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Fuzzy ideal graphs of a semigroup

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Abstract. The main objective of this paper is to connect fuzzy theory, graph theory and fuzzy graph theory with algebraic structure. We introduce the notion of fuzzy graph of semigroup, the notion of fuzzy ideal graph of semigroup as a generalization of fuzzy ideal of semigroup, intuitionistic fuzzy ideal of semigroup, fuzzy graph and graph, the notion of isomorphism of fuzzy graphs of semigroups and regular fuzzy graph of semigroup and we study some of their properties.

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1. Introduction

Semigroups are basic algebraic structures in many branches of engineering like automata, formal languages, coding theory, finite state machines. The formal study of semigroups begin in the early 20th century. In 1965, Zadeh [25] introduced the fuzzy theory. The aim of this theory is to develop theory which deals with problem of uncertainty. Atanassov [5] introduced the concept of intuitionistic fuzzy sets as extensions of Zadeh's fuzzy set theory for representing vagueness and uncertainty. The concept of fuzzy set was applied to theory of subgroups by Rosenfeld [22]. After that Kuroki [11] studied theory of fuzzy semigroups. Jun et al. [9] studied theory of fuzzy semirings. Murali Krishna Rao [14, 15, 16, 17, 18, 19, 20] studied fuzzy ideals of $\Gamma-$semiring. Mordeson et al. [13] studied fuzzy semigroup theory and its applications in coding, finite state machine, automata and formal languages.

In 1736, Euler first introduced the concept of graph theory. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. We know that a graph is a symmetric binary relation on a non-empty set V. The major role of graph theory in computer applications is the development of graph algorithms. A number of algorithms are used to solve problems that are modeled in the form of graphs. These algorithms are used to solve the graph theoretical concepts, which in turn are used to solve the corresponding computer science application problems. Several computer programming languages support the graph theory concepts.

In 1975, Rosenfeld [23] introduced fuzzy graph to model real life situations. The first definition of fuzzy graph was introduced by Kauffman [10] in 1973 based on Zadeh's fuzzy relations. If there is a vagueness in the description of objects or in its relationships or in both, then we need to a assign a fuzzy graph model. Fuzzy graphs are useful to represent relationships which deal with uncertainty. Rosenfeld considered fuzzy relations on fuzzy subsets and developed the theory of fuzzy graphs. Yeh and Bang introduced the various concepts in fuzzy graphs. Rosenfeld developed the fuzzy graph theory as a generalization of Euler's graph theory obtaining analogs of several graph theoretical concepts. Fuzzy graph theory has been finding an increasing number of applications in modeling real time systems. Fuzzy models are becoming useful because of their aim to reduce the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. Mordeson and Peng [12] defined the concept of complement of fuzzy graph and described some operations on fuzzy graphs. Murali Krishna Rao [21] studied fuzzy graph of semigroup. Akram et al. [1, 2, 3, 4] introduced many new concepts including bipolar fuzzy graphs, interval-valued line fuzzy graphs, and strong intuitionistic fuzzy graphs. Bhattacharya [6], Sunitha and Kumar [24] studied fuzzy graphs.

In this paper, we introduce the notion of fuzzy ideal graph of semigroup as a generalization of fuzzy ideal of semigroup, intuitionistic fuzzy ideal of semigroup, fuzzy graph and a graph. We study some of their properties.

2. Preliminaries

In this section, we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. A graph is a pair (V, E), where V is a non-empty set and E is a set of unordered pairs of elements of V. The graph (V, E) is denoted by $G(V, E)$.

Definition 2.2. The number of vertices in a graph $G(V, E)$ is called an order of $G(V, E)$ and it is denoted by $|V|$.

For simplicity an edge $\{x, y\}$ will be denoted by xy.

Definition 2.3. The number of edges in a graph $G(V, E)$ is called a size of $G(V, E)$ and it is denoted by $|E|$.

Definition 2.4. Two vertices x and y in a graph $G(V, E)$ are said to be adjacent or neighbors, if $\{x, y\}$ is an edge of $G(V, E)$.

Definition 2.5. The neighbor set of a vertex x of a graph $G(V, E)$ is the set of all elements in V which are adjacent to x and it is denoted by $N(x)$.

Definition 2.6. A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

Definition 2.7. A graph $G(V, E)$ is said to be $k-$regular graph, if $deg(v) = k$, for all $v \in V$.

Definition 2.8. A graph $G(V, E)$ is connected, if there exists a path between every two vertices a and b of $G(V, E)$

Definition 2.9. Let $G(V_1, E_1)$ and $G(V_2, E_2)$ be graphs. Then there exists a map $h : V_1 \to V_2$ such that

(i) h is a bijection
(ii) if two vertices v_1 and v_2 of V_1 are adjacent in $G(V_1, E_1)$, then $f(v_1)$ and $f(v_2)$ are adjacent in $G(V_2, E_2)$

if and only if h is said to be isomorphism of graphs.

Definition 2.10. A semigroup is an algebraic system $(M, .)$ consisting of a non-empty set M together with an associative binary operation $'' \cdot ''$.

Definition 2.11. A subsemigroup T of a semigroup M is a non-empty subset T of M such that $TT \subseteq T$.

Definition 2.12. A non-empty subset T of a semigroup M is called a left (right) ideal of M, if $MT \subseteq T$ $(TM \subseteq T)$.

Definition 2.13. A non-empty subset T of a semigroup M is called an ideal of M, if it is both a left ideal and a right ideal of M.

Definition 2.14. Let M be a non-empty set. Then a mapping $f : M \to [0, 1]$ is called a fuzzy subset of M.

Definition 2.15. Let M be a semigroup. A fuzzy subset μ of M is said to be fuzzy subsemigroup of M, if it satisfies $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in M$.

Definition 2.16. A fuzzy subset μ of a semigroup M is called a fuzzy left (right) ideal of M, if $\mu(xy) \geq \mu(y)$ $(\mu(x))$, for all $x, y \in M$.

Definition 2.17. A fuzzy subset μ of a semigroup M is called a fuzzy ideal of M, if $\mu(xy) \geq \max\{\mu(x), \mu(y)\}$, for all $x, y \in .M$

Example 2.18. Let $V = \{a, b, c\}$ and binary operation $'\cdot'$ on V is defined by

.	a	b	c
a	a	a	c
b	a	b	c
c	c	c	c

Define $\mu : V \to [0, 1]$ by $\mu(a) = \frac{1}{2}, \mu(b) = \frac{1}{4}, \mu(c) = \frac{2}{3}$. Then μ is a fuzzy subsemigroup and fuzzy ideal of semigroup V.

Definition 2.19. An intuitionistic fuzzy set f of a semigroup M is an object having the form $f = (\mu_f, \lambda_f) = \{x, \mu_f(x), \lambda_f(x) \mid x \in M\}$, where $\mu_f : M \to [0, 1], \lambda_f : M \to [0, 1]$ are membership functions, $\mu_f(x)$ is a degree of membership, $\lambda_f(x)$ is a degree of non membership and $0 \leq \mu_f(x) + \lambda_f(x) \leq 1$, for all $x \in M$.

Definition 2.20. An intuitionistic fuzzy set $f = (\mu_f, \lambda_f)$ of a semigroup M is called an intuitionistic fuzzy ideal, if f satisfies the following conditions

(i) $\mu_f(xy) \geq max\{\mu_f(x), \mu_f(y)\}$,
(ii) $\lambda_f(xy) \leq min\{\lambda_f(x), \lambda_f(y)\}$, for all $x, y \in M$.

Definition 2.21. Let V be a non-empty finite set, σ and μ are fuzzy subsets on V and $V \times V$ respectively. If $\mu(u, v) \leq \min\{\sigma(u), \sigma(v)\}$, for all $\{u, v\} \in E$, then the pair $G = (\mu, \sigma)$ is called a fuzzy graph over the set V. Here μ and σ are called a fuzzy edge and fuzzy vertex of the fuzzy graph G, respectively.

Definition 2.22. The underlying crisp graph of a fuzzy graph $G = (\mu, \sigma)$ is denoted by $G = (\mu^*, \sigma^*)$, where $\mu^* = \{(x, y) \in V \times V \mid \mu(x, y) > 0\}$ and $\sigma^* = \{x \in V \mid \sigma(x) > 0\}$.

Definition 2.23. Let $H = (\delta, \gamma)$ and $G = (\mu, \sigma)$ be fuzzy graphs over the set V. Then H is called a fuzzy subgraph of fuzzy graph G, if $\gamma(x) \leq \sigma(x)$, for all $x \in V$ and $\delta(x, y) \leq \mu(x, y)$, for all $\{x, y\} \in E$.

Definition 2.24. A fuzzy graph $G = (\mu, \sigma)$ is called a strong fuzzy graph, if $\mu(u, v) = \min\{\sigma(u), \sigma(v)\}$, for all $\{u, v\} \in E$.

Definition 2.25. A fuzzy graph $G = (\mu, \sigma)$ is called a complete, if $\mu(u, v) = \min\{\sigma(u), \sigma(v)\}$, for all $u, v \in V$.

Definition 2.26. The order and size of a fuzzy graph $G = (\mu, \sigma)$ are defined as $o(G) = \sum\limits_{x \in V} \sigma(x)$ and $s(G) = \sum\limits_{\{x,y\} \in E} \mu(xy)$, respectively.

The first theorem of graph theory

Let $G(V, E)$ be a graph. Then twice the number of edges of graph $G(V, E)$ is sum of the degrees of all vertices belong to V.

3. Fuzzy ideal graph of semigroup

In this section, we introduce the notion of fuzzy graph of semigroup, fuzzy ideal graph of semigroup, isomorphic fuzzy graphs of semigroups and regular fuzzy graph of semigroup. We study some of their properties. Throughout this paper V is a commutative semigroup.

Definition 3.1. Let $G(V, E)$ be a graph, $(V, \cdot)$ be a finite semigroup and μ be a fuzzy subset of V such that $\mu(uv) \geq \max\{\mu(u), \mu(v)\}$, for all $\{u, v\} \in E$. Then graph $G(V, E)$ is called a fuzzy graph of a semigroup V. It is denoted by $G(V, E, \mu)$.

Definition 3.2. Let $G(V, E)$ be a complete graph. Then fuzzy graph of semigroup $G(V, E, \mu)$ is called a fuzzy ideal graph of a semigroup V.

Remark 3.3. Let $G(V, E, \mu)$ be a fuzzy ideal graph. Define σ as a fuzzy subset of V such that $\sigma(x) = 1 - \mu(x)$, for all $x \in V$. Then $\mu(uv) \geq \max\{\mu(u), \mu(v)\}$, for all $\{u, v\} \in E$. Implies $\mu(uv) \leq \min\{\sigma(u), \sigma(v)\}$, for all $\{u, v\} \in E$.
Thus $G = (\mu, \sigma)$ is a fuzzy graph in the sense of Rosenfeld. So fuzzy ideal graph $G(V, E, \mu)$ is a generalization of fuzzy ideal of semigroup V, intuitionistic fuzzy ideal of semigroup V, fuzzy graph $G(V, \sigma, \mu)$ and a graph $G(V, E)$.

Definition 3.4. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. The order of $G(V, E, \mu)$ is defined as $\sum\limits_{x \in V} \mu(x)$. It is denoted by p.

Definition 3.5. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. The size of $G(V, E, \mu)$ is defined as $\sum\limits_{\{x,y\}\in E} \mu(xy)$. It is denoted by q.

Definition 3.6. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. Then the degree of vertex v of $G(V, E, \mu)$ is defined as $\sum\limits_{u\neq v, \{u,v\}\in E} \mu(uv)$. It is denoted by $D(v)$.

Theorem 3.7. *Let $G(V, E, \mu)$ be a fuzzy ideal graph of a semigroup V with 0 element and 1 element. Then*

(1) $\mu(0) \geq \mu(u)$, *for all* $u \in V$,
(i2) $\mu(0) \geq \mu(1)$.

Proof. Let $G(V, E, \mu)$ be a fuzzy ideal graph of a semigroup V with 0 element and 1 element. Then $\mu(0) = \mu(ov) \geq \mu(v)$, for all $v \in V$ and $\mu(0) = \mu(o1) \geq \mu(1)$. □

Example 3.8. Let $G(V, E)$ be a graph with $V = \{a, b, c\}$ and $E = \{(a, b), (b, c), (c, a)\}$ and binary operation on V, $\cdot$ is defined by

.	a	b	c
a	a	a	c
b	a	b	c
c	c	c	c

Define $\mu : V \to [0, 1]$ by $\mu(a) = \frac{1}{2}, \mu(b) = \frac{1}{4}, \mu(c) = \frac{2}{3}$. Here c is the 0 element and c is the 1 element. $\mu(0) \geq \mu(u)$, for all $u \in V$ and $\mu(0) \geq \mu(1)$. Then μ is a fuzzy ideal of the semigroup V. Thus $G(V, E, \mu)$ is fuzzy ideal graph of the semigroup V.

Order of fuzzy ideal graph $= \sum\limits_{v\in V} \mu(v) = \mu(a) + \mu(b) + \mu(c) = \frac{17}{12}$.

Size of fuzzy ideal graph $= \sum\limits_{\{u,v\}\in E} \mu(uv) = \mu(ab)+\mu(bc)+\mu(ca) = \frac{1}{2}+\frac{2}{3}+\frac{2}{3} = \frac{11}{6}$.

Definition 3.9. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. If $D(v) = k$, for all $v \in V$, then $G(V, E, \mu)$ is said to be regular fuzzy graph.

Definition 3.10. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. Total degree of a vertex $u \in V$ is defined as $D(u) + \mu(u)$. It is denoted by $TD(u)$.

Theorem 3.11. *The size of a $k-$regular fuzzy graph $G(V, E, \mu)$ of a semigroup V is $\frac{|V|k}{2}$.*

Proof. By definition of size of $G(V, E, \mu)$, size of $G(V, E, \mu) = \sum\limits_{\{u,v\}\in E} \mu(uv)$. Then $\sum\limits_{v\in V} D(v) = 2 \sum\limits_{\{u,v\}\in E} \mu(uv) = 2$ size of $G(V, E, \mu)$. Thus $2S(G) = \sum k =| V | k$. So $S(G) = \frac{|V|k}{2}$. □

Definition 3.12. If each vertex of $G(V, E, \mu)$ has the same total degree k, then fuzzy graph $G(V, E, \mu)$ is said to be totally regular fuzzy graph of total degree k.

Definition 3.13. Degree of vertex x of a graph $G(V, E)$ is defined as the number of edges incident on x and it is denoted by $d(x)$ or equivalently $deg(x) = |N(x)|$.

Theorem 3.14. *Let $G(V, E, \mu)$ be a fuzzy graph and $V = \{v_1, v_2, \cdots v_n\}$. Then* $\sum_{v_i \in V} D(v_i) \geq \sum_{v_i \neq v_j, \{v_i, v_j\} \in E} d(v_i)\mu(v_j)$.

Proof. Suppose $G(V, E, \mu)$ is a fuzzy graph and $V = \{v_1, v_2, \cdots v_n\}$. Then

$$D(v_i) \sum_{v_i \neq v_j, \{v_i, v_j\} \in E} \mu(v_i v_j) \geq \sum_{v_i \neq v_j, \{v_i, v_j\} \in E} d(v_i)\mu(v_j).$$

Thus $\sum_{v_i \in V} D(v_i) \geq \sum_{v_i \neq v_j, \{v_i, v_j\} \in E} d(v_i)\mu(v_j)$. □

Corollary 3.15. *Let $G(V, E, \mu)$ be a regular fuzzy graph. Then*

$$\sum_{v_i \in V} D(v_i) \geq d(v_i) \sum \mu(v_j).$$

Theorem 3.16. *Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. Then μ is a constant function if and only if the following are equivalent*

(1) *the fuzzy graph V is regular*
(2) *the fuzzy graph V is totally regular.*

Proof. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V and μ be a constant function.

(1) $\Rightarrow$ (2): Suppose $G(V, E, \mu)$ is a regular fuzzy graph of the semigroup V. Then $D(u) = k$, for all $u \in V$ and $\mu(u) = c$. Thus $TD(u) = D(u) + \mu(u) = k + c$, for all $u \in V$. So the fuzzy graph V is totally regular.

(2) $\Rightarrow$ (1): Suppose $G(V, E, \mu)$ be a totally regular fuzzy graph of the semigroup V and $TD(u) = k$, for all $u \in V$. Then $D(u) + \mu(u) = k$, for all $u \in V$. Thus $D(u) + c = k$, for all $u \in V$. So $D(u) = k - c$, for all $u \in V$. Hence the fuzzy graph V is regular.

□

Example 3.17. Let $G(V, E)$ be a graph with $V = \{a, b, c\}$ and $E = \{(a, b), (b, c), (c, a)\}$ and binary operation $'\cdot'$ on V is defined by

.	a	b	c
a	a	a	c
b	a	b	c
c	c	c	c

Define $\mu : V \to [0, 1]$ by $\mu(a) = \frac{1}{2}, \mu(b) = \frac{1}{2}, \mu(c) = \frac{1}{2}$. Then $D(u) = 1, TD(u) = D(u) + \mu(u) = 1 + \frac{1}{2} = \frac{3}{2}$, for all $u \in V$. Thus $G(V, E, \mu)$ is regular and totally regular fuzzy graph of the semigroup V.

Definition 3.18. Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be fuzzy graphs of semigroups V_1 and V_2. If there exists a map $h : V_1 \to V_2$ such that

(i) h is a isomorphism of semigroups,
(ii) $\mu_1(x) = \mu_2(h(x))$, for all $x \in V_1$,
(iii) $\mu_1(xy) = \mu_2(h(x)h(y))$, for all $\{x, y\} \in E_1$ and $\{h(x), h(y)\} \in E_2$,

then h is said to be isomorphism of fuzzy graphs of semigroups and denoted by

$$G(V_1, E_1, \mu_1) \cong G(V_2, E_2, \mu_2).$$

Theorem 3.19. *Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs of semigroups V_1 and V_2 respectively. Then their orders and sizes are same.*

Proof. Suppose $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ are isomorphic fuzzy graphs. Then there exists an isomorphism $h : V_1 \to V_2$ such that

(i) $\mu_1(x) = \mu_2(h(x))$

(ii) $\mu_1(xy) = \mu_2(h(x)h(y))$, for all $\{x, y\} \in E_1,\ \{h(x), h(y)\} \in E_2$.

Thus

$$\begin{aligned}
\text{order of } G(V_1, E_1, \mu_1) &= \sum_{v \in V} \mu_1(v) \\
&= \sum_{v \in V} \mu_2 h((v)) \\
&= \text{ order of } G(V_2, E_2, \mu_2), \\
\text{size of } \ G(V_1, E_1, \mu_1) &= \sum_{\{x,y\} \in E_1} \mu_1(xy) \\
&= \sum_{\{x,y\} \in E_1} \mu_2(h(xy)) \\
&= \sum_{\{h(x),h(y)\} \in E_2} \mu_2(h(x)h(y)) \\
&= \text{ size of } \ G(V_2, E_2, \mu_2).
\end{aligned}$$

□

Theorem 3.20. *Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs. If $G(V_1, E_1, \mu_1)$ is a regular fuzzy graph of a semigroup V_1, then $G(V_2, E_2, \mu_2)$ is a regular fuzzy graph of a semigroup V_2.*

Proof. Suppose $G(V_1, E_1, \mu_1)$ is a regular fuzzy graph of semigroup and h is the isomorphism of fuzzy graphs $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$. Then there exists an isomorphism $h : V_1 \to V_2$ such that $\mu_1(xy) = \mu_2(h(xy))$, for all $\{x, y\} \in E_1$ and $\{h(x), h(y)\} \in E_2$. Thus

$$\begin{aligned}
D(u) &= \sum_{u \neq v, \{u,v\} \in E_1} \mu_1(uv) \\
&= \sum_{u \neq v, \{h(u),h(v)\} \in E_2} \mu_2(h(u)h(v)) \\
&= D(h(u)).
\end{aligned}$$

So the result holds. □

Theorem 3.21. *Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs. If $G(V_1, E_1, \mu_1)$ is a totally regular fuzzy graph of semigroup then $G(V_2, E_2, \mu_2)$ is a totally regular fuzzy graph of semigroup.*

Definition 3.22. Let $G(V, E, \mu)$ be a fuzzy graph of a semigroup V. Then the complement of $G(V, E, \mu)$ is defined as $G(V, E, \bar{\mu})$, where $\bar{\mu}(xy) = \mu(xy) - \max\{\mu(x), \mu(y)\}$, for all $\{x, y\} \in E$.

Theorem 3.23. *$G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ fuzzy graphs of semigroups are isomorphic if and only if their complements are isomorphic.*

Proof. Suppose $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ are isomorphic fuzzy graphs of semigroups V_1 and V_2. Then there exists an isomorphism $h : V_1 \to V_2$ such that

$$\mu_1(x) = \mu_2(h(x)), \quad \text{for all} \quad x \in V$$

and

$$\mu_1(xy) = \mu_2(h(x)h(y)), \text{ for all } \{x, y\} \in E_1 \text{ and } \{h(x), h(y)\} \in E_2.$$

Thus

$$\begin{aligned}\bar{\mu_1}(xy) &= \mu_1(xy) - \max\{\mu_1(x), \mu_1(y)\}\\ &= \mu_2(h(x)h(y)) - \max\{\mu_2(h(x)), \mu_2(h(y))\}\\ &= \bar{\mu_2}(h(x)h(y)), \text{ for all } \{x, y\} \in E_1.\end{aligned}$$

So $G(V_1, E_1, \bar{\mu_1}) \cong G(V_2, E_2, \bar{\mu_2})$.

Similarly, we can prove the converse part of the theorem. □

4. Conclusion

We introduced the notion of fuzzy graph of semigroup, the notion of isomorphism of fuzzy graphs of semigroup, regular fuzzy graph of semigroup and the notion of fuzzy ideal graph of semigroup as a generalization of fuzzy ideal of semigroup, intuitionistic fuzzy ideal of semigroup, fuzzy graph and graph. We studied some of their properties.

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