_3) \in {}^*\mathbf{NCSet}$, $A_1 \subset A_3^c \subset A_2$. Then $A_1 \cap A_3^c = \phi$. Thus $G(X, (T, I, F)) = (X, (T, F)) \in \mathbf{ISet}$, for each $(X, (T, I, F)) \in {}^*\mathbf{NCSet}$.

Let $(X, (A_{X,1}, A_{X,2}, A_{X,3})), (Y, (A_{Y,1}, A_{Y,2}, A_{Y,3})) \in \mathbf{NCSet}^*$ and let $f : (X, (A_{X,1}, A_{X,2}, A_{X,3})) \to (Y, (A_{Y,1}, A_{Y,2}, A_{Y,3}))$ be an ***NCSet**-mapping. Then $A_{X,1} \subset f^{-1}(A_{Y,1}), A_{X,2} \subset f^{-1}(A_{Y,2}), A_{X,3} \supset f^{-1}(A_{Y,3})$. Thus $G(f) = f$ is an **ISet**-mapping. So $G : {}^*\mathbf{NCSet} \to \mathbf{ISet}$ is a functor. □

Lemma 5.5. *Define* $F : \mathbf{ISet} \to {}^*\mathbf{NCSet}$ *by: for each* $(X, (T, F)) \in \mathbf{ISet}$,

$$F(X, (T, F)) = (X, (T, F^c, F)) \text{ and } F(f) = f.$$

Then F *is a functor.*

Proof. Let $(X, (T, F)) \in \mathbf{ISet}$. Then $T \cap F = \phi$. Thus $T \subset F^c$. So $F(X, (T, F)) = (X, (T, F^c, F)) \in {}^*\mathbf{NCSet}$.

Let $(X,(A_{X,T},A_{X,F})),(Y,(A_{Y,T},A_{Y,F}))\in$ **ISet** and let $f:(X,(A_{X,T},A_{X,F}))\to(Y,(A_{Y,T},A_{Y,F}))$ be an **ISet**-mapping. Consider the mapping

$$F(f)=f:F(X,(A_{X,T},A_{X,F}))\to F(Y,(A_{Y,T},A_{Y,F})),$$

where

$$F(X,(A_{X,T},A_{X,F}))=(X,(A_{X,T},A^c_{X,F},A_{X,F}))$$

and

$$F(Y,(A_{Y,T},A_{Y,F}))=(Y,(A_{Y,T},A^c_{Y,F},A_{Y,F})).$$

Since $f:(X,(A_{X,T},A_{X,F}))\to(Y,(A_{Y,T},A_{Y,F}))$ is an **ISet**-mapping,

$$A_{X,T}\subset f^{-1}(A_{Y,T}) \text{ and } A_{X,F}\supset f^{-1}(A_{Y,F}).$$

Then $A^c_{X,F}\subset f^{-1}(A^c_{Y,F})$. Thus
$F(f)=f:(X,(A_{X,T},A^c_{X,F},A_{X,F}))\to(Y,(A_{Y,T},A^c_{Y,F},A_{Y,F}))$ is an ***NCSet**-mapping. Hence F is a functor. □

Theorem 5.6. *The functor* F : **ISet** $\to$ ***NCSet** *is a left adjoint of the functor* G : ***NCSet** $\to$ **ISet**.

Proof. For each $(X,(T,F))\in$ **ISet**, $1_X:(X,(T,F))\to GF(X,(T,F))=(X,(T,F))$ is an **ISet**-mapping. Let $(Y,(A_{Y,1},A_{Y,2},A_{Y,3}))\in$ ***NCSet** and let $f:(X,(T,F))\to G(A_{Y,1},A_{Y,2},A_{Y,3}))=(Y,(A_{Y,1},A_{Y,3}))$ be an **ISet**-mapping. We will show that $f:F(X,(T,F))=(X,(T,F^c,F))\to(Y,(A_{Y,1},A_{Y,2},A_{Y,3}))$ is an ***NCSet**-mapping. Since $f:(X,(T,F))\to(Y,(A_{Y,1},A_{Y,3}))$ is an **ISet**-mapping,

$$T\subset f^{-1}(A_{Y,1}) \text{ and } F\supset f^{-1}(A_{Y,3}).$$

Since $(Y,(A_{Y,1},A_{Y,2},A_{Y,3}))\in$ ***NCSet**, $A_{Y,1}\subset A^c_{Y,3}\subset A_{Y,2}$. Since $F\supset f^{-1}(A_{Y,3})$, $F^c\subset f^{-1}(A^c_{Y,3})$. Then $F^c\subset f^{-1}(A_{Y,2})$. Thus $f:F(X,(T,F))=(X,(T,F^c,F))\to(Y,(A_{Y,1},A_{Y,2},A_{Y,3}))$ is an ***NCSet**-mapping. Hence 1_X is a G-universal mapping for $(X,(T,F))\in$ **ISet**. This completes the proof. □

For each $(X,(T,F))\in$ **ISet**, $F(X,(T,F))=(X,(T,F^c,F))$ is called a neutrosophic crisp space induced by $(X,(T,F))$. Let us denote the category of all induced neutrosophic crisp spaces and ***NCSet**-mappings as ***NCSet**$_*$. Then clearly, ***NCSet**$_*$ is a full subcategory of ***NCSet**.

Theorem 5.7. *Two categories* **ISet** *and* ***NCSet**$_*$ *are isomorphism.*

Proof. From Lemma 5.6, it is clear that F : **ISet** $\to$ ***NCSet** is a functor. Consider the restriction G : ***NCSet**$_*$ $\to$ **ISet** of the functor G in Lemma 5.4. Let $(X,(T,F))\in$ **ISet**. Then by Lemma 5.6, $F_1(X,(T,F))=(X,(T,F^c,F))$. Thus $GF(X,(T,F))=G_1(X,(T,F^c,F))=(X,(T,F))$. So $G\circ F=1_{\mathbf{ISet}}$.

Now let $(X,(A_{X,1},A_{X,2},A_{X,3}))\in$ ***NCSet**$_*$. Then by definition of ***NCSet**$_*$, there exists $(X,(T,F^c,F))$ such that

$$F(X,(T,F))=(X,(T,F^c,F))=(X,(A_{X,1},A_{X,2},A_{X,3})).$$

Thus by Lemma 5.4,

$$\begin{aligned}G(X,(A_{X,1},A_{X,2},A_{X,3}))&=G(X,(T,F^c,F))\\&=(X,(T,F)).\end{aligned}$$

So

$$FG(X,(A_{X,1},A_{X,2},A_{X,3})) = F(X,(T,F)) \\ = (X,(A_{X,1},A_{X,2},A_{X,3})).$$

Hence $F \circ G = 1_{^*\mathbf{NCSet}_*}$. Therefore $F : \mathbf{ISet} \to {}^*\mathbf{NCSet}_*$ is an isomorphism. This completes the proof. □

6. Conclusions

By forming the category **ISet** consisting of intuitionistic crisp sets and morphisms between them, we prove that final episinks in **ISet** are preserved by pullbacks(See Theorem 4.8) and the category **ISet** is Cartesian closed over **Set** (See Theorem 4.10). Furthermore, we show that two categories **ISet** and $^*\mathbf{NCSet}_*$ are isomorphism (See Theorem 5.7). In the future, we think that the category **ISet** can be studied in another view-point.

Acknowledgements. Text of the acknowledgements, if any

References

[1] K. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia (September, 1983) (in Bugaria).
[2] K. Atanassov and S. Stoeva, Intuitionistic fuzzy sets, Polish Symp. on Interval and Fuzzy Mathematics, Poznan (August, 1983), Proceeding: 23–26.
[3] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[4] K. Atanassov, Review and new results on intuitionistic fuzzy sets, Preprint IM-MFaIS-1-88, Sofia 1–8.
[5] K. Atanassov, Intuitionistic Fuzzy Sets, Springer, Heideberg 1999.
[6] K. Atanassov, On Intuitionistic Fuzzy Sets Theory, Springer, Berlin 1999.
[7] Sadik Bayhan and D. Coker, Pairwise separation axioms in intuitionistic topological spaces, Hacettepe Journal of Mathematics and Statistics 34 S (2005) 101–114.
[8] J. C. Carrega, The category $Set(H)$ and $Fzz(H)$, Fuzzy sets and systems 9 (1983) 327–332.
[9] D. Coker A note on intuiyionistic sets and intuitionistic points, Tr. J. of Mathematics 20 (1996) 343–351.
[10] D. Coker An introduction to intuitionistic topological spaces, VUSEFAL 81 (2000) 51–56.
[11] E. J. Dubuc, Concrete quasitopoi Applications of Sheaves, Proc. Dunham1977, Lect. Notes in Math. 753 (1979) 239–254.
[12] M. Eytan, Fuzzy sets:a topological point of view, Fuzzy sets and systems 5 (1981) 47–67.
[13] J. A. Goguen, Categories of V-sets, Bull. Amer. Math. Soc. 75 (1969) 622–624.
[14] C. Gunduz and B. Davvaz, The universal coefficient theorem in the category of intuitionistic fuzzy modules, Utilitas Mathematica 81 (2010) 131–156.
[15] H. Herrlich, Catesian closed topological categories, Math. Coll. Univ. Cape Town 9 (1974) 1–16.
[16] K. Hur, A Note on the category $Set(H)$, Honam Math. J. 10 (1988) 89–94.
[17] K. Hur, H. W. Kang and J. H. Ryou, Intutionistic H-fuzzy sets, J. Korea Soc. Math. Edu. Ser. B:Pure Appl. Math. 12 (1) (2005) 33–45.
[18] K. Hur, P. K. Lim, J. G. Lee, J. Kim, The category of neutrosophic crisp sets, Ann. Fuzzy Math. Inform. 14 (1) (2017) 43–54.
[19] C. Y. Kim, S. S. Hong, Y. H. Hong and P. H. Park, Algebras in Cartesian closed topological categories, Lecture Note Series Vol. 1 1985.
[20] S. J. Lee and J. M. Chu, Categorical properties of intuitinistic topological spaces, Commun. Korean Math. Soc. 24 (4) (2009) 595–603.

[21] P. K. Lim, S. R. Kim and K. Hur, The category $VSet(H)$, International Journal of Fuzzy Logic and Intelligent Systems 10 (1) (2010) 73–81.
[22] L. D. Nel, Topological universes and smooth Gelfand Naimark duality, mathematical applications of category theory, Prc. A. M. S. Spec. Sessopn Denver,1983, Contemporary Mathematics 30 (1984) 224–276.
[23] A. M. Pittes, Fuzzy sets do not form a topos, Fuzzy sets and Systems 8 (1982) 338–358.
[24] D. Ponasse, Some remarks on the category $Fuz(H)$ of M. Eytan, Fuzzy sets and Systems 9 (1983) 199–204.
[25] D. Ponasse, Categorical studies of fuzzy sets, Fuzzy sets and Systems 28 (1988) 235–244.
[26] A. A. Salama and F. Smarandache, Neutrosophic Crisp Set Theory, The Educational Publisher Columbus, Ohio 2015.
[27] A. A. Salama, Mohamed Abdelfattah and S. A. Alblowi, Some Intuitionistic Topological Notions of Intuitionistic Region, Possible Application to GIS Topological Rules, International Journal of Enhanced Research in Management and Computer Applications 3 (5) (2014) 4–9.
[28] F. Smarandache, Neutrosophy Neutrisophic Property, Sets, and Logic, Amer Res Press, Rehoboth, USA 1998.
[29] W. Yang and X. Li, Category of generalized intuitionistic fuzzy sets, Chinese Quarterly Journal of Mathematics 20 (2) (2005) 192–199.
[30] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

J. Kim (`junhikim@wku.ac.kr`)
Department of Mathematics Education, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea

P. K. Lim (`pklim@wku.ac.kr`)
Division of Mathematics and Informational Statistics, Institute of Basic Natural Science, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea

J. G. Lee (`jukolee@wku.ac.kr`)
Division of Mathematics and Informational Statistics, Institute of Basic Natural Science, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea

K. Hur (`kulhur@wku.ac.kr`)
Division of Mathematics and Informational Statistics, Institute of Basic Natural Science, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea