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A fixed point theorem on fuzzy locally convex spaces

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Abstract. Let X be a linear space over a field $\mathbb{K}$ and $(X, \rho, *)$ a fuzzy seminorm space where $(\rho, *)$ a fuzzy seminorm with $*$ a continuous t-norm. We established a version of fixed point theorem for Fuzzy Locally Convex Space and prove that there exists a unique fixed point for a spherically complete fuzzy locally convex space.

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1. Introduction

Fuzzy set theory is an extension of what one might call classical set theory. In classical set theory, the membership of an element belonging to the set is based upon two valued Boolean logic. An element either belongs or does not belong to that set. For example, for the set of integers, either an integer is even or it is not (it is odd).

But unlike classical set theory, fuzzy set theory permits the gradual assessment of the membership of elements in a set, this is described with the aid of a membership function valued in the real unit interval $[0, 1]$. Fuzzy set generalizes classical sets, since the indicator functions (characteristics functions) of classical sets are special cases of the membership functions of fuzzy sets, if the latter only takes values 0 or 1. Hence, we have what is called Crisp set.

The concept of fuzzy vectors, fuzzy topological spaces were introduced and well elucidated by Kastaras in his famous works [1],[2] and [3]. Other invariants of these abound in literature [4]. Sadeqi and Solaty Kia [5] considered fuzzy seminormed spaces with an example of one, which is fuzzy normable but is not classical normable. More general properties and results on fuzzy seminorms can be seen in [2].

The importance and applications of fixed point theorem cannot be overemphasized. Athaf [6] established a fixed point theorem on a fuzzy metric spaces while

Egwe [7] proved the existence of a fixed point on a nonarchimedean fuzzy normed space. A modern approach to fuzzy analysis is can be seen in [8].

In this paper, we establish a version of fixed point theorem given by Sehgal in [9] and in fact prove that there exists a unique fixed point for a spherically complete fuzzy locally convex space.

2. Preliminaries

Let X be any arbitrary set and A a subset of X such that there exists a function

$$\begin{aligned} \mu_A : & X \longrightarrow [0,1] \\ & x \longmapsto \mu_A(x) \in [0,1] \end{aligned}$$

which assigns to every $x \in X$, a real number $\mu_A(x)$ between 0 and 1 which represent the degree or grade of membership or belongingness of x to A. Thus, the nearer the value of $\mu_A(x)$ to unity the higher the degree or belongingness of x to A.

Hence a fuzzy subset A of X has the following representation: $A = \{(x, \mu_A(x)) : x \in A\}$.

Definition 2.1. Let X be a set. Then a mapping $A : X \longrightarrow [0,1]$ is called a *fuzzy subset* A in X, where $[0,1]$ is the membership space.

General theory on the basic operations and operators on fuzzy sets can be seen in the books of [10] and [11].

We shall now define the concepts of Norms and Metrics in on fuzzy sets in what follows.

Definition 2.2 ([12, 13])**.** A triangular norm, $t-$norm for short is a mapping $* : [0,1] \times [0,1] \longrightarrow [0,1]$, where $*$ is a binary operation such that the following axioms are satisfied: $\forall\, u, v, w \in [0,1]$,

(i) $*(u,v) = *(v,u)$,
(ii) $*(u, *(v,w)) = *(*(u,v), w)$,
(iii) $*(u,v) \leq *(u,w) \quad$ where $v \leq w$,
(iv) $*(u,1) = u * 1 = u, \quad *(u,0) = u * 0 = 0$.

The following t-norms are well-known and frequently used.

(1) $u * v = \min(u,v)$ (Standard intersection)
(2) $u * v = uv$ (Algebraic product)
(3) $u * v = \max(0, u + v - 1)$ (Bounded difference).

We remark here that we shall adopted the first option above in this paper.

Definition 2.3 ([5])**.** Let X be a vector space over a field $\mathbb{K}$ and $*$ a continuous t-norm and M a function (distance function with respect to $t \in (0,\infty)$) on $X^2 \times (0,\infty)$, that is,

$$\begin{aligned} M : & X \times X \times (0,\infty) \longrightarrow [0,1] \\ & (x,y,t) \longmapsto M(x,y,t) \end{aligned}$$

satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,

(FM1) $M(x,y,t) > 0 \ \forall\, t > 0$,

(FM2) $M(x, y, t) = 1 \quad$ if and only if $\quad x = y, \forall\ t > 0$,
(FM3) $M(x, y, t) = M(y, x, t)$,
(FM4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$,
(FM5)$M(x, y, \cdot) : (0, \infty) \longrightarrow [0, 1]$ is continuous.
Then $(X, M, *)$ is called a *fuzzy metric space*, where $M(x, y, t)$ represents the degree of nearness of x and y with respect to t.

Definition 2.4. Let $(X, M, *)$ be a fuzzy metric space.
(i) A sequence $x_n \in X$ is said to be *convergent to a point* $x \in X$, if

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \ \ \forall\ t > 0.$$

(ii) A sequence $x_n \in X$ is called a *Cauchy sequence*, if for each $0 < \epsilon < \ 1$ and $t > 0$, there exist $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$.
(iii) A fuzzy metric space in which every Cauchy sequence is convergent to a limit in the space is said to be *complete*.

Definition 2.5. Let Y be a vector space over a field $\mathbb{K}$ and let $*$ be a continuous t-norm. Let $p : Y \times \mathbb{R} \longrightarrow [0, 1]$ be a mapping satisfying the following conditions:
(i) $p(y, t) = 0$ when $t \leq 0$,
(ii) $p(y, t) = p\left(vy, \frac{t}{|v|}\right)$ when $\ t > 0, v \neq 0$,
(iii) $p(y + z, t + s) \geq p(y, t) * p(z, s)$, where $t,\ s \in \mathbb{R},\ y,\ z \in Y$,
(iv) $p(y, \cdot)$ is an increasing function of $\mathbb{R}$ and $\lim_{t\to\infty} p(y, t) = 1$.
Then $(p, *)$ is called a *fuzzy seminorm* on Y and $(Y, p, *)$ is called a *fuzzy seminorm space*.

Definition 2.6. A family $\mathscr{P}$ of fuzzy seminorms on Y is called *separating*, if to each $y_\circ \neq 0$, there is least one $p \in \mathscr{P}$ and $t \in \mathbb{R}$ such that $p(y, t) \neq 1$.

Definition 2.7. Let $\mathfrak{D}$ be a separated fuzzy locally convex topological vector space, $\mathfrak{A}$ a nonempty subset of $\mathfrak{D}$ and $\mathcal{B}$ be a neighbourhood basis of the origin consisting of absolutely fuzzy convex open subsets of $\mathfrak{D}$. For each $B \in \mathcal{B}$, let φ_B be the Minkowski's functional of B and p a fuzzy seminorm on $\mathfrak{A}$. For each $y, z \in \mathfrak{A}$, $t \in \mathbb{R}$ and $\alpha \in (0, 1)$, we have
(i) $\varphi_B(y - z) = inf\{t > 0 : p(y - z) < t\}$,
(ii) $\varphi_B(y - z, t) = sup\{\alpha \in (0, 1) : p(y - z) < t\}$,
(iii) $B(0, \alpha, t) = \{y - z : p(y - z, t) > 1 - \alpha\}$,
(iv) $B(y, \alpha, t) = \{z : p(y - z, t) > 1 - \alpha\}$.

Definition 2.8. A mapping $F : \mathfrak{A} \longrightarrow \mathfrak{D}$ is called a *fuzzy B-contraction* ($B \in \mathcal{B}$), provided that for each $\varepsilon > 0,\ \alpha \in (0, 1)$, there is a $\delta = \delta(\varepsilon, B, \alpha) > 0$ and $\beta = \beta(\varepsilon, B, \alpha) \in (0, 1)$ such that if $y, z \in \mathfrak{A}$ and if

(2.1)
$$1 - \alpha \geq \varphi_B(y - z, \varepsilon + \delta) > 1 - (\alpha + \beta), \ \text{ then } \ \varphi_B((F(y) - F(z)), \varepsilon) > 1 - \alpha.$$

In what follows, we now give our maim results.

3. Main Results

If $F:\mathfrak{A}\longrightarrow\mathfrak{D}$ is a fuzzy B- Contraction for each $B\in\mathcal{B}$, then F is a fuzzy $\mathcal{B}$-Contraction.
Note that if F is a fuzzy $\mathcal{B}$- Contraction, then F is fuzzy continuous.

The following Lemma shall be needed in the sequel.

Lemma 3.1. *Let $F:\mathfrak{A}\longrightarrow\mathfrak{D}$ be a fuzzy $\mathcal{B}$-contraction. Then F is fuzzy $\mathcal{B}-$contractive, that is, for each $B\in\mathcal{B}$, $\varphi_B(F(y)-F(z),\varepsilon)>\varphi_B(y-z,\varepsilon+\delta)$, if $\varphi_B(y-z,\varepsilon+\delta)\neq 1$ and 1 otherwise.*

Proof. Let $y,z\in\mathfrak{A}$ and suppose $\varphi_B=\varphi$, $\varphi(y-z,\varepsilon+\delta)=1-\alpha<1$ for $\varepsilon>0$ and $\alpha\in(0,1)$. Then $\varphi(y-z,\varepsilon+\delta)>1-(\alpha+\beta)$ for each $\delta>0$. In particular $\varphi(y-z,\varepsilon+\delta_0)>1-(\alpha+\beta_0)$, where $\delta_0=\delta(\varepsilon,B,\alpha)$, $\beta_0=\beta(\varepsilon,B,\alpha)$. Thus by (2.1), $\varphi(F(y)-F(z),\varepsilon)>1-\alpha$. Since B is open, this implies that

$$\varphi(F(y)-F(z),\varepsilon)>1-\alpha=\varphi(y-z,\varepsilon+\delta).$$

If $1-\alpha=1$, then $\varphi(y-z,\varepsilon+\delta)>1-\alpha$ for each $\varepsilon>0$, $\alpha\in(0,1)$ and thus by (2.1), $\varphi(F(y)-F(z),\varepsilon)>1-\alpha$. So $\varphi(F(y)-F(z),\varepsilon)=1$. □

Theorem 3.2. *Let $\mathfrak{A}$ be a sequentially complete fuzzy subset of $\mathfrak{D}$, μ be the membership function on $\mathfrak{A}$ and $F:\mathfrak{A}\longrightarrow\mathfrak{D}$ be a fuzzy $\mathcal{B}$-contraction. Suppose F satisfies the condition:*
for each $y\in\mathfrak{A}$, $\alpha\in(0,1)$, $\mu(y)=\alpha$ with $\mu(F(y))>\alpha$, there is a $\mu_{((y,F(y))\Lambda\mathfrak{A})}(w)=\mu_{(y,F(y))}(w)\star\mu_{\mathfrak{A}}(w)$ such that $\mu(F(w))\leq\mu(w)$.
Then F has a unique fixed point in $\mathfrak{A}$.

Proof. Let $y_0\in\mathfrak{A}$, $t>0$, $\alpha\in(0,1)$ with $\mu(y_0)=\alpha$ and choose a sequence $\mu_{y_n}(y_{n_i})\leq\mu_A(y_{n_i})$ for all $y_{n_i}\in\mathfrak{D}$, $i\in I$ defined (inductively) as follows for each $n\in I$ (positive integers):

If $\mu(F(y_0)\leq\mu(y_0)$, then set $(y_1)=F(y_0)$. Thus $\mu(y_1)\leq\mu(y_0)$ which implies $\varphi(y_1-y_0,t)\longrightarrow 1$, i.e., $y_1-y_0\longrightarrow 0$.

If $\mu(F(y_0))>\mu(y_0)$ and let $\mu_{((y_0,F(y_0))\Lambda\mathfrak{A})}(y_1)=\mu_{(y_0,F(y_0))}(y_1)\ \star\ \mu_{\mathfrak{A}}(y_1)$ such that $\mu(F(y_1))\leq\mu(y_1)$. Then $\varphi(F(y_1)-y_1,t)\longrightarrow 1$, i.e., $F(y_1)-y_1\longrightarrow 0$.
Since we have chosen the sequence $\{y_n\}$, we have:

Suppose $\mu F(y_n)\leq\mu(y_n)$, and let $y_{n+1}=F(y_n)$. Then $\mu(y_{n+1})\leq\mu(y_n)$. Thus

$$\varphi(y_{n+1}-y_n,t)\longrightarrow 1.$$

Suppose $\mu(F(y_n))>\mu(y_n)$ and let

$$\mu_{((y_n,F(y_n))\Lambda\mathfrak{A})}(y_{n+1})=\mu_{(y_n,F(y_n))}(y_{n+1})\ \star\ \mu_{\mathfrak{A}}(y_{n+1})$$

such that $\mu(F(y_{n+1}))\leq\mu(y_{n+1})$. Then $\varphi(F(y_{n+1})-y_{n+1},t)\longrightarrow 1$, i.e.,

$$F(y_{n+1})-y_{n+1}\longrightarrow 0.$$

Thus it follows that for each $n\in I$, there is a $\lambda_n\in[0,1)$ satisfying

$$y_{n+1}=\lambda_n y_n+(1-\lambda_n)F(y_n). \tag{3.1}$$

We show that the fuzzy sequence $\{y_n\}$ so constructed satisfies

$$(a)\ \ y_{n+1}-y_n\longrightarrow 0\quad (b)\ \ y_n-F(y_n)\longrightarrow 0. \tag{3.2}$$

To establish (3.2), note that by (3.1)

$$y_{n+1} - y_n = (1-\lambda_n)(F(y_n) - y_n), \tag{3.3}$$

$$F(y_n) - y_{n+1} = \lambda_n(F(y_n) - y_n). \tag{3.4}$$

Then for $B \in \mathcal{B}$ with $\varphi_B = \varphi$, it follows by the Lemma 3.1 that

$$\begin{aligned} \varphi(F(y_{n+1}) - y_{n+1}, \epsilon) &\geq \varphi\left(F(y_{n+1}) - y_{n+2}, \epsilon\right) \star \varphi\left(y_{n+2} - y_{n+1}, \epsilon\right) \\ &\geq \varphi\left(F(y_n) - y_{n+1}, \epsilon\right) \star \left(y_{n+1} - y_n, \epsilon\right) \\ &\geq \varphi\left(\lambda_n(F(y_n) - y_n), \epsilon\right) \star \left((1-\lambda_n)(F(y_n) - y_n, \epsilon\right) \\ &\geq 1 \star \varphi(F(y_n) - y_n, \epsilon) \\ &\geq \varphi(F(y_n) - y_n, \epsilon). \end{aligned}$$

Thus by (3.3), $\varphi(F(y_{n+1}) - y_{n+1}, \epsilon) \geq \varphi(F(y_n) - y_n, \epsilon)$ for each $n \in I$, that is, $\{\varphi(F(y_n) - y_n, \epsilon)\}$ is an increasing sequence of non negative reals. So for each $\varphi = \varphi_B,\ B \in \mathcal{B}$, there is an $r > 0$ and $0 < \alpha < 1$ with

$$1 - \alpha \geq \varphi(F(y_n) - y_n, r) \longrightarrow 1 - \alpha \leq 1. \tag{3.5}$$

We claim that $1 - \alpha \equiv 1$. Suppose $1 - \alpha > 1$. Choose $\delta = \delta(r, B, \alpha) > 0$ and $\beta = \beta(r, B, \alpha) \in (0.1)$ satisfying (2.1). Then by (3.5), there is a $n_0 \in I$ such that

$$\varphi(F(y_n) - y_n, r + \delta) > 1 - (\alpha + \beta) \text{ for all } n \geq n_0.$$

Now choose an $m \in I$, $m \geq n_0 \ni y_{m+1} = F(y_m)$, (let $m = n_0$ if $\mu(F(y_{n_0})) \leq \mu(y_{n_0}), \alpha \in (0,1)$ *with* $\mu(y_m) = \alpha$) otherwise let $m = n_0 + 1$, then $\mu(F(y_{n_0+1})) \leq \mu(y_{n_0+1})$. Thus for this m,

$$\varphi(y_m - y_{m+1}, r + \delta) = \varphi(y_m - F(y_m), r + \delta) > 1 - (\alpha + \beta)$$

and so by (2.1),

$$\varphi(y_{m+1} - F(y_{m+1}), r) = \varphi(F(y_m) - F(y_{m+1}), r) > 1 - \alpha$$

which contradicts (3.5). Hence $1 - \alpha = 1$ for each $B \in \mathcal{B}$ and this implies that the sequence $y_n - F(y_n) \to 0$. Therefore (b) holds. Now let us show that (a) holds. By (3.3), $y_{n+1} - y_n = (1-\lambda_n)(F(y_n) - y_n)$. Since it is a known fact that $F(y_n)$ is shifting towards y_n, $y_{n+1} \longrightarrow y_n$ as $\lambda_n \longrightarrow 1$. since $y_n - F(y_n) \longrightarrow 0$, we are sure λ_n is moving to 1. Then we can conclude that $y_{n+1} - y_n \longrightarrow 0$.

We assert that $\{y_n\}$ is a Cauchy sequence in A. Suppose not. Let for each $i \in I$, $A_i = \{y_n : n \geq i\}$. Then by the assumption, there is $B \in \mathcal{B} \ni \varphi(y_n - y_m, \varepsilon + \delta) \leq 1 - (\alpha + \beta)$ for any $i \in I$. Choose an ε with $0 < \varepsilon < 1$, $0 < \alpha < 1$ and a δ with $0 < \delta < \delta(\varepsilon, B, \alpha)$, $0 < \beta < \beta(\varepsilon, B, \alpha) < 1$ satisfying $\varepsilon + \delta < 1$, $\alpha + \beta < 1$. Then it follows that $\varphi(y_n - y_m, \varepsilon + \frac{\delta}{2}) \leq 1 - (\alpha + \frac{\beta}{2})$ for any $i \in I$. Thus for each $i \in I$, there exist integers $n(i)$ and $m(i)$ with $i \leq n(i) < m(i)$ such that

$$\varphi(y_{n(i)} - y_{m(i)}, (\epsilon + \frac{\delta}{2})) \leq 1 - (\alpha + \frac{\beta}{2}). \tag{3.6}$$

Let $m(i)$ be the least integer exceeding $n(i)$ satisfying (3.6).Then by (3.6),

(3.7)

$$\begin{aligned}1-(\alpha+\beta) \geq \varphi(y_{n_i}-y_{m_i},\varepsilon+\delta) &= \varphi(y_{n(i)}-y_{m(i)-1}+y_{m(i)-1}-y_{m(i)},\varepsilon+\delta)\\ &\geq \varphi(y_{n(i)}-y_{m(i)-1},\varepsilon+\frac{\delta}{2})\star\varphi(y_{m(i)-1}-y_{m(i)},\frac{\delta}{2})\\ &\geq 1-(\alpha+\frac{\beta}{2})\star\varphi(y_{m(i)-1}-y_{m(i)},\frac{\delta}{2})\\ &\geq 1-(\alpha+\frac{\beta}{2})\star 1\\ &\geq 1-(\alpha+\frac{\beta}{2})\\ &> 1-(\alpha+\beta).\end{aligned}$$

Now by (3.2), there is a $i_0 \in I \ni \varphi(y_i - F(y_i), {}^{\delta}/_{4}) > 1-({}^{\beta}/_{4})$ and $\varphi(y_{i-1}-y_i, {}^{\delta}/_{4}) > 1-({}^{\beta}/_{4})$ whenever $i \geq i_0$. Thus by (3.7), $\varphi(y_{n(i)}-y_{m(i)},\varepsilon+\delta) > 1-(\alpha+\beta)$. It follows from (2.1) that for all i $\geq i_0$, $\varphi(F(y_{n(i)})-F(y_{m(i)}),\varepsilon) > 1-\alpha$. However, for all i $\geq i_0$,

$$\begin{aligned}1-(\alpha+\frac{\beta}{2}) > \varphi(y_{n(i)}-x_{m(i)},\epsilon+\frac{\delta}{2}) &\geq \varphi\left(y_{n_(i)}-F(y_{n_{(i)}}),\frac{\delta}{4}\right)\\ &\star\varphi\left(F(y_{n(i)})-F(y_{m(i)}),\epsilon\right)\\ &\star\varphi\left(F(y_{m(i)})-y_{m(i)},\frac{\delta}{4}\right)\\ &\geq 1*(1-\alpha)\star 1\\ &\geq 1-\alpha\\ &> 1-\left(\alpha+\frac{\beta}{2}\right),\end{aligned}$$

which contradicts (3.6). So $\{y_n\}$ is a Cauchy sequence in A and the sequential completeness implies that there is a $U \in A \ni$

$$\lim_{n\to\infty}\varphi(y_n-U,t)=1 \quad \forall \quad t>0.$$

Now we are required to check if the limit is unique. Suppose there exists

$$V \in A \ni \lim_{n\to\infty}\varphi(y_n-V,t)=1 \ \forall \ t>0 \text{ and } V \neq U.$$

Then we have

$$\begin{aligned}\varphi(U-V,t) &\geq \varphi\left(U-y_n,{}^{t}/_{2}\right)\star\varphi\left(y_n-V,{}^{t}/_{2}\right)\\ &\geq \left(U-U,{}^{t}/_{2}\right)\star\varphi\left(V-V,{}^{t}/_{2}\right) \textit{ taking limit as } n\to\infty\\ &\geq 1\star 1\\ &\geq 1\\ &= 1\end{aligned}$$

which is indicating that the U is same as V. Thus we have a contradiction. So $U = V$. Thus the limit U is unique.

Next, we find out if F has a fixed point. Since F is fuzzy continuous, let us consider

$$\varphi(y_{n+1} - y_n, t) = \varphi(F(y_n) - F(y_{n-1}), t) \geq \varphi(y_n - y_{n-1}, t),$$

i.e.,

$$\varphi(y_n - y_{n+1}, t) \geq \varphi(y_{n-1} - y_n, t)..$$

Taking limit as $n \to \infty$, then we get

$$\begin{aligned}
&\varphi(U - F(U), t) \geq \varphi(U - U, t) \\
&\varphi(U - F(U), t) \geq 1 \\
&\varphi(U - F(U), t) = 1 \text{ from } \ (b) \ \text{ in } (3.2).
\end{aligned}$$

Thus $U = F(U)$. So U is a fixed point in $\mathfrak{A}$. Hence there exists a fixed point for F in fuzzy locally convex space $\mathfrak{A}$.

Finally, we shall establish that this fixed point is Unique. To do this, assume that q is another fixed point in $\mathfrak{A}$. Then $q = F(q)$. Thus $\varphi(q - F(q), t) = 1, \quad t > 0$ such that $q \neq U$. So we get

$$\begin{aligned}
1 > \varphi(U - q, t) &\geq \varphi\left(U - F(U), {}^{t}/_{2}\right) \star \varphi\left(F(U) - q, {}^{t}/_{2}\right) \\
&\geq 1 \star \varphi\left(U - F(U), {}^{t}/_{4}\right) \star \varphi\left(F(U) - q, {}^{t}/_{4}\right) \\
&\geq 1 \star 1 \star \varphi\left(U - F(U), {}^{t}/_{8}\right) \star \varphi\left(F(U) - q, {}^{t}/_{8}\right) \\
&\geq 1 \star 1 \star 1 \star \varphi\left(U - F(U), {}^{t}/_{16}\right) \star \varphi\left(F(U) - q, {}^{t}/_{16}\right) \\
&\vdots \\
&\geq 1 \star 1 \star 1 \star 1 \star \cdots \star \varphi\left(F(U) - q, {}^{t}/_{2^j}\right) \\
&= 1 \text{ as } j \longrightarrow \infty.
\end{aligned}$$

Hence $U = q$. Therefore U is a unique fixed point of the fuzzy locally convex space $\mathfrak{A}$. This completes the proof. □

4. Conclusion

In this paper, we have established that for any Fuzzy Locally Convex Space which is spherically complete, a fixed point exists and is unique. This has opened many other questions as relating to some function spaces and applications. This will be addressed in our next investigation.

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