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Fuzzy multiset operations in applications

B. O. Onasanya, A. S. Sholabomi

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Abstract. This paper provides a case in which multiset operation can be used in database query. It also establishes that it is not always necessary to rearrange the fuzzy multisets before the operations of intersection and union are performed on fuzzy multisets as opposed to the claim by Syropoulos (2001). This fact is supported with some applications and examples of fuzzy multisets in medical record.

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Corresponding Author: B. O Onasanya (babtu2001@yahoo.com)

1. Introduction

Medical fitness of personnels in any organisation is key to optimal performance. The health status of such personnels is a kind of fuzzy multiset.

Set theory formulated by George Cantor is important in mathematics yet cannot effectively model so many real life problems, including the ones involving vagueness and multiplicity which are both in fuzzy multisets.

Fuzzy sets was introduced by Zadeh [25] to handle the problem of vagueness. He proposed a set in which a membership function assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of 'belongingness' to the set under consideration. More on fuzzy sets can be found in [12]. A multiset is a collection of elements in which elements are allowed to repeat; it may contain a finite number of indistinguishable copies of a particular element. More information on multisets can be obtained from [2, 3, 4, 7, 11, 19, 23]. Some applications of multisets can be found in [10, 18, 22]. As a generalization of multiset, Yager [24] introduced the concept of fuzzy multiset. Fuzzy multisets are the extensions and generalizations of multisets and fuzzy sets.

Tools that can make specific information available, out of the crowded and fuzzy lot, in shortest possible time such as structural query language (SQL), is important. This SQL can only work well if the fuzzy multiset operation is appropriately defined.

We defined such operation here and applied Corsini's hyperoperation in [5] to obtain a hypergroupoid and a multihypergtoupoid associated with it to demonstrate the correctness of the fuzzy multiset so constructed. Hence, we conclude that the reordering of fuzzy membership values to have a "membership sequence" as in [13] and "graded sequence" as in [20] is not necessary for accurate predictions. Meanwhile, some further works on the link between fuzzy sets and hyperstructure can also be found in [1, 8, 9].

2. Preliminaries

Definition 2.1 ([25])**.** Let X be a nonempty set. A Fuzzy subset A of a nonempty set X is characterized by the membership function $\mu_A : X \to [0,1]$ as $A = \{(x, \mu_A(x)) : x \in X\}$.

Definition 2.2 ([14])**.** A multiset M drawn from the set X is represented by a count function $C_M : X \to N$, where N represents the set of non-negative integers. $C_M(x)$ is the number of occurrence of the element x in the multiset M. The multiset M drawn from $X = \{x_1, x_2, ..., x_n\}$ will be represented by $M = \{x_1/m_1, x_2/m_2, \cdots, x_n/m_n\}$ where m_i is the number of occurrence of the element $x_i, (i = 1, 2, \cdots, n)$ in the multiset M.

Definition 2.3 ([20, 16])**.** Let X be a nonempty set. A fuzzy multiset A drawn from X is characterised by a "count membership" function of A denoted by CM_A such that $CM_A : X \to Q$, where Q is the set of all crisp multisets drawn from the unit interval [0, 1].

Example 2.4. Let $X = \{w, x, y, z\}$. A crisp multiset M from X is expressed as [w, x, z, w, w, x], with $C_M(w) = 3, C_M(x) = 2, C_M(y) = 0$ and $C_M(z) = 1$. A fuzzy multiset A drawn from X, where $CM_A(w) = \{0.1, 0.2\}, CM_A(x) = \{0.5\}$ and $CM_A(y) = \{0.6, 0.6, 0.8\}$, is given by $A = \{(w, 0.1, 0.2), (x, 0.5), (y, 0.6, 0.6, 0.8)\}$ which means that w with a membership 0.1, and w with 0.2, x with the membership 0.5, y with membership 0.8 and y's with 0.6 are contained in A.

Remark 2.5. For each $x \in X$, Miyamoto [13] and Syropoulos [20] claim that the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$ and is denoted by

$$\{\mu_A^1(x), \mu_A^2(x), \cdots, \mu_A^p(x)\}; \mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \mu_A^p(x).$$

It was also noted that this arrangement was necessary before the union, intersection, equality and subset of fuzzy multisets be considered. But, as shall be shown later, this approach is only necessary when comparatively studying the nature of the fuzzy multisets with some common attributes.

Definition 2.6 ([13])**.** Let A be a fuzzy multiset over a nonempty set X. The length $L(x; A)$, of $\mu_A^j(x)$ is defined by $L(x; A) = max\{j : \mu_A^j(x) \neq 0\}$ and is simply denoted $L(x)$.

Example 2.7. In **Example** 2.4 above, $L(x)$ and $L(y) = 3$.

Definition 2.8 ([20, 16])**.** Let $A, B \in FM(X)$,where $FM(X)$ refers to the set of all fuzzy multisets over a nonempty set X. The following are basic relations and operations for fuzzy multisets :

(1) **Inclusion**

$$A \subseteq B \Leftrightarrow \mu_A^j(x) \leq \mu_B^j(x), j = 1, \cdots, L(x) \text{ for all } x \in X.$$

(2) **Equality**

$$A = B \Leftrightarrow \mu_A^j(x) = \mu_B^j(x), j = 1, \cdots, L(x) \text{ for all } x \in X.$$

(3) **Union**

$$\mu_{A\cup B}^j(x) = \mu_A^j(x) \vee \mu_B^j(x), j = 1, \cdots, L(x) \text{ for all } x \in X,$$

where $\vee$ is the maximum operation.

(4) **Intersection**

$$\mu_{A\cap B}^j(x) = \mu_A^j(x) \wedge \mu_B^j(x), j = 1, \cdots, L(x) \text{ for all } x \in X,$$

where $\wedge$ is the minimum operation.

Example 2.9. $A = \{(a, 0.2), (b, 0.5), (c, 1), (c, 0.2), (a, 0.3), (c, 0.7)\}$ can be written as $A = \{(a, 0.3, 0.2), (b, 0.5), (c, 1.0, 0.7, 0.2)\}$

Definition 2.10 ([6])**.** Let H be a non empty set. The operation $\circ : H \times H \to P^*(H)$ is called a hyperoperation and $(H, \circ)$ is called a hypergroupoid, where $P^*(H)$ is the collection of all non empty subsets of H. In this case, for $A, B \subseteq H$, $A \circ B = \cup\{a \circ b \mid a \in A, b \in B\}$, where the notations $a \circ A$ and $A \circ a$ are used for $\{a\} \circ A$ and $A \circ \{a\}$ respectively. A hypergroupoid $(H, \circ)$ is called a semihypergroup if

$$a \circ (b \circ c) = (a \circ b) \circ c, \text{ for all } a, b, c \in H \text{ (Associativity)}$$

A hypergroupoid $(H, \circ)$ is called a quasihypergroup if

$$H \circ a = a \circ H = H \text{ for all } a \in H \text{ (Reproduction Axiom)}$$

3. APPLICATION TO HEALTH

Most times, there are problems in real life possessing both fuzziness and multiplicity. Such cases can be studied using fuzzy multiset theory. While it is appropriate to "rearrange" the multiset $\{\mu_{11}, \cdots, \mu_{1l}\}$ so that the elements appear in decreasing order for such operations as $\subseteq$ and $=$, we point out that such rearrangement is not always required for such operations as $\cap$ and $\cup$ in real life applications. Such rearrangement should be based on the nature of the data being handled in applications. The following examples suffice.

Example 3.1. Let $A = \{(x, 0.4, 0.5, 0.1), (y, 0.5, 0.5, 0.6), (z, 0.6, 0.5, 0.8)\}$ be a fuzzy multiset of individuals $\{x, y, z\}$ of degree of possibility of having an ailment due to changing weather in the four quarters of the year 2016 and $B = \{(x, 0.4, 0.4, 0.2),$ $(y, 0.6, 0.5, 0.5), (z, 0.4, 0.6, 0.7)\}$ for the following year. This data could even be collected for more years. Hence, we may wish to know, for each individual, in which quarter(s) are they less likely or more likely to be prone to the ailment? The intersection of these fuzzy multisets without any rearrangement gives the minimum degrees of possibility of the ailment at any quarter for each individual over the years of concern. Such result would not have been possible if there is a re-ordering of the multiset of membership values of x, y and z.

Range	Linguistic Fuzzy Sets
≤ 60	Low
60 - 100	Normal
≥ 100	High

TABLE 1. Range of values and linguistic fuzzy sets of having heart attack due to heart rate

	Q_1	Q_2	Q_3	Q_4
E_1	90bpm	100bpm	90bpm	110bpm
E_2	80bpm	90bpm	85bpm	60bpm
E_3	40bpm	100bpm	94bpm	90bpm

TABLE 2. Employee's quarterly maximum heart rate record,say in year 2016

	Q_1	Q_2	Q_3	Q_4
E_1	0.75	1	0.75	0.25
E_2	0.5	0.75	0.63	0
E_3	0	1	0.85	0.75

TABLE 3. Employee's degree of having heart attack due to heart rate for different quarters in the year 2016

Remark 3.2. To be more specific, let $E = \{E_1, E_2, E_3\}$ be the set of employees working in a company and $A = \{HeartRate, Systolic\ Blood\ Pressure\}$ be the set of some major factors that causes heart attack. The membership function could be construction and used to generate the needed fuzzy multisets. Similar to what we earlier, the degree of possibility of heart attack could be deduced at any particular quarter of the year.

Membership functions of Heart Rate

Heart rates are the speed at which the heart beats per minute(bpm). The average resting heart rate is 60-100 bpm. However, a lower resting heart rate (Bradycardia) is a sign of good heath. If on the other hand the heart rate is higher (Tarchycadic), it is a sign of bad heath and the closer it is for the attack of the heart to occur.

Sikchi *et al* [17] gave fuzzy sets of some of these factors but not in a way that relates them to how they cause heart attack. But here, fuzzy multiset membership functions were constructed in a way that they relate to the degrees of possibility of occurrence of heart attack. Each data is assumed to be the maximum in each quarter.

	Q_1	Q_2	Q_3	Q_4
E_1	100bpm	102bpm	96bpm	84bpm
E_2	83bpm	95bpm	90bpm	58bpm
E_3	45bpm	100bpm	100bpm	100bpm

TABLE 4. Employees' quarterly maximum heart rate record, say in year 2017

	Q_1	Q_2	Q_3	Q_4
E_1	1	1	0.9	0.6
E_2	0.56	0.88	0.75	0
E_3	0	1	1	1

TABLE 5. Employees' degree of having heart attack due to heart rate for different quarters in year 2017

Range	Linguistic Fuzzy sets
≤ 90	Low
90 - 120	Normal
120 - 140	Pre-High
≥ 140	High

TABLE 6. Range of values and linguistics fuzzy sets of having heart attack due to systolic blood pressure

Membership functions of degree of possibility of heart attack due to heart rate

$$\mu(x) = \begin{cases} 0, \ \ x \leq 60 \\ \dfrac{x-60}{40}, \ \ 60 < x < 100 \\ 1, \ \ x \geq 100 \end{cases} \tag{3.1}$$

Table 3 shows the degree of possibility to which an employee $E_j(i = 1, 2, 3)$ can have heart attack due to heart rate in $Q_i(i = 1, 2, 3)$ quarter.

Membership functions of Systoblic Blood Pressure

Blood pressure is one important parameter in diagnosing heart disease. It is categorized in *systolic* and *diastolic*. It suffices to use the systolic blood pressure. Any blood pressure from 140 mmHg (for systolic) or more than 90 (for diastolic) is considered high.

	Q_1	Q_2	Q_3	Q_4
E_1	100mmHG	125mmHG	110mmHG	110mmHG
E_2	130mmHG	110mmHG	130mmHG	120mmHG
E_3	150mmHG	134mmHG	140mmHG	130mmHG

TABLE 7. Employees' quarterly maximum systolic blood pressure record in the year 2016

	Q_1	Q_2	Q_3	Q_4
E_1	0	0.25	0	0
E_2	0.5	0	0.5	0
E_3	1	0.7	1	0.5

TABLE 8. Employees' degree of having heart attack due to systolic blood pressure for different quarters in 2016

	Q_1	Q_2	Q_3	Q_4
E_1	105mmHG	120mmHG	130mmHG	100mmHG
E_2	125mmHG	109mmHG	100mmHG	80mmHG
E_3	135mmHG	106mmHG	75mmHG	110mmHG

TABLE 9. Employees' quarterly maximum systolic blood pressure, say in the 2017

Membership functions for degree of possibility of heart attack due to systolic blood pressure

$$\mu(x) = \begin{cases} 0, \;\; x \leq 120 \\ \dfrac{x-120}{20}, \;\; 120 < x < 140 \\ \dfrac{x-140}{180-140} \;\; 140 \leq x < 180 \\ 1, \;\; x \geq 140 \end{cases} \tag{3.2}$$

Table 8 shows the degree of possibility to which an employee E_j can have heart attack due to systolic blood pressure in Q_i Quarters.

Table 10 shows the degree of possibility to which an employee E_j can have heart attack due to systolic blood pressure in Q_i Quarters.

All the tables of the employees and their respective membership values for systolic blood pressure and heart rate are connected together through Structural Query Language (SQL). Further operations could be performed on the retrieved data. The

	Q_1	Q_2	Q_3	Q_4
E_1	0	0	0.5	0
E_2	0.25	0	0	0
E_3	0.75	0	0	0

Table 10. Employees' degree of having heart attack due to systolic blood pressure for different quarters in 2017

	HEART RATE	BLOOD PRESSURE
E_1	(0.75, 1, 0.75, 0.25)	(0, 0.25, 0, 0)
E_2	(0.5, 0.75, 0.63, 0)	(0.5, 0, 0.5, 0)
E_3	(0, 1, 0.85, 0.75)	(1, 0.75, 1, 0.5)

Table 11. Fuzzy multisets of employees' major attributes that lead to heart attack in the year 2016

	HEART RATE	BLOOD PRESSURE
E_1	(1, 1, 0.9, 0.6)	(0, 0, 0.5, 0)
E_2	(0.56, 0.88, 0.75, 0)	(0.25, 0, 0, 0)
E_3	(0, 1, 1, 1)	(0.75, 0, 0, 0)

Table 12. Fuzzy multisets of employees' major attributes that lead to heart attack in the year 2017

fuzzy multisets for heart rate and blood pressure respectively for year 2016 is given below:

$Y_{2016hr} = \{(E_{1hr}, 0.75, 1, 0.75, 0.25), (E_{2hr}, 0.5, 0.75, 0.63, 0), (E_{3hr}, 0.1, 0.85, 0.75)\}$ and $Y_{2016bp} = \{(E_{1bp}, 0, 0.25, 0, 0), (E_{2bp}, 0.5, 0, 0.5, 0), (E_{3bp}, 1, 0.75, 1, 0.5)\}$.

The fuzzy multisets for Heart Rate and Blood Pressure respectively for year 2017 is given below:

$$Y_{2017hr} = \{(E_{1hr}, 1, 1, 0.9, 0.6), (E_{2hr}, 0.56, 0.88, 0.75, 0), (E_{3hr}, 0, 1, 1, 1)\}$$

and $Y_{2017bp} = \{(E_{1bp}, 0, 0, 0.5, 0), (E_{2hr}, 0.25, 0, 0, 0), (E_{3hr}, 0.75, 0, 0, 0)\}$. Therefore, intersection of Y_{2016hr} and Y_{2017hr} is given by

$$\{(E_{1hr}, 0.75, 1, 0.75, 0.25), (E_{2hr}, 0.5, 0.75, 0.63, 0), (E_{3hr}, 0, 1, 0.85, 0.75)\}$$

and that of Y_{2016br} and Y_{2016br} is

$$\{(E_{1br}, 0, 0, 0, 0), (E_{2br}, 0.25, 0, 0, 0), (E_{3br}, 0.75, 0, 0, 0)\}.$$

4. HYPERSTRUCTURES FROM FUZZY MULTISET

In this section, attention is drawn to demonstrating the effectiveness of the fuzzy multisets constructed in this work by comparing the associated hyperstructures with those obtained from the hyperoperations constructed by [5].

$\bigotimes_\mu$	110	120	130
110	110	110, 120	110, 120, 130
120	110, 120	120	110, 120, 130
130	110, 120, 130	110, 120, 130	130

TABLE 13. Hypergroupoid due to $\bigotimes_\mu$

$\bigotimes_\mu^p$	110	120	130
110	110	110, 120	110, 130, 130
120	110, 120	120	110, 120, 130, 130
130	110, 120, 130, 130	110, 120, 130, 130	110, 130, 130

TABLE 14. Multihypergroupoid due to $\bigotimes_\mu^p$

Definition 4.1 ([5])**.** Given a fuzzy multiset $(\mathcal{H}, \mathfrak{A})$, where $\mathfrak{A}$ is a function $\mathfrak{A} : \mathcal{H} \to I \times I\!N$ such that for any $x \in \mathcal{H}$ there is the pair $(\mu(x), m(x))$, where $\mu(x) \in [0, 1]$ and $m(x) \in I\!N$. For any $x, y, z \in H$, define operations:

(i) $x \bigotimes_\mu y = \{z : min\{\mu(x), \mu(y)\} \leq \mu(z) \leq max\{\mu(x), \mu(y)\}\}$;

(ii) $x \oslash_m y = \{z : min\{m(x), m(y)\} \leq m(z) \leq max\{m(x), m(y)\}\}$;

(iii) $x_m \boxtimes_\mu y = x \bigotimes_\mu y \cap x \oslash_m y$.

Then, $(\mathcal{H}, \bigotimes_\mu)$, $(\mathcal{H}, \oslash_m)$ and $(\mathcal{H},_m \boxtimes_\mu)$ are hypergroupoids.

Also, for any of the hyperoperations $\bigotimes_\mu$, $\oslash_m$ and ${}_m\boxtimes_\mu$, use an arbitrary operation "$\circ$" and define $x \circ^p y = \{z/p(z) : z \in x \circ y\}$.

Definition 4.2 ([5])**.** For any $x, y, z \in \mathcal{H}$, define operations:

(i) $x \bigotimes_\mu^p y = \{z/p(z) : z \in x \bigotimes_\mu y\}$;

(ii) $x \bigotimes_m^p y = \{z/p(z) : z \in x \oslash_m y\}$;

(iii) $x \boxtimes_\mu^p y = \{z/p(z) : z \in x \bigotimes_\mu y \cap x \oslash_m y\}$.

Then, $(\mathcal{H}, \bigotimes_\mu^p), (\mathcal{H}, \oslash_m^p)$ and $(\mathcal{H}, \boxtimes_\mu^p)$ are multihypergroupoids.

Example 4.3. Consider the multiset $\mathcal{M} = \{130, 110, 130, 120\}$ of blood pressure of employer E_2 in the year 2016 from Table 7. Using the fuzzy set in Table 8, the results in Tables 13 and 14 are respectively the hypergroupoid and multihypergroupoid obtained respectively from the hyperoperations $\bigotimes_\mu$ and $\bigotimes_\mu^p$.

Indeed, $(\mathcal{M}, \bigotimes_\mu)$ is a hypergroup. In this particular case, it can be shown as regards the hyperoperations $\oslash_m$ and $\oslash_m^p$ respectively that the hypergroupoid and multihypergroupoid are such that $(\mathcal{M}, \bigotimes_\mu) = (\mathcal{M}, \oslash_m)$ and $(\mathcal{M}, \bigotimes_\mu^p) = (\mathcal{M}, \oslash_m^p)$

5. Conclusions

The intersection obtained without rearranging the multisets will give the minimum degree of possibility to which an employee can have heart attack in each quarter for the two years. It is easier to conclude that, for the two consecutive years (or more years as the case may be), a high chance of heart attack due to heart rate occurs for E_1 and E_2 within the first three quarters, while it occurs for E_3 within the last three quarters. Also, E_1 and E_2 have no threat to have heart attack due to blood pressure throughout the year but E_3 has it within the first quarter.

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ONASANYA Babatunded Oluwaseun (babtu2001@yahoo.com) – Department of Mathematics, University of Ibadan, Nigeria.

SHOLABOMI Adetunji Samuel (tunji86ng@gmail.com) – Department of Mathematics, University of Ibadan, Nigeria.