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Real-life decision making based on a new correlation coefficient in Pythagorean fuzzy environment

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Abstract. Pythagorean fuzzy set (PFS) is a generalized version of intuitionistic fuzzy set (IFS) with the capacity to manage the situation that cannot be captured by IFS. PFS is characterized by three grades namely; membership grade, non-membership grade and hesitancy grade with the property that the square of sum of the grades is equal to one. The idea of correlation coefficients for measuring the interrelationship between PFSs have been proposed in literature. Nonetheless, these sort of correlation coefficients for PFSs lack precision. Due to this weakness, a new correlation coefficient for PFSs is introduced in this paper. In this study, the Garg's correlation coefficient for PFSs is generalized and modified for better accuracy. Some interesting properties of the proposed correlation coefficient for PFSs are characterized with some results. A set of numerical examples are given to demonstrate the efficiency of the introduced correlation coefficient for PFSs with regard to the existing ones. It appears that the proposed correlation coefficient for PFSs outperforms the ones hitherto studied in literature. Subsequently, some real-life decision-making (RLDM) problems such as pattern recognition problem (e.g., classification of mineral fields) and diagnostic medicine in the framework of Pythagorean fuzzy pairs are discoursed with the aid of the new correlation coefficient. This proposed measuring tool could be exploited in multi-criteria decision-making problems via object oriented approach.

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1. Introduction

Most of the Real-life decision-making (RLDM) problem are multi-criteria in nature. Multi-criteria decision-making (MCDM) is a discipline in decision science that deals with decisions involving the choice of a best alternative from several potential options, subject to several criteria or attributes that may be concrete or vague. MCDM methods are used to help decision-makers make their decision according to their preferences to enhance optimal choice among the alternatives, in cases where there is more than one conflicting criterion [48]. The ultimate target of RLDM problems is to choose the foremost desirable alternative among limited options concurring to the preference values of the criteria given by distinctive decision makers. In real-life issues, we encounter many decision-making (DM) problems, involving complex uncertainties and hence, beyond the capacity of fuzzy sets. Fuzzy set [62] has a membership function, μ that assigns to each element of the universe of discourse, a number from the unit interval, $[0, 1]$ to indicate the degree of membership to the set under consideration.

Fuzzy set theory could not precisely handle the complex imprecisions imbedded in decision-making because it considers only membership degree. As a result, a number of generalizations of fuzzy sets such intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PFSs), etc. were proposed. To resolve the limitation of fuzzy set, Atanassov [1, 2] proposed the construct of IFSs by integrating both the membership function, μ and non-membership function, ν with hesitation margin, π such that their sum is one (i.e., $\mu+\nu+\pi=1$) with the property that $\mu+\nu\leq 1$. IFS provides a better way to manage the inaccuracy, dubiousness, and vulnerabilities in imprecise information and in tackling DM problems. IFS is way better equipped to deal with uncertainties because it also factors in the hesitancy of the decision maker, a feature that is not possible in the fuzzy sets. IFS delivers a formidable framework to reasonably curb uncertainties and consequently, very pertinent in modelling many real-life problems such medical diagnosis [6, 21, 51], electoral process [19, 29], research questionnaire [25], etc. Some applications of IFSs via distance measures were discussed in [5, 23, 26, 44]. New ranking method in normal intuitionistic fuzzy environment with application to decision-making was presented in [35].

Though IFS is equipped with the facility to tackle uncertainties, there are times when $\mu+\nu\geq 1$, which is beyond the ability of IFSs. For example, if a decision maker expresses preference about the degree of alternative that satisfies a criterion as 0.7, whereas the degree of alternative that dissatisfies the criterion is 0.5. Clearly, $0.7+0.5\nleq 1$, and as such, beyond the ability of IFS. This rationalizes the reason why Yager [61] proposed a framework called Pythagorean fuzzy sets (PFSs) also known as intuitionistic fuzzy set of second type [2], which is equipped with the capacity to capture such cases. The Pythagorean fuzzy set, as a new extension of intuitionistic fuzzy set, seek to manage the complex imprecisions in practical decision-making problems. In PFS, the sum of squares of its membership, μ and non-membership, ν degrees is either less than or equal to 1, or greater than or equal to 1(i.e., $\mu+\nu\leq 1$ or $\mu+\nu\geq 1$), with the property that $\mu^2+\nu^2+\pi^2=1$, where π is the Pythagorean fuzzy set index or hesitation margin. Thus, every IFS is a PFS but the inverse is certainly not true. PFS possesses the power to handle uncertain information more sufficiently and accurately when compare to IFS. An elaborate description on the fundamentals of PFSs such as modal operators on PFSs [10], properties of continuous Pythagorean fuzzy information [40], Pythagorean fuzzy power average operators [41] and some results on PFSs [50] have been done. The idea of composite relations on Pythagorean fuzzy sets with applications have been studied [12, 13]. The application of PFSs in solving multi-criteria decision-making (MCDM) problems were presented in [60, 63]. Similarly, some novel methods for solving MCDM and multi-attribute decision-making (MADM) problems in the environment of interval-valued Pythagorean fuzzy sets have been discussion [7, 32]. Some multi-parametric similarity measures for PFSs were studied with applications in [18, 11, 49]. In the same vein, some distance measure operators on PFSs and their applications have been discussed [15, 20, 45]. The concept of Pythagorean fuzzy aggregation operators with applications have been studied [31, 33, 34, 56, 57]. By extension, q-rung orthopair fuzzy sets (crisp or inter-valued), their aggregation operators with applications were elaborated in [36, 37, 38].

Correlation coefficient plays a vital role in RLDM problems. In correlation analysis, the joint relationship of two variables can be verified with the aid of a measure of interdependency of the two variables. The notion of correlation coefficient have been extended to fuzzy, intuitionistic fuzzy and Pythagorean fuzzy settings to enable it applications in tackling cases of uncertainties which are rife in RLDM problems. Correlation coefficient was first studied in fuzzy environment [4, 8, 9] and extended to intuitionistic fuzzy context [39]. In [59], the correlation coefficient method for IFSs [39] was modified for better efficiency. Some correlation coefficient techniques in intuitionistic fuzzy environment which improved the technique in [59] were presented in [16, 17, 22, 28]. Some correlation coefficient techniques based on integral functions were studied in [47, 58]. Hung [42] first proposed a method of computing correlation coefficient of IFSs from statistical viewpoint. Subsequently, improved versions were presented in [46, 52, 53, 55]. Hung and Wu [43] presented an approach of measuring correlation coefficient of IFSs based on centroid method. In Pythagorean fuzzy environment, correlation coefficient was proposed by Garg [30] via triparametric approach to measure the interrelation between PFSs, and the measure was applied to pattern recognition and medical diagnostic problems. Ejegwa [14] proposed a novel correlation coefficient of PFSs and applied the approach to MCDM problems. A detail explication of an algorithmic approach of computing correlation coefficient of PFSs and its application in diagnosis was discussed in [27]. The concept of correlation coefficients from a statistical viewpoint have been discussed [24, 54].

By examining the veracity of the researches on the concept of correlation coefficient measures in Pythagorean fuzzy environment [14, 30, 54], we discover that the approaches in [14, 30] cannot reliably measure the correlation coefficient of PFSs with precision. To remedy this limitation, we are motivated to propose a new triparametric correlation coefficient for PFSs that modifies the approaches in [14, 30] with better interpretation and output. Crisply, this paper generalizes and modifies the correlation coefficient approaches in Pythagorean fuzzy environment [14, 30], numerically validates its superiority over the existing ones, and illustrates its applications in some selected RLDM problems. The rest of the article are outlined thus;

Section 2 provides some mathematical preliminaries and discusses the correlation coefficient measures in [14, 30], Section 3 presents the new correlation coefficient method for PFSs with numerical verifications/comparisons, Section 4 dwells on the applications of the proposed method in pattern recognition problem and diagnostic medicine, and Section 5 concludes the paper and provides direction for further studies.

2. Preliminaries

This section presents the concept of PFSs with some properties and reiterates some existing correlation coefficient measures of PFSs.

2.1. **Pythagorean fuzzy sets.** Suppose X is a non-empty set that is fixed, then the following definitions follow.

Definition 2.1 ([1])**.** An IFS A of X is an object having the form

$$A = \{\langle \frac{\mu_A(x), \nu_A(x)}{x} \rangle \mid x \in X\}, \tag{2.1}$$

where the functions

$$\mu_A(x) : X \to [0,1] \text{ and } \nu_A(x) : X \to [0,1]$$

are the degree of membership and the degree of non-membership, respectively of the element $x \in X$ to A, and for every $x \in X$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For each A of X,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of non-determinacy of $x \in X$, to A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus, $\mu_A(x)+\nu_A(x)+\pi_A(x) = 1$. An IFS A can also be represented by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}.$$

Definition 2.2 ([61])**.** A Pythagorean fuzzy set A of X is of the form

$$A = \{\langle \frac{\mu_A(x), \nu_A(x)}{x} \rangle \mid x \in X\}, \tag{2.2}$$

where the functions

$$\mu_A(x) : X \to [0,1] \text{ and } \nu_A(x) : X \to [0,1]$$

define the degree of membership and the degree of non-membership, respectively of the element $x \in X$ to A, and for every $x \in X$,

$$0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1. \tag{2.3}$$

Supposing $(\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by

$$\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]} \text{ and } \pi_A(x) \in [0,1]. \tag{2.4}$$

Thus $(\mu_A(x))^2+(\nu_A(x))^2+(\pi_A(x))^2=1$. Otherwise $\pi_A(x)=0$, whenever $(\mu_A(x))^2+(\nu_A(x))^2=1$. We can also write a PFS A as

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\}.$$

The set of all $PFSs$ of X is denoted by $PFS(X)$.

Definition 2.3 ([63]). Let $A\in PFS(X)$. Then, the score function, s and the accuracy function, a of A are defined by

$$s(A)=(\mu_A(x))^2-(\nu_A(x))^2 \text{ and} \tag{2.5}$$

$$a(A)=(\mu_A(x))^2+(\nu_A(x))^2, \tag{2.6}$$

where $s(A)\in[-1,1]$ and $a(A)\in[0,1]$.

It follows immediately from Eq. 2.6 that the degree of indeterminacy of $x\in X$ to A is

$$\pi_A(x)=\sqrt{1-a(A)}. \tag{2.7}$$

Example 2.4. Assume $A\in PFS(X)$, $\mu_A(x)=0.7$ and $\nu_A(x)=0.5$ for $X=\{x\}$. Clearly, $0.7+0.5\not\leq 1$, but $0.7^2+0.5^2\leq 1$. Then $\pi_A(x)=0.5099$ and thus $(\mu_A(x))^2+(\nu_A(x))^2+(\pi_A(x))^2=1$.

Table 1 explains the difference between PFSs and IFSs [11].

TABLE 1. PFSs and IFSs

IFSs	PFSs
$\mu+\nu\leq 1$	$\mu+\nu\leq 1$ or $\mu+\nu\geq 1$
$0\leq\mu+\nu\leq 1$	$0\leq\mu^2+\nu^2\leq 1$
$\pi=1-(\mu+\nu)$	$\pi=\sqrt{1-[\mu^2+\nu^2]}$
$\mu+\nu+\pi=1$	$\mu^2+\nu^2+\pi^2=1$

Definition 2.5 ([61]). Suppose $A,B\in PFS(X)$. Then we have the following:

(i) $\bar{A}=\{\langle x,\nu_A(x),\mu_A(x)\rangle|x\in X\}$,
(ii) $A\cup B=\{\langle x,\max(\mu_A(x),\mu_B(x)),\min(\nu_A(x),\nu_B(x))\rangle|x\in X\}$,
(iii) $A\cap B=\{\langle x,\min(\mu_A(x),\mu_B(x)),\max(\nu_A(x),\nu_B(x))\rangle|x\in X\}$,
(iv) $A\oplus B=\{\langle x,\sqrt{(\mu_A(x))^2+(\mu_B(x))^2-(\mu_A(x))^2(\mu_B(x))^2},\nu_A(x)\nu_B(x)\rangle|x\in X\}$,
(v) $A\otimes B=\{\langle x,\mu_A(x)\mu_B(x),\sqrt{(\nu_A(x))^2+(\nu_B(x))^2-(\nu_A(x))^2(\nu_B(x))^2}\rangle|x\in X\}$.

Definition 2.6 ([61]). Let A and B be PFSs of X. Then

$$A=B\Leftrightarrow\mu_A(x)=\mu_B(x) \text{ and } \nu_A(x)=\nu_B(x)\ \forall x\in X$$

and

$$A\subseteq B\Leftrightarrow\mu_A(x)\leq\mu_B(x) \text{ and } \nu_A(x)\geq\nu_B(x) \text{ or } \nu_A(x)\leq\nu_B(x)\ \forall x\in X.$$

We say $A\subset B\Leftrightarrow A\subseteq B$ and $A\neq B$. Also A and B are comparable to each other if $A\subseteq B$ and $B\subseteq A$.

Definition 2.7 ([11])**.** Suppose $A \in PFS(X)$. Then the level/ground set or support of A is defined by

$$A_* = \{x \in X | \mu_A(x) > 0,\ \nu_A(x) < 1\}$$

and the set A^* is defined by

$$A^* = \{x \in X | \mu_A(x) \geq 0,\ \nu_A(x) \leq 1\}.$$

Certainly, A_* and A^* are subsets of X.

Definition 2.8 ([11])**.** Pythagorean fuzzy pairs (PFPs) or Pythagorean fuzzy values (PFVs) is an object in the form $\langle a, b\rangle$, where $a, b \in [0, 1]$, and $a^2 + b^2 \leq 1$. PFPs are used for the evaluation of objects or processes and which components (a and b) are interpreted as degrees of membership and non-membership or degrees of validity and non-validity or degrees of correctness and non-correctness.

Additional properties and examples of PFSs which are not IFSs can be found in [61]. For clarity, in a PFS $A = \{\langle \frac{\mu_A(x), \nu_A(x)}{x} \rangle | x \in X\}$, if $\mu_A(x) = 0$ (the element $x \notin A$ in the ordinary sense), then $\nu_A(x) = 1$. Again, if $\mu_A(x) = 1$ ($x \in X$ in the ordinary sense), then $\nu_A(x) = 0$.

2.2. **Correlation coefficient for PFSs.** This section studies correlation coefficient for PFSs. Firstly, we recall the concept of correlation coefficient in [30] in the framework of PFSs. Although the idea of correlation coefficient for PFSs has been studied in [54], but the idea do not capture the three fundamental or orthodox parameters of PFSs and as such, the output cannot be reliably trusted. Now, we give the axiomatic definition of correlation coefficient for PFSs as follows.

Definition 2.9 ([14])**.** Let $A, B \in PFS(X)$. Then, the correlation coefficient denoted by $K(A, B)$ is a measuring function $K : PFS \times PFS \to [0, 1]$ satisfying the following conditions;

(i) $K(A, B) \in [0, 1]$,
(ii) $K(A, B) = K(B, A)$,
(iii) $K(A, B) = 1$ if and only if $A = B$.

2.2.1. *Some existing techniques of correlation coefficient for PFSs.* Recall the correlation coefficients for PFSs in [14, 30] as follows. Let $A, B \in PFS(X)$ for $X = \{x_1, x_2, ..., x_n\}$. Then, the correlation coefficients for A and B are given as:

$$K_1(A, B) = \frac{C(A, B)}{\max[T(A), T(B)]} \tag{2.8}$$

and

$$K_2(A, B) = \frac{C(A, B)}{\sqrt{T(A)T(B)}}, \tag{2.9}$$

where the informational energies and correlation for the PFSs are

$$\left.\begin{aligned} T(A) &= \sum_{i=1}^{n} [\mu_A^4(x_i) + \nu_A^4(x_i) + \pi_A^4(x_i)] \\ T(B) &= \sum_{i=1}^{n} [\mu_B^4(x_i) + \nu_B^4(x_i) + \pi_B^4(x_i)] \end{aligned}\right\}, \tag{2.10}$$

$$C(A,B)=\sum_{i=1}^{n}[\mu_A^2(x_i)\mu_B^2(x_i)+\nu_A^2(x_i)\nu_B^2(x_i)+\pi_A^2(x_i)\pi_B^2(x_i)]. \tag{2.11}$$

In [14], a correlation coefficient for PFSs in [30] was generalized. The generalized correlation coefficient for A and B is

$$\mathbf{K}(A,B)=\frac{\mathbf{C}(A,B)}{\max[\mathbf{C}(A,A),\mathbf{C}(B,B)]}=\frac{\mathbf{C}(A,B)}{\max[\mathbf{T}(A),\mathbf{T}(B)]}, \tag{2.12}$$

where $\mathbf{C}(A,B)$, $\mathbf{T}(A)$ and $\mathbf{T}(B)$ are defined as

$$\left.\begin{aligned}\mathbf{T}(A)&=\sum_{i=1}^{n}[\mu_A^k(x_i)+\nu_A^k(x_i)+\pi_A^k(x_i)]\\ \mathbf{T}(B)&=\sum_{i=1}^{n}[\mu_B^k(x_i)+\nu_B^k(x_i)+\pi_B^k(x_i)]\end{aligned}\right\}, \tag{2.13}$$

$$\mathbf{C}(A,B)=\sum_{i=1}^{n}[\mu_A^{\frac{k}{2}}(x_i)\mu_B^{\frac{k}{2}}(x_i)+\nu_A^{\frac{k}{2}}(x_i)\nu_B^{\frac{k}{2}}(x_i)+\pi_A^{\frac{k}{2}}(x_i)\pi_B^{\frac{k}{2}}(x_i)], \tag{2.14}$$

where $k \leq 4$ or strictly $k=3$.

These correlation coefficient measures for PFSs have been properly characterized and approved to be reasonable measuring tools for measuring the interrelation between PFSs (see [14, 30] for details).

3. New correlation coefficient for PFSs

The novel correlation coefficient for PFSs modify the generalized correlation coefficient for PFSs discussed in [14].

Definition 3.1. Let $A,B \in PFS(X)$ for $X=\{x_1,x_2,...,x_n\}$. Then, the modified generalized correlation coefficient for A and B is

$$\tilde{\mathbf{K}}(A,B)=\frac{\mathbf{C}(A,B)}{Aver[\mathbf{C}(A,A),\mathbf{C}(B,B)]}=\frac{\mathbf{C}(A,B)}{Aver[\mathbf{T}(A),\mathbf{T}(B)]}, \tag{3.1}$$

where $\mathbf{C}(A,B)$, $\mathbf{T}(A)$ and $\mathbf{T}(B)$ are as in Definitions 2.13 and 2.14.

Proposition 3.2. *Suppose $A,B \in PFS(X)$. Then*

(1) $\boldsymbol{T}(A)=\boldsymbol{T}(\bar{A})$,
(2) $\boldsymbol{C}(A,B)=\boldsymbol{C}(B,A)$.

Proof. Straightforward. □

Proposition 3.3. *Let $A,B \in PFS(X)$. Then the following statements are equivalent:*

(1) $\boldsymbol{C}(A,B)=\boldsymbol{C}(\bar{A},\bar{B})$.
(2) $\boldsymbol{C}(\bar{A},\bar{B})=\boldsymbol{C}(B,A)$.

Proof. Straightforward. □

Theorem 3.4. *Suppose $A,B \in PFS(X)$. If $A=B$, then*

(1) $\boldsymbol{C}(A,B) = \boldsymbol{T}(A)$ *or* $\boldsymbol{C}(A,B) = \boldsymbol{T}(B)$,
(2) $\boldsymbol{C}(A,B) = Aver[\boldsymbol{T}(A), \boldsymbol{T}(B)]$,
(3) $\dfrac{\boldsymbol{C}(A,B)}{Aver[\boldsymbol{T}(A), \boldsymbol{T}(B)]} = 1$.

Proof. Suppose $A = B$. Then

(1)

$$\begin{aligned}\mathbf{C}(A,B) &= \sum_{i=1}^{n}[\mu_A^{\frac{k}{2}}(x_i)\mu_B^{\frac{k}{2}}(x_i) + \nu_A^{\frac{k}{2}}(x_i)\nu_B^{\frac{k}{2}}(x_i) + \pi_A^{\frac{k}{2}}(x_i)\pi_B^{\frac{k}{2}}(x_i)]\\ &= \sum_{i=1}^{n}[\mu_A^{k}(x_i) + \nu_A^{k}(x_i) + \pi_A^{k}(x_i)]\\ &= \mathbf{T}(A).\end{aligned}$$

The second alternative follows from the first.

(2) Since $\mathbf{C}(A,B) = \mathbf{C}(A,A) = \mathbf{T}(A)$ and

$$\begin{aligned}Aver[\mathbf{T}(A), \mathbf{T}(B)] &= Aver[\mathbf{T}(A), \mathbf{T}(A)]\\ &= \mathbf{T}(A),\end{aligned}$$

the result follows.

(3) It is easy to see from (2) that $\dfrac{\mathbf{C}(A,B)}{Aver[\mathbf{T}(A), \mathbf{T}(B)]} = 1$.

□

Theorem 3.5. *Suppose $A, B \in PFS(X)$. Then $\tilde{\boldsymbol{K}}(A,B)$ is a correlation coefficient between A and B.*

Proof. The function $\tilde{\mathbf{K}}(A,B)$ is a correlation coefficient between A and B, if the conditions in Definition 2.9 are satisfied.

Firstly, we show that $\tilde{\mathbf{K}}(A,B) \in [0,1]$, i.e., $0 \leq \boldsymbol{K}(A,B) \leq 1$. But $\tilde{\mathbf{K}}(A,B) \geq 0$ is trivial since $\mathbf{C}(A,B) \geq 0$ and $[\mathbf{T}(A), \mathbf{T}(B)] > 0$.

To show that $\tilde{\mathbf{K}}(A,B) \leq 1$, we make the following assumptions, i.e., let

$$\sum_{i=1}^{n}\mu_A^k(x_i) = a, \quad \sum_{i=1}^{n}\mu_B^k(x_i) = b,$$

$$\sum_{i=1}^{n}\nu_A^k(x_i) = c, \quad \sum_{i=1}^{n}\nu_B^k(x_i) = d,$$

$$\sum_{i=1}^{n}\pi_A^k(x_i) = e, \quad \sum_{i=1}^{n}\pi_B^k(x_i) = f.$$

Recall that $\tilde{\mathbf{K}}(A,B) = \dfrac{\mathbf{C}(A,B)}{Aver[\mathbf{T}(A),\mathbf{T}(B)]}$. Applying the principle of Cauchy-Schwarz's inequality, we have

$$\begin{aligned}
\tilde{\mathbf{K}}(A,B) &= \frac{\sum_{i=1}^{n}\left[\mu_A^{\frac{k}{2}}(x_i)\mu_B^{\frac{k}{2}}(x_i)+\nu_A^{\frac{k}{2}}(x_i)\nu_B^{\frac{k}{2}}(x_i)+\pi_A^{\frac{k}{2}}(x_i)\pi_B^{\frac{k}{2}}(x_i)\right]}{Aver\left[\sum_{i=1}^{n}\left(\mu_A^k(x_i)+\nu_A^k(x_i)+\pi_A^k(x_i)\right),\sum_{i=1}^{n}\left(\mu_B^k(x_i)+\nu_B^k(x_i)+\pi_B^k(x_i)\right)\right]}\\
&\leq \frac{\sum_{i=1}^{n}\left[\left(\mu_A^k(x_i)\mu_B^k(x_i)\right)^{\frac{1}{2}}+\left(\nu_A^k(x_i)\nu_B^k(x_i)\right)^{\frac{1}{2}}+\left(\pi_A^k(x_i)\pi_B^k(x_i)\right)^{\frac{1}{2}}\right]}{Aver\left[\sum_{i=1}^{n}\left(\mu_A^k(x_i)+\nu_A^k(x_i)+\pi_A^k(x_i)\right),\sum_{i=1}^{n}\left(\mu_B^k(x_i)+\nu_B^k(x_i)+\pi_B^k(x_i)\right)\right]}\\
&= \frac{(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}}{Aver[(a+c+e),(b+d+f)]}.
\end{aligned}$$

But

$$\begin{aligned}
\tilde{\mathbf{K}}(A,B)-1 &\leq \frac{(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}}{Aver[(a+c+e),(b+d+f)]}-1\\
&= \frac{(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}-Aver[(a+c+e),(b+d+f)]}{Aver[(a+c+e),(b+d+f)]}\\
&= \frac{-\{Aver[(a+c+e),(b+d+f)]-[(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}]\}}{Aver[(a+c+e),(b+d+f)]}\\
&= -\frac{\{Aver[(a+c+e),(b+d+f)]-[(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}]\}}{Aver[(a+c+e),(b+d+f)]}\\
&\leq 0.
\end{aligned}$$

Thus $\tilde{\mathbf{K}}(A,B)\leq 1$. So $\tilde{\mathbf{K}}(A,B)\in[0,1]$.

Again, $\tilde{\mathbf{K}}(A,B)=1 \Leftrightarrow A=B \Rightarrow$

$$\tilde{\mathbf{K}}(A,B)=\frac{\mathbf{C}(A,A)}{Aver[\mathbf{T}(A),\mathbf{T}(A)]}=\frac{\mathbf{T}(A)}{\mathbf{T}(A)}=1.$$

Clearly, $\tilde{\mathbf{K}}(A,B)=\tilde{\mathbf{K}}(B,A)$. These complete the proof. □

Theorem 3.6. *Suppose $A,B\in PFS(X)$. Then $\tilde{\boldsymbol{K}}(A,B)=\boldsymbol{K}(A,B)$ if and only if $\boldsymbol{T}(A)=\boldsymbol{T}(B)$.*

Proof. Straightforward. □

3.1. **Numerical comparison of the proposed correlation coefficient for PFSs with existing methods.** In this section, we present reliability analysis of the proposed correlation coefficient for PFSs in comparison to other triparametric correlation coefficients for PFSs.

3.1.1. *Numerical experiments.* Now, we give examples of PFSs and then compute their correlation coefficient using the methods in [14, 30] and the proposed method to enhance juxtaposition.

Example 3.7. Suppose A and B are PFSs in $X = \{x, y, z\}$, where $A_* = B_*$ for

$$A = \{\langle \frac{0.8, 0.2}{x} \rangle, \langle \frac{0.3, 0.1}{y} \rangle, \langle \frac{0.7, 0.4}{z} \rangle\}$$

$$B = \{\langle \frac{0.6, 0.3}{x} \rangle, \langle \frac{0.7, 0.3}{y} \rangle, \langle \frac{0.9, 0.1}{z} \rangle\}.$$

Using K_1, K_2, $\mathbf{K}$ and $\tilde{\mathbf{K}}$, we obtain the following values of correlation coefficient between A and B:

$$K_1(A, B) = 0.7526 \text{ and } K_2(A, B) = 0.7920.$$

The values of the generalized correlation coefficient for $k = 1, 2, 3, 4$ are

$$\mathbf{K}(A, B) = 0.9537 \text{ for } k = 1,$$
$$\mathbf{K}(A, B) = 0.9118 \text{ for } k = 2,$$
$$\mathbf{K}(A, B) = 0.8359 \text{ for } k = 3,$$
$$\mathbf{K}(A, B) = 0.7526 \text{ for } k = 4.$$

The values of the modified generalized correlation coefficient for $k = 1, 2, 3, 4$ are

$$\tilde{\mathbf{K}}(A, B) = 0.9648 \text{ for } k = 1,$$
$$\tilde{\mathbf{K}}(A, B) = 0.9118 \text{ for } k = 2,$$
$$\tilde{\mathbf{K}}(A, B) = 0.8548 \text{ for } k = 3,$$
$$\tilde{\mathbf{K}}(A, B) = 0.7909 \text{ for } k = 4.$$

Example 3.8. Assume we have two PFSs defined in $X = \{a, b, c\}$ as follow:

$$A_1 = \{\langle \frac{0.8, 0.1}{a} \rangle, \langle \frac{0.7, 0.3}{b} \rangle, \langle \frac{0.7, 0.1}{c} \rangle\},$$

$$A_2 = \{\langle \frac{0.5, 0.4}{a} \rangle, \langle \frac{0.0, 1.0}{b} \rangle, \langle \frac{1.0, 0.0}{c} \rangle\}.$$

Clearly, $A_* \neq B_*$. Using the correlation coefficients in [30], we get

$$K_1(A_1, A_2) = 0.3892, \; K_2(A_1, A_2) = 0.5050.$$

The values of the generalized correlation coefficient for $k = 1, 2, 3, 4$ are

$$\mathbf{K}(A_1, A_2) = 0.6221 \text{ for } k = 1,$$
$$\mathbf{K}(A_1, A_2) = 0.6315 \text{ for } k = 2,$$
$$\mathbf{K}(A_1, A_2) = 0.4986 \text{ for } k = 3,$$
$$\mathbf{K}(A_1, A_2) = 0.3892 \text{ for } k = 4.$$

The values of the modified generalized correlation coefficient for $k = 1, 2, 3, 4$ are

$$\tilde{\mathbf{K}}(A_1, A_2) = 0.6953 \text{ for } k = 1,$$
$$\tilde{\mathbf{K}}(A_1, A_2) = 0.6315 \text{ for } k = 2,$$
$$\tilde{\mathbf{K}}(A_1, A_2) = 0.5603 \text{ for } k = 3,$$
$$\tilde{\mathbf{K}}(A_1, A_2) = 0.4883 \text{ for } k = 4.$$

From Examples 3.7 and 3.8, we obtain the following table.

Table 2. Results of correlation coefficients

Methods	Example 3.7	Example 3.8
Garg [30] I	0.7526	0.3892
Garg [30] II	0.7920	0.5050
Ejegwa [14]	0.9537 for $k=1$	0.6221 for $k=1$
	0.9118 for $k=2$	0.6315 for $k=2$
	0.8359 for $k=3$	0.4986 for $k=3$
	0.7526 for $k=4$	0.3892 for $k=4$
New method	0.9648 for $k=1$	0.6953 for $k=1$
	0.9118 for $k=2$	0.6315 for $k=2$
	0.8548 for $k=3$	0.5603 for $k=3$
	0.7909 for $k=4$	0.4883 for $k=4$

3.1.2. *Discussion.* From Table 2, the following observations are gathered:

(i) In Example 3.7, we see that the proposed method gives a better correlation coefficient when compare to the existing measures for $k \leq 3$. That is, $\tilde{\mathbf{K}} > \mathbf{K} > K_2 > K_1$ for $k \leq 3$. Similarly, in Example 3.8, we see that $\tilde{\mathbf{K}} > \mathbf{K} > K_2 > K_1$ for $k \leq 2$. Also, K_1 is recovered from $\mathbf{K}$ if $k = 4$, which proves that $\mathbf{K}$ is the generalized version of K_1.

(ii) Since $\mathbf{K} = \tilde{\mathbf{K}}$ for $k = 2$ in both examples, we infer that the informational energies of the PFSs are equal. This agrees to Theorem 3.6.

(iii) The proposed method shows the true relationship that exists between the PFSs under consideration. Because the proposed method has varieties of form makes it a choice correlation coefficient measure for PFSs.

(iv) The fact that the proposed correlation coefficient has the greatest correlation coefficient value makes it more suitable to solve RLDM problems more accurately than the existing ones.

4. Applicative examples in Pythagorean fuzzy decision-making based on correlation coefficients

RLDM problems in form of MCDM are faced in many real-life issues, and they pose a huge challenge to decision-maker. In this section, some RLDM problems in pattern recognition (that is, classification of mineral fields) and medical diagnosis are discussed using the studied correlation coefficients for PFSs. The cases consider are drawn from [30].

4.1. **Pattern recognition: classification of mineral fields.** We consider a set of some known mineral fields, $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \tilde{C}_3\}$ represented by the following PFSs in a given finite universe $X = \{y_1, y_2, y_3\}$ as

$$\tilde{C}_1 = \{\frac{\langle 1.0, 0.0\rangle}{y_1}, \frac{\langle 0.8, 0.0\rangle}{y_2}, \frac{\langle 0.7, 0.1\rangle}{y_3}\},$$

$$\tilde{C}_2 = \{\frac{\langle 0.8, 0.1\rangle}{y_1}, \frac{\langle 1.0, 0.0\rangle}{y_2}, \frac{\langle 0.9, 0.1\rangle}{y_3}\},$$
$$\tilde{C}_3 = \{\frac{\langle 0.6, 0.2\rangle}{y_1}, \frac{\langle 0.8, 0.0\rangle}{y_2}, \frac{\langle 1.0, 0.0\rangle}{y_3}\}.$$

Also, consider an unknown mineral field, $\tilde{P} \in PFS(X)$ represented by

$$\tilde{P} = \{\frac{\langle 0.5, 0.3\rangle}{y_1}, \frac{\langle 0.6, 0.2\rangle}{y_2}, \frac{\langle 0.8, 0.1\rangle}{y_3}\}$$

that is supposed to be classified into any of the aforementioned mineral fields.

The aim of this exercise is to classify the unknown mineral field, $\tilde{P}$ into one of the classes $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$. Using the correlation coefficient measures in [30], $\mathbf{K}$ and $\tilde{\mathbf{K}}$ for $k = 3$, we compute the correlation coefficient for $\tilde{P}$ and $\tilde{C}_i$(for $i = 1, 2, 3$) as follows:

$$K_1(\tilde{C}_1, \tilde{P}) = 0.5864,\ K_1(\tilde{C}_2, \tilde{P}) = 0.6004,\ K_1(\tilde{C}_3, \tilde{P}) = 0.7762,$$

$$K_2(\tilde{C}_1, \tilde{P}) = 0.6741,\ K_2(\tilde{C}_2, \tilde{P}) = 0.7235,\ K_2(\tilde{C}_3, \tilde{P}) = 0.8953,$$

$$\mathbf{K}(\tilde{C}_1, \tilde{P}) = 0.6982,\ \mathbf{K}(\tilde{C}_2, \tilde{P}) = 0.7100,\ \mathbf{K}(\tilde{C}_3, \tilde{P}) = 0.8454.$$

Similarly,

$$\tilde{\mathbf{K}}(\tilde{C}_1, \tilde{P}) = 0.7489,\ \tilde{\mathbf{K}}(\tilde{C}_2, \tilde{P}) = 0.7760,\ \tilde{\mathbf{K}}(\tilde{C}_3, \tilde{P}) = 0.9053.$$

Table 3. Results for pattern recognition

Methods	$(\tilde{C}_1, \tilde{P})$	$(\tilde{C}_2, \tilde{P})$	$(\tilde{C}_3, \tilde{P})$
Garg [30] I	0.5864	0.6004	**0.7762**
Garg [30] II	0.6741	0.7235	**0.8953**
Ejegwa [14]	0.6982	0.7100	**0.8454**
New method	0.7489	0.7760	**0.9053**

Thus from the computations (see Table 3), we conclude that the unknown mineral field, $\tilde{P}$ belongs to the mineral field $\tilde{C}_3$ since the correlation coefficient between $\tilde{P}$ and $\tilde{C}_3$ is the greatest. The modified correlation coefficient, $\tilde{\mathbf{K}}$ gives the best measure.

4.2. **Medical diagnosis.** Here, we present a scenario of medical diagnosis. Assume a patient, $\check{P}$ visits a given laboratory for medical diagnosis. The patient diagnosis shows the following symptoms viz; temperature, headache, stomach pain, cough, and chest pain. That is, the set of symptoms S is

$$S = \{x_1, x_2, x_3, x_4, x_5\},$$

where x_1 = temperature, x_2 = headache, x_3 = stomach pain, x_4 = cough, and x_5 = chest pain.

After the sample collected from $\check{P}$ was analyzed, the following result in PFS setting is obtained as

$$\check{P} = \{\frac{\langle 0.8, 0.1\rangle}{x_1}, \frac{\langle 0.6, 0.1\rangle}{x_2}, \frac{\langle 0.2, 0.8\rangle}{x_3}, \frac{\langle 0.6, 0.1\rangle}{x_4}, \frac{\langle 0.1, 0.6\rangle}{x_5}\}.$$

Let the set of diseases, $\check{D}_i$ (for $i = 1, 2, 3, 4, 5$) that $\check{P}$ is suspected to be suffering from be

$$\check{D}_i = \{\check{D}_1, \check{D}_2, \check{D}_3, \check{D}_4, \check{D}_5\},$$

where $\check{D}_1$ =viral fever, $\check{D}_2$ =malaria fever, $\check{D}_3$ =typhoid fever, $\check{D}_4$ =stomach problem, and $\check{D}_5$ =heart problem.

The diseases, $\check{D}_i$ (for $i = 1, 2, 3, 4, 5$) are represented by the following PFSs:

$$\check{D}_1 = \{\frac{\langle 0.4, 0.0\rangle}{x_1}, \frac{\langle 0.3, 0.5\rangle}{x_2}, \frac{\langle 0.1, 0.7\rangle}{x_3}, \frac{\langle 0.4, 0.3\rangle}{x_4}, \frac{\langle 0.1, 0.7\rangle}{x_5}\},$$
$$\check{D}_2 = \{\frac{\langle 0.7, 0.0\rangle}{x_1}, \frac{\langle 0.2, 0.6\rangle}{x_2}, \frac{\langle 0.0, 0.9\rangle}{x_3}, \frac{\langle 0.7, 0.0\rangle}{x_4}, \frac{\langle 0.1, 0.8\rangle}{x_5}\},$$
$$\check{D}_3 = \{\frac{\langle 0.3, 0.3\rangle}{x_1}, \frac{\langle 0.6, 0.1\rangle}{x_2}, \frac{\langle 0.2, 0.7\rangle}{x_3}, \frac{\langle 0.2, 0.6\rangle}{x_4}, \frac{\langle 0.1, 0.9\rangle}{x_5}\},$$
$$\check{D}_4 = \{\frac{\langle 0.1, 0.7\rangle}{x_1}, \frac{\langle 0.2, 0.4\rangle}{x_2}, \frac{\langle 0.8, 0.0\rangle}{x_3}, \frac{\langle 0.2, 0.7\rangle}{x_4}, \frac{\langle 0.2, 0.7\rangle}{x_5}\},$$
$$\check{D}_5 = \{\frac{\langle 0.1, 0.8\rangle}{x_1}, \frac{\langle 0.0, 0.8\rangle}{x_2}, \frac{\langle 0.2, 0.8\rangle}{x_3}, \frac{\langle 0.2, 0.8\rangle}{x_4}, \frac{\langle 0.8, 0.1\rangle}{x_5}\}.$$

Our goal is to determine the disease that the patient, $\check{P}$ is suffering from with regards to the suspected diseases

$$\check{D}_i = \{\check{D}_1, \check{D}_2, \check{D}_3, \check{D}_4, \check{D}_5\}.$$

Using the correlation coefficients in [30], $\mathbf{K}$ and $\tilde{\mathbf{K}}$ for $k = 3$, we get the following outputs:

$$K_1(\check{P}, \check{D}_1) = 0.8328,\ K_1(\check{P}, \check{D}_2) = 0.8895,\ K_1(\check{P}, \check{D}_3) = 0.7485,$$

$$K_1(\check{P}, \check{D}_4) = 0.6229,\ K_1(\check{P}, \check{D}_5) = 0.5075,$$

$$K_2(\check{P}, \check{D}_1) = 0.8622,\ K_2(\check{P}, \check{D}_2) = 0.9047,\ K_2(\check{P}, \check{D}_3) = 0.7808,$$

$$K_2(\check{P}, \check{D}_4) = 0.6233,\ K_2(\check{P}, \check{D}_5) = 0.5080,$$

$$\mathbf{K}(\check{P}, \check{D}_1) = 0.8877,\ \mathbf{K}(\check{P}, \check{D}_2) = 0.9125,\ \mathbf{K}(\check{P}, \check{D}_3) = 0.8235,$$

$$\mathbf{K}(\check{P}, \check{D}_4) = 0.6628,\ \mathbf{K}(\check{P}, \check{D}_5) = 0.5682.$$

Also

$$\tilde{\mathbf{K}}(\check{P}, \check{D}_1) = 0.9660,\ \tilde{\mathbf{K}}(\check{P}, \check{D}_2) = 0.9902,\ \tilde{\mathbf{K}}(\check{P}, \check{D}_3) = 0.8989,$$

$$\tilde{\mathbf{K}}(\check{P}, \check{D}_4) = 0.7123,\ \tilde{\mathbf{K}}(\check{P}, \check{D}_5) = 0.6116.$$

Table 4. Results for medical diagnosis

Methods	$(\check{P}, \check{D}_1)$	$(\check{P}, \check{D}_2)$	$(\check{P}, \check{D}_3)$	$(\check{P}, \check{D}_4)$	$(\check{P}, \check{D}_5)$
Garg [30] I	0.8328	**0.8895**	0.7485	0.6229	0.5075
Garg [30] II	0.8622	**0.9047**	0.7808	0.6233	0.5080
Ejegwa [14]	0.8877	**0.9125**	0.8235	0.6628	0.5682
New method	0.9660	**0.9902**	0.8989	0.7123	0.6116

From the computations (see Table 4), we conclude that patient, $\check{P}$ is suffering from malaria fever since the correlation coefficient between them shows the greatest interrelationship in each of the correlation coefficient measures.

5. Conclusions

In this paper, new correlation coefficient for PFSs which generalized and modified the correlation coefficients of PFSs in [14, 30] have been proposed and some of its properties were discussed. The weakness of the existing correlation coefficients for PFSs have also been emphasized in the paper. With this, the correlation coefficient, K_1 for PFSs in [30] can be effectively recovered from the correlation coefficient, $\mathbf{K}$ in [14] by replacing $k = 4$, and $\mathbf{K}$ and $\tilde{\mathbf{K}}$ are equal if they have the same informational energies. Some examples that authenticate the reliability of $\tilde{\mathbf{K}}$ over $\mathbf{K}$ and the existing ones in [30] have been given. Besides ameliorating the existing methods, the new correlation coefficient measure for PFSs has a better performance index in comparison to the approaches in [14, 30], as presented in Tables 2, 3 and 4. In fact, the new approach modifies and generalizes the methods in [14, 30] with an improved output. To validate the application of the proposed method, some cases of RLDM problems such as classification of mineral fields and medical diagnosis were considered as PFPs. From the study, we conclude that the modified version of the generalized correlation coefficient in Pythagorean fuzzy context gives a reliable output when compare to the existing ones in Pythagorean fuzzy environment and hence, can appropriately resolve RLDM problems effectively. In a nutshell, this paper generalized and modified the correlation coefficient approaches in Pythagorean fuzzy environment [14, 30], numerically validated its superiority over the existing ones, and illustrated its applications in some selected RLDM problems. The new correlation coefficient measure could be applied in some MCDM problems using cluster algorithm. Exploiting the novel correlation coefficient measure in interval-valued PFSs and q-rung orthopair fuzzy environments (crisp or inter-valued) [36, 37, 38] could yield some exciting results.

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