Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Generalized difference sequence space of fuzzy numbers

Abdul Hamid Ganie, Sameer Ahmad Gupkari

Received 10 October 2020; Revised 7 December 2020 ; Accepted 31 December 2020

Abstract. In the present paper, we wish to introduce and study some new techniques of difference sequence spaces of fuzzy numbers by employing sequence of modulus functions.

2010 AMS Classification: 40A05, 26A03, 11B05

Keywords: Difference sequences, Fuzzy numbers.

Corresponding Author: A.H.Ganie (a.ganie@seu.edu.sa)

1. Introduction

A sequence space is defined to be a linear space of real or complex sequences. Throughout the paper $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denotes the set of non-negative integers, the set of real numbers and the set of complex numbers respectively. Let ω denote the space of all sequences (real or complex); ℓ_∞ and c respectively, denotes the space of all bounded sequences, the space of convergent sequences.

Throughout the paper $p = (p_k)$ is a sequence of positive real numbers. The notion of paranormed sequences was studied at the initial stage by Simons [33]. It was further investigated by Maddox [27], Tripathy et al [35, 38, 39, 41], Ganie et al [13, 20, 21, 22] and many others.

Following Ganie [14, 16] and Maddox [27], a modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that
(i) $f(x) = 0$ if and only if $x = 0$,
(ii) $f(x + y) \leq f(x) + f(y)$ $\forall x, y \geq 0$
(iii) f is increasing,
(iv) f if continuous from right at $x = 0$.

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [42] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [28] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties. Matloka [28] also has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Nanda [30], Altin [1], Altinok [2], Başarir and Mursaleen [3], Ganie and Sheikh [21], Bilgin [4], Chaudhury and Das [5], Çlock [6, 7, 8], Diamond and Kloeden [9], Esi [10, 11], Fang and Fang [12], Hamid and Neyaz [19], Hazarika [18], Kelava [25], Saves [31, 32], Tripathy et al [34, 36, 37, 40], etc.

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on the real line $\mathbb{R}$. For X, $Y \in D$ we define

$d(X : Y) = \max(|a_1 - b_1|, |a_2 - b_2|)$ where $X = [a_1, a_2]$, $Y = [b_1, b_2]$.

It is known that (D, d) is a complete metric space.

Let $I = [0, 1]$. A fuzzy real number X is a fuzzy set on $\mathbb{R}$ and is a mapping $X : \mathbb{R} \to I$ associating each real number t with its grade membership $X(t)$.

A fuzzy real number X is called *convex*, if

$$X(t) \geq X(s) \wedge X(r) = \min(X(s), X(s)), \text{ where } s < t < r.$$

A fuzzy real number X is called *normal*, if there exists $t_0 \in \mathbb{R}$ such that $X(t_0) = 1$.

A fuzzy real number X is called $upper\ semi - continuous$, if for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$ for all $a \in I$ and given $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$ is open in the usual topology of $\mathbb{R}$.

The set of all $upper-semi\ continuous$, $normal$, $convex$ fuzzy numbers is denoted by $R(I)$. The α-level set of a fuzzy real number X for $0 < \alpha \leq 1$ denoted by X^α is defined by $X^\alpha = \{t \in \mathbb{R} : X(t) \geq \alpha\}$. The 0-level set is the closure of strong 0-cut.

For each $r \in \mathbb{R}$, $\bar{r} \in \mathbb{R}(I)$ is defined by

$$\bar{r} = \begin{cases} \bar{r}, & \text{if } t = r, \\ 0, & \text{if } t \neq r. \end{cases}$$

The absolute value of $|X|$ of $X \in \mathbb{R}(I)$ is defined by (See for instance Kaleva and Seikkla [25])

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}$$

Let $\bar{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha).$$

Then $\bar{d}$ defines a metric on $\mathbb{R}(I)$ (Matloka [28]). The additive identity and multiplicative identity in $\mathbb{R}(I)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

Throughout the article ω^F, c^F, c_0^F and ℓ_∞^F denote the classes of all, convergent, null, bounded sequence spaces of fuzzy real numbers.

A fuzzy real valued sequence $\{X_n\}$ is said to be convergent to fuzzy real number X, if for $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $\bar{d}(X, Y) < \varepsilon$ for all $k \geq n_0$.

A fuzzy real valued sequence $\{X_n\}$ is said to be solid (normal) if $(X_k) \in E^F$ implies that $(\alpha_k X_k) \in E^F$ for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$.

Let $K = \{k_1 < k_2 < ...\} \subseteq \mathbb{N}$ and E^F be a sequence space. A k-step space of E^F is a sequence space $\lambda_K^{E^F} = \{(X_{k_n}) \in \omega^F : (X_n) \in E^F\}$.

A canonical preimage of a sequence $\{X_k\} \in \lambda_K^{E^F}$ is a sequence $\{Y_n\} \in \omega^F$ defined as

$$Y_n = \begin{cases} X_n, & \text{if} \quad k \in K, \\ \bar{0}, & \quad\text{otherwise.} \end{cases}$$

A canonical preimage of a step space $\lambda_K^{E^F}$ is a set of all elements in $\lambda_K^{E^F}$, i.e., Y is in canonical preimage of $\lambda_K^{E^F}$ if and only if Y is canonical preimage of some $X \in \lambda_K^{E^F}$.

A sequence space E^F is said to be monotone, if it contains the canonical preimages of its step spaces.

A sequence space E^F is said convergence free, if $(Y_k) \in E^F$ whenever $(X_k) \in E^F$ and $Y_k = \bar{0}$ whenever $X_k = \bar{0}$.

The difference sequence spaces, $Z(\Delta) = \{ x = (x_k) : \Delta x \in Z\}$,where $Z = \ell_\infty , c$ and c_0, were studied by Kizmaz [26].

It was further studied in [15, 17], [24] as follows. Let $m \geq 0$ be an integer then $H(\Delta^m) = \{ x = (x_k) : \Delta^m x \in Z\}$, for $Z = \ell_\infty , c$ and c_0, where $\Delta^m x_k = x_k - x_{k+m}$. Moreover, in [36] Tripathy et al generalized the above notions and unified these as follows:

$$\Delta_n^m x_k = \{ x \in \omega : (\Delta_n^m x_k) \in Z\},$$

where

$$\Delta_n^m x_k = \sum_{\mu=0}^{n} (-1)^\mu \begin{pmatrix} n \\ r \end{pmatrix} x_{k+m\mu},$$

and

$$\Delta_n^0 x_k = x_k \forall\ k \in \mathbb{N}.$$

The idea of Kizmaz [26] was applied by Saveas [31, 32] for introducing the notion of difference sequences for fuzzy real numbers and study their different properties. The difference sequence space were further studies by Çlock [7, 8], Ganie and Sheikh [20, 23, 24], Ganie, Sheikh and Sen [21], Mursaleen [29] and many others.

For (a_k) and (b_k) be two sequence with complex terms and $p = (p_k) \in$ ell_∞, we have the following known inequality:

$$|a_k + b_k|^{p_k} \leq B \left(|a_k|^{p_k} + |b_k|^{p_k}\right)$$

where $B = \max\{1, 2^{M-1}\}$ and $M = \sup_k p_k$.

2. New types of sequences of fuzzy numbers

Let $X = (X_k)$ be a sequence of fuzzy numbers and $\Lambda = (f_k)$ be a sequence of moduli. In this article, we define the following classes of difference sequences of fuzzy numbers:

$$c_0(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) : \lim_k k^{-s}[f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k} = 0 \right\},$$
$$c(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) : \lim_k k^{-s}[f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} = 0 \right\},$$
$$\ell_\infty(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) : \sup_k k^{-s}[f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k} < \infty \right\},$$

for some X_0, $s \geq 0$ and $p = (p_k)$ is a sequence of real numbers such that $p_k > 0$ for all k and $\sup_k p_k = M < \infty$.

Note that for $s = 0$, these spaces are reduced to the spaces introduced by Neyaz and Hamid [21]. For $m = 1 = n, s = 0,\ f_k(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then these spaces are reduced to $c_0(F, \triangle)$, $c(F, \triangle)$ and $\ell_\infty(F, \triangle)$, introduced by Mursaleen and Başarir [29]. Again if we take $m = 0,\ n = 1,\ s = 0,\ f_k(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then these spaces are respectively reduced to $c_0(F)$, $c(F)$ and $\ell_\infty(F)$ introduced by Nanda [30].

Theorem 2.1. *If $\bar{d}$ is a translation invariant metric, then $c_0(F, \Lambda, \triangle_n^m, p)$, $c(F, \Lambda, \triangle_n^m, p)$ and $\ell_\infty(F, \Lambda, \triangle_n^m, p)$ are closed under the operation of addition and scalar multiplication.*

Proof. As $\bar{d}$ is translation invariant metric, it implies that

$$\bar{d}(\triangle^m X_k + \triangle^m Y_k, X_0 + Y_0) \leq \bar{d}(\triangle_n^m X_k, X_0) + \bar{d}(\triangle_n^m Y_k, Y_0) \tag{2.1}$$

and

$$\bar{d}(\triangle_n^m \lambda X_k, \lambda X_0) \leq |\lambda| \bar{d}(\triangle_n^m X_k, X_0), \tag{2.2}$$

where λ is a scalar and $|\lambda > 1$. We shall prove only for $c(F, \Lambda, \triangle_n^m, p)$. The others can be treated similarly. Suppose that $X = (X_k)$, $Y = (Y_k) \in c(F, \Lambda, \triangle_n^m, p)$. Then,

$$\begin{aligned} &[f_k(\bar{d}(\triangle_n^m X_k + \triangle_n^m Y_k, X_0 + Y_0))]^{p_k} \\ &\leq [f_k(\bar{d}(\triangle_n^m X_k, X_0) + \bar{d}(\triangle_n^m Y_k, Y_0))]^{p_k} \text{ [by (2.1)]} \\ &\leq [f_k(\bar{d}(\triangle_n^m X_k, X_0)) + f_k(\bar{d}(\triangle_n^m Y_k, Y_0))]^{p_k} \text{ [by (ii)]} \\ &\leq K^M [f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} + K^M [f_k(\bar{d}(\triangle_n^m Y_k, Y_0))]^{p_k} \text{ [by (2.2)]}. \end{aligned}$$

Then $X + Y \in c(F, \Lambda, \triangle_n^m, p)$. Let $X = (X_k) \in c(F, \Lambda, \triangle_n^m, p)$. For $\lambda \in \mathbb{R}$ there exists an integer K such that $|\lambda| \leq K$. Then by taking into account the property (2.2) and the modulus functions f_k for all $k \in \mathbb{N}$, we have

$$[f_k(\bar{d}(\lambda \triangle_n^m X_k, \lambda X_0))]^{p_k} \leq [f_k|\lambda|(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} \leq K^M [f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k}.$$

This implies that $\lambda X \in c(F, \Lambda, \triangle_n^m, p)$. 2.1. ☐

Theorem 2.2. *Let $p = (p_k) \in \ell_\infty$. Then the classes of sequences $c_0(F, \Lambda, \triangle_n^m, p)$, $c(F, \Lambda, \triangle_n^m, p)$ and $\ell_\infty(F, \Lambda, \triangle_n^m, p)$, are paranormed spaces, paranormed by g defined*

$$g(x) = \sup_k (f(\bar{d}(\triangle_n^m(\alpha_k X_k), \bar{0})))^{\frac{p_k}{M}},$$

where $M = \max(1, \sup_k p_k)$ *and* $X = (X_k)$.

Proof. The proof is an easy exercise so is left as an easy exercise. □

Theorem 2.3. *Let* $\Lambda = (f_k)$ *be a sequence of moduli. Then*

$$c_0\left(F, \Lambda, \triangle_n^m, p\right) \subset c\left(F, \Lambda, \triangle_n^m, p\right) \subset \ell_\infty\left(F, \Lambda, \triangle_n^m, p\right).$$

Proof. $c_0\left(F, \Lambda, \triangle_n^m, p\right) \subset c\left(F, \Lambda, \triangle_n^m, p\right)$ is trivial. Let $X = (X_K) \in c\left(F, \Lambda, \triangle_n^m, p\right)$. Then there is some fuzzy number X_0 such that $\lim_k [f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k} = 0$. Now, from (2.2), we have

$$[f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k} \le K[f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} + K[f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k}.$$

As $X = (X_k) \in c\left(F, \Lambda, \triangle_n^m, p\right)$, we obtain $X = (X_k) \in \ell_\infty\left(F, \Lambda, \triangle_n^m, p\right)$ and this proves the result. □

Theorem 2.4. *The classes* $\left(F, \Lambda, \triangle_n^m, p\right)$ *and* $\ell_\infty\left(F, \Lambda, \triangle_n^m, p\right)$ *are neither solid nor monotone (in general).*

Proof. Let $f(x) = x$, for all $x \in x \in [0, \infty)$, $m = 2,\ n = 1,\ \lambda_k = 2$ for all $k \in \mathbb{N}$

$$p_k = \begin{cases} 1 & \text{for } k = \text{odd}, \\ 2 & \text{for } \quad k = \text{even}. \end{cases}$$

Consider the sequence (X_k) defined by $(X_k) = H$ for all $k \in \mathbb{N}$, where

$$H(t) = \begin{cases} t + 1, & \text{if } -1 \le t \le 0, \\ 1 - t, & \text{if } \ \ 0 \le t \le 1. \\ 0, & \quad \text{otherwise}. \end{cases}$$

Then clearly, $(X_k) \in c\left(F, \Lambda, \triangle_n^m, p\right)$. For, N, a class of sequences, consider its J-step space N_j defined as follows:

when $(X_k) \in N_j$, then its canonical pre-image $(Y_k) \in N_j$ is given by

$$Y_k = \begin{cases} X_k, & \text{if } k = \text{even}, \\ \bar{0}, & \text{if } k = \text{odd}. \end{cases}$$

Then $(Y_k) \notin c\left(F, \triangle_1^2, p\right)$. Thus the class of sequences $c\left(F, \triangle_1^2, p\right)$ is not monotone. So it is not solid. Hence the class of sequences $c\left(F, \triangle_n^m, p\right)$ is not monotone in general. □

Theorem 2.5. *The spaces* $c_0\left(F, \Lambda, \triangle_n^m, p\right),\ c\left(F, \Lambda, \triangle_n^m, p\right)$ *and* $\ell_\infty\left(F, \Lambda, \triangle_n^m, p\right)$ *are not symmetric in general.*

Proof. We only consider the case $c\left(F, \Lambda, \triangle_n^m, p\right)$. To prove the result we consider the following example.

Let $f(x) = x$, for all $x \in x \in [0, \infty)$, $m = 2,\ n = 1,\ \lambda_k = 3$ and

$$p_k = \begin{cases} 2 & \text{for } k = \text{odd}, \\ 3 & \text{for } \quad k = \text{even}. \end{cases}$$

for all $k \in \mathbb{N}$. Consider the sequence $(X_k) = (H, N, H, N, ...)$, where the fuzzy number H is defined as follows:

$$H(t)=\begin{cases} t+1, & \text{if } -1\leq t\leq 0,\\ 1-t, & \text{if } \ 0\leq t\leq 1.\\ 0, & \text{otherwise.}\end{cases}$$

and the fuzzy number N is defined by

$$N(t)=\begin{cases} \frac{t}{2}+1, & \text{if } -2\leq t\leq 0,\\ 1-\frac{t}{2}, & \text{if } \ 0\leq t\leq 2.\\ 0, & \text{otherwise.}\end{cases}$$

Then $(X_k)\in c\left(F,\triangle_1^2,p\right)$. Consider its rearrangement (Y_k) of (X_k) defined by $(Y_k)=(H,N,N,H,H,N,N,H,H,...)$. Then $(Y_k)\notin c\left(F,\triangle_1^2,p\right)$. Hence, the class of sequences $c\left(F,\Lambda,\triangle_n^m,p\right)$ is not symmetric, and the result follows. □

3. Conclusions

The manuscript is dealing with the introducing the new spaces viz, $c_0\left(F,\Lambda,\triangle_n^m,p\right)$, $c\left(F,\Lambda,\triangle_n^m,p\right)$ and $\ell_\infty\left(F,\Lambda,\triangle_n^m,p\right)$. Some basic properties concerning these spaces like symmetry, monotonicity etc have been analyzed. The consequences of the results obtained in this article are more general and extensive than the existing known results.

Acknowledgements. We thank the anonymous referee(s) for the careful reading, valuable suggestions which improved the presentation of the paper.

References

[1] Y. Altin, M. Et and M. Başarir, On some generalized difference sequences of fuzzy numbers, Kuwait J. Sci. Engrg. 34 (1A) (2007) 1–14.
[2] H. Altinok and M. Mursaleen, A-statistical boundedness for sequences of fuzzy numbers, Tainwanse J. Math. 15 (5) (2011) 2081–2093.
[3] M. Başarir and M. Mursaleen, Some difference sequence spaces of fuzzy numbers, J. Fuzzy Math. 11 (3)(2003) 1–7.
[4] T. Bilgin, D-statistical and strong D-Cesàro convergence of sequences of fuzzy numbers, Math. Commun. 8 (2003) 95–100.
[5] A. K. Chaudhury and P. Das, Some results on fuzzy topology on fuzzy sets, Fuzzy Sets and Systems, 56 (1993) 331–336.
[6] R. Çolak, Y. Altin and M. Mursaleen, On some sets of difference sequences of fuzzy numbers, Soft Comput. 15 (2011) 787–793.
[7] R. Çolak, H. Altinok and M. Et, Generalized difference sequences of fuzzy numbers, Chaos, Solitons and Fractals 40 (2009) 1106–1117.
[8] R. Çolak and M. Et, On some generalized difference sequence spaces and related matrix transformations. Hokkaido Math J. 26 (3)(1997) 483–492.
[9] P. Diamond and P. Kloeden, Metric spaces of fuzzy sets, Fuzzy Sets Syst. 35 (1990) 241–249.
[10] A. Esi, On some new paranormed sequence spaces of fuzzy numbers defined by Orlicz functions and statistical convergence, Math. Model. Anal. 11 (4) (2006) 379–386.
[11] A. Esi and B. Hazarika, Some new generalized classes of sequences of fuzzy numbers defined by an Orlicz function, Ann. Fuzzy Math. Inform. 4 (2) (2012) 401–406.
[12] J. X. Fang and H. Hung, On the level convergence of a sequence of fuzzy numbers, Fuzzy Sets Syst. 147 (2004) 417–435.
[13] A. H. Ganie, Sigma bounded sequence and some matrix transformations, Algebra Letters 3 (2013) 1–7.
[14] A. H. Ganie, Riesz spaces using modulus function, Int. jour. Math. Models and Methods in Appl. Sci. 14 (2020) 20–23.

[15] A. H. Ganie, Some new approach of spaces of non-integral order, J. Nonlinear Sci, Appl. 14 (2) (2021) 89–96.
[16] A. H. Ganie, New spaces over modulus function, Boletim da Sociedade Paranaense de Matemßtica (accepted).
[17] A. H. Ganie and A. Antesar, Certain sequence spaces using Δ-operator, Adv. Stud. Contemp. Math. (Kyungshang) 30 (1) (2020) 17–27.
[18] A. H. Ganie and D. Fathima, Almost convergence property of generalized Riesz spaces, Journal of Applied Mathematics and Computation 4 (4) (2020) 249–253.
[19] A. H. Ganie and S. A. Lone, Some sequence spaces of sigma means defined by Orlicz function, Appl. Math. Inf. Sci. 6 (14) (2020) 1–4.
[20] A. H. Ganie and N. A. Sheikh, On some new sequence spaces of non-absolute type and matrix transformations, Jour. Egyp. Math. Soc. 21 (2013) 34–40.
[21] A. H. Ganie and N. A. Sheikh, Generalized difference sequence spaces of fuzzy numbers, New York J. Math. 19 (2013) 431–438.
[22] A. H. Ganie and N. A. Sheikh, Infinite matrices and almost convergence, Filomat 29 (6) (2015) 1183–1188.
[23] A. H. Ganie, Mobin Ahmad and N. A. Sheikh, New generalized difference sequence spaces of fuzzy numbers, Ann. Fuzzy Math. Inform. 12 (2) (2016) 255–263.
[24] A. H. Ganie, B. C. Tripathy, N. A. Sheikh and M. Sen, Invariant means and matrix transformations, Functional Analysis: Theory, Methods & Applications 2 (2016) 28–33.
[25] O. Kelava and S. Seikkala, On fuzzy metric spaces, Fuzzy Sets Syst. 12 (1984) 215–229.
[26] H. Kizmaz, On certain sequence spaces, Canad Math. Bull. 24 (2)(1981) 169–175.
[27] I. J. Maddox, Sequence spaces defined by a modulus, Math . Proc. Cam. Philos. Soc. 100 (1) (1986) 161–166.
[28] M. Matloka, Sequences of fuzzy numbers, BUSEFAL 28 (1986) 28–37.
[29] M. Mursaleen and M. Başarir, On some new sequence spaces of fuzzy numbers, Indian J. Pure Appl. Math. 34 (9) (2003) 1351–1357.
[30] S. Nanda, On sequences of fuzzy numbers, Fuzzy Sets Syst. 33 (1989) 123–126.
[31] E. Savaş, A note on sequence of fuzzy numbers, Inf. Sci. 124 (2000) 297–300.
[32] E. Saveş, On some A_I-convergent difference sequence spaces of fuzzy numbers defined by the sequence of Orlicz functions, Jou. Ineq. Appl. 261 (2012) 1–13.
[33] S. Simmons, The sequence spaces $l(p_v)$ and $m(p_v)$, Proc. London Math. Soc. 15 (3) (1965) 422–436.
[34] B. C. Tripathy and A. Baruah, Lacunary statistically convergent and lacunary strongly convergent generalized difference sequences of fuzzy real numbers, Kyungpook Math. Journal 50 (4) (2010) 565–574.
[35] B. C. Tripathy and H. Dutta: On some lacunary difference sequence spaces defined by a sequence of Orlicz functions and q-lacunary statistical convergence, Analele Stiintifice ale Universitatii Ovidius, Seria Matematica 20 (1) (2012) 417–430.
[36] B. C. Tripathy and A. Esi, A new type of difference sequence spaces, Int. J. Sci. Tech. 1 (1) (2006) 11–14.
[37] B. C. Tripathy and S. Mahanta : On a class of generalized lacunary difference sequence spaces defined by Orlicz function; Acta Math. Applicata Sinica (Eng. Ser.) 20 (2) (2004) 231–238.
[38] B. K. Tripathy and S. Nanda, Absolute value of fuzzy real numbers and fuzzy sequence spaces, J. Fuzzy Math. 8 (4) (2000) 883–892.
[39] B. C. Tripathy and B. Sarma, Sequence spaces of fuzzy real numbers defined by Orlicz functions, Math. Slovaca 58 (5)(2008) 621–628.
[40] B. C. Tripathy and M. Sen, On generalised statistically convergent sequences, Indian J. Pure and App. Maths. 32 (11) (2001) 1689–1694.
[41] B. C. Tripathy, B. Hazarika and B. Choudhary, Lacunary I-convergent sequences, Kyungpook Math. Journal 52 (4) (2012) 473–482.
[42] L. A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.

Abdul Hamid Ganie (a.ganie@seu.edu.sa)
Basic Science Department, College of Science and Theoretical Studies, Abha-M, SEU-KSA

Department of Applied Sciences and Humanities, SSM College of Engineering, Pattan, Kashmir-India

Sameer Ahmad Gupkari (samath.ein@gmail.com)
J and K Institute of Mathematical Sciences, Kashmir-India