Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

$\tilde{I}$-proximity spaces based on soft sets

A. Kandil, S. A. El-Sheikh, E. Said

Received 1 November 2017; Revised 4 December 2017; Accepted 11 December 2017

Abstract. In this paper, we present a new structure of basic proximity of soft sets based on the notion of soft ideal. For $\tilde{I} = \{\tilde{\Phi}\}$, we have the basic proximity of soft sets and for other types of $\tilde{I}$ we obtain many types of basic proximity structure of soft sets. Also we redefine this structure by using soft ideals. Some results of these spaces are: if (X, E, τ) is $\tilde{I}$-soft normal space and (X, E, τ^*) is $R_0^{'}$-space, then there exists $\tilde{I}$-Lodato proximity of soft sets $\delta_{\tilde{I}}$ such that $\tau^* = \tau_{\delta_{\tilde{I}}}$. Also the soft topology generated by $\tilde{I}$-basic proximity of soft sets is finer than the soft topology generated by $R_0^{'}$-$\check{C}$ech closure operator of soft sets. Finally, for a bijective soft map $f : (X, E_1) \rightarrow (Y, E_2, \delta_{f(\tilde{I})})$, we construct the largest $\tilde{I}$-Lodato proximity of soft sets $\delta_{\tilde{I}}$ on (X, E_1) such that f is $\tilde{I}$-proximally soft continuous mapping.

2010 AMS Classification: Primary, 54A10 Secondary 54H99

Keywords: Soft set, Soft ideal, $\tilde{I}$- basic proximity of soft sets, $\tilde{I}$-Lodato proximity of soft sets, $\tilde{I}$-soft normal space.

Corresponding Author: E. Said (emansaid_30@yahoo.com)

1. Introduction

Soft set theory has been applied successfully in many complicated problems such as economics, engineering, environment, social science, medical science, etc., that we cannot successfully use classical methods to solve it because of various types of uncertainties present in these problems. There are several theories, for example, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets, theory of internal mathematics, and theory of rough sets which can be considered as a mathematical tools for dealing with uncertainties. Soft set theory is a new mathematical tool for dealing with uncertainties which is free from the difficulties of the above theories. In 1999, Molodtsov [28] initiated the theory of soft sets. In [28, 29] Molodtsov showed several applications of soft set theory such as smoothness of functions, game theory, operation research, Rieman integration, Perron integration, probability theory, measurement theory, etc. Maji et. al. [24] introduced an application of soft set theory to a decision making proplem. D. Pei et. al. [31] studied the relationships between soft sets and information system. Research on soft set theory has been accelerated [3, 5, 6, 7, 13, 23, 25]. Soft set theory has been applied in many topics like algebra, topology, etc. H. Aktas and N. Cagman [1] introduced a notion of soft group. Also E. İnan [12] introduced a new algebraic structure in proximity. Kharal et. al. [17] and Majumdar et. al. [27] introduced the notion of mappings of soft sets. several authors like Shabir et. al. [32], Hazara et. al. [8] studied the notion of soft topological spaces. Hussain et. al. [11] introduced some properties on soft topological spaces. Lashin et. al. [18] investigated rough set theory in the frame work of topological spaces. Also Aygunoglu et. al. [2] studied soft product topologies and soft compactness. B. Tany et. al. [33] introduced the topological structure of fuzzy soft sets. The notion of soft ideal was introduced by Kandil et. al. [14]. Proximity structure was studied by Efremovič, Lodato, and others [4, 19, 20, 21, 22, 30]. Hazara et. al. [9, 10] introduced the notion of proximity in soft setting for the first time and a different notion of basic proximity based on soft sets. Recently, Kandil et. al. [15, 16] introduced a new structure of proximity of soft sets and a new structure of soft proximity based on the ideal notion. In this paper, we introduce a new approach of basic proximity of soft sets based on the notion of soft ideal. For $\tilde{I} = \{\tilde{\Phi}\}$, we have the basic proximity of soft sets which is defined by Hazara et. al. [10]. Also we redefine this structure by using soft ideals. The notion of $\tilde{I}$-Lodato proximities of soft sets is also defined. The notion of $\tilde{I}$-proximally soft continuous and $\tilde{I}$-proximally soft neighbourhood are also introduced. Some results of these spaces are: if (X, E, τ) is $\tilde{I}$-soft normal space and (X, E, τ^*) is $R_0^{'}$-space, then there exist $\tilde{I}$-Lodato proximity of soft sets $\delta_{\tilde{I}}$ such that $\tau^* = \tau_{\delta_{\tilde{I}}}$. Also we study the relation between the topology generated by $\tilde{I}$-basic proximity and the topology generated by $R_0^{'}$-$\check{C}$ech closure operator in soft setting . Finally, for a bijective soft map $f : (X, E_1) \to (Y, E_2, \delta_{f(\tilde{I})})$, we establish the largest $\tilde{I}$-Lodato proximity of soft sets $\delta_{\tilde{I}}$ which makes f $\tilde{I}$-proximally soft continuous mapping.

2. Preliminaries

In this section we recall basic definitions, results and examples.

Definition 2.1 ([28])**.** Let X be a universe set, E be a set of parameters, $P(X)$ denote the power set of X and let $A \subseteq E$. A pair (F, A) is called a soft set over X, where F is a mapping defined by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A). The soft set (F, E) will be denoted by F. The family of soft sets over a universe X with the same parameters E denoted by $P(X)^E$.

Definition 2.2 ([25])**.** Let (H, A) and (K, B) be two soft sets over a common universe X. Then (H, A) is a soft subset of (K, B), denoted by $H_A \tilde{\subseteq} K_B$, if
(i) $A \subseteq B$,
(ii) $\forall e \in A$, $H(e) \subseteq K\ (e)$.

Definition 2.3 ([25]). Let (H, A) and (K, B) be two soft sets over a common universe X. Then (H, A) and (K, B) are said to be soft equal, if (H, A) is a soft subset of (K, B) and (K, B) is a soft subset of (H, A).

Definition 2.4 ([26]). The complement of a soft set (H, A) , denoted by $(H, A)^c$ is $(H, A)^c = (H^c, A)$, where $H^c : A \to P(X)$ is a mapping defined by $H^c(e) = X - H(e), \forall\ e \in A$.

Definition 2.5 ([25]). Let (H, A) be a soft set. Then (H, A) is called null soft set denoted by $\tilde{\Phi}_A$, if $\forall e \in A$, $H(e) = \phi$.

Definition 2.6 ([25]). Let (H, A) be a soft set. Then (H, A) is called absolute soft set denoted by $\tilde{A}$, if $\forall e \in A$, $H(e) = X$.

Definition 2.7 ([25]). The union of two soft sets (F, A) and (G, B) over a common universe X is the soft set $(H, C) = (F, A)\tilde{\cup}(G, B)$, where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & , if\ \ e \in A - B \\ G(e) & , if\ e \in B - A \\ F(e) \cup G(e) & , if\ e \in A \cap B. \end{cases}$$

Definition 2.8 ([25]). The intersection of two soft sets (F, A) and (G, B) over a common universe X is the soft set $(H, C) = (F, A)\tilde{\cap}(G, B)$, where $C = A \cap B$ and $H(e) = F(e) \cap G(e)\ \forall e \in C$.

Definition 2.9 ([8]). Let $\tilde{f} : X \to Y$ and $\hat{f} : E_1 \to E_2$ be two mappings. Then a soft mapping $f = (\tilde{f}, \hat{f}) : P(X)^{E_1} \to P(Y)^{E_2}$ is defined as:

(i) for all $H \in P(X)^{E_1}$, the soft image of H under f, denoted by $f(H)$ is a soft set in $P(Y)^{E_2}$ such that

$$f(H) = \begin{cases} \tilde{f}(\cup_{e \in\ \hat{f}^{-1}(e')} H(e))\ \text{,if}\ \hat{f}^{-1}(e') \neq \phi \\ \phi,\ \text{if}\ \hat{f}^{-1}(e') = \phi, e' \in E_2. \end{cases}$$

(ii) for all $K \in P(Y)^{E_2}$, the soft inverse image of K under f, denoted by $f^{-1}(K)$ is a soft set in $P(X)^{E_1}$ such that

$$f^{-1}(K)(e) = \tilde{f}^{-1}(K(\hat{f}(e))), \forall e \in E_1.$$

Definition 2.10 ([33]). A non empty collection of soft sets over a universe X with a fixed set of parameters E is said to be a soft topology of soft subsets over (X, E), denoted by τ, if

(i) $\tilde{X}, \tilde{\Phi} \in \tau$,

(ii) $F_i \in \tau \Rightarrow \tilde{\cup}_i F_i \in \tau$,

(iii) $F_1, F_2 \in \tau \Rightarrow F_1 \tilde{\cap} F_2 \in \tau$.

The space (X, τ, E) is called soft topological space over X. The members of τ are called open soft sets in X.

Definition 2.11 ([11]). Let (X, τ, E) be a soft topological space. A soft set $H \in P(X)^E$ is said to be closed soft set in X, if its complement H^c is an open soft set. The family of closed soft sets is denoted by τ^c.

Definition 2.12 ([34]). A soft set $F \in P(X)^E$ is said to be soft point, provided that $\exists\ x \in X$, $e \in E$ such that $F(\alpha) = \{x\}$, if $\alpha = e$ and $F(\alpha) = \phi$, if $\alpha \neq e$, for each $\alpha \in E$. The soft point denoted by x_e.

Definition 2.13 ([34]). The soft point x_e is said to be belong to the soft set $G \in P(X)^E$, denoted by $x_e \tilde{\in} G$, if for $e \in E$, $\{x\} \subseteq G(e)$.

Definition 2.14 ([14]). A non empty collection of soft sets over a universe X with the same set of parameters E is said to be a soft ideal on X with the same set E, denoted by $\tilde{I}$, if

(i) $H \in \tilde{I}$ and $K \in \tilde{I} \Rightarrow H \tilde{\cup} K \in \tilde{I}$,
(ii) $H \in \tilde{I}$ and $K \tilde{\subseteq} H \Rightarrow K \in \tilde{I}$.

Definition 2.15 ([14]). Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal of soft sets over X with the same set of parameters E. Then

$$F^*(\tilde{I}, \tau) = \tilde{\cup}\{x_e : O_{x_e} \tilde{\cap} F \notin \tilde{I}\ \forall O_{x_e} \in \tau\}$$

is called the soft local function of F with respect to $\tilde{I}$ and τ, where O_{x_e} is τ-open soft set containing x_e.

Theorem 2.16 ([14]). *Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal of soft sets over X with the same set of parameters E. Then the operator*

$$C^* : P(X)^E \to P(X)^E$$

defined by

$$C^*(F) = F \tilde{\cup} F^*$$

satisfies Kuratowski's axioms and induces a soft topology on X with the same set of parameters E called τ^* given by

$$\tau^* = \{F \in P(X)^E : C^*(F^c) = F^c\}.$$

Definition 2.17 ([10]). A subset δ of $P(X)^E \times P(X)^E$ is said to be a basic proximity of soft sets on (X, E), if

(i) $F\delta G \Leftrightarrow G\delta F$,
(ii) $F\delta(G \tilde{\cup} H) \Leftrightarrow F\delta G$ or $F\delta H$,
(iii) $F \tilde{\cap} G \neq \tilde{\Phi} \Rightarrow F\ \delta G$,
(iv) $F\delta G\ \Rightarrow F, G \neq \tilde{\Phi}$.

Definition 2.18 ([9]). A mapping $C : P(X)^E \to P(X)^E$ is said to be a Čech closure operator of soft sets on (X, E), if it satisfies the following axioms:

(i) $C(\tilde{\Phi}) = \tilde{\Phi}$,
(ii) $F \tilde{\subseteq} C(F), \forall F \in P(X)^E$,
(iii) $C(F \tilde{\cup} G) = C(F) \tilde{\cup} C(G), \forall F, G \in P(X)^E$.

Moreover if C satisfies the additional condition $C(C(F)) = C(F), \forall F \in P(X)^E$, then C is said to be a Kuratowski closure operator of soft sets on (X, E). If C is a Čech closure operator of soft sets, then (X, E, C) is called closure space of soft sets.

Čech closure operator induces a soft topology on X with the same set of parameters E defined by

$$\tau_c = \{F \in P(X)^E : C(F^c) = F^c\},$$

where F^c is the complement of the soft set F.

Theorem 2.19 ([10])**.** *Let δ be a basic proximity of soft sets on (X, E), $F \in P(X)^E$, $e \in E$. Define*

$$C^{\delta}(F)(e) = \{x \in X : x_e \delta F\}$$

Then C^{δ} is a Čech closure operator of soft sets on (X, E).

Definition 2.20 ([10])**.** Let C be a Čech closure operator of soft sets on (X, E). Then C is said to be $R_0^{'}$, if for all $x, y \in X$ and for all $e_1, e_2 \in E$, $x \in C(y_{e_2})(e_1) \Leftrightarrow y \in C(x_{e_1})(e_2)$. The triple (X, E, C) is called $R_0^{'}$ space.

Definition 2.21 ([10])**.** A basic proximity δ of soft sets on (X, E) is said to be Lodato proximity of soft sets on (X, E), if for each $F, G \in P(X)^E$

$$F\delta G \Leftrightarrow F\delta C^{\delta}(G)$$

3. New structure of proximity of soft sets

Definition 3.1. Let $\tilde{I}$ be a soft ideal on $P(X)^E$. An $\tilde{I}$-proximity soft space $(X,\ E,\ \delta_{\tilde{I}}\ )$ is a set X, a set of parameters E and a binary relation $\delta_{\tilde{I}}$ of soft sets on (X, E) satisfying the following conditions: For all $F, G, H \in P(X)^E$,

(P_1) $F\delta_{\tilde{I}}G \Leftrightarrow G\delta_{\tilde{I}}\ F$,
(P_2) $F\delta_{\tilde{I}}\ (G\tilde{\cup}H) \Leftrightarrow F\delta_{\tilde{I}}\ G$ or $F\delta_{\tilde{I}}\ H$,
(P_3) $F\tilde{\cap}G \notin \tilde{I} \Rightarrow F\delta_{\tilde{I}}\ G$,
(P_4) $F\delta_{\tilde{I}}G\ \Rightarrow F, G \notin \tilde{I}$.

We will write $F\delta_{\tilde{I}}G$ if $(F, G) \in \delta_{\tilde{I}}$, otherwise will write $F\bar{\delta}_{\tilde{I}}G$.

Proposition 3.2. *If $\tilde{I} = \{\tilde{\Phi}\}$, then the $\tilde{I}$-basic proximity of soft sets on (X, E) is a basic proximity of soft sets* [10].

Example 3.3. Let $\tilde{I}$ be a soft ideal on $P(X)^E$ and $\delta_{\tilde{I}}$ be a binary relation on $P(X)^E$ defined as:

$$F\delta_{\tilde{I}}G \Leftrightarrow F, G \notin\ \tilde{I}$$

Then It is clear that $\delta_{\tilde{I}}$ satisfies the conditions (P_1)-(P_4). Thus $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity relation on $P(X)^E$.

Example 3.4. Let $\tilde{I}$ be a soft ideal on $P(X)^E$ and $\delta_{\tilde{I}}$ be a binary relation on $P(X)^E$ defined as:

$$F\delta_{\tilde{I}}G \Leftrightarrow F\tilde{\cap}G \notin \tilde{I}$$

Then It is obvious that $\delta_{\tilde{I}}$ satisfies the conditions (P_1)-(P_4). Thus $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity relation on $P(X)^E$

Lemma 3.5. *Let $(X, E, \delta_{\tilde{I}}\ )$ be an $\tilde{I}$-basic proximity soft space, $F\delta_{\tilde{I}}G$, $F\tilde{\subseteq}H$ and $G\tilde{\subseteq}K$. Then $H\delta_{\tilde{I}}K$.*

Proof. It follows directly from P_1 and P_2. □

Theorem 3.6. *Let $(X, E, \delta_{\tilde{I}})$ be an $\tilde{I}$-basic proximity soft space, $F \in P(X)^E$ and $e \in E$. Then the $\delta_{\tilde{I}}$ operator*

$$\delta_{\tilde{I}} : P(X)^E \to P(X)^E$$

defined by

$$F^{\delta_{\tilde{I}}}(e) = \{x \in X : x_e \delta_{\tilde{I}} F\}$$

satisfies the following conditions:

(1) $F \in \tilde{I} \Rightarrow F^{\delta_{\tilde{I}}} = \tilde{\Phi}$,
(2) $F \tilde{\subseteq} G \Rightarrow F^{\delta_{\tilde{I}}} \tilde{\subset} G^{\delta_{\tilde{I}}}$,
(3) $(F \tilde{\cup} G)^{\delta_{\tilde{I}}} = F^{\delta_{\tilde{I}}} \tilde{\cup} G^{\delta_{\tilde{I}}}$,
(4) $(F \tilde{\cap} G)^{\delta_{\tilde{I}}} \tilde{\subseteq} F^{\delta_{\tilde{I}}} \tilde{\cap} G^{\delta_{\tilde{I}}}$,
(5) $F \tilde{\nsupseteq} F^{\delta_{\tilde{I}}}$, *in general.*

Proof. (1) let $F^{\delta_{\tilde{I}}} \neq \tilde{\Phi}$. Then $\exists\ e \in E$ such that $F^{\delta_{\tilde{I}}}(e) \neq \tilde{\Phi}(e) = \phi$. Thus $\exists$ $x \in F^{\delta_{\tilde{I}}}(e)$ such that $x_e \delta_{\tilde{I}} F$. But by P_4 and the fact $F \in \tilde{I}$, we have $x_e \bar{\delta}_{\tilde{I}} F$. So, we have a contradiction. Hence $F^{\delta_{\tilde{I}}} = \tilde{\Phi}$.

(2) Let $x \in F^{\delta_{\tilde{I}}}(e)$. Then $x_e \delta_{\tilde{I}} F$. Since $F \tilde{\subseteq} G$, $x_e \delta_{\tilde{I}} G$, by Lemma 3.5. Thus $x \in G^{\delta_{\tilde{I}}}(e)$. So $F^{\delta_{\tilde{I}}} \tilde{\subseteq} G^{\delta_{\tilde{I}}}$.

(3) By part (2), $(F^{\delta_{\tilde{I}}} \tilde{\cup} G^{\delta_{\tilde{I}}}) \tilde{\subseteq} (F \tilde{\cup} G)^{\delta_{\tilde{I}}}$. To prove the other inclusion, let $x \in (F \tilde{\cup} G)^{\delta_{\tilde{I}}}(e)$. Then $x_e \delta_{\tilde{I}} (F \tilde{\cup} G)$. By ($P_2$), $x_e \delta_{\tilde{I}} F$ or $x_e \delta_{\tilde{I}} G$. Thus $x \in F^{\delta_{\tilde{I}}}(e)$ or $x \in G^{\delta_{\tilde{I}}}(e)$. So $x \in F^{\delta_{\tilde{I}}}(e) \cup G^{\delta_{\tilde{I}}}(e)$. Hence $(F \tilde{\cup} G)^{\delta_{\tilde{I}}} \tilde{\subseteq} F^{\delta_{\tilde{I}}} \tilde{\cup} G^{\delta_{\tilde{I}}}$. Therefore $(F \tilde{\cup} G)^{\delta_{\tilde{I}}} = F^{\delta_{\tilde{I}}} \tilde{\cup} G^{\delta_{\tilde{I}}}$.

(4) It is clear by part (2).

(5) Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$, $\delta_{\tilde{I}}$ be an $\tilde{I}$-basic proximity of soft sets which is defined in Example 3.4 and let $\tilde{I} = \{\tilde{\Phi}, F_1, F_2, F_3\}$, where
$F_1(e_1) = h_2, F_1(e_2) = h_2$,
$F_2(e_1) = \phi, F_2(e_2) = h_2$,
$F_3(e_1) = h_2, F_3(e_2) = \phi$.
If $F = F_1$, then $F^{\delta_{\tilde{I}}} = \tilde{\Phi}$. Thus $F \tilde{\nsupseteq} F^{\delta_{\tilde{I}}}$, in general. □

Theorem 3.7. *Let $(X, E, \delta_{\tilde{I}})$ be an $\tilde{I}$-basic proximity soft space, $F \in P(X)^E$ and $e \in E$. Then the operator*

$$C^{\delta_{\tilde{I}}} : P(X)^E \to P(X)^E$$

defined by

$$C^{\delta_{\tilde{I}}}(F)(e) = F(e) \cup F^{\delta_{\tilde{I}}}(e) \tag{2.1}$$

is R_0'-Čech closure operator of soft sets on (X, E) and induces a soft topology on X with the same set of parameters E defined by

$$\tau_{\delta_{\tilde{I}}} = \{F \in P(X)^E : C^{\delta_{\tilde{I}}}(F^c) = F^c\}.$$

Proof. (i) Since $\tilde{\Phi} \in \tilde{I}$, $\tilde{\Phi}^{\delta_{\tilde{I}}} = \tilde{\Phi}$, by Theorem 3.6 (1). Then $C^{\delta_{\tilde{I}}}(\tilde{\Phi}) = \tilde{\Phi}$.

(ii) Formula (2.1) implies that $F \tilde{\subseteq} C^{\delta_{\tilde{I}}}(F)$.

(iii) Let $F, G \in P(X)^E$ and $e \in E$. Then by Theorem 3.6 (3), $C^{\delta_{\tilde{I}}}(F \tilde{\cup} G) = C^{\delta_{\tilde{I}}}(F) \tilde{\cup} C^{\delta_{\tilde{I}}}(G)$. Thus $C^{\delta_{\tilde{I}}}$ is a Čech closure operator of soft sets on (X, E). Let

$x, y \in X$ and $e_1, e_2 \in E$. Then $x \in C^{\delta_{\tilde{I}}}(y_{e_2})(e_1) \Leftrightarrow x_{e_1}\delta_{\tilde{I}}\, y_{e_2} \Leftrightarrow y_{e_2}\delta_{\tilde{I}}\, x_{e_1} \Leftrightarrow y \in (x_{e_1})^{\delta_{\tilde{I}}}(e_2) \Leftrightarrow y \in C^{\delta_{\tilde{I}}}(x_{e_1})(e_2)$. Thus $C^{\delta_{\tilde{I}}}$ is an R_0'-Čech closure operator of soft sets on (X, E). □

Theorem 3.8. *If C is an R_0'-Čech closure operator of soft sets on (X, E), then there is an $\tilde{I}$-basic proximity $\delta_{\tilde{I}}$ of soft sets on (X, E) such that $C^{\delta_{\tilde{I}}} \subseteq C$.*

Proof. Define

$$F\delta_{\tilde{I}}G \Leftrightarrow (C(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}C(G)) \notin \tilde{I} \text{ and } F, G \notin \tilde{I}.$$

(P_1) It is clear that $F\delta_{\tilde{I}}G \Leftrightarrow G\delta_{\tilde{I}}\, F$, by the definition of $\delta_{\tilde{I}}$.

(P_2) Let $F, G, H \in P(X)^E$. Then

$F\delta_{\tilde{I}}\,(G\tilde{\cup}H) \Leftrightarrow (C(F)\tilde{\cap}(G\tilde{\cup}H))\tilde{\cup}(F\tilde{\cap}C(G\tilde{\cup}H)) \notin \tilde{I}$ and $F, G\tilde{\cup}H \notin \tilde{I}$

$\Leftrightarrow ((C(F)\tilde{\cap}G)\tilde{\cup}(C(F)\tilde{\cap}H))\tilde{\cup}((F\tilde{\cap}C(G))\tilde{\cup}(F\tilde{\cap}C(H))) \notin \tilde{I}$, $F \notin \tilde{I}$ and $G \notin \tilde{I}$ or $H \notin \tilde{I}$

$\Leftrightarrow (C(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}C(G)) \notin \tilde{I}$ and $F, G \notin \tilde{I}$ or $(C(F)\tilde{\cap}H)\tilde{\cup}(F\tilde{\cap}C(H)) \notin \tilde{I}$ and $F, H \notin \tilde{I}$

$\Leftrightarrow F\delta_{\tilde{I}}G$ or $F\delta_{\tilde{I}}\, H$.

(P_3) Let $F, G \in P(X)^E$ such that $F\tilde{\cap}G \notin \tilde{I}$. Then by Definition 2.14 (ii), $(C(F)\tilde{\cap}G)\tilde{\cup}(F\tilde{\cap}C(G)) \notin \tilde{I}$ and $F, G \notin \tilde{I}$. Thus $F\delta_{\tilde{I}}G$.

(P_4) By the definition of $\delta_{\tilde{I}}$, $F\bar{\delta}_{\tilde{I}}G \ \ \forall F \in \tilde{I}$ or $G \in \tilde{I}$. Then $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity of soft sets on (X, E). To prove that $C^{\delta_{\tilde{I}}} \subseteq C$. Let $F \in P(X)^E, e \in E$ and let $x \in C^{\delta_{\tilde{I}}}(F)(e)$. Then $x \in F(e)$ or $x \in F^{\,\delta_{\tilde{I}}}(e)$. If $x \in F(e)$, then the result holds. If $x \in F^{\delta_{\tilde{I}}}(e)$, then $x_e\delta_{\tilde{I}}\, F$ and thus $(C(x_e)\tilde{\cap}F)\tilde{\cup}(x_e\tilde{\cap}C(F)) \notin \tilde{I}$ and x_e, $F \notin \tilde{I}$. So $C(x_e)\tilde{\cap}F \notin \tilde{I}$ or $x_e\tilde{\cap}C(F) \notin \tilde{I}$. If $x_e\tilde{\cap}C(F) \notin \tilde{I}$, then $x_e\tilde{\cap}C(F) \neq \tilde{\Phi}$ and thus $x \in C(F)(e)$. If $C(x_e)\tilde{\cap}F \notin \tilde{I}$, then $C(x_e)\tilde{\cap}F \neq \tilde{\Phi}$. So $\exists\ e_1 \in E$ such that$C(x_e)(e_1) \cap F(e_1) \neq \phi$. Hence $\exists\ y \in X$ such that $y \in C(x_e)(e_1)$ and $y \in F(e_1)$. Since C is an R_0'-Čech closure operator of soft sets on (X, E), $x \in C(y_{e_1})(e)$. Since $y \in F(e_1)$, $y_{e_1}\tilde{\subseteq}F$. Then $C(y_{e_1})\tilde{\subseteq}C(F)$. Thus $x \in C(F)(e)$. So $C^{\delta_{\tilde{I}}}(F)(e) \subseteq C(F)(e)$. Hence $C^{\delta_{\tilde{I}}}(F)\tilde{\subseteq}C(F)$. □

Definition 3.9. Let $\delta_{\tilde{I}}$ be an $\tilde{I}$-basic proximity of soft sets on (X, E). Then $\delta_{\tilde{I}}$ is said to be an $\tilde{I}$-Lodato proximity of soft sets on (X, E), if for each $F, G \in P(X)^E$,

$$F\delta_{\tilde{I}}G \Leftrightarrow F\delta_{\tilde{I}}C^{\delta_{\tilde{I}}}(G).$$

Theorem 3.10. *If $\delta_{\tilde{I}}$ is an $\tilde{I}$-Lodato proximity of soft sets on (X, E), then $C^{\delta_{\tilde{I}}}$ is a Kuratowski closure operator of soft sets on (X, E).*

Proof. Let $\delta_{\tilde{I}}$ be an $\tilde{I}$-Lodato proximity of soft sets on (X, E). Then $C^{\delta_{\tilde{I}}}$ is a Čech closure operator of soft sets on (X, E). Let $F \in P(X)^E$, $e \in E$. Then by Theorem 3.7 (2), $C^{\delta_{\tilde{I}}}(F)\tilde{\subseteq}C^{\delta_{\tilde{I}}}(C^{\delta_{\tilde{I}}}(F))$. To prove the reverse inclusion , let $x \in C^{\delta_{\tilde{I}}}(C^{\delta_{\tilde{I}}}(F))(e)$. Then $x \in C^{\delta_{\tilde{I}}}(F)(e)$ or $x \in (C^{\delta_{\tilde{I}}}(F))^{\delta_{\tilde{I}}}(e)$. Thus $x \in F(e)$ or $x \in F^{\delta_{\tilde{I}}}(e)$ or $x_e\delta_{\tilde{I}}C^{\delta_{\tilde{I}}}(F)$. So $x \in F(e)$ or $x_e\delta_{\tilde{I}}F$ or $x_e\delta_{\tilde{I}}C^{\delta_{\tilde{I}}}(F)$. Since $\delta_{\tilde{I}}$ is an $\tilde{I}$-Lodato proximity of soft sets on (X, E), $x \in F(e)$ or $x_e\delta_{\tilde{I}}F$. Thus $x \in C^{\delta_{\tilde{I}}}(F)(e)$. So $C^{\delta_{\tilde{I}}}(C^{\delta_{\tilde{I}}}(F))\tilde{\subseteq}C^{\delta_{\tilde{I}}}(F)$. Hence $C^{\delta_{\tilde{I}}}(F) = C^{\delta_{\tilde{I}}}(C^{\delta_{\tilde{I}}}(F))$. □

Definition 3.11. A soft topological space(X, E, τ) is called $\tilde{I}$-soft normal space, if $\forall\ F_1,\ F_2 \in \tau^{*c}$ such that $F_1 \tilde{\cap}\ F_2 \in \tilde{I}$, $\exists\ H, G \in \tau$ such that $F_1 \tilde{\subseteq} H$, $F_2 \tilde{\subseteq} G$ and $H \tilde{\cap} G \in \tilde{I}$, where τ^{*c} is the family of all τ^*-soft closed sets.

Theorem 3.12. *Let $\tilde{I}$ be a soft ideal on (X, E), $(X, E,\ \tau)$ be an $\tilde{I}$-soft normal space and $(X, E,\ \tau^*)$ be an $R_0^{'}$-space. Then there is an $\tilde{I}$-Lodato proximity of soft sets on (X, E) such that $C^{\delta_{\tilde{I}}} = C^*$.*

Proof. Define

$$F \delta_{\tilde{I}}\, G \Leftrightarrow C^*(F) \tilde{\cap} C^*(G) \notin \tilde{I}\ .$$

(P_1) It is clear that $F \delta_{\tilde{I}}\, G \Leftrightarrow G \delta_{\tilde{I}}\, F$, by the definition of $\delta_{\tilde{I}}$.

(P_2) Let $F, G, H \in P(X)^E$. Then

$$\begin{aligned} F \delta_{\tilde{I}}\, (G \cup H) &\Leftrightarrow C^*(F) \tilde{\cap} C^*(G \tilde{\cup} H) \notin \tilde{I} \\ &\Leftrightarrow C^*(F) \tilde{\cap} (C^*(G) \tilde{\cup} C^*(H)) \notin \tilde{I} \\ &\Leftrightarrow (C^*(F) \tilde{\cap} C^*(G)) \tilde{\cup} (C^*(F) \tilde{\cap} C^*(H) \notin \tilde{I} \\ &\Leftrightarrow C^*(F) \tilde{\cap} C^*(G) \notin \tilde{I} \text{ or } C^*(F) \tilde{\cap} C^*(H) \notin \tilde{I} \\ &\Leftrightarrow F \delta_{\tilde{I}} G \text{ or } F \delta_{\tilde{I}} H. \end{aligned}$$

(P_3) Let $F, G \in P(X)^E$ such that $F \cap G \notin \tilde{I}$. Then by Definition 2.14 (ii), $C^*(F) \tilde{\cap} C^*(G) \notin \tilde{I}$. Thus $F \delta_{\tilde{I}} G$.

(P_4) By the definition of $\delta_{\tilde{I}}$, $F \bar{\delta}_{\tilde{I}} G\ \forall F \in \tilde{I}$ or $G \in \tilde{I}$. Let $F \in P(X)^E$, $e \in E$ and $x \in C^{\delta_{\tilde{I}}}(F)(e)$. Then $x \in F(e)$ or $x \in F^{\delta_{\tilde{I}}}(e)$. If $x \in F(e)$, then the result holds. If $x \in F^{\delta_{\tilde{I}}}(e)$, then $x_e\ \delta_{\tilde{I}}\ F$ and thus $C^*(x_e) \tilde{\cap} C^*(F) \notin \tilde{I}$. So $C^*(x_e) \tilde{\cap} C^*(F) \neq \tilde{\Phi}$. Then $\exists\ e_1 \in E$ such that $C^*(x_e)(e_1) \cap C^*(F)(e_1) \neq \phi$. Thus $\exists\ y \in X$ such that $y \in C^*(x_e)(e_1)$ and $y \in C^*(F)(e_1)$. Since C^* is an $R_0^{'}$ closure operator of soft sets of (X, E), $x \in C^*(y_{e_1})(e)$. Since $y \in C^*(F)(e_1)$, $y_{e_1} \tilde{\subseteq} C^*(F)$. So $C^*(y_{e_1}) \tilde{\subseteq} C^*(C^*(F))$. Hence $x \in C^*(F)(e)$, i.e., $C^{\delta_{\tilde{I}}}(F)(e) \subseteq C^*(F)(e)$. Therefore $C^{\delta_{\tilde{I}}}(F) \tilde{\subseteq} C^*(F)$.

To prove the reverse inclusion, suppose that $x \notin C^{\delta_{\tilde{I}}}(F)(e)$. Then $x \notin F(e)$ and $x \notin F^{\delta_{\tilde{I}}}(e)$. Thus $x_e \bar{\delta}_{\tilde{I}} F$. So $C^*(x_e) \tilde{\cap} C^*(F) \in \tilde{I}$, by the definition of $\delta_{\tilde{I}}$. Since $(X, E,\ \tau)$ is $\tilde{I}$-soft normal space, $\exists\ H, G \in \tau$ such that $C^*(x_e) \tilde{\subseteq} H$ and $C^*(F) \tilde{\subseteq} G$ and $H \tilde{\cap} G \in \tilde{I}$. This result and Definition 2.14 (ii) imply $\exists\ H \in \tau$ such that $x_e \tilde{\in} H$ and $H \tilde{\cap} F \in \tilde{I}$. Then $x \notin C^*(F)(e)$. Thus $C^*(F) \tilde{\subseteq} C^{\delta_{\tilde{I}}}(F)$. So $C^{\delta_{\tilde{I}}} = C^*$.

To prove that $\delta_{\tilde{I}}$ is an $\tilde{I}$-Lodato proximity of soft sets on (X, E), let $F \delta_{\tilde{I}} C^{\delta_{\tilde{I}}}(G)$. Then $C^*(F) \cap C^*(C^{\delta_{\tilde{I}}}(G) \notin \tilde{I}$. Thus $C^*(F) \cap C^*(C^*(G)) \notin \tilde{I}$. Since C^* is a kuratowski closure operator, $C^*(F) \cap C^*(G) \notin \tilde{I}$. So $F \delta_{\tilde{I}} G$. Also if $F \delta_{\tilde{I}} G$, then $F \delta_{\tilde{I}} C^{\delta_{\tilde{I}}}(G)$. □

Definition 3.13. Let $\delta_{\tilde{I}}$ be an $\tilde{I}$-basic proximity of soft sets on (X, E) and $F, G \in P(X)^E$. Then G is said to be $\delta_{\tilde{I}}$ -soft neighbourhood of F (in symbols, $F \ll_{\tilde{I}} G$), if $F \bar{\delta}_{\tilde{I}} G^c$.

The set of all $\delta_{\tilde{I}}$ - soft neighbourhood of F will be denoted by $N(\delta_{\tilde{I}}, F)$. When there is no ambiguity we will write $N_{\delta_{\tilde{I}}}(F)$ for $N(\delta_{\tilde{I}}, F)$.

Proposition 3.14. *Let $\delta_{\tilde{I}}$ be an $\tilde{I}$-basic proximity of soft sets on (X, E), $F \in P(X)^E$, $e \in E$ and $x_e \tilde{\in} F$. Then $x_e \ll_{\tilde{I}} F \Leftrightarrow F$ is a neighbourhood of x_e in the closure space $(X, E, C^{\delta_{\tilde{I}}})$.*

Proof. $x_e \ll_{\tilde{I}} F$
$\Leftrightarrow x_e \bar{\delta}_{\tilde{I}} F^c \Leftrightarrow x \notin (F^c)^{\delta_{\tilde{I}}}(e) \Leftrightarrow x \notin C^{\delta_{\tilde{I}}}(F^c)(e)$
$\Leftrightarrow x_e \tilde{\cap} C^{\delta_{\tilde{I}}}(F^c) = \tilde{\Phi}$
$\Leftrightarrow F$ is a $\delta_{\tilde{I}}$ −neighbourhood of x_e in the closure space $(X, E, C^{\delta_{\tilde{I}}})$. □

Lemma 3.15. *For each $F, G \in P(X)^E$, the following results hold:*
(1) $G \in N_{\delta_{\tilde{I}}}(F) \Rightarrow F^c \in N_{\delta_{\tilde{I}}}(G^c)$,
(2) $N_{\delta_{\tilde{I}}}(F \tilde{\cup} G) = N_{\delta_{\tilde{I}}}(F) \tilde{\cap} N_{\delta_{\tilde{I}}}(G)$.

Proof. (1) Let $F, G \in P(X)^E$ such that $G \in N_{\delta_{\tilde{I}}}(F)$. Then $F \bar{\delta}_{\tilde{I}} G^c$. Thus $G^c \bar{\delta}_{\tilde{I}} (F^c)^c$, by (P_1). So $G^c \ll_{\tilde{I}} F^c$. Hence $F^c \in N(\delta_{\tilde{I}}, G^c)$.

(2) Let $H \in N_{\delta_{\tilde{I}}}(F \tilde{\cup} G) \Leftrightarrow (F \tilde{\cup} G) \bar{\delta}_{\tilde{I}} H^c$
$\Leftrightarrow F \bar{\delta}_{\tilde{I}} H^c$ and $G \bar{\delta}_{\tilde{I}} H^c$
$\Leftrightarrow F \ll_{\tilde{I}} H$ and $G \ll_{\tilde{I}} H$
$\Leftrightarrow H \in N_{\delta_{\tilde{I}}}(F)$ and $H \in N_{\delta_{\tilde{I}}}(G)$
$\Leftrightarrow H \in N_{\delta_{\tilde{I}}}(F) \tilde{\cap} N_{\delta_{\tilde{I}}}(G)$. □

Lemma 3.16. *Let $\delta_{\tilde{I}}$ be an $\tilde{I}$-basic proximity of soft sets on (X, E). Then*

$$F^{\delta_{\tilde{I}}} = \tilde{\cap}\{G : G \in N_{\delta_{\tilde{I}}}(F)\}.$$

Proof. Let $x \in F^{\delta_{\tilde{I}}}(e)$. Then $x_e \delta_{\tilde{I}} F$. Thus $x_e \tilde{\not\supseteq} G^c$, for all $G \in N_{\delta_{\tilde{I}}}(F)$. So $x \in G(e)$, for every $G \in N_{\delta_{\tilde{I}}}(F)$. Hence $F^{\delta_{\tilde{I}}}(e) \subseteq \tilde{\cap}\{G : G \in N_{\delta_{\tilde{I}}}(F)\}(e)$.

To prove the reverse inclusion, suppose $x \notin F^{\delta_{\tilde{I}}}(e)$. Then $x_e \, \bar{\delta} F$. Thus $(x_e)^c \in N_{\delta_{\tilde{I}}}(F)$. So $\tilde{\cap}\{G : G \in N_{\delta_{\tilde{I}}}(F)\} \tilde{\subseteq} (x_e)^c$. Hence $x \notin \tilde{\cap}\{G : G \in N_{\delta_{\tilde{I}}}(F)\}(e)$. Therefore $F^{\delta_{\tilde{I}}} = \tilde{\cap}\{G : G \in N_{\delta_{\tilde{I}}}(F)\}$. □

Lemma 3.17. *Let $F, G \in P(X)^E$ such that $F \tilde{\subseteq} G$. Then*

$$N_{\delta_{\tilde{I}}}(G) \tilde{\subseteq} N_{\delta_{\tilde{I}}}(F).$$

Proof. Let $H \in N_{\delta_{\tilde{I}}}(G)$. Then $G \bar{\delta}_{\tilde{I}} H^c$. Since $F \tilde{\subseteq} G$, $F \bar{\delta}_{\tilde{I}} H^c$. Thus $H \in N_{\delta_{\tilde{I}}}(F)$. So $N_{\delta_{\tilde{I}}}(G) \tilde{\subseteq} N_{\delta_{\tilde{I}}}(F)$. □

Definition 3.18. A soft mapping $f = (\tilde{f}, \hat{f}) : (X_1, E_1, \delta_{\tilde{I}_1}) \to (X_2, E_2, \delta_{\tilde{I}_2})$ is said to be $\tilde{I}$-proximally soft continuous, if

$$F \delta_{\tilde{I}_1} G \Rightarrow f(F) \delta_{\tilde{I}_2} f(G) \ \forall F, G \in P(X)^E.$$

Theorem 3.19. *Every proximally soft continuous mapping is soft continuous.*

Proof. Let $f = (\tilde{f}, \hat{f}) : (X_1, E_1, \delta_{\tilde{I}_1}) \to (X_2, E_2, \delta_{\tilde{I}_2})$ be an $\tilde{I}$-proximally soft continuous mapping and let $F \in P(X_1)^{E_1}$, $e' \in E_2$ such that $\hat{f}^{-1}(e') \neq \phi$. Then,

$$\begin{aligned} f(F^{\delta_{\tilde{I}_1}})(e') &= \tilde{f}(\cup_{e \in \hat{f}^{-1}(e')} F^{\delta_{\tilde{I}_1}}(e)) \\ &= \tilde{f}(\cup_{e \in \hat{f}^{-1}(e')}\{x \in X_1 : x_e \ \delta_{\tilde{I}_1} F\}) \\ &\subseteq \tilde{f} \ (\cup_{e \in \hat{f}^{-1}(e')}\{x \in X_1 : f(x_e) \ \delta_{\tilde{I}_2} \ f(F)\}) \\ &= \cup_{e \in \hat{f}^{-1}(e')}\{\tilde{f}(x) \in X_1 : f(x_e) \ \delta_{\tilde{I}_2} \ f(F)\} \end{aligned}$$

$= \{\tilde{f}(x) \in X_2 : (\tilde{f}(x))_{e'}\ \delta_{\tilde{I}_2}\ f(F)\}.$

Thus $f(F^{\delta_{\tilde{I}_1}})(e') \subseteq \{\tilde{f}(x) \in X_2 : (\tilde{f}(x))_{e'}\ \delta_{\tilde{I}_2}\ f(F)\}$, i.e.,

$$f(F^{\delta_{\tilde{I}_1}})(e') \subseteq (f(F))^{\delta_{\tilde{I}_2}}(e').$$

So $f(F^{\delta_{\tilde{I}_1}})\tilde{\subseteq}(f(F))^{\delta_{\tilde{I}_2}}$. Hence

$$f(C^{\delta_{\tilde{I}_1}}(F)) = f(F\tilde{\cup}F^{\delta_{\tilde{I}_1}}) = f(F)\tilde{\cup}f(F^{\delta_{\tilde{I}_1}})\tilde{\subseteq}f(F)\tilde{\cup}(f(F))^{\delta_{\tilde{I}_2}} = C^{\delta_{\tilde{I}_2}}(f(F)).$$

Therefore f is soft continuous. □

Lemma 3.20. *Let $(X, \delta_{\tilde{I}})$ be an $\tilde{I}$-basic proximity of soft sets on (X, E), F, G and $H \in P(X)^E$ such that $F\bar{\delta}_{\tilde{I}}G$ and $G^c\tilde{\cap}H \in \tilde{I}$. Then $F\bar{\delta}_{\tilde{I}}H$.*

Proof. Since $G^c\tilde{\cap}H \in \tilde{I}$, $F\bar{\delta}_{\tilde{I}}(G^c\tilde{\cap}H)$, by ($P_4$). This result and $F\bar{\delta}_{\tilde{I}}G$ imply $F\bar{\delta}_{\tilde{I}}H$, by ($P_2$). □

Lemma 3.21. *Let $\tilde{I}$ be a soft ideal on X with the same set of parameters E and $f : P(X)^E \to P(Y)^{E'}$ be a bijective soft mapping. Then $f(\tilde{I})$ is a soft ideal on Y with the same set of parameters E', where $f(\tilde{I}) = \{f(F) : F \in \tilde{I}\}$.*

Proof. Straightforward. □

Lemma 3.22. *Let $\tilde{I}$ be a soft ideal of soft sets on (X, E) and $F \in P(X)^E$. Then*

$$\tilde{I}_F = \{H : F\tilde{\cap}H \in \tilde{I}\}$$

is a soft ideal on (X, E).

Proof. Straightforward. □

Theorem 3.23. *Let $f : (X, E_1, \delta_{\tilde{I}}) \to (Y, E_2, \delta_{f(\tilde{I})})$ be a bijective soft map such that $\delta_{f(\tilde{I})}$ is an $\tilde{I}$-lodato proximity of soft sets on (Y, E_2). Then the largest $\tilde{I}$-lodato proximity of soft sets on (X, E_1) which makes f $\tilde{I}$-proximally soft continuous defined by*

$$F\bar{\delta}_{\tilde{I}}H \Leftrightarrow \exists\ G \in P(Y)^{E_2} \text{ such that } f(F)\bar{\delta}_{f(\tilde{I})}G^c \text{ and } f(H)\tilde{\cap}G \in f(\tilde{I}).$$

Proof. We prove that $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity of soft sets on (X, E_1).

(P_1) Suppose $F\bar{\delta}_{\tilde{I}}H$. Then $\exists\ G \in P(Y)^{E_2}$ such that $f(F)\bar{\delta}_{\tilde{I}}G^c$ and $f(H)\tilde{\cap}G \in f(\tilde{I})$. Thus by Lemma 3.20, $f(F)\bar{\delta}_{\tilde{I}}f(H)$. Let $K = (f(F))^c$. Since $f(H)\bar{\delta}_{\tilde{I}}f(F)$ and $f(F)\tilde{\cap}(f(F))^c$, $H\bar{\delta}_{\tilde{I}}F$.

(P_2) Suppose $(F\tilde{\cup}H)\bar{\delta}_{\tilde{I}}G$. Then $\exists\ K \in P(Y)^{E_2}$ such that $(f(F)\tilde{\cup}f(H))\bar{\delta}_{\tilde{I}}K^c$ and $f(G)\tilde{\cap}K \in f(\tilde{I})$. Then by ($P_3$), $F\bar{\delta}_{\tilde{I}}G$ and $H\bar{\delta}_{\tilde{I}}G$.

Conversely, Suppose $F\bar{\delta}_{\tilde{I}}G$ and $H\bar{\delta}_{\tilde{I}}G$. Then $\exists\ K_1, K_2 \in P(Y)^{E_2}$ such that $f(F)\bar{\delta}_{f(\tilde{I})}K_1^c$, $f(H)\bar{\delta}_{f(\tilde{I})}K_2^c$, $f(G)\tilde{\cap}K_1 \in f(\tilde{I})$ and $f(G)\tilde{\cap}K_2 \in f(\tilde{I})$. Thus by ($P_2$) and Definition 2.14 (i) , $(f(F)\tilde{\cup}f(H))\bar{\delta}_{f(\tilde{I})}(K_1\tilde{\cup}K_2)^c$ and $f(G)\tilde{\cap}(K_1\tilde{\cup}K_2) \in f(\tilde{I})$. So $(F\tilde{\cup}H)\bar{\delta}_{\tilde{I}}G$.

(P_3) Suppose $F\bar{\delta}_{\tilde{I}}H$. Then $\exists\ G \in P(Y)^{E_2}$ such that $f(F)\bar{\delta}_{f(\tilde{I})}G^c$ and $f(H)\tilde{\cap}G \in f(\tilde{I})$. Thus by Lemma 3.20, $f(F)\bar{\delta}_{f(\tilde{I})}f(H)$. So $f(F)\tilde{\cap}f(H) \in f(\tilde{I})$, by (P_3). Since $f(F\tilde{\cap}H)\tilde{\subseteq} f(F)\tilde{\cap}f(H)$, $f(F\tilde{\cap}H) \in f(\tilde{I})$, by Definition 2.14 (ii). So $F\tilde{\cap}H \in \tilde{I}$.

(P_4) Suppose $F \in \tilde{I}$ and let $G = \tilde{\Phi} \in P(Y)^{E_2}$. Since $f(F)\bar{\delta}_{f(\ \tilde{I})}Y$ and $f(H)\tilde{\cap}G = \tilde{\Phi} \in f(\tilde{I})$, $F\bar{\delta}_{\tilde{I}}H$.

To prove that $f : (X, E_1, \delta_{\tilde{I}}) \to (Y, E_2, \delta_{f(\tilde{I})})$ is $\tilde{I}$-proximally soft continuous, suppose that $F, H \in P(X)^{E_1}$ such that $f(F)\bar{\delta}_{f(\tilde{I})}f(H)$. Since $\delta_{f(\tilde{I})}$ is an $\tilde{I}$-lodato proximity of soft sets on (X, E_2), $f(F)\bar{\delta}_{f(\tilde{I})}C^{\delta_{f(\tilde{I})}}(f(H))$ and we have

$$(C^{\delta_{f(\tilde{I})}}(f(H)))^c\tilde{\cap}f(H) = \tilde{\Phi} \in f(\tilde{I}).$$

Then $F\bar{\delta}_{\tilde{I}}H$.

Now, we prove that $\delta_{\tilde{I}}$ is the largest $\tilde{I}$-basic proximity of soft sets on (X, E_1). Let $\alpha_{\tilde{I}}$ be any $\tilde{I}$-basic proximity of soft sets on (X, E_1) such that $f : (X, E_1, \alpha_{\tilde{I}}) \to (Y, E_2, \delta_{f(\tilde{I})})$ is $\tilde{I}$-proximally soft continuous and $F\bar{\delta}_{\tilde{I}}H$. Then $\exists\ G \in P(Y)^{E_2}$ such that $f(F)\bar{\delta}_{f(\tilde{I})}G^c$ and $f(H)\tilde{\cap}G \in f(\tilde{I})$. Thus by Lemma 3.20, $f(F)\bar{\delta}_{f(\tilde{I})}f(H)$. Since f is $\tilde{I}$-proximally soft continuous, $F\bar{\alpha}H$. So the result holds.

Finally, we prove that $\delta_{\tilde{I}}$ is an $\tilde{I}$-lodato proximity of soft sets on (X, E_1). Let $F\bar{\delta}_{\tilde{I}}H$. Then $\exists\ G \in P(Y)^{E_2}$ such that $f(F)\bar{\delta}_{f(\tilde{I})}G^c$ and $f(H)\tilde{\cap}G \in f(\tilde{I})$. Thus by Lemma 3.20, $f(F)\bar{\delta}_{f(\tilde{I})}f(H)$. Since $\delta_{f(\tilde{I})}$ is an $\tilde{I}$-lodato proximity of soft sets on (X, E_2), $f(F)\bar{\delta}_{f(\tilde{I})}C^{\delta_{f(\tilde{I})}}(f(H))$ and we have

$$(C^{\delta_{f(\tilde{I})}}(f(H)))^c\tilde{\cap}C^{\delta_{f(\tilde{I})}}(f(H)) = \tilde{\Phi} \in f(\tilde{I}).$$

Since f is $\tilde{I}$-proximally soft continuous, $f(C^{\delta_{\tilde{I}}}(H))\tilde{\subseteq}C^{\delta_{f(\tilde{I})}}(f(H))$. So

$$(C^{\delta_{f(\tilde{I})}}(f(H)))^c\tilde{\cap}f(C^{\delta_{\tilde{I}}}(H)) = \tilde{\Phi} \in f(\tilde{I}).$$

Hence $F\bar{\delta}_{\tilde{I}}C^{\delta_{\tilde{I}}}(H)$. □

4. $\tilde{I}$-Basic Proximity structure of soft sets and soft ideal

Let $\tilde{I}$ be a soft ideal of soft sets on (X, E) and $\delta_{\tilde{I}}$ be a binary relation on $P(X)^E$ and $F \in P(X)^E$. Define

$$\delta_{\tilde{I}}[F] = \{G \in P(X)^E : F\bar{\delta}_{\tilde{I}}\ G\}.$$

Proposition 4.1. *If $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity of soft sets on (X, E) and $F \in P(X)^E$, then $\delta_{\tilde{I}}[F]$ is a soft ideal on (X, E).*

Proof. Since $\tilde{\Phi} \in \tilde{I}$, $F\bar{\delta}_{\tilde{I}}\ \tilde{\Phi}$. Then $\tilde{\Phi} \in \delta_{\tilde{I}}[F]$. Thus $\delta_{\tilde{I}}[F] \neq \phi$. Let $H \in \delta_{\tilde{I}}[F]$. Then $F\bar{\delta}_{\tilde{I}}\ H$. Since $G\tilde{\subseteq}H$, $F\bar{\delta}_{\tilde{I}}H$, by (P_2). Thus $G \in \delta_{\tilde{I}}[F]$. Let $H, G \in \delta_{\tilde{I}}[F]$. Then $F\bar{\delta}_{\tilde{I}}G$ and $F\bar{\delta}_{\tilde{I}}H$. By (P_2), $F\bar{\delta}_{\tilde{I}}\ (G\tilde{\cup}H)$. Thus $G\tilde{\cup}H \in \delta_{\tilde{I}}\ [F]$. So $\delta_{\tilde{I}}[F]$ is a soft ideal on (X, E). □

Definition 4.2. Let $\tilde{I}$ be a soft ideal of soft sets on (X, E). A binary relation $\delta_{\tilde{I}}$ on $P(X)^E$ is said to be an $\tilde{I}-$basic proximity of soft sets on (X, E), if $\delta_{\tilde{I}}$ satisfies the following conditions:
(i) $F \in \delta_{\tilde{I}}[H] \Rightarrow H \in \delta_{\tilde{I}}[F]$,
(ii) $F \in \delta_{\tilde{I}}[H]$ and $G \in \delta_{\tilde{I}}[H] \Leftrightarrow F \tilde{\cup} G \in \delta_{\tilde{I}}[H]$,
(iii) $F \in \delta_{\tilde{I}}[H] \Rightarrow F \cap H \in \tilde{I}$,
(iv) $F \in \delta_{\tilde{I}}[H]$ $\forall F \in \tilde{I}$, $H \in P(X)^E$.

Theorem 4.3. *A binary relation $\delta_{\tilde{I}}$ on $P(X)^E$ is an $\tilde{I}$-basic proximity of soft sets on (X, E) if and only if it satisfies the following conditions:*
(1) $F \in \delta_{\tilde{I}}[H] \Rightarrow H \in \delta_{\tilde{I}}[F]$,
(2) *$\delta_{\tilde{I}}[F]$ is a soft ideal on (X, E) $\forall F \in P(X)^E$ and $\tilde{I} \tilde{\subseteq} \delta_{\tilde{I}}[F]$,*
(3) $\delta_{\tilde{I}}[F] \subseteq \tilde{I}_F$.

Proof. Suppose that $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity of soft sets on (X, E). It is clear that (1) and (2) are satisfied. To prove (3), let $H \in \delta_{\tilde{I}}[F]$. Then $F \tilde{\cap} H \in \tilde{I}$. Thus $H \in \tilde{I}_F$. So $\delta_{\tilde{I}}[F] \subseteq \tilde{I}_F$.

Conversely, it follows directly from (1), (2) and (3) that (i), (ii), (iv) hold. To prove (iii), let $H \in \delta_{\tilde{I}}[F]$. Then by (3) $H \in \tilde{I}_F$. Therefore $H \cap F \in \tilde{I}$. Consequently $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity of soft sets on (X, E) . □

Proposition 4.4. *If $\delta_{\tilde{I}}$ is an $\tilde{I}$-basic proximity of soft sets on (X, E) and F, H and $G \in P(X)^E$ such that $F \in \delta_{\tilde{I}}[H]$ and $G \tilde{\subseteq} H$, then $F \in \delta_{\tilde{I}}[G]$.*

Proof. Let $F \in \delta_{\tilde{I}}[H]$ and $G \tilde{\subseteq} H$. Suppose that $F \notin \delta_{\tilde{I}}[G]$. Then $G \delta_{\tilde{I}} F$, but $G \tilde{\subseteq} H$, then by Lemma 3.5, $H \delta_{\tilde{I}} F$. Thus $F \notin \delta_{\tilde{I}}[H]$. Which is a contradiction. □

5. Conclusions

In this paper, we have introduced a new description of basic proximity of soft sets by using the soft ideal notion. Also we have studied the relation between the soft topology generated by $\tilde{I}$-Lodato proximity of soft sets and the soft topology generated by a Kuratowski closure operator of soft sets. Also we have studied the relation between the soft topology generated by $\tilde{I}$-basic proximity of soft sets and R_0'-Čech closure operator of soft sets. For a bijective soft map $f : (X, E_1) \rightarrow (Y, E_2, \delta_{f(\tilde{I})})$, we have obtained the largest $\tilde{I}$-Lodato proximity of soft sets $\delta_{\tilde{I}}$ on (X, E_1) such that f is $\tilde{I}$-proximally soft continuous mapping. Finally, we redefine this structure by using soft ideals.

Acknowledgements. The authors are thankful to the Editors-in-Chief for their important comments which helped to improve the presentation of the paper. The authors also thankful to the Referees for their valuable comments which helped in rewriting the paper in the present form.

References

[1] H. Aktas and N. Cagman, Soft sets and soft group, Inform. Sci. 177 (2007) 2726–2735.

[2] A. Aygünoğlu and H. Aygun, Introduction to fuzzy soft groups, Computers and Mathematics with Applications 58 (2009) 1279–1286.

[3] D. Chen, E. E. C. Tsong, D. S. Young and X. Wong, The parametrization reduction of soft sets and its applications, Comput. Math. Appl. 49 (2005) 757–763.

[4] V. A. Efremovič, The geometry of proximity, Math. Sb. 31 (1952) 189–200.

[5] F. Feng, C. X. Li, B. Davvaz and M. I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing 14 (2010) 899–911.

[6] F. Feng, X. Y. Liu, V. Leoreanu-Fotea and Y. B. Jun, Soft sets and soft rough sets, Inform. Sci. 181 (2011) 1125–1137.

[7] R. Gowi and G. Jegadeesan, On soft *Č*ech clousure spaces, International Journal of Mathematics Trends and Technology 9 (2) (2014) 122-127.

[8] H. Hazara, P. Majumdar and S. K. Samanta, Soft Topology, Fuzzy information and Engineering 3 (1) (2012) 105–115.

[9] H. Hazara, P. Majumdar and S. K. Samanta, Soft proximity, Ann. Fuzzy Math. Inform. 7 (6) (2014) 867–877.

[10] H. Hazara, P. Majumdar and S. K. Samanta, proximity of soft sets, Ann. Fuzzy Math. Inform. 8 (1) (2014) 113–123.

[11] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62 (2011) 4058–4067

[12] E. İnan, Approximately semigroups and ideals: an algebraic view of digital images, Afyon Kocatepe University Journal of Science and Engineering 17 (2017) 479–487.

[13] Y. Jiang, Y. Tang, Q. Chen, J. Wang and S. Tang, Extending soft sets with description logics, Comput. Math. Appl. 59 (2010) 2087–2096.

[14] A. Kandil, O. A. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory soft local function and generated soft topological spaces, Appl. Math. Inf. Sci. 8 (4) (2014) 1595–1603.

[15] A. Kandil, O. A. Tantawy, S. A. El-Sheikh and A. Zakaria, New strucures of proximity spaces, Information Science Letters 3 (2014) 85–89.

[16] A. Kandil, O. A. Tantawy, S. A. El-Sheikh and A. Zakaria, Soft I-proximity spaces, Ann. Fuzzy Math. Inform. 9 (2015) 675–682.

[17] A. Kharal and B. Ahmed, Mapping on soft classes, New Math. and Natural Comput. 7 (3) (2011) 471–481.

[18] E. F. Lashin, A. M. Kozae, A. A. Abo Khadra and T. Medhat, Rough set for topological spaces, Internat. J. Approx. Reason. 40 (2005) 35–43.

[19] S. Leader, Local proximity spaces, Math. Ann. 169 (1976) 275–281.

[20] M. W. Lodato, On topologically induced generalized proximity relations, Ph.D. thesis, Rutgers University, 1962.

[21] M. W. Lodato, On topologically induced generalized proximity relations I, Proc. Amer. Math. Soc. 15 (1964) 417–422.

[22] M. W. Lodato, On topologically induced generalized proximity relations II, Pacific J. Mat. 17 (1966) 131–135.

[23] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589–602.

[24] P. K. Maji, R. Biswas and A. R. Roy, An application of soft sets in a decision making proplem, Comput. Math. Appl. 44 (2002) 1077–1083.

[25] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.

[26] P. Majumdar and S. K. Samanta, Similarity measure of soft sets, New Math and Nat. Comput. 4 (1) (2008) 1–12.

[27] P. Majumdar and S. K. Samanta, On soft mapping, Computers and Math. With Appl. 60 (9) (2010) 2666–2672.

[28] D. Molodtsov, Soft set theory - first results, Comput. Math. With Appl. 37 (1999) 19–31.

[29] D. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (2006) 8–39.

[30] S. A. Naimpally and B. D. Warrack, Proximity spaces, Cambridge Tracts in Math. (1970).

[31] D. Pei and D. Miao, From soft sets to information systems, in proceeding of the IEEE International Conference on Granular Computing, 2 (2005) 617–621.
[32] M. Shabir and M. Naz, On soft topological spaces, Comput. Math with Appl. 61 (7) (2011) 1786–1799.
[33] B. Tany and M. B. Kandemir, Topological structure of fuzzy soft sets, Comput. Math with Appl. 61 (2011) 2952–2957.
[34] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3 (2012) 171–185.

Author 1 (dr.ali_kandil@yahoo.com)
Department of Mathematics, Faculty of Science, Helwan university, Egypt

Author 2 (elsheikh33@hotmail.com)
Department of Mathematics, Faculty of Education, Ain Shams university,Cairo 11341, Egypt

Author 3 (emansaid_30@yahoo.com)
Department of Mathematics, Faculty of Education, Ain Shams university,Cairo 11341, Egypt