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A study on intuitionistic fuzzy graphs of second type

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Abstract. In this paper, we define the concept of neighborhood of a vertex, order, size of a graph and the regular intuitionistic fuzzy graphs of second type. Also establish some of their properties and apllications.

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Keywords: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Sets of Second Type, Intuitionistic Fuzzy Graphs, Intuitionistic Fuzzy Graphs of Second Type, Neighborhood, Order, Size, Regular.

1. Introduction

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical(crisp)sets. Further the fuzzy sets are generalised by Krassimir. T. Atanassov [2] in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets [IFS] and their extensions like Intuitionistic Fuzzy sets of second type [IFSST], Intuitionistic L-Fuzzy sets [ILFS], Temporal Intuitionistic Fuzzy sets [TIFS] and also he introduced the concept of intuitionistic fuzzy relations. P. Bhattacharya [3] has discussed some properties of fuzzy graphs. R. Parvathi and M. G. Karunambigai [6, 7] introduced IFG elaborately and analyzed their components also established some of their operations. A. Nagoor Gani and S. Shajitha Begum [4, 5] studied IFG and introduced the concept of neighborhood degree of intuitionistic fuzzy graphs and the regular intuitionistic fuzzy graphs also order and size of IFG. Muhammad Akram and Rabia Akmal [1] studied the operations on Intuitionistic Fuzzy Graph Structures. The present authors [8, 9] introduced the extension of Intuitionistic Fuzzy Graphs namely Intuitionistic Fuzzy Graphs of Second Type [IFGST] and defined the concept of complete intuitionistic fuzzy graphs of second type. In section 2, we give some basic definitions and in section 3, we define the concept of neighborhood degree, order, size of IFGST and the regular intuitionistic fuzzy graphs of second type. Also establish some of their properties. In section 4 we propose the applications of IFG and their extensions.The paper is concluded in section 5 .

2. Preliminaries

In this section, we give some basic definitions.

Definition 2.1 ([2])**.** An intuitionistic fuzzy set [IFS] A in a universal set E is defined as an object of the form,

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

where $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in E$ respectively, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2 ([2])**.** An intuitionistic fuzzy sets of second type [IFSST] A in a universal set E is defined as an object of the form,

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

where $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in E$ respectively, satisfying $0 \leq \mu_A(x)^2 + \nu_A(x)^2 \leq 1$.

Definition 2.3 ([6])**.** An intuitionistic fuzzy graph [IFG] is of the form $G = [V, E]$, where (i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \longrightarrow [0,1]$ and $\nu_1 : V \longrightarrow [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, $(i = 1, 2, ...n)$,
(ii) $E \subseteq V \times V$,
where $\mu_2 : V \times V \longrightarrow [0,1]$ and $\nu_2 : V \times V \longrightarrow [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq min[\mu_1(v_i), \mu_1(v_j)],$$

$$\nu_2(v_i, v_j) \leq max[\nu_1(v_i), \nu_1(v_j)]$$

and

$$0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1,$$

for every $(v_i, v_j) \in E$, $(i, j = 1, 2, ...n)$.

Definition 2.4 ([4])**.** Let $G = [V, E]$ be an IFG. The neighbourhood of any vertex v is defined as:

$$N(v) = (N_\mu(v), N_\nu(v)),$$

where

$$N_\mu(v) = \{w \in V : \mu_2(v, w) = \mu_1(v) \wedge \mu_1(w)\}$$

and

$$N_\nu(v) = \{w \in V : \nu_2(v, w) = \nu_1(v) \vee \nu_1(w)\}.$$

Definition 2.5 ([4])**.** The neighbourhood degree of a vertex is defined as:

$$d_N(v) = (d_{N\mu}(v), d_{N\nu}(v)),$$

where $d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N\nu}(v) = \sum_{w \in N(v)} \nu_1(w)$.

Definition 2.6 ([4])**.** The minimum neighbourhood degree is defined as:

$$\delta_N(G) = (\delta_{N\mu}(G), \delta_{N\nu}(G)),$$

where $\delta_{N\mu}(G) = \wedge\{d_{N_\mu}(v) : v \in V\}$ and $\delta_{N\nu}(G) = \wedge\{d_{N_\nu}(v) : v \in V\}$.

Definition 2.7 ([4])**.** The maximum neighbourhood degree is defined as:

$$\Delta_N(G) = (\Delta_{N\mu}(G), \Delta_{N\nu}(G)),$$

where $\Delta_{N\mu}(G) = \vee\{d_{N_\mu}(v) : v \in V\}$ and $\Delta_{N\nu}(G) = \vee\{d_{N_\nu}(v) : v \in V\}$.

Definition 2.8 ([4])**.** The closed neighbourhood degree of a vertex is defined as:

$$d_N[v] = (d_{N\mu}[v], d_{N\nu}[v]),$$

where $d_{N\mu}[v] = \sum\limits_{w \in N(v)} \mu_1(w) + \mu_1(v)$ and $d_{N\nu}[v] = \sum\limits_{w \in N(v)} \nu_1(w) + \nu_1(v)$.

Definition 2.9 ([4])**.** The closed minimum neighbourhood degree is defined as:

$$\delta_N[G] = (\delta_{N\mu}[G], \delta_{N\nu}[G]),$$

where $\delta_{N\mu}[G] = \wedge\{d_{N_\mu}[v] : v \in V\}$ and $\delta_{N\nu}[G] = \wedge\{d_{N_\nu}[v] : v \in V\}$.

Definition 2.10 ([4])**.** The closed maximum neighbourhood degree is defined as:

$$\Delta_N[G] = (\Delta_{N\mu}[G], \Delta_{N\nu}[G]),$$

where $\Delta_{N\mu}[G] = \vee\{d_{N_\mu}[v] : v \in V\}$ and $\Delta_{N\nu}[G] = \vee\{d_{N_\nu}[v] : v \in V\}$.

Definition 2.11 ([4])**.** Let $G = [V, E]$ be an IFG. Then the order of G is defined as:

$$O(G) = (O_\mu(G), O_\nu(G)),$$

where $O_\mu(G) = \sum_{v \in V} \mu_1(v)$ and $O_\nu(G) = \sum_{v \in V} \nu_1(v)$.

Definition 2.12 ([4])**.** Let $G = [V, E]$ be an IFG. Then the size of G is defined as:

$$S(G) = (S_\mu(G), S_\nu(G)),$$

where $S_\mu(G) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $S_\nu(G) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$.

Definition 2.13 ([4])**.** An Intuitionistic fuzzy graph $G = [V, E]$ is said to be regular, if all the vertices have the same closed neighborhood degree, i.e.,

$$\delta_{N\mu}[G] = \Delta_{N\mu}[G] \text{ and } \delta_{N\nu}[G] = \Delta_{N\nu}[G].$$

Definition 2.14 ([9])**.** An intuitionistic fuzzy graphs of second type [IFGST] is of the form $G = [V, E]$,
where (i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \longrightarrow [0, 1]$ and $\nu_1 : V \longrightarrow [0, 1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i)^2 + \nu_1(v_i)^2 \leq 1$, for every $v_i \in V$, $(i = 1, 2, ...n)$,
(ii) $E \subseteq V \times V$,
where $\mu_2 : V \times V \longrightarrow [0, 1]$ and $\nu_2 : V \times V \longrightarrow [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq min[\mu_1(v_i)^2, \mu_1(v_j)^2],$$

$$\nu_2(v_i, v_j) \leq max[\nu_1(v_i)^2, \nu_1(v_j)^2]$$

and

$$0 \leq \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \leq 1,$$

for every $(v_i, v_j) \in E$, $(i, j = 1, 2, ...n)$.

Definition 2.15 ([8]). An IFGST, $G = [V, E]$ is called the complete IFGST, if for every $v_i, v_j \in V$, $\mu_{2ij} = min(\mu_{1i}^2, \mu_{1j}^2)$ and $\nu_{2ij} = max(\nu_{1i}^2, \nu_{1j}^2)$.

3. Regular intuitionistic fuzzy graphs of second type

In this section, we define the concept of neighborhood degree, order, size of IFGST and define the regular intuitionistic fuzzy graphs of second type. Also establish some of their properties.

Definition 3.1. Let $G = [V, E]$ be an IFGST then the neighborhood of a vertex $v \in V$ is defined by:

$$N(v) = (N\mu(v), N\nu(v)),$$

where $N\mu(v) = \{w \in V : \mu_2(v, w) = \min(\mu_1^2(v), \mu_1^2(w))\}$
and

$$N\nu(v) = \{w \in V : \nu_2(v, w) = \max(\nu_1^2(v), \nu_1^2(w))\}.$$

Definition 3.2. Let $G = [V, E]$ be an IFGST then the neighborhood degree of a vertex $v \in V$ is defined by:

$$d_N(v) = (d_{N\mu}(v), d_{N\nu}(v)),$$

where $d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N\nu}(v) = \sum_{w \in N(v)} \nu_1(w)$.

Definition 3.3. Let $G = [V, E]$ be an IFGST then the minimum neighborhood degree of G is defined by:

$$\delta_N(G) = (\delta_{N\mu}(G), \delta_{N\nu}(G)),$$

where $\delta_{N\mu}(G) = \min\{d_{N\mu}(v) : v \in V\}$ and $\delta_{N\nu}(G) = \min\{d_{N\nu}(v) : v \in V\}$.

Definition 3.4. Let $G = [V, E]$ be an IFGST then the maximum neighborhood degree of G is defined by:

$$\Delta_N(G) = (\Delta_{N\mu}(G), \Delta_{N\nu}(G)),$$

where $\Delta_{N\mu}(G) = \max\{d_{N\mu}(v) : v \in V\}$ and $\Delta_{N\nu}(G) = \max\{d_{N\nu}(v) : v \in V\}$.

Definition 3.5. Let $G = [V, E]$ be an IFGST then the closed neighborhood degree of a vertex $v \in V$ is defined by:

$$d_N[v] = (d_{N\mu}[v], d_{N\nu}[v]),$$

where $d_{N\mu}[v] = \sum_{w \in N(v)} \mu_1(w) + \mu_1(v)$ and $d_{N\nu}[v] = \sum_{w \in N(v)} \nu_1(w) + \nu_1(v)$.

Definition 3.6. Let $G = [V, E]$ be an IFGST then the minimum closed neighborhood degree of G is defined by:

$$\delta_N[G] = (\delta_{N\mu}[G], \delta_{N\nu}[G]),$$

where $\delta_{N\mu}[G] = \min\{d_{N\mu}[v] : v \in V\}$ and $\delta_{N\nu}[G] = \min\{d_{N\nu}[v] : v \in V\}$.

Definition 3.7. Let $G = [V, E]$ be an IFGST then the maximum closed neighborhood degree G is defined by:

$$\Delta_N[G] = (\Delta_{N\mu}[G], \Delta_{N\nu}[G]),$$

where $\Delta_{N\mu}[G] = \max\{d_{N\mu}[v] : v \in V\}$ and $\Delta_{N\nu}[G] = \max\{d_{N\nu}[v] : v \in V\}$.

Example 3.8.

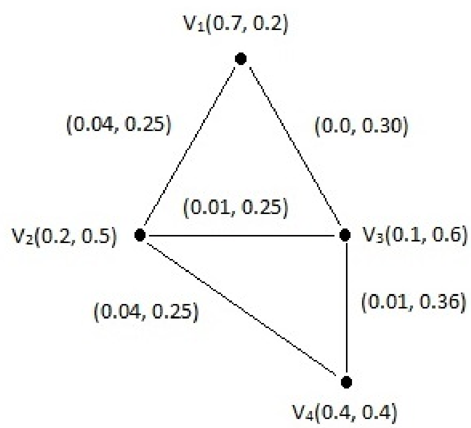

Figure 1.

In the above Figure 1, we have,

(i) the neighbourhood of the vertices are:

$$N(v_1) = \{v_2\}, N(v_2) = \{v_1, v_3, v_4\}, N(v_3) = \{v_2, v_4\}, N(v_4) = \{v_2, v_3\},$$

(ii) the neighbourhood degree of the vertices are:

$$d_N(v_1) = (0.2, 0.5), d_N(v_2) = (1.2, 1.2), d_N(v_3) = (0.6, 0.9), d_N(v_4) = (0.3, 1.1),$$

(iii) the minimum neighbourhood degree is $\delta_N(G) = (0.2, 0.5)$ and the maximum neighbourhood degree is $\Delta_N(G) = (1.2, 1.2)$,

(iv) the closed neighbourhood degree of the vertices are:

$$d_N[v_1] = (0.9, 0.7), d_N[v_2] = (1.4, 1.7), d_N[v_3] = (0.7, 1.5), d_N[v_4] = (0.7, 1.5),$$

(v) the minimum closed neighbourhood degree is $\delta_N[G] = (0.7, 0.7)$ and the maximum closed neighbourhood degree is $\Delta_N[G] = (1.4, 1.7)$.

Definition 3.9. An IFGST G is said to be regular, if all the vertices of G have the same closed neighborhood degree,i.e, $d_N[v_i] = d_N[v_j]$, for all $v_i, v_j \in V$.

If $d_{N\mu[v]} = k_1$ and $d_{N\nu[v]} = k_2$, for every $v \in V$, then G is called a (k_1, k_2) regular IFGST.

Example 3.10.

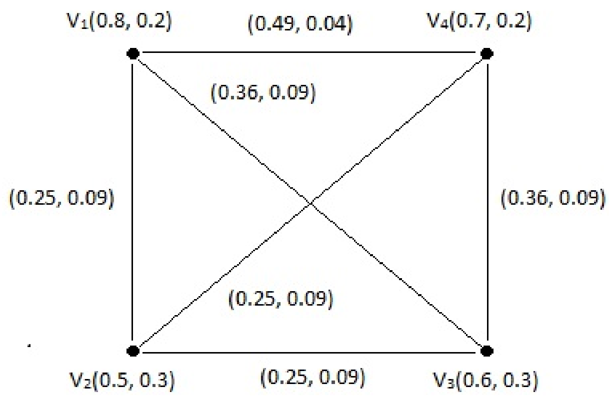

FIGURE 2. Regular IFGST

Theorem 3.11. *For every $u, v \in V$, we have*
(1) $\mu_2(u,v) = \mu_2(v,u)$,
(2) $\nu_2(u,v) = \nu_2(v,u)$.

Proof. Let $G = [V, E]$ be an IFGST. Suppose u is neighborhood of v in G. Then We have
$\mu_2(u,v) = \min(\mu_1^2(u), \mu_1^2(v))$ and $\nu_2(u,v) = \max(\nu_1^2(u), \nu_1^2(v))$,
$\mu_2(u,v) = \min(\mu_1^2(v), \mu_1^2(u))$ and $\nu_2(u,v) = \max(\nu_1^2(v), \nu_1^2(u))$.
Thus $\mu_2(u,v) = \mu_2(v,u)$ and $\nu_2(u,v) = \nu_2(v,u)$. This completes the proof. □

Theorem 3.12. *Every complete intuitionistic fuzzy graphs of second type is regular.*

Proof. Let $G = [V, E]$ be a complete IFGST. Then by the definition of complete IFGST, we have $\mu_{2ij} = \min(\mu_{1i}^2, \mu_{1j}^2)$ and $\nu_{2ij} = \max(\nu_{1i}^2, \nu_{1j}^2)$, for every $v_i, v_j \in V$. By the definition of closed neighborhood, μ degree each vertex is the sum of the membership values of the vertices and itself and the closed neighborhood ν degree each vertex is the sum of the non-membership values of the vertices and itself.
Thus all the vertices in G will have the same closed neighborhood μ degree and closed neighborhood ν degree. So minimum closed neighborhood degree is equal to maximum closed neighborhood degree, i.e, $\delta_{N\mu}[G] = \Delta_{N\mu}[G]$ and $\delta_{N\nu}[G] = \Delta_{N\nu}[G]$. Hence G is regular. This completes the proof. □

Definition 3.13. Let $G = [V, E]$ be an $IFGST$. Then the order of G is defined by:

$$O(G) = (O_\mu(G), O_\nu(G)),$$

where $O_\mu(G) = \sum_{v \in V} \mu_1(v)$ and $O_\nu(G) = \sum_{v \in V} \nu_1(v)$.

Definition 3.14. Let $G = [V, E]$ be an IFG. Then the size of G is defined by:

$$S(G) = (S_\mu(G), S_\nu(G)),$$

where $S_\mu(G) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $S_\nu(G) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$.

Example 3.15. In Figure 1. we have $O(G) = (1.4, 1.7)$ and $S(G) = (0.09, 1.16)$.

Theorem 3.16. *The order of a complete IFGST is same as the closed neighborhood degree of each vertex,i.e.,* $O_\mu(G) = (d_{N\mu}[v] : v \in V)$ *and* $O_\nu(G) = (d_{N\nu}[v] : v \in V)$.

Proof. Let $G = [V, E]$ be a complete IFGST. Then $O_\mu(G)$ is the sum of the membership value of all the vertices and the $O_\nu(G)$ is the sum of the non-membership value of all the vertices. Since G is a complete IFGST, the closed neighborhood μ-degree of each vertex is the sum of the membership value of vertices and the closed neighborhood ν-degree of each vertex is the sum of the non-membership value of vertices. This completes the proof. □

4. Applications

The newly defined IFGST has important applications in image processing, neural network, medical diagnosis, etc. We will construct the modal to represent a traffic network system by using IFG and their extensions.

5. Conclusion

In this paper, we defined the concept of neighborhood of a vertex, order, size of IFGST and the regular intuitionistic fuzzy graphs of second type. Also established some of their properties and proposed some applications of IFG and their extensions. In future we will study some more properties and applications of IFGST.

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