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Soft semi-pre-Baire spaces on fuzzy soft topological spaces

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Abstract. The main aim of this paper is to initiate and explore the properties of fuzzy soft semi-pre-Baire spaces. We introduce and investigate fuzzy soft semi-pre-nowhere dense set, fuzzy soft semi-pre first category set, fuzzy soft semi-pre-second category set in fuzzy soft topological spaces.

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1. Introduction

A number of real life problems in engineering, social and medical sciences, economics etc. involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Some of these problems are essentially humanistic and thus subjective in nature (e.g. human understanding and vision systems), while others are objective, yet they are firmly embedded in an imprecise environment. The concept of fuzzy sets was first introduced by Zadeh [1] in the year 1965. This concept provides a natural foundation for treating mathematically the fuzzy soft phenomena which exist pervasively in our heal world and for new branches of fuzzy soft mathematics. Thenceforth, Chang [2] made tremendous growth of the numerous fuzzy soft topological concepts. Molodtsov [3] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. Later other authors like Maji et al. [4] further studied the theory of soft sets and to solve some decision making problems. The concept of fuzzy soft set is introduced and studied [5, 6, 7, 8, 9] as a more generalized concept which is a combination of fuzzy set and fuzzy soft set. Since then much attention has been paid to generalize the basic concept of fuzzy topology in soft setting and this modern theory of FST has been developed. In recent years, FST has been found to be very useful in solving any practical problem. The aim of this paper is to introduce the concept of fuzzy soft semi-pre-nowhere dense sets, FSSPFC, FSSPSC, FSSPR and FSSPBS in FSTS. In section 3, we have presented a brief note on the preliminaries related to soft sets definitions around our characterizations. In section 4 deals with FSSPNDS and their characterizations are established in FSTS. In section 5, FSSPBS are introduced. Finally, we have some conclusion in concluding section 6.

2. Review of Literature

In this paper ,the major work for fuzzy soft semi-pre-nowhere dense sets are introduced and further countable union of fuzzy soft semi-pre-nowhere dense sets are zero are called fuzzy soft semi-pre-Baire spaces.

3. Preliminaries

Now we give some basic notions and results that are used in the sequel. In this work by using a FSTS we shall mean a non-empty set U together with a FST U (in the sense of Chang) and denote it by (U, E, ψ). The interior, closure and the complement of a fuzzy soft set F_A will be denoted by $int^{fs}(F_A)$, $cl^{fs}(F_A)$ and $1-F_A$ respectively.

Definition 3.1 ([6])**.** The fuzzy soft set $F_\phi \in FS(U, E)$ is said to be *null fuzzy soft set* and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\overline{0}$ of U, where $\overline{0}(x) = 0$ for all $x \in U$.

Definition 3.2 ([6])**.** Let $F_E \in FS(U, E)$ and $F_E(e) = \overline{1}$ all $e \in E$, where $\overline{1}(x) = 1$ for all $x \in U$. Then F_E is called an *absolute fuzzy soft set*. It is denoted by $\overline{E}$.

Definition 3.3 ([6])**.** A fuzzy soft set F_A is said to be a *fuzzy soft subset* of a fuzzy soft set G_B over a common universe U, if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, i.e., if $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \breve{\subseteq} G_B$.

Definition 3.4 ([6])**.** Two fuzzy soft sets F_A and G_B over a common universe U are said to be *fuzzy soft equal*, if F_A is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A.

Definition 3.5 ([6])**.** The *union* of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \breve{\bigvee} G_B$.

Definition 3.6 ([6])**.** Let F_A and G_B be two fuzzy soft sets. Then the *intersection* of F_A and G_B is a fuzzy soft set H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \breve{\wedge} G_B$.

Definition 3.7 ([10])**.** Let $F_E \in FS(U, E)$ be a fuzzy soft set. Then the *complement* of F_A, denoted by F^c_A, defined by

$$F^c_A(e) = \begin{cases} \overline{1} - \mu^e_{F_A}, \text{ if } e \in A \\ \overline{1}, \text{ if } e \notin A \end{cases}$$

Definition 3.8 ([11]). Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a *fuzzy soft topology* on U, if ψ satisfies the following axioms:

(i) Φ, $\overline{E}$ belong to ψ,
(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ,
(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ.

The triplet (U, E, ψ) is called a *fuzzy soft topological space* over U. The members of ψ are called *fuzzy soft open sets* in U and complements of them are called *fuzzy soft closed sets* in U.

Definition 3.9 ([11]). The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the *interior* of F_A and is denoted by $int^{fs}(F_A)$.

Definition 3.10 ([11]). Let $F_A \in FS(U, E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the *closure* of F_A and is denoted by $cl^{fs}(F_A)$.

Remark 3.11. [11]

(1) For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $(cl^{fs}(F_A))^c = int^{fs}(F_A^c)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^c)$.
(2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ), we define the fuzzy soft subspace topology ψ on F_A by $K_D \in \psi_{F_A}$ if $K_D = F_A \breve{\wedge} G_B$ for some $G_B \in \psi$.
(3) For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively.

Definition 3.12 ([12]). A fuzzy soft set F_A in a FSTS (U, E, ψ) is called a *fuzzy soft neighborhood* (briefly, FS nbd) *set*, if there exists no non-zero fuzzy soft open set G_B in (U, E, ψ) such that $G_B < cl^{fs}(F_A)$. i.e., $int^{fs}cl^{fs}(F_A) = 0$.

Definition 3.13 ([12]). A fuzzy soft set F_A in a FSTS (U, E, ψ) is called *fuzzy soft dense*, if there exists no fuzzy soft closed set G_B in (U, E, ψ) such that $F_A < G_B < 1$. i.e., $cl^{fs}(F_A) = 1$.

Definition 3.14 ([12]). Let (U, E, ψ) be a fuzzy soft topology. A fuzzy soft set F_A in (U, E, ψ) is called *fuzzy soft first category*, if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft nowhere dense sets in (U, E, ψ). Any other fuzzy soft set in (U, E, ψ) is said to be of fuzzy soft second category.

Definition 3.15 ([13]). Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A | Fcl(Fint(f_A))$, then f_A is called fuzzy semi open soft set. We denote the set of all fuzzy semi open soft sets by $FSOS(X, T, E)$, or $FSOS(X)$ and the set of all fuzzy semi closed soft sets by $FSCS(X, T, E)$, or $FSCS(X)$.

Theorem 3.16 ([13]). *Let* (X, T, E) *be a fuzzy soft topological space and* $f_A \in FSOS(X)$. *Then*

(1) *arbitrary fuzzy soft union of fuzzy semi open soft set is fuzzy semi open soft,*
(2) *arbitrary fuzzy soft intersection of fuzzy semi closed soft set is fuzzy semi closed soft.*

Theorem 3.17 ([13]). *Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then*

1. $f_A \in FSOS(X)$ *if and only if* $Fcl(f_A) = Fcl(Fint(f_A))$,
2. *if* $g_B \in T$, *then* $g_B Fcl(f_A)|Fcl(g_B g_B)$.

Theorem 3.18 ([13]). *Let (X, T, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then*

1. $f_A \in FSOS(X)$ *if and only if there exists* $g_B \in T$ *such that* $g_B \hat{o} f_A \hat{o} Fcl(g_B)$,
2. *if* $f_A FSOS(X)$ *and* $f_A \hat{o} h_D \hat{o} Fcl(f_A)$, *then* $h_D \in FSOS(X)$.

Definition 3.19 ([13]). Let (X, T, E) be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $f_e \in FSS(X)_E$. Then

(i) f_e is called *fuzzy semi interior soft point* of f_A, if there exists $g_B \in FSOS(X)$ such that $f_e \in g_B \hat{o} f_A$. The set of all fuzzy semi interior soft points of f_A is called the fuzzy semi soft interior of f_A and is denoted by $FSint(f_A)$. Consequently, $FSint(f_A) = \{g_B : g_B|f_A, g_B \in FSOS(X)\}$.

(ii) f_e is called *fuzzy semi cluster soft point* of f_A, if $f_A h_D \notin \hat{o}_E, \forall h_D \in FSOS(X)$. The set of all fuzzy semi cluster soft points of f_A is called fuzzy semi soft closure of f_A and denoted by $FScl(f_A)$. Consequently, $FScl(f_A) = \{h_D : h_D \in FSCS(X), f_A\ h_D\}$.

Theorem 3.20 ([13]). *Let (X, T, E) be fuzzy soft topological space and f_A, $g_B \in FSS(X)_E$. Then the following properties are satisfied for the fuzzy semi interior operator, denoted by $FSint$.*

1. $FSint(\tilde{1}_E) = \tilde{1}_E$ *and* $FSint(\tilde{0}_E) = \tilde{0}_E$.
2. $FSint(f_A)\hat{o}f_A$.

Definition 3.21 ([14]). A fuzzy set λ in a fuzzy topological space (X, T) is called a *fuzzy F_σ-set* in (X, T), if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(1 - \lambda_i) \in T$ for $i \in I$.

4. Fuzzy soft semi-pre-nowhere dense set

Motivated by the concept of fuzzy soft semi-pre-nowhere dense sets on FSTS.

Definition 4.1. A fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ) is called a *fuzzy soft semi-pre- nowhere dense set*, if there exists no non-zero fuzzy soft semi-pre-open set G_B in (U, E, ψ) such that $G_B < spcl^{fs}(F_A)$. That is,

$$spint^{fs} spcl^{fs}(F_A) = 0.$$

Example 4.2. Let $U = \{x_1, x_2, x_3\}$. Consider F_E, G_E, H_E are the fuzzy sets on U defined as follows:

$$F_E = \begin{bmatrix} 7 & 7 & 4 \\ 6 & 5 & 7 \\ 7 & 5 & 6 \end{bmatrix}, G_E = \begin{bmatrix} 10 & 5 & 6 \\ 7 & 6 & 7 \\ 8 & 5 & 8 \end{bmatrix}, H_E = \begin{bmatrix} 7 & 7 & 6 \\ 8 & 72 & 62 \\ 56 & 82 & 62 \end{bmatrix}.$$

Then $\psi = \{0, F_E, G_E, H_E, 1\}$ is clearly fuzzy soft topology on (U, E, ψ). Now consider the fuzzy soft semi set in (U, E, ψ) are $int^{fs}cl^{fs}(G_E) \leq G_E \leq (1 - G_E)$. $int^{fs}cl^{fs}(H_E) \leq H_E \leq (1 - F_E \wedge G_E)$. Then $int^{fs}cl^{fs}(H_E) = int^{fs}(1 - F_E \wedge G_E) = 0$. Thus (U, E, ψ) is a fuzzy soft semi-pre-nowhere dense sets.

Proposition 4.3. *The complement of a fuzzy soft semi-pre-nowhere dense set in a FSTS* (U, E, ψ) *need not be a fuzzy soft semi-pre-nowhere dense set.*

Proof. In Example 4.2, $(1 - F_E)$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ), where as $F_E = 1 - (1 - F_E)$ is not a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.4. *If* F_A *and* G_B *are fuzzy soft semi-pre-nwd sets in a FSTS* (U, E, ψ), *then* $F_A \vee G_B$ *need not be a fuzzy soft semi-pre-nwd set in* (U, E, ψ).

Proof. For in Example 4.2, $1 - F_E$, $1 - F_A$ are fuzzy soft semi-pre-nwd sets in (U, E, ψ). But $(1 - F_E) \vee (1 - G_E) = H_E$ implies that $spint^{fs} spcl^{fs}(H_E) \neq 0$. Then union of fuzzy soft semi-pre-nowhere dense sets need not be fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.5. *If the fuzzy soft sets* F_A *and* G_B *are fuzzy soft semi-pre-nwd sets in a FSTS* (U, E, ψ), *then* $F_A \wedge G_B$ *is a fuzzy soft semi-pre-nowhere dense set in* (U, E, ψ).

Proof. Let the fuzzy soft sets F_A and G_B be fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $spint^{fs} spcl^{fs}(F_A \wedge G_B) \leq spint^{fs} spcl^{fs}(F_A) \wedge spint^{fs} spcl^{fs}(G_B) \leq 0 \wedge 0 = 0$ [since $spint^{fs} spcl^{fs}(F_A) = 0$ and $spint^{fs} spcl^{fs}(G_B) = 0$]. That is, $spint^{fs} spcl^{fs}(F_A \wedge G_B) = 0$. Then $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.6. *If* F_A *is a fuzzy soft semi-pre-nowhere dense set in a FSTS* (U, E, ψ), *then* $spint^{fs}(F_A) = 0$.

Proof. Let F_A be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then we have $spint^{fs} spcl^{fs}(F_A) = 0$. Now, $F_A \leq spcl^{fs}(F_A)$. Thus we have

$$spint^{fs}(F_A) \leq spint^{fs} spcl^{fs}(F_A) = 0.$$

So $spint^{fs}(F_A) = 0$. □

Proposition 4.7. *If* F_A *is a fuzzy soft semi-pre-nowhere dense set and* G_B *is any fuzzy soft set in a FSTS* (U, E, ψ), *then* $F_A \wedge G_B$ *is a fuzzy soft semi-pre-nowhere dense set in* (U, E, ψ).

Proof. Let F_A be a fuzzy softsemi-pre-nowhere dense set in (U, E, ψ). Then we get

$$spint^{fs} spcl^{fs}(F_A) = 0.$$

On the other hand,

$$\begin{aligned} spint^{fs} spcl^{fs}(F_A \wedge G_B) &\leq spint^{fs} spcl^{fs}(F_A) \wedge spint^{fs} spcl^{fs}(G_B) \\ &\leq 0 \wedge spint^{fs} spcl^{fs}(G_B) = 0. \end{aligned}$$

Thus $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Definition 4.8. A fuzzy soft set F_A in a FSTS (U, E, ψ) is called *fuzzy soft Semi-pre-dense*, if there exists no fuzzy soft semi-pre- closed set G_B in (U, E, ψ) such that $F_A < G_B < 1$. That is, $spcl^{fs}(F_A) = 1$.

Proposition 4.9. *If* F_A *is a fuzzy soft semi-pre-dense and fuzzy soft semi-pre-open set in a fuzzy soft topological space* (U, E, ψ) *and if* $G_B \leq 1 - F_A$, *then* G_B *is a fuzzy soft semi-pre-nowhere dense set in* (U, E, ψ).

Proof. Let F_A be a fuzzy soft semi-pre-dense set in (U, E, ψ). Then $spcl^{fs}(F_A) = 1$ and $spint^{fs}(F_A) = F_A$. Now $G_B \leq 1 - F_A$ implies that $spcl^{fs}(G_B) \leq spcl^{fs}(1 - F_A)$. Thus $spcl^{fs}(G_B) \leq 1 - spint^{fs}(F_A) = 1 - F_A$. So $spcl^{fs}(G_B) \leq (1 - F_A)$, which implies that $spint^{fs}spcl^{fs}(G_B) \leq spint^{fs}(1 - F_A) = 1 - spcl^{fs}(F_A) = 1 - 1 = 0$. That is, $spint^{fs}spcl^{fs}(G_B) = 0$. Hence G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.10. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ). Then $1 - F_A$ is a fuzzy soft semi-pre-dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then we get

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

Now $F_A \leq spcl^{fs}(F_A)$ implies that $spint^{fs}(F_A) \leq spint^{fs}spcl^{fs}(F_A) = 0$. Thus $spint^{fs}(F_A) = 0$ and $spcl^{fs}(1 - F_A) = 1 - spint^{fs}(F_A) = 1 - 0 = 1$. So $1 - F_A$ is a fuzzy soft semi-pre-dense set in (U, E, ψ). □

Proposition 4.11. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ), then $spcl^{fs}(F_A)$ is also a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere set in (U, E, ψ). Then we have

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

Now $spcl^{fs}spcl^{fs}(F_A) = spcl^{fs}(F_A)$. Thus we get

$$spint^{fs}spcl^{fs}(spcl^{fs}(F_A)) = spint^{fs}spcl^{fs}(F_A) = 0.$$

So $spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Proposition 4.12. *If F_A is a fuzzy soft semi-pre-nowhere dense set in a FSTS (U, E, ψ), then $1 - spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then by proposition 4.11, $spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Also by proposition 4.11, $1 - spcl^{fs}(F_A)$ is a fuzzy soft semi-pre-dense set in (U, E, ψ). □

Proposition 4.13. *Let F_A be a fuzzy soft semi-pre-dense set in a FSTS (U, E, ψ). If G_B is any fuzzy soft set in (U, E, ψ), then G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ) if and only if $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let G_B be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then we have

$$spint^{fs}spcl^{fs}(G_B) = 0.$$

On the other hand,

$$\begin{aligned} spint^{fs}spcl^{fs}(F_A \wedge G_B) &\leq spint^{fs}spcl^{fs}(F_A) \wedge spint^{fs}spcl^{fs}(G_B) \\ &\leq spint^{fs}spcl^{fs}(F_A) \wedge 0 = 0. \end{aligned}$$

That is, $spint^{fs}spcl^{fs}(F_A \wedge G_B) = 0$. Thus $F_A \wedge G_B$ is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).

Conversely, let $F_A \wedge G_B$ be a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). Then $spint^{fs}spcl^{fs}(F_A \wedge G_B) = 0$. Thus we get

$$spint^{fs}spcl^{fs}(F_A) \wedge spint^{fs}spcl^{fs}(G_B) = 0.$$

Since F_A is a fuzzy softsemi-pre-dense set in (U, E, ψ), $spcl^{fs}(F_A) = 1$. So we get

$$spint^{fs}(1) \wedge spint^{fs}spcl^{fs}(G_B) = 0.$$

That is, $(1) \wedge spint^{fs}spcl^{fs}(G_B) = 0$. Hence we have

$$spint^{fs}spcl^{fs}(G_B) = 0.$$

Therefore G_B is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Definition 4.14. Let (U, E, ψ) be a FSTS. A fuzzy soft set F_A in (U, E, ψ) is called *fuzzy soft semi-pre-first category*, if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Any other fuzzy soft set in (U, E, ψ) is said to be of fuzzy soft semi-pre-second category.

Definition 4.15. Let F_A be a fuzzy soft semi-pre-first category set in a FSTS (U, E, ψ). Then $1 - F_A$ is called a *fuzzy soft semi-pre-residual set* in (U, E, ψ).

Proposition 4.16. *If F_A is a fuzzy soft semi-pre-first category set in a fuzzy soft topological space (U, E, ψ), then $1 - F_A = \wedge_{i=1}^{\infty}(F_{A_i})$, where $spcl^{fs}(G_{B_i}) = 1$.*

Proof. Let F_A be a fuzzy soft semi-pre-first category set in (U, E, ψ). Then we have $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $1 - F_A = \wedge_{i=1}^{\infty}(F_{A_i})$. Let $G_{B_i} = 1 - (F_{A_i})$. Then $1 - F_A = \wedge_{i=1}^{\infty}(F_{A_i})$. Since (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ), by proposition 4.13, $(1 - F_{A_i})$'s are fuzzy soft semi-pre-dense set in (U, E, ψ). Thus we get

$$spcl^{fs}(G_{B_i}) = spcl^{fs}(1 - (F_{A_i})) = 1.$$

So $1 - F_A = \wedge_{i=1}^{\infty}(F_{A_i})$. Where $spcl^{fs}(G_{B_i}) = 1$. □

Proposition 4.17. *If F_A be a fuzzy soft semi-pre-closed set in a fuzzy soft topological space (U, E, ψ) and if $spint^{fs}(F_A) = 0$, then F_A is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).*

Proof. Let F_A be a fuzzy soft semi-pre-closed set in (U, E, ψ). Then $spcl^{fs}(F_A) = F_A$. Now $spint^{fs}spcl^{fs}(F_A) = spcl^{fs}F_A$ and $spint^{fs}(F_A) = 0$. Thus we have

$$spint^{fs}spcl^{fs}(F_A) = 0.$$

So F_A is a fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). □

Definition 4.18. A Fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ) is called a *fuzzy soft semi-pre-F_σ-set* in (U, E, ψ), if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-closed sets in (U, E, ψ).

5. Fuzzy soft semi-pre-Baire spaces

Motivated by the concept of fuzzy soft semi-pre-Baire spaces and their characterizations on FSTS.

Definition 5.1. Let (U, E, ψ) be a FSTS. Then (U, E, ψ) is called a *fuzzy soft semi-pre-Baire space*, if $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense set in (U, E, ψ).

Example 5.2. Let $F_E = \begin{bmatrix} .7 & .7 & .6 \\ .8 & .72 & .62 \\ .44 & .82 & .62 \end{bmatrix}, G_E = \begin{bmatrix} .7 & .7 & .4 \\ .6 & .5 & .7 \\ .7 & .55 & .6 \end{bmatrix}, H_E = \begin{bmatrix} .10 & .5 & .6 \\ .7 & .6 & .7 \\ .8 & .5 & .8 \end{bmatrix}$.
Now consider the fuzzy soft sets. Then we have

$$int^{fs}cl^{fs}(F_E) \leq F_E \leq 1 - (F_E \wedge G_E)int^{fs}cl^{fs}(F_E) = int^{fs}(1 - F_E \wedge G_E) = 0.$$

Thus $\vee_{i=1}^{\infty} int^{fs}cl^{fs}(F_E \vee G_E) = 0$ is a fuzzy soft semi-pre-Baire space. So we get

$$spint^{fs}cl^{fs}(F_E \vee G_E) = spint^{fs}(1 - F_E \vee G_E) = 0.$$

Hence (U, E, ψ) is a fuzzy soft semi-pre-Baire space.

Proposition 5.3. *If $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$ and (F_{A_i})'s are fuzzy soft semi-pre-closed sets in a FSTS (U, E, ψ), then (U, E, ψ) is a fuzzy soft semi-pre-Baire space.*

Proof. Let (F_{A_i})'s be a fuzzy soft semi-pre-closed set in (U, E, ψ). Since

$$spint^{fs}(F_{A_i}) = 0,$$

(F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Then we have $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Thus (U, E, ψ) is a fuzzy soft semi-pre-Baire space. □

Proposition 5.4. *If $spcl^{fs}[\wedge_{i=1}^{\infty}(F_{A_i})] = 1$, where (F_{A_i})'s are fuzzy soft semi-pre-dense and fuzzy soft semi-pre-open sets in a FSTS (U, E, ψ). Then (U, E, ψ) is a fuzzy soft semi-pre-Baire space.*

Proof. Now $spcl^{fs}[\wedge_{i=1}^{\infty}(F_{A_i})] = 1$. This implies that $1 - spcl^{fs}[\wedge_{i=1}^{\infty}(F_{A_i})] = 0$. Then we have $spint^{fs}[1 - \wedge_{i=1}^{\infty}(F_{A_i})] = 0$. Thus $spint^{fs}[\vee_{i=1}^{\infty}(1 - F_{A_i})] = 0$. Since (F_{A_i})'s are fuzzy soft semi-pre-dense sets in (U, E, ψ), $spcl^{fs}(F_{A_i}) = 1$ and $spint^{fs}(1 - F_{A_i}) = 1 - spcl^{fs}(F_{A_i}) = 1 - 1 = 0$. So we have

$$spint^{fs}[\vee_{i=1}^{\infty}(1 - F_{A_i})] = 0,$$

where $spint^{fs}(1 - F_{A_i}) = 0$ and $(1 - F_{A_I})$'s are fuzzy soft semi-pre-closed sets in (U, E, ψ). Hence by proposition 5.3, (U, E, ψ) is a fuzzy soft semi-pre-Baire space. □

Proposition 5.5. *Let (U, E, ψ) be a FSTS. Then the following are equivalent:*

(1). (U, E, ψ) *is a fuzzy soft semi-pre-Baire space,*
(2). $spint^{fs}(F_A) = 0$, *for every fuzzy soft semi-pre-first category set* F_A *in* (U, E, ψ),
(3). $spcl^{fs}(G_B) = 1$, *for every fuzzy soft semi-pre residual set* G_B *in* (U, E, ψ).

Proof. (1) ⇒ (2): Let F_A be a fuzzy soft semi-pre-first category set in (U, E, ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $spint^{fs}(F_A) = spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$ ((U, E, ψ) is a fuzzy soft semi-pre-Baire space). Thus $spint^{fs}(F_A) = 0$.

(2) ⇒ (3): Let G_B be a fuzzy soft semi-pre-residual set in (U, E, ψ). Then $1 - G_B$ is a fuzzy soft semi-pre-first category set in (U, E, ψ). Thus by hypothesis, we get

$$spint^{fs}(1 - G_B) = 0,\ 1 - spcl^{fs}(G_B) = 0.$$

So we have $spcl^{fs}(G_B) = 1$.

(3) ⇒ (1): Let F_A be a fuzzy soft semi-pre-first category set in (U, E, ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now $1 - F_A$ is a fuzzy soft semi-pre-residual set in (U, E, ψ). Since F_A is a fuzzy soft semi-pre-first category set in (U, E, ψ). By hypothesis, we have $spcl^{fs}(1 - F_A) = 1$. Thus $1 - spint^{fs}(F_A) = 1$, $spint^{fs}(F_A) = 0$. So $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Hence (U, E, ψ) is a fuzzy soft semi-pre-Baire space. □

Proposition 5.6. *IF a FSTS (U, E, ψ) is a fuzzy soft semi-pre-Baire space, then (U, E, ψ) is a fuzzy soft semi-pre-second category space.*

Proof. Let (U, E, ψ) be a fuzzy soft semi-pre-Baire space. Then $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Then $\vee_{i=1}^{\infty}(F_{A_i}) \neq 1_X$ (suppose $\vee_{i=1}^{\infty}(F_{A_i}) = 1_X$ $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = spint^{fs}(1_X)$, implies $0 = 1$, a contradiction. Thus (U, E, ψ) is a fuzzy soft semi-pre-second category space. □

Remark 5.7. The converse of the above proposition need not be true. A fuzzy soft semi-pre-second category space need not be fuzzy soft semi-pre-Baire space.

In Example 5.2, the fuzzy soft sets $1 - F_A$, $1 - G_B$, $1 - F_A \vee G_B$, $1 - F_E$ are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Now

$$(1 - F_A) \vee (1 - G_B) \vee (1 - F_A \vee G_B) \vee (1 - F_E) = 1 - F_A \wedge G_B \neq 1_X$$

and $int(1 - F_A \wedge G_B) \neq 0$. Then the fuzzy soft topological space (U, E, ψ) is a fuzzy soft semi-pre-second category space but not a fuzzy soft semi-pre-Baire space.

Proposition 5.8. *If a FSTS (U, E, ψ) is a fuzzy soft semi-pre-Baire space, then no non-zero fuzzy soft semi-pre-open set in (U, E, ψ) is a fuzzy soft semi-pre-first category set in (U, E, ψ).*

Proof. Suppose that F_A is a non-zero fuzzy soft semi-pre-open set in (U, E, ψ) such that $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense sets in (U, E, ψ). Then we have $spint^{fs}(F_A) = spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})]$. Since F_A is a non-zero fuzzy soft semi-pre-open set in (U, E, ψ), we have $spint^{fs}(F_A) = F_A$. Thus $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = F_A \neq 0$. But this is a contradiction to (U, E, ψ) being a fuzzy soft semi-pre-Baire space in which $spint^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 0$, where (F_{A_i})'s are fuzzy soft semi-pre-nowhere dense set in (U, E, ψ). So we must have $F_A \neq \vee_{i=1}^{\infty}(F_{A_i})$. Hence no non-zero fuzzy soft semi-pre-open set in (U, E, ψ) is a fuzzy soft semi-pre-first category set in (U, E, ψ). □

6. Conclusions

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [3] and easily applied to many problems having uncertainties from social life. In this paper, we gave the definition of the FSTS and applying this basics for characterizations and properties in soft semi pre Baire spaces with examples .Hence this paper may be helpful for finding a base to researches who want to work in the field of FSSPBS..

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