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Measuring entropy values of QRS-complexes before and after training program of sport horses with ECG

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Abstract. In the present paper, QRS- complexes in ECG for sport horses are transformed to fuzzy sets. By means of this transformation, we calculate entropy and similarity measure values by using the amplitude and the duration of QRS- complexes of sport horses. In addition, the results before and after 5 months training program are calculated and some significant comments are given. Besides, Riesz entropy notion is introduced. Finally, Riesz entropy values of QRS- complexes are calculated for sport horses before and after 5 months training program.

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1. Introduction

The theoretical and practical applications of fuzzy sets have been extended day by day with Zadeh. Fuzzy set theory leads for handling uncertainties and vagueness in real world observations. In recent times, entropy of the fuzzy sets are seen an important concept for engineering areas, application of mathematics, theormodynamic systems and medicine. Recently, Şengönül et. al [17] and Şengönül [18] have done researches by taking consideration ECG (Electrocardiagram), fuzzy set and entropy concepts. Additionally, in [17], P and T wave entropies are investigated and the consequences of before and after training programs are compared for sport horses. Moreover, in [17] medical predictions for some cardiac problems are observed by using P and T waves of ECG.

Each year, heart attack kills million of people in the world. Unfortunately, heart disease is the leading cause of death for both human and animals. The warning signs and symptons of a heart attack is key to preventing death. Graphical range of pulse rates are demonstrated by ECG without damaging the tissues. Ranges obtained by using the height and width of ECG waves are observed and interpreted by health care professionals and veterinarians. By this way, the most appropriate method should be determined by following the diagnosis and treatment of diseases. Besides, it is known that experts spend a lot of time for interpreting ECG ranges. In addition, there are number of erroneous findings in the diagnosis phase of the disease, and some notable informations are overlooked. In this case, we can say that obtaining numerical results from these diagrams is much more reliable and simple than just regarding wave ranges. ECG test is used to measure the electrical activity in the heart without damaging the tissues. By this way, ECG is used to identify people at risk of a heart attack or other symptons, elderly. This test also used in heart checkups.

I would also like to point out that ECG is not a definitive diagnosis method of cardiac patients and does not give information about heart's pumping ability, but it helps to diagnosis with patient history and physical examination with regard to other laboratory tests.

The following data can be obtained by ECG:

- It shows the shape of the heart muscle contraction.
- Heart rhythm and conduction disturbances are determined.
- Coronary insufficiency or myocardial infarction can be diagnosed.
- Thickening of the heart muscle and enlargement of the heart cavity may be determined.
- Cardiac effects of non-cardiac diseases will be investigated.

The heart which is a pump four-paned provides the blood circulation in the body. After ventricles fill with blood in the diastolic phase, heart makes the pompole renovation job with the contraction in systole phase. The contraction of the heart muscle occurs with some electrical stimuli as being in all muscles. The stimulation of the heart muscle cell is called depolarization. Returning to rest after the stimulation is called repolarization. Resting heart muscle cells also called polarized. Polarized cell that is electrically balanced, charged positive outside of the cell and negative inside of the cell.

When the heart muscle cell is warned then it begins to be depolarized. Where the warning occurs, extracellular becomes negative, intracellular becomes positive. There is an electrical stimulation between the outer surface of warned cells and unstimulated cells. And this electrical stimulation diffuse through all the cell, until all the cells being depolarized. The transmission of alert occurs transition from cell to cell, that is, a domino effect occurs. After a while, fully warned cells become repolarized.

All the heart muscle cells are in harmony and contract after warned. In each cell, depolarization and repolarization proceed in the same direction. Spreading of the stimulus throughout atriums and ventricles and return to the resting state cause the recorded electrical currents generated in the ECG. The phases of ECG may be introduced as in the following.

1.1. **Reading ECG.** ECG consists of waves and intervals that are located upper and lower of the horizontal straight line and indicate the time when there is no electrical activity.

1.2. **ECG waves.** ECG records the electrical currents formed in the atrium and ventricular cells. There are two main events for ECG. First is depolarization:dissemination of warnings of the heart muscle, the second is repolarization:return to its normal resting state of the stimulated heart muscle. Because depolarization and repolarization occur simultaneously, this electrical current is saved in the waves. Each phase of the electrical activity of the heart creates a wave or complex as given below:

- P wave: Atrium depolarization
- QRS- complex: Ventricular depolarization
- T wave: Ventricular repolarization.

In the present study, firstly, some basic definition and notations on fuzzy sets, entropies of fuzzy sets and ECG are given. Later, entropies of QRS- complex of sport horses before and after five months training program are computed. By this way, obtained numerical values are commented and compared with regard to these training programs. We refer to source [6] for numerical values used in this study.

Let us give some background information on fuzzy sets used many various different disciplines (see, [1, 2, 3, 4, 5], [7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 23]) and entropy of the fuzzy sets.

Now, we give some necessary concepts to explain our idea. For brevity in notation, through all the text, we shall write $\sum_n$, sup_n, and $\lim_n$ instead of $\sum_{n=0}^{\infty}$, $sup_{n\in\mathbb{N}}$ and $\lim_{n\to\infty}$ where $\mathbb{N}=\{0,1,2,...\}$. Furthermore, we write $\mathbb{R}$ and $\mathbb{C}$ for the set of real and complex valued numbers, respectively. Let X be a non empty set. A fuzzy subset A of X is a set of ordered pairs:

$$A=\{(x,A(x)):x\in X\}$$

where $A(x)$ is called the membership function of $x\in A$, that function is in the form $A:X\to[0,1]$, [22].

Let us define triangular fuzzy set A on the set $\mathbb{R}$ with membership function as follows:

$$A(x)=\begin{cases}\frac{h_A}{u_1-u_0}(x-u_0), & x\in[u_0,u_1)\\ \frac{-h_A}{u_2-u_1}(x-u_1)+h_A, & x\in[u_1,u_2]\\ 0, & others,\end{cases} \tag{1.1}$$

where the notations h_A denotes height of the fuzzy set A and $u_0,u_1,u_2\in\mathbb{R}$. For brief, we write triple $(u_0,u_1:h_A,u_2)$ for fuzzy set A. Let the notation F be the set of all fuzzy sets in the form $(u_0,u_1:h_A,u_2)$ on the set of real numbers.

Definition 1.1. The support of a fuzzy set A, is the crisp set of all $x\in X$ such that membership function of A is bigger than 0 for all $x\in A$.

Because of the fact that it is necessary for the text, we can give some knowledge on sequences of fuzzy sets.

We represent by $w(F)$ the sequence of fuzzy sets as in the following:

$$w(F)=\{u=(u^k):u:\mathbb{N}\to F,(u^k)=((u_0^k,u_1^k:h_{u^k},u_2^k))\}.$$

Each subspace of $w(F)$ is called sequence space of F. Here, $u_1^k:h_{u^k}$ shows k^{th} term of the sequence $u=(u^k)$. Besides, (u^k) has the highest membership degree at u_1^k.

Assume that there are two sequences of triangular fuzzy numbers, then degree of similarity $\mathscr{S}(u^n, v^n)$ between the sequences of triangular fuzzy numbers $u = (u^n)$ and $v = (v^n)$ can be calculated as in the following,

$$\mathscr{S}(u^n, v^n) = \frac{\inf\{h_{u^n}, h_{v^n}\}}{\sup\{h_{u^n}, h_{v^n}\}} \left[1 - \frac{1}{3} \lim_n \sum_{k=0}^{2} |u_k^n - v_k^n|\right] = \lambda, \tag{1.2}$$

where $\mathscr{S}(u^n, v^n) \in [0,1]$, $\lambda \in \mathbb{R}$ and the notion of $h(u)$ shows the highest membership degree of sequence of fuzzy number u. For all $k \in \mathbb{N}$, $h(u)$ be considered as 1 because of the validity of fuzzy number conditions, through all the text. The function $\mathscr{S} : w(F) \times w(F) \to \mathbb{R}$ is called similarity degree between sequences of fuzzy sets u and v. If $\mathscr{S}(u^n, v^n) = 1$ then we say that u is completely similar to the sequence v, if $0 < \mathscr{S}(u^n, v^n) = \alpha < 1$ then we say that the sequence u is α- similar to the sequence v, if $\alpha \leq 0$ we can say that u is not similar to v, [24].

Fuzzy problems involve a major subject such as measuring the fuzziness. To detect this fuzziness, many methods have been introduced. To explain this issue in more depth, we give the definition of entropy by means of [25].

Definition 1.2. Let $A(x)$ be the membership function of the fuzzy set A for $x \in X$. If the function $e : F \to \mathbb{R}$, actualize the following conditions, then e is called entropy of the fuzzy set A:
(i) $e(A) = 0$, if A is crisp set in X,
(ii) $e(A)$ has a unique maximum, if $A(x) = \frac{1}{2}$, for all $x \in X$,
(iii) $e(B) \leq e(A)$, if B is crisper than A,
(iv) $e(A^c) = e(A)$, where A^c is the complement of A.

Let $A = A(x)$ be the membership function of the fuzzy set A and the function $h : [0,1] \to [0,1]$ satisfies the following properties:

(1) Monotonically increasing at $[0, \frac{1}{2}]$ and decreasing at $[\frac{1}{2}, 1]$,
(2) $h(x) = 0$ if $x = 0$ and $h(x) = 1$ if $x = \frac{1}{2}$.

Then the function h is called entropy function and equality $H(A(x)) = h(A(x))$ holds for all $x \in R$. Some well known entropy functions are given as follows:

$h_1(x) = 4x(1-x)$, $h_2(x) = -x \ln x - (1-x)\ln(1-x)$, $h_3(x) = \min\{2x, 2-2x\}$ and

$$h_4(x) = \begin{cases} 2x, & x \in [0, \frac{1}{2}] \\ 2(1-x), & x \in [\frac{1}{2}, 1] \end{cases}.$$

Note that the functions h_1, h_2 and h_3 are called logistic, Shannon and tent functions, respectively.

Let h be an entropy function and u is a sequence of fuzzy sets. Then total entropy of u is given as below:

$$e(u) = \int_{x \in \mathbb{R}} h(u^k(x)) p_k(x) dx. \tag{1.3}$$

Here, $p_k(x)$ is the probability density function in $\mathbb{R}$, [17].

One of the best known regular matrix is $\mathscr{R} = (r_{nk})$, the Riesz matrix which is a lower triangular matrix defined by

$$r_{nk} = \begin{cases} \frac{r_k}{R_n} & , \quad 0 \le k \le n \\ 0 & , \quad k > n \end{cases}$$

for all $n, k \in \mathbb{N}$, where (r_k) is real sequence with $r_0 > 0$, $r_k \ge 0$ and $R_n = \sum_{k=0}^{n} r_k$. The Riesz matrix R is regular if and only if $R_n \to \infty$ as $n \to \infty$, [16].

Definition 1.3 ([17]). Let suppose that $u = (u^k)$ be a sequence of fuzzy sets and $M = (m_{nk})$ be a lower triangular infinite matrix of real or complex numbers. Besides, $p_k(x) = c_k \in [0, 1]$ for all $k \in \mathbb{N}$ and following equations hold:

$$\begin{aligned} \lim_n \sum_k m_{nk} \int_{x\in R} h(u^k(x))p_k(x)dx \\ = \lim_n \sum_k m_{nk} c_k \left(2h_{u^k} - \frac{4}{3}h_{u^k}^2\right) \ell(u^k) \quad = L. \end{aligned} \tag{1.4}$$

Then, real number L is called total M- entropy of the sequence (u^k) of fuzzy sets.

If we write Riesz matrix, $\mathscr{R}$, instead of M then we obtain

$$\lim_n \frac{1}{R_k} \sum_{k=0}^{n} r_k \int_{x\in\mathbb{R}} h(u^k(x))p_k(x)dx \tag{1.5}$$

called as Riesz total entropy of the sequence $u = (u^k)$ of fuzzy sets and denoted by $E_{\mathscr{R}}(u^k)$.

We can transform QRS- complex of ECG to fuzzy set, easily. Furthermore, if we determine infinite number of ECG values for organism, then it will be considered as a sequence of ECG.

Every fuzzy set is a reflection of the blurred ideas belonging to the individual. Thus, any sequence of fuzzy sets can be thought of as a sequence of individual ideas. This fuzzy information sequence should contain the appropriate data to obtain a practical intuition. Hence, the terms of this sequence can be used to get significant values. Furthermore, calculations were made taking into account the unit of mm, along the tex.

2. Determination of entropy values of QRS-complex for sporting horses

QRS- complex demonstrate the rapid depolarization of the right and left ventricles. Firstly, in this section, we introduce entropy of QRS- complex of the ECG for sport horses as below:

$$e(QRS) = \int_{x\in R} h_1(QRS(x))r(x)dx \tag{2.1}$$

where, $QRS(x)$ is membership function of QRS and $r(x)$ is conductivity function of the organism.

Secondly, the research managed on 24 sport horses (12 females and 12 males), healthy and aged between 3.5-8 years is given. By using the maximum height 1.420mm= 0.142mV and maximum width 0.100 of QRS- complex of sport horses, we determine its membership function showed by QRS^b, by considering before training program as given in the following:

$$QRS^b(x) = \begin{cases} 28.4x, & x \in [0, 0.05] \\ 2.84 - 28.4x, & x \in (0.05, 0.100] \\ 0, & \text{otherwise.} \end{cases} \tag{2.2}$$

In addition this, by using the maximum height 1.565mm= 0.1565mV and maximum width 0.116 of QRS- complex, we calculate the membership function of fuzzy set QRS of sport horses after 5 months training program as given below:

$$QRS^a(x) = \begin{cases} 26.98x, & x \in [0, 0.058] \\ 3.13 - 26.98x, & x \in (0.058, 0.116] \\ 0, & \text{otherwise.} \end{cases} \tag{2.3}$$

It is easy to see that support of the fuzzy set QRS is duration of the complex QRS and height is maximum height of complex QRS. QRS- complex in ECG for sport horses can be turned into a fuzzy set. By using this method, we obtain a sequence of QRS- complexes.

Let us take $suppQRS^b \approx]0, 0.100[$ and $suppQRS^a \approx]0, 0.116[$. Closure of the $suppQRS^b$ and $suppQRS^a$ are equal to $[0, 0.100]$ and $[0, 0.116]$, respectively. By considering these situations, we can write entropy values of fuzzy sets QRS^b and QRS^a as in the following:

$$h_1^b(QRS(x)) = \begin{cases} 113.6x - 3.226x^2, & x \in [0, 0.05] \\ 531.648x - 3.226x^2 - 20.9024, & x \in (0.05, 0.100] \\ 0, & \text{otherwise.} \end{cases} \tag{2.4}$$

In the same way, we calculate $h_1^a(QRS(x))$ as below:

$$h_1^a(QRS(x)) = \begin{cases} 107.92x - 2.911x^2, & x \in [0, 0.058] \\ 567.6592x + 2.911x^2 - 26.6676, & x \in (0.058, 0.116] \\ 0, & \text{otherwise.} \end{cases} \tag{2.5}$$

If we choose $r(x) = c$, then we calculate $e^b(QRS)$ entropy value of QRS- complex for sport horses before training program as in the following:

$$e^b(QRS) = 1.0894c. \tag{2.6}$$

Similar to above calculation, we see that normal entropy value $e_a(QRS)$ for sport horses after training program is equal to

$$e^a(QRS) = 1.5027c. \tag{2.7}$$

Now, we will make a comment in light of the results of $e^b(QRS)$ and $e^a(QRS)$. Let us compute the distance $\triangle_e(QRS)$ by:

$$\triangle e(QRS) = |e^a(QRS) - e^b(QRS)|.$$

By taking into consideration (2.6) and (2.7), we find $\triangle_e(QRS) = 0.4133c$. We can evaluate this value as, the entropy for sport horses is increasing after movement or training program.

Definition 2.1. Let $(QRS)^i$ be finite sequence of QRS- complex and resistance of the dry skin be stable. Then, total Riesz entropy of $(QRS)^i$ is defined as follows:

$$E_{\mathscr{R}}((QRS)^i) = \frac{c}{R_k}\sum_{i=0}^{k} r_i \ell(QRS^i)(2h_{u_1^i} - \frac{4}{3}h^2_{u_1^i})S(QRS^i, QRS).$$

Here, $S(QRS^i, QRS)$ shows similarity degree between QRS- complex and components of the $(QRS)^i$.

Now, taking into account the data given in Table I, we will investigate total $\mathscr{R}$-entropy of the electrocardiogram for sport horses and give some comments. When the necessary calculations are made it is obvious that the total $\mathscr{R}$- entropy result is as below:

$$E_{\mathscr{R}}(QRS)^i = \tfrac{c}{R_k}[r_1 0.0022 + r_2 0.0081 + r_3 0.0229 + r_4 0.0280 + r_5 0.0108 + r_6 0.0276 + r_7 0.0313 + r_8 0.0229],$$

where c is resistance of the dry skin in the i^{th} time.

The total $\mathscr{R}$- entropy value calculated by using this Riesz matrix is smaller than the known entropy value with the right choice of Riesz matrix. Thus, the total $\mathscr{R}$-entropy definition reduces the degree of uncertainty. This will be more useful in calculating the real value and in correct diagnosis.

3. Comments

As it is well known, entropy is a value that varies according to the degree of uncertainty. After training program, substantial rises derived from entropy values of sport horses means that some irregularities arised in the pulse of sport horse. These irregularities do not indicate that sport horses are unhealthy. It is generally accepted that rapid heart beats are sign of the disease. But, contrary to above idea, we justify that the increase in heart beats has meant that the training program has been successful.

The QRS- complex expanded after the training program. However, this expansion was measured by < 0.12 sec. and does not constitute an abnormal condition. But if the QRS- complex expansion is bigger than 0.12 sec, then a life threatening arrhythmia will occur. Also, in this case, acute myocardial infarction or heart failure may be observed.

If the values $e^b(QRS)$ and $e^a(QRS)$ smaller than $1.0894c$ and $1.5027c$ for sport horses then it means that there is a problem on the health of sport horses. For instance, hypertrophy may be arised.

4. Section of tables

Table 1. Non-clinical QRS- complex waves data for sport horse before training program.

Gender:xx	Age:xx	Weight:xx						
Days	1	2	3	4	5	6	7	8
${h^b}_{u_1^k}$	0.5	0.8	1	1.2	1.4	1	1.1	1
$d^b(u_1^k)$	0.01	0.02	0.05	0.07	0.06	0.06	0.07	0.05
$e^b(QRS^k)$	0,0067	0,0150	0,0333	0,0418	0,0577	0,0400	0,0977	0,0333
$S^b(QRS^k, QRS)$	0,3363	0,5409	0,6866	0,8324	0,9662	0,6901	0,7630	0,6866

Table 2. Non-clinical QRS- complex waves data for sport horse after training program.

Gender:xx	Age:xx	Weight:xx						
Days	1	2	3	4	5	6	7	8
${h^a}_{u_1^k}$	0.4	0.5	1	1.2	1.4	1.5	1.4	1.2
$d^a(u_1^k)$	0.02	0.04	0.05	0.058	0.1	0.1	0.11	0.75
$e^a(QRS^k)$	0,0117	0,0266	0,0333	0,0278	0,0186	0,0346	0,0205	0,3600
$S^a(QRS^k, QRS)$	0,2433	0,3074	0,6179	0,7445	0,8874	0,8241	0,8919	0,5237

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