

Some results on inequalities of intuitionistic fuzzy matrices

S. SRIRAM, J. BOOBALAN

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ABSTRACT. In this paper, we study certain inequalities of intuitionistic fuzzy matrices with respect to algebraic sum and algebraic product.

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Corresponding Author: J. Boobalan (jboobalan@hotmail.com)

1. INTRODUCTION

In 1983 Atanassov [1] introduced the new concept of intuitionistic fuzzy set (IFS) which is an extension of fuzzy set (FS) initiated by Zadeh [30]. Meenakshi [9] studied the theoretical developments of fuzzy matrices. Using the concept of IFS, Im et.al [5] studied intuitionistic fuzzy matrix (IFM). Simultaneously Khan et.al [8] defined the intuitionistic fuzzy matrix and Pal [19] introduced the intuitionistic fuzzy determinant, studied some properties on it. IFM is a generalization of Fuzzy matrix introduced by Thomson [27] and has been useful in dealing with areas such as decision making, clustering analysis, relational equations etc. Atanassov [3], using the definition of index matrix, has paved way for intuitionistic fuzzy index matrix and has further extending it to temporal intuitionistic fuzzy index matrix. IFM is also very useful in the discussion of intuitionistic fuzzy relation [4, 10]. Xu [29] studied intuitionistic fuzzy value and also IFMs. He defined intuitionistic fuzzy similarity relation and also utilized it in clustering analysis. A lot of research activities have been carried out over the years on IFMs in [6, 12, 20, 21, 28]. Mondal et.al [11] initiated a study on the similarity relations, invertability and eigenvalues of IFM. In [7], a research was carried out on how a transitive IFM decomposed into a sum of nilpotent IFM and symmetric IFM. Atanassov introduced model operators in [2] which are meaningless in fuzzy set theory and found a promising direction in research. The above operators for IFMs and some results are obtained in [13]. Muthuraji et.al [14] studied some properties of model operators in IFM. Also they obtained

a decomposition of an IFM. Shyamal and Pal [26] introduced two binary operators for fuzzy matrices and studied the algebraic properties. Also, they extended the binary operators for IFMs [25]. Padder and Murugadas [15, 16, 17, 18] studied the convergence and transitivity of IFM. Sriram and Boobalan [22, 23] investigated the algebraic properties and studied the properties of IFMs in the case where the operations are combined with the well known operations. In this paper, we formulate and prove certain inequalities connected with algebraic operations of intuitionistic fuzzy matrices.

2. PRELIMINARIES

In this section, some basic definitions related to intuitionistic fuzzy matrix are presented.

Definition 2.1 ([8, 11]). An intuitionistic fuzzy matrix(IFM) is a matrix of pairs $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$ of a non negative real numbers satisfying $\mu_{a_{ij}} + \nu_{a_{ij}} \leq 1$, for all i, j .

Definition 2.2 ([24]). Let A and B are two intuitionistic fuzzy matrices such that $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$, $B = (\langle \mu_{b_{ij}}, \nu_{b_{ij}} \rangle)$. Then

$$A \vee B = (\langle \max(\mu_{a_{ij}}, \mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{b_{ij}}) \rangle),$$

$$A \wedge B = (\langle \min(\mu_{a_{ij}}, \mu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{b_{ij}}) \rangle).$$

Definition 2.3 ([24]). Let A and B be two IFMs such that $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$, $B = (\langle \mu_{b_{ij}}, \nu_{b_{ij}} \rangle)$. Then we write $A \leq B$, if $\mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$ for all i, j .

Definition 2.4 ([24]). The $m \times n$ zero IFM O is an IFM all of whose entries are $\langle 0, 1 \rangle$.

The $m \times n$ universal IFM J is an IFM all of whose entries are $\langle 1, 0 \rangle$.

Definition 2.5 ([8]). Let $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$ and $B = (\langle \mu_{b_{ij}}, \nu_{b_{ij}} \rangle)$ be two intuitionistic fuzzy matrices. Then

(i)

$$A \oplus B = (\mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}} \cdot \mu_{b_{ij}}, \nu_{a_{ij}} \cdot \nu_{b_{ij}})$$

is called the algebraic sum of A and B ,

(ii)

$$A \otimes B = (\mu_{a_{ij}} \mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} \nu_{b_{ij}})$$

is called the algebraic product of A and B .

Definition 2.6 ([22]). The complement of an intuitionistic fuzzy matrix A which is denoted by A^c and is defined by $A^c = (\langle \nu_{a_{ij}}, \mu_{a_{ij}} \rangle)$.

3. MAJOR SECTION

In this section, we prove some inequalities connected with algebraic operations on intuitionistic fuzzy matrices.

Theorem 3.1. *If A, B and C are three intuitionistic fuzzy matrices of same order and if $A \leq C$ and $B \leq C$, then $A \vee B \leq C$.*

Proof. If $A \leq C$, then $\mu_{a_{ij}} \leq \mu_{c_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{c_{ij}}$, for all i, j .

If $B \leq C$, then $\mu_{b_{ij}} \leq \mu_{c_{ij}}$ and $\nu_{b_{ij}} \geq \nu_{c_{ij}}$ for all i, j . Thus

$$\max(\mu_{a_{ij}}, \mu_{b_{ij}}) \leq \mu_{c_{ij}} \text{ and } \min(\nu_{a_{ij}}, \nu_{b_{ij}}) \geq \nu_{c_{ij}}, \text{ for all } i, j.$$

So $A \vee B \leq C$. □

Theorem 3.2. *If A, B and C are three intuitionistic fuzzy matrices of same order and if $A \leq B$, then $A \vee C \leq B \vee C$.*

Proof. If $A \leq B$, then $\mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$, for all i, j . Then

$$\max(\mu_{a_{ij}}, \mu_{c_{ij}}) \leq \max(\mu_{b_{ij}}, \mu_{c_{ij}}) \text{ and } \min(\nu_{a_{ij}}, \nu_{c_{ij}}) \geq \min(\nu_{b_{ij}}, \nu_{c_{ij}}), \text{ for all } i, j.$$

For $a, b, c \in \mathbb{R}$, if $a \leq b$, then $\max(a, c) \leq \max(b, c)$ and $\min(a, c) \leq \min(b, c)$. Thus $A \vee C \leq B \vee C$. □

Theorem 3.3. *If A, B and C are three intuitionistic fuzzy matrices of same order and if $C \leq A$ and $C \leq B$, then $C \leq A \wedge B$.*

Proof. If $C \leq A$, then $\mu_{c_{ij}} \leq \mu_{a_{ij}}$ and $\nu_{c_{ij}} \geq \nu_{a_{ij}}$, for all i, j .

If $C \leq B$, then $\mu_{c_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{c_{ij}} \geq \nu_{b_{ij}}$, for all i, j .

For $a, b \in \mathbb{R}$, if $a \leq b$ and $a \leq c$, then $a \leq \min(b, c)$. Also if $c \geq a, c \geq b$, then $c \geq \max(a, b)$. Thus $\mu_{c_{ij}} \leq \min(\mu_{a_{ij}}, \mu_{b_{ij}}), \nu_{c_{ij}} \geq \max(\nu_{a_{ij}}, \nu_{b_{ij}})$, for all i, j . So $C \leq A \wedge B$. □

Corollary 3.4. *If A, B and C are three intuitionistic fuzzy matrices of same order and if $A \leq B, A \leq C$ and $B \wedge C = O$, then $A = O$.*

Proof. If $A \leq B$, then $\mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$, for all i, j .

If $A \leq C$, then $\mu_{a_{ij}} \leq \mu_{c_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{c_{ij}}$, for all i, j . Since $B \wedge C = O$, by using the Theorem 3.3, $\min(\mu_{a_{ij}}, \mu_{c_{ij}}) = 0$ and $\max(\nu_{a_{ij}}, \nu_{c_{ij}}) = 1$ for all i, j . Since $\mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\mu_{a_{ij}} \leq \mu_{c_{ij}}$, we have $\mu_{a_{ij}} \leq \min(\mu_{b_{ij}}, \mu_{c_{ij}})$. Thus $\mu_{a_{ij}} \leq 0$, for all i, j . So $\mu_{a_{ij}} = 0$, for all i, j .

Since $\nu_{a_{ij}} \geq \nu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{c_{ij}}$ we have $\nu_{a_{ij}} \geq \max(\nu_{b_{ij}}, \nu_{c_{ij}})$, for all i, j . Thus $\nu_{a_{ij}} = 1$, for all i, j . So $A = (\langle 0, 1 \rangle)$, for all i, j . Hence $A = O$. □

Theorem 3.5. *If A, B and C are three intuitionistic fuzzy matrices of same order and if $A \leq B$, then $A \wedge C \leq B \wedge C$.*

Proof. If $A \leq B$, then $\mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$, for all i, j . Thus for all i, j ,

$$\min(\mu_{a_{ij}}, \mu_{c_{ij}}) \leq \min(\mu_{b_{ij}}, \mu_{c_{ij}})$$

and

$$\max(\nu_{a_{ij}}, \nu_{c_{ij}}) \geq \max(\nu_{b_{ij}}, \nu_{c_{ij}}).$$

So $A \wedge C \leq B \wedge C$. □

Theorem 3.6. *Let A, B and C be three intuitionistic fuzzy matrices of same order.*

- (1) *If $(A \wedge B) \vee (A \wedge C) = A$, then $A \leq (B \vee C)$.*
- (2) *If $(A \vee B) \wedge (A \vee C) = A$, then $A \geq (B \wedge C)$.*

Proof. (1) Since $A = (A \wedge B) \vee (A \wedge C)$,

$$\begin{aligned} & (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle) \\ &= (\langle \max(\min(\mu_{a_{ij}}, \mu_{b_{ij}}), \min(\mu_{a_{ij}}, \mu_{c_{ij}})), \min(\max(\nu_{a_{ij}}, \nu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{c_{ij}})) \rangle) \\ &= (\langle \min(\mu_{a_{ij}}, \max(\mu_{b_{ij}}, \mu_{c_{ij}})), \max(\nu_{a_{ij}}, \min(\nu_{b_{ij}}, \nu_{c_{ij}})) \rangle). \end{aligned}$$

Then $\min(\mu_{a_{ij}}, \max(\mu_{b_{ij}}, \mu_{c_{ij}})) = \mu_{a_{ij}}$ and $\max(\nu_{a_{ij}}, \min(\nu_{b_{ij}}, \nu_{c_{ij}})) = \nu_{a_{ij}}$. Thus

$$(3.1) \quad \mu_{a_{ij}} \leq \max(\mu_{b_{ij}}, \mu_{c_{ij}}), \text{ for all } i, j$$

and

$$(3.2) \quad \nu_{a_{ij}} \geq \min(\nu_{b_{ij}}, \nu_{c_{ij}}), \text{ for all } i, j.$$

Thus from (3.1) and (3.2), $A \leq (B \vee C)$.

(2) The proof is similar to (1). □

Theorem 3.7. *If A, B and C are three intuitionistic fuzzy matrices of same order and if $A \leq B$ and $B \wedge C = O$, then $A \wedge C = O$.*

Proof. If $A \leq B$, then $\mu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \nu_{b_{ij}}$. Thus by Theorem 3.5, $A \wedge C \leq B \wedge C$. So $A \wedge C = O$. □

Theorem 3.8. *Suppose A and B are two intuitionistic fuzzy matrices of same order. Then $A \leq B$ if and only if $B^c \leq A^c$.*

Proof. Let us assume that $A \leq B$. Then we have $\mu_{a_{ij}} \leq \mu_{b_{ij}}, \nu_{a_{ij}} \geq \nu_{b_{ij}}$, for all i, j . Thus $\nu_{b_{ij}} \leq \nu_{a_{ij}}, \mu_{b_{ij}} \geq \mu_{a_{ij}}$, for all i, j . So $B^c \leq A^c$.

The converse can be proved by the similar arguments. □

Theorem 3.9. *If A and B are two intuitionistic fuzzy matrices of same order and if $A \leq B^c$, then $B \leq A^c$.*

Proof. If $A \leq B^c$, then we have $\mu_{a_{ij}} \leq \nu_{b_{ij}}$ and $\nu_{a_{ij}} \geq \mu_{b_{ij}}$, for all i, j . Thus $\mu_{b_{ij}} \leq \nu_{a_{ij}}, \nu_{b_{ij}} \geq \mu_{a_{ij}}$, for all i, j . So $B \leq A^c$. □

Theorem 3.10. *If A and B are two intuitionistic fuzzy matrices of same order and if $A^c \leq B$, then $B^c \leq A$.*

Proof. If $A^c \leq B$, then $\nu_{a_{ij}} \leq \mu_{b_{ij}}$ and $\mu_{a_{ij}} \geq \nu_{b_{ij}}$, for all i, j . Thus $\nu_{b_{ij}} \leq \mu_{a_{ij}}, \mu_{b_{ij}} \geq \nu_{a_{ij}}$, for all i, j . So $B^c \leq A$. □

Theorem 3.11. *For any two intuitionistic fuzzy matrices A and B ,*

$$(1) A \vee (A \oplus B) = A \oplus B,$$

$$(2) A \wedge (A \oplus B) = A.$$

Proof. (1) $A \vee (A \oplus B)$

$$\begin{aligned} &= (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle) \vee (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle) \\ &= (\langle \max(\mu_{a_{ij}}, \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}}) \rangle) \\ &= (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle) \\ &= A \oplus B. \end{aligned}$$

$$\begin{aligned} (2) A \wedge (A \oplus B) &= (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle) \wedge (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle) \\ &= (\min(\mu_{a_{ij}}, \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}})) \\ &= (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle) \\ &= A. \end{aligned}$$

□

Theorem 3.12. For any two intuitionistic fuzzy matrices A and B ,

- (1) $A \vee (A \otimes B) = A$,
- (2) $A \wedge (A \otimes B) = A \otimes B$.

Proof. (1) $A \vee (A \otimes B)$
 $= (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle) \vee (\langle \mu_{a_{ij}} \mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} \nu_{b_{ij}} \rangle)$
 $= (\langle \max(\mu_{a_{ij}}, \mu_{a_{ij}} \mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} \nu_{b_{ij}}) \rangle)$
 $= (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$
 $= A$.

(2) $A \wedge (A \otimes B) = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle) \wedge (\langle \mu_{a_{ij}} \mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} \nu_{b_{ij}} \rangle)$
 $= (\langle \min(\mu_{a_{ij}}, \mu_{a_{ij}} \mu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} \nu_{b_{ij}}) \rangle)$
 $= (\langle \mu_{a_{ij}} \mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}} \nu_{b_{ij}} \rangle)$
 $= A \otimes B. \quad \square$

4. CONCLUSIONS

We formulate and proved certain inequalities connected with algebraic sum and algebraic product operations on intuitionistic fuzzy matrices. These inequalities may be extended to Interval values intuitionistic fuzzy matrices (IVIFMs).

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S. SRIRAM (ssm_3096@yahoo.co.in)

Mathematics Wing, Directorate of Distance Education, Annamalai University,
Annamalainagar - 608 002, India

J. BOOBALAN (jboobalan@hotmail.com)

Department of Mathematics, Manbumigu Dr Puratchithalaivar M.G.R.
Government Arts and Science College, Kattumannarkoil - 608 301, India