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Inverse domination in bipolar fuzzy graphs

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Abstract. In this paper we introduced and investigated the concept of inverse domination in bipolar fuzzy graph G and denoted by $\gamma^{`}(G)$. We obtained many results related to $\gamma^{`}(G)$. Finally we introduced and study the relationship of $\gamma^{`}(G)$ with some others parameters in bipolar fuzzy graph

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1. Introduction

Graph theory has numerous applications to the problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, transportation, etc. In 1973, Kaufmann [10] introduced definition of fuzzy graphs. Rosenfeld [12] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness, etc. The concept of domination in fuzzy graphs was investigated by Somasundaram and Somasundaram [14], and A. Somasundaram [15] presented the concepts of independent domination, total domination, connected domination of fuzzy graphs. Akram introduced the concept of bipolar fuzzy graphs, he discussed the concept of isomorphism of these graphs, and investigated some of their important properties, also defined some operations on bipolar fuzzy graphs [1, 2, 3, 4, 5]. The concepts of domination in bipolar fuzzy graphs was investigated by Karunambigai et al. [9]. The concept of inverse domination in fuzzy graphs was investigated by Ghobadi et al. [6]. Because of the wide application of inverse domination in the real live, such as backup stations for radio broadcasting, reserve stations for electricity networks, computer networks and the internet network we will study and investigate the concept of the inverse domination concept in bipolar fuzzy graphs. The rapid growth of research in this area is due to the following: 1.

Due to its importance in the practical side of electricity and internet networks, etc. 2. The wide variety of domination parameters that can be defined in bipolar fuzzy graphs. 3. Due to the wide variety of application of domination in bipolar fuzzy graphs. 4. Due to the work of domination on fuzzy graph and bipolar fuzzy graph. In this paper we will introduce and investigate the concept of inverse domination in bipolar fuzzy graphs and we will obtain many result related to this concept. The relationship between this concept and the others in bipolar fuzzy graph will be given. Finally, we introduce the concept of inverse domatic number in bipolar fuzzy graph

2. Preliminaries

In this section, we review some basic definitions related to bipolar fuzzy graphs and inverse domination in fuzzy graph. A bipolar fuzzy graph (briefly, BFG), G is of the form $G = (V, E)$, where

(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu^+ : V \to [0, 1]$ and $\mu^- : V \to [-1, 0]$,

(ii) $E \subset V \times V$, where $\rho^+ : V \times V \to [0, 1]$ and $\rho^- : V \times V \to [-1, 0]$ such that

$$\rho^+ = \rho^+(v_i, v_j) \leq \mu^+(v_i) \wedge \mu^+(v_j) \text{ and } \rho^- = \rho^-(v_i, v_j) \geq \mu^-(v_i) \vee \mu^-(v_j),$$

for all $v_i, v_j \in V$.

In a bipolar fuzzy graph G, if $\rho^+ = \rho^- = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise, there exists an edge between v_i and v_j.

A bipolar fuzzy graph, $BFG\ G = (V, E)$ is said to be $semi - \rho^+$ strong bipolar fuzzy graph, if $\rho^+ = min(\mu_i^+, \mu_j^+)$ for every i and j.

A bipolar fuzzy graph $BFG\ G = (V, E)$ is said to be $semi - \rho^-$ strong bipolar fuzzy graph, if $\rho^- = max(\mu_i^-, \mu_j^-)$, for every i and j.

Let $G = (V, E)$ be a bipolar fuzzy graph. Then

(i) the cardinality of G is defined to be

$$|G| = \sum_{v_i \in V} \frac{1 + \mu^+(v_i) + \mu^-(v_i)}{2} + \sum_{(v_i, v_j) \in E} \frac{1 + \rho^+(v_i, v_j) + \rho^-(v_i, v_j)}{2},$$

(ii) the vertex cardinality of G is defined by

$$|V| = \sum_{v_i \in V} \frac{1 + \mu^+(v_i) + \mu^-(v_i)}{2}.$$

The total number of different members in V is called the order of a bipolar fuzzy graph is denoted by $P(G)$.

The edge cardinality of a bipolar fuzzy graph G is defined by

$$|E| = \sum_{(v_i, v_j) \in E} \frac{1 + \rho^+(v_i, v_j) + \rho^-(v_i, v_j)}{2}.$$

The total number of different members in E is called the size of a bipolar fuzzy graph is denoted by $q(G)$.

An edge $e = (x, y)$ of a bipolar fuzzy graph is called an strong edge, if

$$\rho^+(x, y) = min\{\mu^+(x), \mu^+(y)\} \text{ and } \rho^-(x, y) = max\{\mu^-(x), \mu^-(y)\}.$$

The degree of a vertex can be generalized in different ways for a bipolar fuzzy graph $G = (V, E)$. The effective degree of a vertex v in a bipolar fuzzy graph,

$G = (V, E)$ is defined to be sum of the weights of the strong edges incident at v and it is denoted by $d_E(v)$.

The minimum effective edges degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$ and the maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$.

The vertex v is adjacent to a vertex u, if they reach between the strong edge.

Two vertices v_i and v_j are said to be neighbors in a bipolar fuzzy graph $G = (V, E)$, if either one of the following conditions holds:

(i) $\rho^+(v_i, v_j) > 0$ and $\rho^-(v_i, v_j) > 0$,

(ii) $\rho^+(v_i, v_j) = 0$ and $\rho^-(v_i, v_j) < 0$,

(iii) $\rho^+(v_i, v_j) > 0$ and $\rho^-(v_i, v_j) = 0, v_i, v_j \in V$.

And they are strong neighbor, if $\rho^+(v, u) = min\{\mu^+(v), \mu^+(u)\}$ and $\rho^-(v, u) = max\{\mu^-(v), \mu^-(u)\}$. A vertex subset $N(v) = \{u \in V : v \textit{ adjacent to } u\}$ is called the open neighborhood set of a vertex v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v. The neighborhood degree of a vertex v in a bipolar fuzzy graph, $G = (V, E)$ is defined to be sum of the weights of the vertices adjacent to v, and it is denoted by $d_N(v)$, that is mean that $d_N(v) = |N(v)|$.

The minimum neighborhood degree of G is $\delta_N(G) = min\{d_N(v)|v \in V\}$ and the maximum neighborhood degree of G is $\Delta_N(G) = max\{d_N(v)|v \in V\}$.

A bipolar fuzzy graph, $G = (V, E)$ is said to be complete bipolar fuzzy graph, if

$$\rho^+(v_i, v_j) = min\{\mu^+(v_i), \mu^+(v_j)\},\ \rho^-(v_i, v_j) = max\{\mu^-(v_i), \mu^-(v_j)\},$$

for all $v_i, v_j \in V$ and is denoted by K_p.

The complement of a bipolar fuzzy graph, $G = (V, E)$ is a bipolar fuzzy graph, $\overline{G} = (\overline{V}, \overline{E})$ where

(i) $\overline{V} = V$,

(ii) $\overline{\mu^+} = \mu^+$, $\overline{\mu^-} = \mu^-$ for all vertex,

(iii) $\overline{\rho^+} = min\{\mu_i^+, \mu_j^+\} - \rho^+$ and $\overline{\rho^-} = max\{\mu_i^-, \mu_j^-\} - \rho^-$,

for all $i, j = 1, 2, 3, \cdots, n$.

A bipolar fuzzy graph $G = (V, E)$ is said to bipartite, if the vertex set V of G can be partitioned into two non empty sets V_1 and V_2 such that

(i) $\rho^+(v_i, v_j) = 0$ and $\rho^-(v_i, v_j) = 0$, if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$,

(ii) $\rho^+(v_i, v_j) > 0$ and $\rho^-(v_i, v_j) < 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j or $\rho^+(v_i, v_j) = 0$ and $\rho^-(v_i, v_j) < 0$, if $v_i \in V_1$ and $v_j \in V_2$ or $\rho^+(v_i, v_j) > 0, \rho^-(v_i, v_j) = 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j.

A bipartite bipolar fuzzy graph $G = (V, E)$ is said to be complete bipartite bipolar fuzzy graph, if $\rho^+(v_i, v_j) = min\{\mu^+(v_i), \mu^+(v_j)\}$ and $\rho^-(v_i, v_j) = max\{\mu^-(v_i), \mu^-(v_j)\}$ for all $v_i \in V_1$ and $v_j \in V_2$. Its denoted by $K_{m,n}$, where $|V_1| = m, |V_2| = n$.

A vertex $u \in V$ of a bipolar fuzzy graph, $G = (V, E)$ is said to be an isolated vertex, if $\rho^+(v, u) = 0$ and $\rho^-(v, u) = 0$, for all $v \in V$. That is $N(u) = \phi$. Thus an isolated vertex does not dominate any other vertex in G.

A bipolar fuzzy graph, $G = (V, E)$ is said to be strong bipolar fuzzy graph, if $\rho^+ = min(\mu_i^+, \mu_j^+)$, for every i and j and $\rho^- = max(\mu_i^-, \mu_j^-)$, for all $(v_i, v_j) \in E$.

A bipolar fuzzy graph, $H = (X, Y)$, is said to be a bipolar fuzzy subgraph of $G = (V, E)$, if $X \subseteq V$, and $Y \subseteq E$. That is, $\grave{\mu}^+ \leq \mu_i^+$, $\mu^- \geq \mu^-$ and $\grave{\rho}^+ \leq \rho^+$, $\grave{\rho}^- \geq \rho^-$.

A subset $(D \subseteq V)$ of $V(G)$ is called dominating set in bipolar fuzzy graph BFG, if for each $v \in V - D$, there exists $u \in D$ such that (u, v) is a strong edge.

A dominating set D of (BFG) is called minimal dominating set, if $D - u$ is not dominating set for every $u \in D$. A minimal dominating set D with $|D| = \gamma(G)$ is denoted by $\gamma - set$.

Let D be γ-set of G. If $V - D$ contains $\gamma - set$, $D^{\backprime}$ of (G), then $D^{\backprime}$ is called an inverse dominating set with respect to D of G.

An inverse dominating set $D^{\backprime}$ of G, is called minimal inverse dominating set, if $D^{\backprime} - \{u\}$ is not inverse dominating set, for every $u \in D^{\backprime}$.

The minimum cardinality among all minimal inverse dominating sets of G, is called the inverse domination number of G and denoted by $\gamma^{\backprime}(G)$ or $\gamma^{\backprime}$.

3. Inverse domination in bipolar fuzzy graphs

Definition 3.1. let D be $\gamma - set$ of BFG G. If $V - D$ contains $\gamma - set$, $D^{\backprime}$, then $D^{\backprime}$ is called an inverse dominating set with respect to D of G.

Definition 3.2. An inverse dominating set $D^{\backprime}$ of BFG G is called minimal inverse dominating set, if $D^{\backprime} - \{u\}$ is not inverse dominating set, for every $u \in D^{\backprime}$.

Definition 3.3. The minimum cardinality among all minimal inverse dominating sets of BFG G is called the inverse dominating number of BFG G and denoted by $\gamma^{\backprime}(G)$ or $\gamma^{\backprime}$.

Definition 3.4. An inverse dominating set of BFG G with $|D^{\backprime}| = \gamma^{\backprime}(G)$, is denoted by $\gamma^{\backprime} - set$.

Remark 3.5. For any complete BFG G, $\gamma^{\backprime}(G) = min\{|v|; v \in V - D\}$.

Example 3.6. For the bipolar fuzzy graph G in Figure 3.1 such that every edge is strong edge

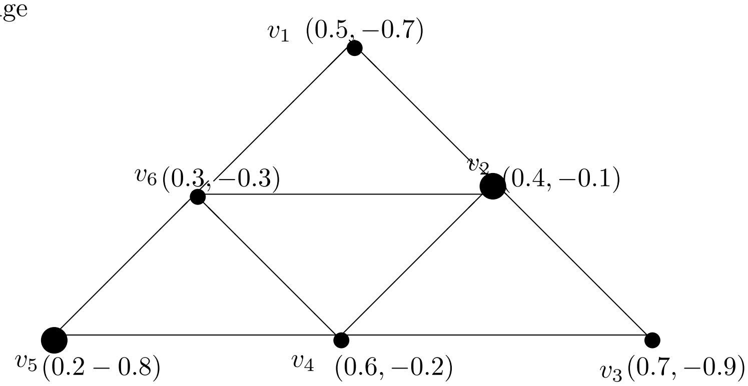

Figure 3.1

$D = \{v_2, v_5\}$ is dominating set. Then $V - D = \{v_1, v_3, v_4, v_6\}$. Thus $D^{\backprime} = \{v_3, v_6\}$ is minimal dominating set with minimum fuzzy cardinality. So $\gamma^{\backprime}(G) = 0.9$.

Theorem 3.7. *For any BFG G with at least one isolated vertex, $\gamma^{\backprime}(G) = 0$.*

Proof. Let $G = (V, E)$ be any BFG, x be an isolated vertex in G and let D be $\gamma - set$ of G. Then $x \in D$. Now, Suppose that G has an inverse dominating set say $D^{\backprime}$ and suppose that $D^{\backprime} \neq \phi$. Then every vertex in $D^{\backprime}$ is not isolated. Since $D^{\backprime} \subseteq V - D$, for every vertex $y \in D^{\backprime}$, there exists at least one vertex $x \in D$, adjacent to a vertex y in $D^{\backprime}$, which is a contradiction. □

Theorem 3.8. *For any BFG G, with* $\gamma^{\backprime} - set$*,*

$$\gamma^{\backprime}(G) + \gamma(G) \leq p.$$

Furthermore, equality holds if $V - D$ *is an independent.*

Proof. Let G be BFG, let D be $\gamma - set$ of G and $D^{\backprime}$ be $\gamma^{\backprime} - set$ with respect to D of G. Then $D^{\backprime} \subseteq V - D$. Thus $|D^{\backprime}| \leq |V - D|$. So $\gamma^{\backprime}(G) \leq p - \gamma(G)$. Hence $\gamma^{\backprime}(G) + \gamma(G) \leq p$. □

Theorem 3.9. *For any strong BFG,* $\gamma^{\backprime} \leq \frac{p}{2}$.

Proof. Since $\gamma \leq \frac{p}{2}$ and $\gamma(G) \leq \gamma^{\backprime}$, by Theorem 3.8, $\gamma^{\backprime}(G) \leq p - \frac{p}{2} = \frac{p}{2}$. □

Example 3.10. For the strong bipolar fuzzy graph G in Figure 3.2.

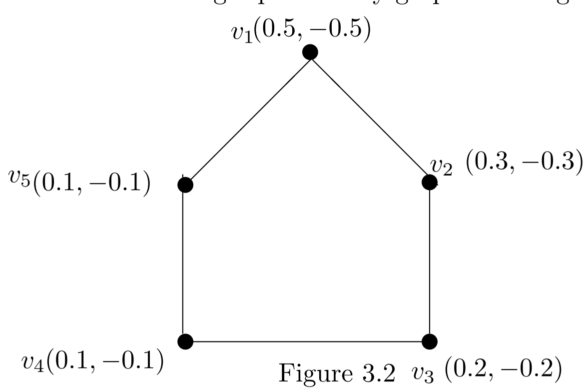

Figure 3.2

Then clearly, $D^{\backprime} \leq \frac{P}{2}$.

The condition in above theorem is convincing because if $V - D$ independent then this theorem not realized.

Example 3.11. For the strong bipolar fuzzy graph G in Figure 3.3.

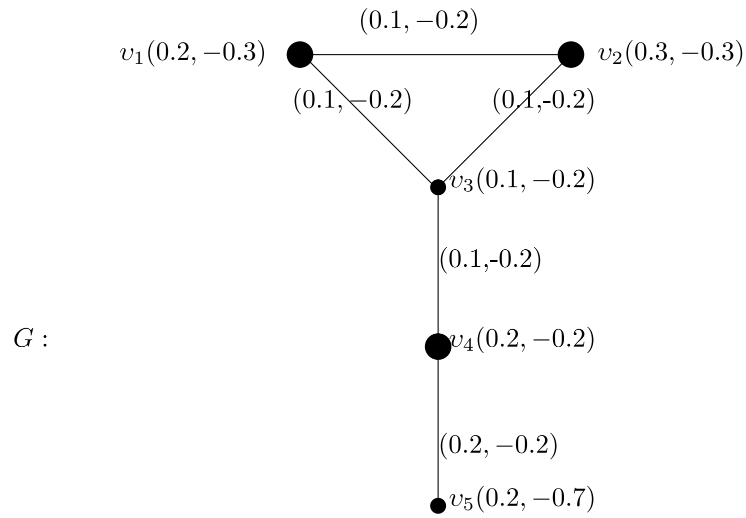

Figure 3.3

$D = \{v_3, v_5\}$ is a $\gamma - set$ of G, Then $D' = V - D = \{v_1, v_2, v_4\}$ is independent. Thus $\gamma' \nleq \frac{P}{2}$.

Theorem 3.12. *An inverse dominating set D' of BFG G, is minimal inverse dominating set if and only if one of the following condition holds:*

(1) $N(v) \cap D' = \phi$,
(2) *there is a vertex $u \in V - D'$ such that $N(u) \cap D' = \{v\}$.*

Proof. Let G be BFG, let D' be $\gamma' - set$ of G and let $v \in D'$. Then $D' - \{v\}$ is not dominating set of G and there exists a vertex $u \in V - \{D' - \{v\}\}$ such that u is not dominated by any vertex of $D' - \{v\}$. If $u = v$, then $N(v) \cap D' = \phi$. If $u \neq v$, then $N(u) \cap D' = \{v\}$. Thus (1) or (2) holds.

Conversely, Suppose that D' is an inverse dominating set and for each a vertex $v \in D'$, one of two conditions holds. Suppose that D' is not minimal inverse dominating set. Then there is a vertex $v \in D'$ such that $D' - \{v\}$ is an inverse dominating set. Thus v is adjacent to at least one vertex in $D' - \{v\}$. So the condition (1) dose not hold. If $D' - \{v\}$ is inverse dominating set, then every vertex in $V - D'$ is adjacent to at least one vertex in $D' - \{v\}$. Thus the condition (2) dose not hold, which a contradiction. Hence D' is minimal inverse dominating set. □

Theorem 3.13. *For any BFG G without isolated vertices, $\gamma'(G) \leq \beta_0(G)$*

Proof. let G be BFG and D be a vertex covering set of G. Then D is dominating set of G. Thus $V - D$ is an inverse dominating set in BFG. So $\gamma'(G) \leq |V - D| = P - \alpha_0(G) = \beta_0(G)$ □

Theorem 3.14. *For any BFG G, $\gamma' \leq p - \Delta_N$.*

Proof. Let G be BFG and let v be a vertex in G with $d_N(v) = |N(v)| = \Delta_N$. Then $V - N(v)$ is $\gamma - set$ and $D' \subseteq V - N(v)$,then $|D'| \leq |V - N(v)| = p - |N(v)|$. Thus $\gamma'(G) \leq p - \Delta_N \leq p - \delta_E$. □

Corollary 3.15. *$\gamma^{\backprime} \leq p - \Delta_E$ and $\gamma^{\backprime} \leq p - \delta_N$.*

Proof. Since $\Delta_E \leq \Delta_N$ for any fuzzy graph, by Theorem 3.14, $\gamma^{\backprime} \leq p - \Delta_E$. Similarly, since $\delta_N \leq \Delta_N$, $p - \Delta_N \leq p - \delta_N$. Thus $\gamma^{\backprime} \leq p - \delta_N$. □

Theorem 3.16. *Let $G = (V, E)$ be a complete bipartite bipolar fuzzy graph, then $\gamma^{\backprime}(G) = min\{|v|; v \in V_1 - D\} + min\{|u|; u \in V_2 - D\}$.*

Proof. Let D be dominating set of G. Then $\gamma(G) = min\{|v|; v \in V_1\} + min\{|u|; u \in V_2\}$. Let $x \in V_1$ and $y \in V_2$ such that $x, y \notin D$. Then by Definition 3.1 and the definition of complete bipolar bipartite, we get

$$\gamma^{\backprime}(G) = min\{|x|; x \in V_1 - D\} + min\{|y|; y \in V_2 - D\}.$$

□

Theorem 3.17. *For any BFG G, $\gamma^{\backprime}(G) \leq \Gamma(G)$. Furthermore, the equality holds if G is a path.*

Proof. The inequality is trivial. Now let P be a path in $BFG\ G = (V, E)$ and let D be $\gamma^{\backprime} - set$. Since every path contains only two dominating sets say D and $V - D$. Then $D^{\backprime} = V - D$. Thus $\gamma^{\backprime}(P) = |V - D| = \Gamma(P)$, □

Definition 3.18. A bipolar fuzzy graph G is called wheel, if $V(G)$ can be partitioned in to two vertex subsets $V_1 = \{v_1, v_2, \ldots, v_n\}$ and $V_2 = \{u\}$ such that V_1 is independent and u adjacent to every vertex v_i in V_1, and is denoted by $W_P = W_{t,P-1}$, where $t = |u|$. In this case, u is called a root vertex.

Theorem 3.19. *Let G be a wheel BFG. Then $\gamma^{\backprime}(W_P) = \gamma(C_{p-t})$.*

Proof. Let $D^{\backprime}$ be $\gamma^{\backprime} - set$ of G with respect to D. Then $D^{\backprime} \subset V - D$. Since $W_P = C_{P-t} \cup \{u\}$ such that u is a root vertex, $D = \{|u| = t\}$ and $\gamma(W_P) = |D| = |u|$. Thus $D^{\backprime} \subseteq V(C) + \{u\} - \{u\} = V(C)$. So $D^{\backprime} \subseteq V(C_{P-t})$. Let S be $\gamma - set$ of C_{P-t}. Then $\gamma(C_{P-t}) = |S|$. Since $D^{\backprime}$ is minimal dominating set of G with minimum fuzzy cardinality and $D^{\backprime} \subseteq V(C_{P-t})$, $D^{\backprime} = S$. Thus $|D^{\backprime}| = |S|$. □

Theorem 3.20. *For any BFG G with at least one inverse dominating set,*

$$\gamma(G) \leq \frac{p + \gamma^{\backprime}(G)}{3}.$$

Proof. Let $G = (V, E)$ be BFG with $\gamma^{\backprime} - set$, since $\gamma \leq \gamma^{\backprime}(G)$ and $\gamma \leq \frac{p}{2}$. Then the result holds. □

Theorem 3.21. *Every inverse dominating set of BFG is inverse dominating set of a crisp graph G^*, but the converse is not true.*

Proof. Let $G = (V, E)$ be BFG on $V(G)$, with inverse dominating set $D^{\backprime}$ with respect to D of G. Since $\gamma - set$ D of a fuzzy graph is $\gamma - set$ of a crisp G^* (See Theorem 5.2.1 in [11]), $\gamma - set$ of BFG is $\gamma - set$ in G^*. Then D is dominating set of G^*. By Definition 3.1, $D^{\backprime} \subseteq V - D$. Thus for every vertex $u \in V - D$, $u \in V - D$ in G^*. Also any vertex $x \in D^{\backprime}$, $x \in V - D$. So $x \in V - D$ in G^*. Hence $D^{\backprime}$ is an inverse dominating set in (G^*). □

In the following example, we show that the conversely Theorem 3.21 is not true.

Example 3.22. For the bipolar fuzzy graph G in Figure 3.4.

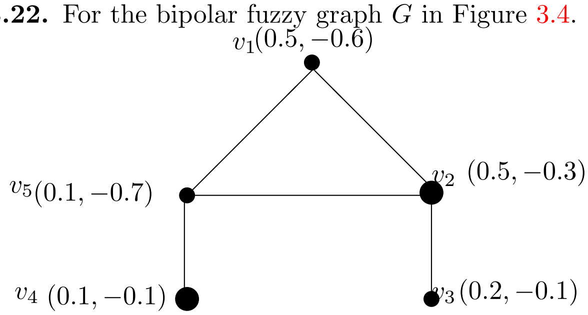

Figure 3.4

Then $D = \{v_2, v_5\}$ is dominating of G^* and $D'_1 = \{v_1, v_4, v_3\}$ is inverse dominating set of G^*. But $D' = \{v_2, v_4\}$ is inverse dominating set of BFG.

Theorem 3.23. *Let G be BFG, with inverse dominating of G. Then $\gamma'(G) \leq \gamma'(G^*)$. Furthermore, the equality holds, if $|v| = 1$, $\forall v \in V(G)$.*

Proof. Let G be BFG with γ'-set D' of G with respect to D. Then $\gamma(G) \leq \gamma'(G)$. Since $\gamma(G) \leq \gamma(G^*)$, by Theorem 5.2.1 in [14], $\gamma'(G) \leq \gamma(G^*)$. Since $\gamma(G^*) \leq \gamma'(G^*)$, $\gamma'(G) \leq \gamma'(G^*)$. □

Theorem 3.24. *For any BFG G, $\gamma'(G) + \gamma'(\bar{G}) \leq p$.*

Proof. Since $\gamma(G) + \gamma'(G) \leq p$, $\gamma'(G) \leq p - \gamma(G)$, and also $\gamma'(\bar{G}) \leq p - \gamma(\bar{G})$. Then $\gamma'(G) + \gamma'(\bar{G}) \leq 2p - (\gamma(G) + \gamma(\bar{G}))$. Since $\gamma(G) \leq \frac{p}{2}$ and $\gamma(\bar{G}) \leq \frac{p}{2}$,

$$\gamma'(G) + \gamma'(\bar{G}) \leq 2p - p = p.$$

□

Definition 3.25. Let $G = (V, E)$ be BFG and let D' be inverse dominating set of G with respect to D, if the induced subgraph $< V - D' >$ satisfies the following:

(i) if $< V - D' >$ is disconnected, then D' is called split inverse dominating set of G,

(ii) if $< V - D' >$ is connected, then D' is called non-split inverse dominating set of G.

Definition 3.26. A split (non-split) inverse dominating set D' $\gamma'_s - set$ ($\gamma'_{ns} - set$) of G are said to be minimal split (non-split) inverse dominating sets, if $D' - \{v\}$ is not split (not non-split) dominating set,$\forall \{v\} \in D$.

Definition 3.27. (i) The minimum fuzzy cardinality among over all minimal split inverse dominating sets is called an inverse split domination number and is denoted by $\gamma'_s(G)$ or γ'_s.

(ii) The inverse non-split dominating number is the minimum fuzzy cardinality among over all minimal inverse non-split dominating sets and is denoted by $\gamma'_{ns}(G)$ or γ'_{ns}.

Theorem 3.28. *For any BFG G,* $\gamma^{\backprime}(G) \leq \gamma^{\backprime}_{s}(G) \leq \gamma^{\backprime}_{ns}(G)$.

Proof. Let $G = (V, E)$ be BFG, and $D^{\backprime}$ be a split inverse dominating set of G. Then $< V - D^{\backprime} >$ is disconnected. Since every split inverse dominating set is inverse dominating set of G, $\gamma^{\backprime}(G) \leq \gamma^{\backprime}_{s}(G)$. Sinnce $\gamma^{\backprime}_{s}(G) \leq \gamma^{\backprime}_{ns}(G)$, $\gamma^{\backprime}(G) \leq \gamma^{\backprime}_{s}(G) \leq \gamma^{\backprime}_{ns}(G)$. □

Theorem 3.29. *For any BFG* $G = (V, E)$*,*

$$\gamma^{\backprime}(G) \leq \frac{P.\Delta_N(G)}{\Delta_N(G) + 1}.$$

Proof. Let G be BFG and $D^{\backprime}$ be an inverse dominating set of G with respect to D. Then by Theorem 3.2.5 in [11],

$$\gamma(G) \geq \frac{P}{\Delta_N(G) + 1}.$$

Since $\gamma(G) \leq P - \gamma^{\backprime}(G)$,

$$\frac{P}{\Delta_N(G) + 1} \leq P - \gamma^{\backprime}.$$

Thus

$$\gamma^{\backprime}(G) \leq \frac{P\Delta_N(G)}{\Delta_N(G) + 1}.$$

□

Definition 3.30. Let $G = (V, E)$ be BFG with inverse dominating set $D^{\backprime}$ of G with respect to D. Then a partition $P^{\backprime} = V - D = \{D^{\backprime}_1, D^{\backprime}_2, \ldots, D^{\backprime}_n\}$ of $V - D$ is called an inverse dominatic partition of G, if $D^{\backprime}_i$ is an inverse dominating sets of G. The inverse dominatic number is maximum fuzzy cardinality of $||P^{\backprime}||$ and is denoted by $d^{\backprime}(G)$, where

$$||P^{\backprime}|| = \sum_{i=1}^{-} \frac{O(D^{\backprime})}{|D^{\backprime}|}.$$

That is, $d^{\backprime}(G) = max\{||P||\}$. Such that $|D^{\backprime}|$ is the number of element in $D^{\backprime}$ and $O(D^{\backprime})$ is fuzzy cardinality of $D^{\backprime}$.

Example 3.31. For the strong bipolar fuzzy graph G in Figure 3.5.

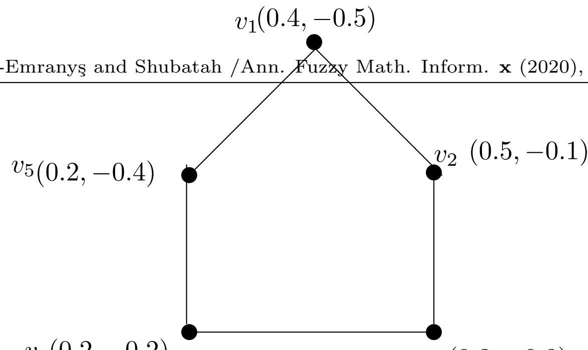

Figure 3.5.

$D^{\backprime}_1 = P^{\backprime}_1 = \{v_2, v_4, v_5\}, D^{\backprime}_2 = P^{\backprime}_2 = \{(v_2, v_4), (v_2, v_4)\}, D^{\backprime}_3 = P^{\backprime}_3 = \{v_2, v_5\}$ with $||P^{\backprime}_1|| = \frac{1.6}{3} = 0.53$, $||P^{\backprime}_2|| = \frac{0.7+0.5}{2} + \frac{0.4+0.5}{2} = 1.05$, $||P^{\backprime}_3|| = \frac{0.4+0.5}{2} = 0.45$. Then $d^{\backprime}(G) = 1.05$.

Theorem 3.32. *Let T be bipolar fuzzy tree (BFT). For any end vertex adjacent at least one non-end vertex in T, $\gamma(T) + \gamma^{\backprime}(T) = P$.*

Proof. Let D be the minimal dominating set of T and let u be any non-end vertex. Then $u \in D$, Therefore $u \notin V - D$. Since D is dominating set, $V - D$ is dominating set of T. Thus $V - D$ is inverse dominating set with respect to D in T not contain u. So every two vertices in $V - D$ are not adjacent. Hence $V - D$ is an independent. Therefore $|D| + |V - D| = P$. □

Remark 3.33. If H is any connected bipolar fuzzy spanning subgraph of BFG G, then $\gamma^{\backprime}(G) \leq \gamma^{\backprime}(H)$.

4. Conclusion

In this paper, an inverse domination number and inverse dominatic number are defined on bipolar fuzzy graphs and also applied for the various types of bipolar fuzzy graphs and suitable examples have given. We have done some results with examples and Relations of inverse domination number and known parameters in bipolar fuzzy graph were discussed with the suitable examples. Further, we introduced and investigated some result of inverse dominatic number in bipolar fuzzy graph and some suitable examples have given.

References

[1] M. Akram, Bipolar fuzzy graphs, Inform. Sci. 181 (2011) 5548–5564.

[2] M. Akram and W. A. Dudek, Regular bipolar fuzzy graphs, Neural Computing and Applications 1 (2012) 197–205.

[3] M. Akram, Bipolar fuzzy graphs with applications, Knowledge Based Systems 39 (2013) 1–8.

[4] M. Akram and Adeel Farooq, Bipolar fuzzy trees, NTMSCI (3) (2016) 58–72.

[5] M. Akram and Rabia Akmal, Certain Operations on bipolar fuzzy graph, Structures applications and applied Mathematics An International Journal 11 (1) (2016) 443–468.

[6] S. Ghobadi, N. D. Soner and Q. M. Mahyoub, Inverse dominating Set in fuzzy graphs, Journal of Proceedings of the Jangjeon Mathematical Soci ety. 11 (1) (2008) 75–81.

[7] F. Haray, Graph theory, Addison Wesley, Third printing, October 1972.

[8] V. R. Kulli and S. C. Sigarkanti, Inverse domination in graphs, reprint extract from: Nat. acad. Sci. Letters 14 (1991) 473–475.

[9] M. G. Karunambigai, M. Akram and K. Palanivel, Domination in bipolar fuzzy graphs, International conference on fuzzy systems (2013) 7–10.
[10] A. Kaufman, Introduction a La Theorie des Sous- emsembles Flous, Paris, Masson etcie Editeurs 1973.
[11] Q. M. Mahiuob, Astudy on some topics in the theory of fuzzy graph, P. H. D, thesis, University of Mysore, kranatak, India 2008.
[12] A. Rosenfeld, Fuzzy graphs, In. L. A. Zadeh, K. S. Fu and M. Shimura, (Eds). Fuzzy Sets and their Applications, Academic Press 1975. 77–95.
[13] H. Rashmanloua, Sovan Samanta, Madhumangal Pal and Rajab Ali Borzooeid, A study on bipolar fuzzy graphs, Journal of Intelligent fuzzy systems 28 (16) (2015) 571–580.
[14] A. Somasundaram and S. Somasundaran, Domination in fuzzy graphs-I, Pattern Recognition Letters 19 (1998) 787–791.
[15] A. Somasundaram, Domination in fuzzy graphs-II, Journal of Fuzzy Mathematics 20 (2000) 281–289.
[16] A. Somasundaram, Domination in products of fuzzy graphs, International Journal of Uncertainty fuzziness and Knowledge-Based systems 13 (2) (2005) 195–204.
[17] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.
[18] W. R. Zhang, Bipolar fuzzy sets and relations, A computational framework for cognitive modeling and multi agent decision analysis, Proceedings of IEEE. Conf. (1994) 305–309.

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