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Two notes on "On soft Hausdorff spaces"

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Abstract. One of the well known results in general topology says that every compact subset of a Hausdorff space is closed. This result in soft topology is not true in general as demonstrated throughout this note. We begin this investigation by showing that [Theorem 3.34, p.p.23] which proposed by Varol and Aygün [7] is invalid in general, by giving a counterexample. Then we derive under what condition this result can be generalized in soft topology. Finally, we evidence that [Example 3.22, p.p. 20] which introduced in [7] is false, and we make a correction for this example to satisfy a condition of soft Hausdorffness.

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1. Introduction

To handle problems which contain uncertainties and vagueness, Molodtsov [4] in 1999, proposed a new mathematical tool, namely soft sets. Shabir and Naz [6] in 2011, formulated the concept of soft topological spaces based on the soft sets notion. They initiated some soft topological notions such as some soft operators and soft separation axioms. In [2], the authors introduced a concept of soft compactness and investigated main features. Zorlutuna et al. [8] gave a notion of soft points and then the authors [3, 5] modified this notion in order to investigate some results related to soft limit points and soft metric spaces.

In 2013, Varol and Aygün [7] characterized a soft Hausdorff space and discussed some results related to it. However, they made two errors, as we observe, the first one was in [Theorem 3.34, p.p. 23] which investigated a relationship between soft closed set and soft compact soft Hausdorff space. To show this mistake, we provide a counterexample and then we present the correct form of this result. The second error was in [Example 3.22, p.p. 20]. We explain why this example did not satisfy the second axiom of a soft topology, hence it is not a soft Hausdorff space. To remove an error of this example, we consider the given collection as a soft base. This treatment constructs a soft topology satisfying a soft T_2-axiom.

2. Preliminaries

In what follows, we recall some definitions which will be needed throughout this investigation.

Definition 2.1 ([4])**.** A pair (G, A) is said to be a soft set over X, if G is a mapping of A into 2^X.

Definition 2.2 ([1])**.** The relative complement of a soft set (G, A), denoted by (G^c, A), where $G^c : A \to 2^X$ is the mapping defined by $G^c(a) = X \setminus G(a)$, for each $a \in A$.

Definition 2.3 ([6])**.** For a soft set (G, A) over X and $x \in X$, we say that:
(i) $x \in (G, A)$, if $x \in G(a)$, for each $a \in A$,
(ii) $x \notin (G, A)$, if $x \notin G(a)$, for some $a \in A$.

Definition 2.4 ([6])**.** A collection τ of soft sets over a non-empty set X with a fixed set of parameters A is called a soft topology on X, if it satisfies the following three axioms:
(i) the null soft set $\widetilde{\Phi}$ and the absolute soft set $\widetilde{X}$ are members of τ,
(ii) the union of an arbitrary number of soft sets in τ is also a member of τ,
(iii) the intersection of a finite number of soft sets in τ is also a member of τ.
The triple (X, τ, A) is called a soft topological space. Each soft set in τ is called soft open and its relative complement is called soft closed.

Definition 2.5 ([2])**.** (i) A family $\{(G_i, A) : i \in I\}$ of soft open sets is called a soft open cover of (X, τ, A), if $\widetilde{X} = \widetilde{\bigcup}_{i\in I}(G_i, A)$.

(ii) A soft topological space (X, τ, A) is called soft compact (resp. soft Lindelöf), provided that every soft open cover of $\widetilde{X}$ has a finite (resp. countable) subcover.

(iii)]A family $\mathcal{B}$ of soft open subsets of (X, τ, A) is called a soft base of τ if every member of τ can be expressed as a union of members of $\mathcal{B}$.

Definition 2.6 ([3, 5])**.** A soft subset (P, A) of $\widetilde{X}$ is called soft point, if there is $a \in A$ and there is $x \in X$ satisfies that $P(a) = \{x\}$ and $P(e) = \varnothing$, for each $e \in A \setminus \{a\}$.

A soft point will be shortly denoted by P_a^x.

Definition 2.7 ([6])**.** A soft topological space (X, τ, A) is said to be a soft T_2-space (or a soft Hausdorff space), if for every $x \neq y \in X$, there are two disjoint soft open sets (G, A) and (F, A) such that $x \in (G, A)$ and $y \in (F, A)$.

3. Main Results

In [7], the authors introduced the following result which is numbered as Theorem 3.34 in their paper.

Theorem 3.1. *Let (X, τ, A) be a soft Hausdorff space. If (F, A) is soft compact subset of $\widetilde{X}$, then (F, A) is soft closed.*

The following example illustrates that the above theorem need not be true in general.

Example 3.2. Let $A = \{a_1, a_2\}$ be a set of parameters and let $X = \{x, y\}$ be the universe set. Then a collection $\tau = \{\widetilde{\Phi}, \widetilde{X}, (G, A), (H, A)\}$ is a soft topology on X, where

$$G(a_1) = \{x\},\ G(a_2) = \{x\}$$

and

$$H(a_1) = \{y\},\ H(a_2) = \{y\}.$$

Obviously, (X, τ, A) is a soft Hausdorff space. On the other hand, a soft set (F, A), which defined as $F(a_1) = \{x\},\ F(a_2) = \{y\}$, is a soft compact subset of $\widetilde{X}$. But it is not soft closed.

Remark 3.3. We think that an error of the proof of Theorem 3.34 in [7] attributed to that the authors incorrectly expected that: If for each $x \in (M, A)$ implies that $x \in (N, A)$, then $(M, A)\widetilde{\subseteq}(N, A)$. This result is true via the set theory, but it need not be true via the soft set theory. We can show this by taking the two soft subsets $(M, A) = \{(a_1, \{x\}), (a_1, X)\}$ and $(N, A) = \{(a_1, X), (a_1, \{x\})\}$ of (X, τ, A) which given in the above example. Now, every $x \in (M, A)$ implies that $x \in (N, A)$ and every $x \in (N, A)$ implies that $x \in (M, A)$. However $(M, A) \not\subseteq (N, A)$ and $(N, A) \not\subseteq (M, A)$.

We correctly formulate this result via the soft set theory by utilizing a soft point notion as illustrated in the following result.

Proposition 3.4. *$(F, A)\widetilde{\subseteq}(G, A)$ if and only if for each $P^x_a \in (F, A)$ implies that $P^x_a \in (G, A)$.*

Definition 3.5. A soft set (G, A) over X is said to be stable, if there exists a subset U of X such that $G(a) = U$, for each $a \in A$.

The following result is the correct form of [Theorem 3.34, p.p.23] in [7].

Theorem 3.6. *Every stable soft compact subset (F, A) of a soft Hausdorff space (X, τ, A) is soft closed.*

Proof. Suppose that (F, A) is a stable soft set. Then (F^c, A) is stable as well. Let $P^x_a \in (F^c, A)$. This means that $x \in (F^c, A)$. Similarly, for each $P^{y_i}_a \in (F, A)$, we get $y_i \in (F, A)$. Thus $x \neq y_i$. By hypothesis, there exist two disjoint soft open sets (G_i, A) and (W_i, A) such that $x \in (G_i, A)$ and $y_i \in (W_i, A)$. It follows that $\{(W_i, A) : i \in I\}$ forms a soft open cover of (F, A). Consequently, $(F, A)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n}(W_i, A)$. Putting $\widetilde{\bigcap}_{i=1}^{i=n}(G_i, A) = (H, A)$ and $\widetilde{\bigcup}_{i=1}^{i=n}(W_i, A) = (V, A)$. Then (H, A) and (V, A) are soft open sets such that $(H, A)\widetilde{\bigcap}(V, A) = \widetilde{\Phi}$. Thus $(H, A)\widetilde{\bigcap}(F, A) = \widetilde{\Phi}$. So $(H, A)\widetilde{\subseteq}(F^c, A)$. Since P^x_a is chosen arbitrary, (F^c, A) is a soft neighborhood of its soft points. Hence it is soft open. Therefore it is soft closed. □

In the rest of this work, we point out that [Example 3.22, p.p.20] in [7] is incorrect. We firstly mention this example as it originally introduced in [7].

Example 3.7. Let $X = E = \mathcal{R}$, where $\mathcal{R}$ is the set of real numbers. Consider $(F, E)_y = \{(x, (x, y]) : x, y \in E$ and $x < y\}$. Then $\mathcal{T} = \{\widetilde{\Phi}, \widetilde{\mathcal{R}}, (F, E)_y : y \in E\}$ defined a soft topology on $\mathcal{R}$. Furthermore, $(\mathcal{R}, \mathcal{T}, \mathcal{R})$ is a soft Hausdorff space.

To show an error of this example, let $B = \{2 - \frac{1}{n} : n = 1, 2, ...\}$. Then $\Lambda = \{(F, E)_y : x = 0$ and $y \in B\}$ is a collection of soft open sets. However, $\bigcup_{y \in B}(0, (0, y]) = (0, (0, 2))$. This implies that $\widetilde{\bigcup}_{y \in B}(F, E)_y \notin \mathcal{T}$. So $\mathcal{T}$ which given in the above example is not a soft topology.

We observe that $\widetilde{\mathcal{R}}$ can be expressed as a union of members of $\mathcal{T}$, and the soft intersection of any two members of $\mathcal{T}$ can be expressed as a union of members of $\mathcal{T}$ as well. So we correct the above example by considering $\mathcal{T}$ as a soft base for a soft topology τ. Then $(\mathcal{R}, \tau, \mathcal{R})$ is a soft Hausdorff space.

Remark 3.8. It worthily noted that a soft topology which has $\mathcal{T}$ as a soft base consider as a version of the upper limit topology via soft topology.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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