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Soft rough topology

M. Y. Bakier, A. A. Allam, SH. S. Abd-Allah

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Abstract. Our goal of this work is to show the deviations between some properties of soft approximation spaces and the same properties of Pawlak approximation space. We also introduce and study the concepts of soft rough topology and a new class of function called soft rough continuous functions. Finally some basic properties of these concepts are explored.

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Corresponding Author: M. Y. Bakier (mybakier@yahoo.com)

1. Introduction

Rough set theory was initiated by Pawlak [6] for dealing with vagueness and granularity in information systems. This theory deals with the approximation of an arbitrary subs et of a universe by two definable or observable subsets called lower and upper approximations. The reference space in rough set theory is the approximation space whose topology is generated by the equivalence classes of R. The notion of rough topology was introduced by Lellis Thivagar et al. [4] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined rough closed sets, rough-interior, rough closure, and rough continuous functions. The concept of soft sets was first introduced by Molodtsov [5] in 1999 as a general mathematical tool for dealing with uncertain objects. In [3] soft set theory is utilized,for the first time, to generalize Pawlaks rough set model. Based on the novel granulation structures called soft approximation spaces,soft rough approximations and soft rough sets are introduced. In this study we show the deviation between some properties of soft approximation spaces and the same properties of Pawlak approximation space. The notions of soft rough topology and soft rough continuous are introduced and studied. We also investigate some related properties of these concepts.

2. Preliminaries

This section presents a review of some fundamental notions of rough sets and soft sets.

Definition 2.1 ([6]). Suppose U be a finite nonempty set called the universe and R be an equivalence relation on U. The pair (U, R) is called the approximation space. Let X be a subset of U and $R(x)$ denotes the equivalence class determined by x.

(i) The lower approximation of a subset X of U is defined as:

$$\underline{R}(X) = \cup_{x\in U} R(x) : R(x) \subseteq X.$$

(ii)The upper approximation of a subset X of U is defined as:

$$\overline{R}(X) = \cup_{x\in U} R(x) : R(x) \cap X \neq \varnothing.$$

(iii) The boundary region of X with respect to R is defined as:

$$B_R(X) = \overline{R}(X) - \underline{R}(X).$$

The set X is said to be rough with respect to R, if $\underline{R}(X) \neq \overline{R}(X)$, i.e., $B_R(X) \neq \varnothing$.

Definition 2.2 ([5]). A pair $S = (F, A)$ is called a soft set over U,where $A \subseteq E$ and F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.3 ([3]). Let $S = (F, A)$ be a soft set over U. Then the pair $P = (U, S)$ is called a soft approximation space. Based on the soft approximation space P,the following two operations are defined as:

$$\underline{R_P}(X) = \{u \in U : \exists a \in A, [u \in F(a) \subseteq X]\},$$

$$\overline{R_P}(X) = \{u \in U : \exists a \in A, [u \in F(a) \cap X \neq \varnothing]\}$$

assigning to every subset $X \subset U$ two sets $\underline{R_P}(X)$ and $\overline{R_P}(X)$,which are called the soft P-lower approximation and the soft P-upper approximation of X, respectively. In general,we refer to $\underline{R_P}(X)$ and $\overline{R_P}(X)$ as soft rough approximations of X with respect to P. If $\underline{R_P}(X) = \overline{R_P}(X)$,$X$ is said to be soft P-definable; otherwise X is called a soft P-rough set.

Definition 2.4 ([3]). Let $S = (F, A)$ be a soft set over U. If $\cup_{a\in A} F(a) = U$, then S is said to be a full soft set.

Definition 2.5 ([2]). Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \subset F_A$. Then,the collection $\tilde{\tau}_{F_B} = \{F_{A_i} \cap F_B : F_{A_i} \in \tilde{\tau}, i \in I \subseteq N\}$ is called a soft subspace topology on F_B. Thus,$(F_B, \tilde{\tau}_{F_B})$ is called a soft topological subspace of $(F_A, \tilde{\tau})$.

3. Some deviations of some properties of Pawlak approximation space and Soft approximation space

The purpose of this article is to list some of the properties of lower and upper approximations in soft set theory. Also we present a comparison between these properties and its corresponding in Pawlak rough set theory. Suppose that $S = (F, A)$ is a soft set over U,$X, Y \subseteq U$,and $P = (U, S)$ is the corresponding soft approximation space. One can verify that soft rough approximation space satisfies most of the properties of Pawlak approximation space,but different in some other as shown by the following examples.

Example 3.1. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}, E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_2, e_3, e_4\} \subseteq E$. Consider the soft approximation $P = (U, S)$,where $S = (F, A)$ is a soft set over U given by: $F(e_2) = \{h_4\}, F(e_3) = \{h_1, h_6\}, F(e_4) = \{h_1, h_2, h_5\}$. For $X = \{h_3, h_4, h_5\}$,we have $\underline{R_P}(X) = \{h_4\}$, and $\overline{R_P}(X) = \{h_1, h_2, h_4, h_5\}$. Thus $\underline{R_P}(X) \neq \overline{R_P}(X)$ and X is a soft P-rough set. Note that $X = \{h_3, h_4, h_5\} \nsubseteq \overline{R_P}(X) = \{h_1, h_2, h_4, h_5\}$ in this case. Moreover, it is easy to see that $\underline{R_P}(U) = \overline{R_P}(U) = \{h_1, h_2, h_4, h_5, h_6\} = \cup_{a\in A} F(a)$. Finally $\overline{R_P}(X) \subseteq \overline{R_P}\,\overline{R_P}(X) = \{h_1, h_2, h_4, h_5, h_6\}$.

Example 3.1 has shown that increasing,normalized and idempotent property does not hold in soft approximation space when a subset X is a soft P-rough. The following example shows that the conjugate property does not hold when $S = (F, A)$ be a full soft set over U.

Example 3.2. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}, E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_2, e_3, e_4, e_5\} \subseteq E$. Consider the soft approximation $P = (U, S)$,where $S = (F, A)$ is a soft set over U given by:$F(e_2) = \{h_4, h_7\}, F(e_3) = \{h_7, h_8\}, F(e_4) = \{h_1, h_5, h_6\}, F(e_5) = \{h_2, h_3, h_4, h_5\}$. For $X = \{h_4, h_7, h_8\}$,we have $\underline{R_P}(X) = \{h_4, h_7, h_8\}$,and $\overline{R_P}(X) = \{h_2, h_3, h_4, h_5, h_7, h_8\}$. Hence $-\underline{R_P}(X) = \{h_1, h_2, h_3, h_5, h_6\}$ and $-\overline{R_P}(X) = \{h_1, h_6\}$. In addition,we have that $\underline{R_P}(-X) = \{h_1, h_5, h_6\}$,and $\overline{R_P}(-X) = \{h_1, h_2, h_3, h_4, h_5, h_6\}$. Easily one can see $-\underline{R_P}(X) \subset \overline{R_P}(-X)$ and $-\overline{R_P}(X) \subset \underline{R_P}(-X)$.

The following table indicates some deviations between Pawlak space and soft space on some properties of lower and upper approximations.

Pawlak space	Soft space
$X \subseteq \overline{R_P}(X)$.	$X \nsubseteq \overline{R}(X)$.
$\underline{R}(U) = \overline{R}(U) = U$.	$\underline{R_P}(U) = \overline{R_P}(U) = \cup_{a\in A} F(a)$.
$\underline{R}(X) = \overline{R}\underline{R}(X)$.	$\underline{R_P}(X) \subseteq \overline{R_P}\underline{R_P}(X)$.
$\overline{R}(X) = \overline{R}\,\overline{R}(X)$.	$\overline{R_P}(X) \subseteq \overline{R_P}\,\overline{R_P}(X)$.
$-\underline{R}(X) = \overline{R}(-X)$	$-\underline{R_P}(X) \subset \overline{R_P}(-X)$
$-\overline{R}(X) = \underline{R}(-X)$.	$-\overline{R_P}(X) \subset \underline{R_P}(-X)$
	when $S = (F, A)$ be a full soft set over U.

*Table*1

4. Soft rough topology

Modern topology depends strongly on the ideas of set theory. This section is attempt to introduce and study the concepts of soft rough topology and some of it's properties.

Definition 4.1. Let U be the universe,$P = (U, S)$ be a soft approximation space and $\tau_{SR}(X) = \{U, \varnothing, \underline{R_P}(X), \overline{R_P}(X), Bnd_P(X)\}$, where $X \subseteq U$. $\tau_{SR}(X)$ satisfies the following axioms:

(i) U and $\varnothing \in \tau_{SR}(X)$,

(ii) the union of the elements of any subcollection of $\tau_{SR}(X)$ is in $\tau_{SR}(X)$,
(iii) The intersection of the elements of any finite subcollection of$\tau_{SR}(X)$ is in $\tau_{SR}(X)$.

$\tau_{SR}(X)$ forms a topology on U called as the soft rough topology on U with respect to X. $(U, \tau_{SR}(X), E)$ is called a soft rough topological space.

Example 4.2. Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Consider the soft approximation $P = (U, S)$,where $S = (F, A)$ is a soft set over U given by: $F(e_1) = \{x_2, x_3\}, F(e_2) = \{x_1, x_4, x_5\}, F(e_3) = \{x_6\}, F(e_4) = \{x_7, x_8\}$. For $X = \{x_2, x_4, x_6\}$,we have $\underline{R_P}(X) = \{x_6\}, \overline{R_P}(X) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$,and $Bnd_P(X) = \{x_1, x_2, x_3, x_4, x_5\}$. Then

$$\tau_{SR}(X) = \{U, \varnothing, \{x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5\}\}$$

is a soft rough topology.

Proposition 4.3. *If $\tau_{SR}(X)$ is a soft rough topology on U with respect to X, then the set $\beta = \{U, \underline{R_P}(X), Bnd_P(X)\}$ is a base for $\tau_{SR}(X)$.*

Proof. (i) $\cup_{A \in \beta} A = U$.
(ii) Let U and $\underline{R_P}(X) \in \beta$ and $W = \underline{R_P}(X)$. Then $\forall x \in U \cap \underline{R_P}(X), \exists W \subset U \cap \underline{R_P}(X), x \in W$. Similarly for $U, Bnd_P(X) \in \beta$ and $\underline{R_P}(X), Bnd_P(X) \in \beta$. Thus β is a base for $\tau_{SR}(X)$. □

Definition 4.4. Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space with respect to X,where $X \subseteq U$ and $A \subseteq U$. The soft rough interior of A is defined as the union of all soft rough-open subsets of A and it is denoted by $SR - Int(A)$. The soft rough closure of A is defined as the intersection of all soft rough closed subsets containing A and it is denoted by $SR - Cl(A)$.

Definition 4.5. Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR'}(Y), E)$ be two soft rough topological spaces with respect to X and Y respectively. $\tau_{SR'}(Y)$ is finer than $\tau_{SR}(X)$, if $\tau_{SR'}(Y) \supseteq \tau_{SR}(X)$.

Theorem 4.6. *Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR'}(Y), E)$ be two soft rough topological spaces,and β , $\beta_{R'}$ be soft rough bases for τ_{SR} and $\tau_{SR'}$,respectively. If $\beta_{R'} \subseteq \beta$,then τ_{SR} is finer than $\tau_{SR'}$.*

Proof. Let $\beta_{R'} \subseteq \beta$. Then, for each $A \in \tau_{SR'}$ and $C \in \beta_{R'}$, $A = \cup_{C \in \beta_{R'}} C = \cup_{C \in \beta} C$. Thus, $\beta \in \tau_{SR}$. So $\tau_{SR'} \subseteq \tau_{SR}$. □

Definition 4.7. Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space and $A \subseteq B \subseteq U$. Then, the collection $\tau_{SR_A} = \{B_i \cap A : B_i \in \tau_{SR}, i \in I \subseteq N\}$ is called a soft rough subspace topology on A. Then, (A, τ_{SR_A}) is called a soft rough topological subspace of (B, τ_{SR}).

Theorem 4.8. *Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space and $A \subseteq B$. Then τ_{SR_A} is a soft rough topology.*

Proof. Indeed, it contains $\varnothing$ and A, because $\varnothing \cap A = \varnothing$ and $B \cap A = A$, where $\varnothing, B \in \tau_{SR}$. Since $\tau_{SR} = \{B_i : B_i \subseteq B, i \in I\}$, it is closed under finite soft intersections and arbitrary soft unions:

$$\bigcap_{i\in I}^{n}(B_i \cap A) = (\bigcap_{i\in I}^{n} B_i) \cap A,$$
$$\bigcup_{i\in I}(B_i \cap A) = (\bigcup_{i\in I} B_i) \cap A.$$

□

5. Continuity of soft rough topological space

Definition 5.1. Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR'}(Y), E)$ be two soft rough topological spaces. The mapping $f : (U, \tau_{SR}(X), E) \to (V, \tau_{SR'}(Y), E)$ is called soft rough continuous on U, if the inverse image of each soft rough-open set in V is soft rough open in U.

Example 5.2. Let $U = \{p_1, p_2, p_3, p_4\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_3, e_4\} \subset E$ and $S = \{(e_1, \{p_1, p_4\}), (e_3, \{p_2\}), (e_4, \{p_3\})\}$. If $X \subset U$ such that $X = \{p_3, p_4\}$, then we have $\overline{R_P}(X) = \{p_1, p_3, p_4\}$, $\underline{R_P}(X) = \{p_3\}$ and $Bnd_P(X) = \{p_1, p_4\}$. Thus $\tau_{SR}(X) = \{U, \varnothing, \{p_1, p_3, p_4\}, \{p_3\}, \{p_1, p_4\}\}$ is a soft rough topology.

Let $V = \{s_1, s_2, s_3, s_4\}$, $S' = \{(e_1, \{s_1\}), (e_3, \{s_2, s_3\}), (e_4, \{s_4\})\}$ and $Y = \{s_3, s_4\} \subset V$. Then we have $\overline{\acute{R}_P}(Y) = \{s_2, s_3, s_4\}$, $\underline{\acute{R}_P}(Y) = \{s_4\}$ and $Bnd_{P_{\acute{R}}}(Y) = \{s_2, s_3\}$. Thus $\tau_{SR'}(Y) = \{V, \varnothing, \{s_2, s_3, s_4\}, \{s_2, s_3\}, \{s_4\}\}$ is a soft rough topology.

Define $f : U \to V$ such that $f(p_1) = s_2$, $f(p_2) = s_1$, $f(p_3) = s_4$,and $f(p_4) = s_3$. Then $f^{-1}(\{s_2, s_3, s_4\}) = \{p_1, p_3, p_4\}$, $f^{-1}(\{s_2, s_3\}) = \{p_1, p_4\}\}$ and $f^{-1}(\{s_4\}) = \{p_3\}$. Thus f is soft rough continuous, since the inverse image for each soft rough-open set in V is soft rough -open in U.

Theorem 5.3. *A function $f : (U, \tau_{SR}(X), E) \to (V, \tau_{SR'}(Y), E)$ is soft rough continuous on U if and only if the inverse image of every soft rough closed set in V is soft rough closed in U.*

Proof. Let f be soft rough continuous and $F \subset V$ be soft rough closed. For a soft open rough set $(V - F)$, we have $f^{-1}(V - F) = U - f^{-1}(F)$ is soft rough open. Thus,$f^{-1}(F)$ is soft rough closed in U.

Conversely, Let $G \subset V$ be soft rough open. Then $(V - G)$ is soft rough closed. Thus $f^{-1}(V - G) = U - f^{-1}(G)$ is a soft rough closed in U. So $f^{-1}(G)$ is a soft rough open in U. Hence, F is a soft rough continuous function. □

Theorem 5.4. *A function $f : (U, \tau_{SR}(X), E) \to (V, \tau_{SR'}(Y), E)$ is soft rough continuous on U if and only if $f^{-1}(SR' - Int(B)) \subset SR - Int(f^{-1}(B))$ for every subset B of V.*

Proof. Let f be soft rough continuous and $B \subset V$. Then $SR - Int(B)$ is soft rough-open in U. But

$$f^{-1}(B) \subset f^{-1}(SR' - Int(B)) = SR - Int[f^{-1}(SR' - Int(B))].$$

Thus, $f^{-1}(SR' - Int(B)) \subset SR - Int(f^{-1}(B))$.

Conversely, let $f^{-1}(SR' - Int(B)) \subset SR - Int(f^{-1}(B))$, for every subset B of V. If B is soft rough-open in V, then $SR-Int(B) = B$. Thus $f^{-1}(SR' - Int(B)) \subset SR-Int(f^{-1}(B))$. That is, $f^{-1}(B) \subset SR-Int(f^{-1}(B))$. But $f^{-1}(SR'-Int(B)) \subset f^{-1}(B)$. So $f^{-1}(B) = SR - Int(f^{-1}(B))$. Hence $f^{-1}(B)$ is soft rough-open in U, for every soft rough-open set B in V. Therefore, f is soft rough continuous. □

Example 5.5. Let $U = \{p_1, p_2, p_3, p_4\}$, $E = \{e_1, e_2, e_3, e_4\}, A = \{e_2, e_3, e_4\} \subset E$ and $S = \{(e_2, \{p_1\}), (e_3, \{p_2, p_4\}), (e_4, \{p_3\})\}$. For $X = \{p_2, p_3\} \subset U$. Then we have $\tau_{SR}(X) = \{U, \varnothing, \{p_2, p_3, p_4\}, \{p_2, p_4\}, \{p_3\}\}$ is a soft rough topology and $(\tau_{SR})^c(X) = \{U, \varnothing, \{p_1\}, \{p_1, p_3\}, \{p_1, p_2, p_4\}\}$.

Let $V = \{s_1, s_2, s_3, s_4\}$ and $S' = \{(e_2, \{s_1\}), (e_3, \{s_2\}), (e_4, \{s_3\})\}$. For $Y = \{s_2, s_3\} \subset V$, we have $\tau_{SR'}(Y) = \{V, \varnothing, \{s_2, s_3\}\}$ is a soft rough topology and $(\tau_{SR'})^c(Y) = \{V, \varnothing, \{s_1, s_4\}\}$.

Define $f : U \to V$ such that $f(p_1) = s_1$, $f(p_2) = s_2$, $f(p_3) = s_4$ and $f(p_4) = s_3$. Then the inverse image of every soft rough-closed set in V is soft rough -closed in U. Thus f is soft rough continuous.

Let $B = \{s_2, s_3, s_4\} \subset V$. Then we have $f^{-1}(SR' - Int(B)) = f^{-1}(\{s_2, s_3\}) = \{p_2, p_4\}$ and $SR-Int(f^{-1}(B)) = SR-Int(\{p_2, p_3, p_4\}) = \{p_2, p_3, p_4\}$. Thus $f^{-1}(SR'-Int(B)) \subset SR - Int(f^{-1}(B))$, for every subset B of V.

Theorem 5.6. *A function $f : (U, \tau_{SR}(X), E) \to (V, \tau_{SR'}(Y), E)$ is soft rough continuous on U if and only if $f(SR - Cl(A)) \subseteq SR - Cl(f(A))$, for every subset A of U.*

Proof. Suppose f be soft rough continuous and let $A \subseteq U$. Then $f(A) \subseteq V$, $SR - Cl(f(A))$ is soft rough closed in V, $f^{-1}(SR' - Cl(f(A)))$ is soft rough closed in U, $f(A) \subseteq SR - Cl(f(A))$, $A \subseteq f^{-1}(SR' - Cl(f(A)))$. Thus $f^{-1}(SR' - Cl(f(A)))$ is a soft rough closed set containing A. So $SR-Cl(A) \subseteq f^{-1}(SR'-Cl(A))$, $f(SR-Cl(A)) \subseteq SR - Cl(f(A))$.

Conversely,let $f(SR - Cl(A)) \subseteq SR - Cl(f(A))$, for every subset A of U and F is a soft rough closed set in V. Then $f^{-1}(F) \subseteq U$, $f(SR - Cl(f^{-1}(F))) \subseteq SR-Cl(f(f^{-1}(F))) \subseteq SR-Cl(F)$. That is, $SR-Cl(f^{-1}(F)) \subseteq f^{-1}(SR'-Cl(F)) = f^{-1}(F)$. Thus $SR - Cl(f^{-1}(F)) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq SR - Cl(f^{-1}(F))$. So $SR - Cl(f^{-1}(F)) = f^{-1}(F)$ and $f^{-1}(F)$ is soft rough closed set in U, for every soft rough closed F in V. Hence f is soft rough continuous. □

Remark 5.7. When f is soft rough continuous,the equality does not hold in the previous theorem. For example, let $U = \{p_1, p_2, p_3, p_4\}$, $E = \{e_1, e_1, e_3, e_4\}, B = \{e_1, e_3, e_4\} \subset E$ and $S = \{(e_1, \{p_1\}), (e_3, \{p_2, p_4\}), (e_4, \{p_3\})\}$. For $X = \{p_1, p_3, p_4\} \subset U$, we have $\tau_{SR}(X) = \{U, \varnothing, \{p_1, p_3\}, \{p_2, p_4\}\}$. Let $V = \{s_1, s_2, s_3, s_4\}$ and $S' = \{(e_1, \{s_1, s_3\}), (e_3, \{s_2\}), (e_4, \{s_4\})\}$. For $Y = \{s_1, s_2\} \subset V$, we have $\tau_{SR'}(Y) = \{V, \varnothing, \{s_2\}, \{s_1, s_3\}, \{s_1, s_2, s_3\}\}$.

Define $f : U \to V$ such that $f(p_1) = s_2$, $f(p_2) = s_1$, $f(p_3) = s_2$,and $f(p_4) = s_1$. Then $f^{-1}(V) = U, f^{-1}(\varnothing) = \varnothing, f^{-1}\{s_2\} = \{p_1, p_3\}$,
$f^{-1}\{s_1, s_3\} = \{p_2, p_4\}$ and $f^{-1}\{s_1, s_2, s_3\} = U$. Thus the inverse image of every soft rough-open set in V is soft rough -open in U. So f is soft rough continuous.

Let $A = \{p_1, p_3\} \subset U$, Then we have $f(SR' - Cl(A)) = f(\{p_1, p_3\}) = \{s_2\}$. But, $SR-Cl(f(A)) = SR-Cl(\{s_2\}) = \{s_2, s_4\}$. Thus $f(SR-Cl(A)) \neq SR-Cl(f(A))$, for every subset A of U, even though f is soft rough continuous. That is,equality does not hold in the previous theorem when f is soft rough continuous.

6. Application

In this section we apply the concept of soft rough topology in Diabetes mellitus (DM). Consider the following information table giving data about 6 patients. The rows of the table represent the attributes (the symptoms for Diabetes) and the columns represent the objects (the patients). Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ and $A = \{e_1, e_2, e_3\}$, where e_1 stands for Increased Hunger, e_2 stands for Frequent Urination, e_3 stands for Increased Thirst. Let $S = (F, A)$ be a soft set over U given by Table 2 and the soft approximation space $P = (U, S)$. For $X = \{p_1, p_2, p_3\}$,the set of patients having diabetes,we have $\tau_{SR}(X) = \{U, \varnothing, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}\}$ is a soft rough topology and its basis, $\beta = \{U, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}\}$.

If the attribute ' Increased Hunger' is removed, then we have

$$\tau_{SR-e_1}(X) = \{U, \varnothing, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}\}$$

is a soft rough topology and its basis, $\beta - e_1 = \{U, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}\} = \beta$.

If the attribute ' Frequent Urination' is removed, then we have

$$\tau_{SR-e_2}(X) = \{U, \varnothing, \{p_1, p_2, p_4, p_5, p_6\}\}$$

is a soft rough topology and its basis, $\beta - e_2 = \{U, \varnothing, \{p_1, p_2, p_4, p_5, p_6\}\} \neq \beta$.

If the attribute ' Increased Thirst' is removed, then we have

$$\tau_{SR-e_3}(X) = \{U, \varnothing, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}\}$$

is a soft rough topology and its basis, $\beta - e_3 = \{U, \{p_1, p_2, p_3\}, \{p_4, p_5, p_6\}\} = \beta$. Thus $CORE(SR) = \{e_2\}$, i.e, 'Frequent Urination' is the key attribute that has close connection to the disease diabetes.

$A \setminus U$	p_1	p_2	p_3	p_4	p_5	p_6
e_1	1	0	0	1	1	1
e_2	1	1	1	0	0	0
e_3	0	1	0	1	1	1
Diabetes	1	1	1	0	0	0

$Table 2$

7. Conclusions

In this work, we have shown that real world problems can be dealt with the soft rough topology. The concept of basis has been applied to find the deciding factors of a chronic disease 'Diabetes'. We could find that frequent urination is the only deciding symptom for diabetes.

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Author 1 (mybakier@yahoo.com) – Mathematics Department, Assiut University, Assiut, Egypt

Author 2 (allam51ahmed@yahoo.com) – Mathematics Department, Assiut University, Assiut, Egypt

Author 3 (shimaaa85@yahoo.com) – Mathematics Department, Assiut University, Assiut, Egypt