Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

On pairwise fuzzy almost P-spaces

G. Thangaraj, E. Roseline Gladis

Received 27 May 2021; Revised 19 August 2021; Accepted 8 November 2021

Abstract. In this paper, the concept of pairwise fuzzy almost P-space is introduced by means of pairwise fuzzy G_δ-sets. Several characterizations of pairwise fuzzy almost P-spaces are obtained. The conditions for the pairwise fuzzy submaximal spaces to become pairwise fuzzy almost P-spaces are established. It is shown that the pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-spaces are pairwise fuzzy irresolvable spaces. The condition for the pairwise fuzzy almost P-spaces to become pairwise fuzzy σ-second category spaces and pairwise fuzzy weakly Volterra spaces are also established.

2020 AMS Classification: 54A40, 03E72, 54G10, 55E55.

Keywords: Pairwise fuzzy dense set, Pairwise fuzzy G_δ-set, Pairwise fuzzy F_σ-set, Pairwise fuzzy submaximal space, Pairwise fuzzy irresolvable space, Pairwise fuzzy strongly irresolvable space.

Corresponding Author: G. Thangaraj (g.thangaraj@rediffmail.com)

1. Introduction

The concept of fuzzy set as a new approach for modelling uncertainties was introduced by Zadeh [1] in his classic paper. Chang [2] introduced the notion of fuzzy topological space. Almost P-spaces in classical topology were introduced by Veksler [3] and were also studied further by R.Levy [4]. Kim [5] studied several characterizations of almost P-spaces. Weak P-spaces and almost P-spaces are the two most popular weakenings of P-spaces and weak P-spaces may fail to be weakly Volterra and every almost P-space in classical topology is dense-hereditarily Volterra and open hereditarily Volterra. In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [7, 8, 9, 10]. Recently, Şenel et al. [6] applied the concept of Octahedron Sets to multi-criteria group decision-making problems.

Kandil [11] introduced the concept of fuzzy bitopological space as a generalization of fuzzy topological space. Since then several bitopological notions are being generalised to the setting of fuzzy bitopological spaces. The concept of pairwise fuzzy P-space was introduced by Thangaraj and Chandiran in [12]. Several characterizations of pairwise fuzzy P-spaces are established by the authors in [13]. In this paper, the concept of pairwise fuzzy almost P-space is introduced by means of pairwise fuzzy G_δ-set. Several characterizations of pairwise fuzzy almost P-spaces are obtained. The conditions for all pairwise fuzzy submaximal spaces and pairwise fuzzy open hereditarily irresolvable spaces to become pairwise fuzzy almost P-spaces are established. Also the conditions for all pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec spaces to become pairwise fuzzy almost P-spaces are established. It is shown that the pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-spaces are pairwise fuzzy irresolvable spaces. The condition for pairwise fuzzy almost P-space to become pairwise fuzzy σ-second category space and pairwise fuzzy weakly Volterra space is also established. There is need and scope of investigation considering different types of pairwise fuzzy P-spaces and pairwise fuzzy Baire and pairwise fuzzy Volterra spaces by applying the notion of pairwise fuzzy G_δ-sets having pairwise fuzzy denseness so that these results may contribute to both theory and application in various areas of sciences.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval $[0, 1]$. A fuzzy set λ in X is a function from X into I. The null set 0_X is the function from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the function from X into I which takes 1 only. By a fuzzy bitopological space (Kandil, 1989), we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1 ([2])**.** Let λ and μ be fuzzy sets in X. Then for all $x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$,
(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$,
(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i \mid i \in I\}$ of fuzzy sets in (X, T), the *union* $\psi = \vee_i(\lambda_i)$ and the *intersection* $\delta = \wedge_i(\lambda_i)$ are defined respectively as

(vi) $\psi(x) = \sup_i\{\lambda_i(x) \mid x \in X\}$,
(vii) $\delta(x) = \inf_i\{\lambda_i(x) \mid x \in X\}$.

Definition 2.2 ([14])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The *interior* and the *closure* of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda,\ \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu,\ 1 - \mu \in T\}$.

Lemma 2.3 ([14])**.** *For a fuzzy set λ of a fuzzy topological space X,*

(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.4 ([12])**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy open set*, if $\lambda \in T_i$ $(i = 1, 2)$.

The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a *pairwise fuzzy closed set*.

Definition 2.5 ([15])**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy dense set*, if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$ in (X, T_1, T_2).

Definition 2.6 ([16])**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy nowhere dense set*, if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$ in (X, T_1, T_2).

Definition 2.7 ([16])**.** Let (X, T_1, T_2) be the fuzzy bitopological space. A fuzzy set λ defined on X is called a *pairwise fuzzy first category set*, if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Any other fuzzy set in (X, T_1, T_2) is said to be a *pairwise fuzzy second category set* in (X, T_1, T_2).

Definition 2.8 ([16])**.** If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a *pairwise fuzzy residual set* in (X, T_1, T_2).

Definition 2.9 ([12])**.** A fuzzy set λ defined on X in a fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy G_δ-set*, if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.10 ([12])**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy F_σ-set*, if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy closed sets in (X, T_1, T_2).

Definition 2.11 ([17])**.** A fuzzy set λ in a fuzzy bitopological space is called a *pairwise fuzzy regular G_δ-set*, if $\lambda = \wedge_{k=1}^{\infty}\left(int_{T_i} cl_{T_j}(\lambda_k)\right)$, $(i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are fuzzy sets defined on X.

Definition 2.12 ([17])**.** A fuzzy set μ in a fuzzy bitopological space is called a *pairwise fuzzy regular F_σ-set*, if $\mu = \vee_{k=1}^{\infty}\left(cl_{T_i} int_{T_j}(\mu_k)\right)$, $(i \neq j$ and $i, j = 1, 2)$, where (μ_k)'s are fuzzy sets defined on X.

Definition 2.13 ([12])**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy somewhere dense set* in (X, T_1, T_2), if $int_{T_i} cl_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$. That is, λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2), if $int_{T_1} cl_{T_2}(\lambda) \neq 0$ and $int_{T_2} cl_{T_1}(\lambda) \neq 0$ in (X, T_1, T_2).

Definition 2.14 ([12])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy P-space*, if every non-zero pairwise fuzzy G_δ-set in (X, T_1, T_2) is a pairwise fuzzy open set in (X, T_1, T_2). That is, if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy open sets in (X, T_1, T_2), then λ is a pairwise fuzzy open set in (X, T_1, T_2).

Definition 2.15 ([18])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy submaximal space*, if each pairwise fuzzy dense set in (X, T_1, T_2) is a pairwise fuzzy open set in (X, T_1, T_2). That is, if λ is a pairwise fuzzy dense set in a fuzzy bitopological space (X, T_1, T_2), then $\lambda \in T_i$ $(i = 1, 2)$.

Definition 2.16 ([18])**.** A fuzzy bitopological space (X, T_1, T_2) is said to be a *pairwise fuzzy strongly irresolvable space*, if $cl_{T_1}int_{T_2}(\lambda) = 1 = cl_{T_2}int_{T_1}(\lambda)$ for each pairwise fuzzy dense set λ in (X, T_1, T_2).

Definition 2.17 ([18])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy nodec space*, if every non-zero pairwise fuzzy nowhere dense set in (X, T_1, T_2) is a pairwise fuzzy closed set in (X, T_1, T_2).

Definition 2.18 ([19])**.** A fuzzy set λ in the fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy σ-nowhere dense set*, if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T_1}int_{T_2}(\lambda) = int_{T_2}int_{T_1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.19 ([15])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy irresolvable space*, if for each pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_1}cl_{T_2}(1 - \lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$ in (X, T_1, T_2).

Definition 2.20 (citegtsr)**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy Baire space*, if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ $(i = 1, 2)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Definition 2.21 ([16])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy first category space*, if the fuzzy set 1_X is a pairwise fuzzy first category set in (X, T_1, T_2). That is, $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Otherwise, (X, T_1, T_2) will be called a *pairwise fuzzy second category space.*

Definition 2.22 ([19])**.** Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a *pairwise fuzzy σ-first category set*, if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2).

Any other fuzzy set in (X, T_1, T_2) is said to be a *pairwise fuzzy σ-second category set* in (X, T_1, T_2).

Definition 2.23 ([19])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy σ-first category space*, if the fuzzy set 1_X is a pairwise fuzzy σ-first category set in (X, T_1, T_2). That is, $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2).

Otherwise, (X, T_1, T_2) will be called a *pairwise fuzzy σ-second category space.*

Definition 2.24 ([20])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy globally disconnected space*, if each pairwise fuzzy semi-open set is a pairwise fuzzy open set in (X, T_1, T_2).

Definition 2.25 ([21])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy open hereditarily irresolvable space*, if $int_{T_1}cl_{T_2}(\lambda) \neq 0$ and $int_{T_2}cl_{T_1}(\lambda) \neq 0$, then $int_{T_1}int_{T_2}(\lambda) \neq 0$ and $int_{T_2}int_{T_1}(\lambda) \neq 0$ for any non-zero fuzzy set λ in (X, T_1, T_2).

Definition 2.26 ([22])**.** A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise hyperconnected space*, if every pairwise fuzzy open set is a pairwise fuzzy dense set in (X, T_1, T_2).

Definition 2.27 ([12]). A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy weakly Volterra space*, if $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where (λ_k)'s are the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2).

Theorem 2.28 ([17]). *If λ is a pairwise fuzzy regular G_δ-set in a fuzzy bitopological space (X, T_1, T_2), then λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Theorem 2.29 (citegtvc2). *If λ is a pairwise fuzzy regular F_σ-set in a fuzzy bitopological space (X, T_1, T_2), then λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2).*

Theorem 2.30 ([22]). *In a fuzzy bitopological space (X, T_1, T_2), the fuzzy set λ is a pairwise fuzzy σ-nowhere dense set in (X, T_1, T_2) if and only if $1 - \lambda$ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Theorem 2.31 ([16]). *If λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2).*

Theorem 2.32 ([3]). *If λ is a pairwise fuzzy residual set in a pairwise fuzzy globally disconnected space (X, T_1, T_2), then λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Theorem 2.33 ([23]). *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space and if $cl_{T_1}int_{T_2}(\lambda) \neq 1$ and $cl_{T_2}int_{T_1}(\lambda) \neq 1$ for a fuzzy set λ in (X, T_1, T_2), then $cl_{T_1}cl_{T_2}(\lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(\lambda) \neq 1$, in (X, T_1, T_2).*

Theorem 2.34 ([18]). *If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy first category set in (X, T_1, T_2).*

Theorem 2.35 ([24]). *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal space and λ is a pairwise fuzzy first category set in (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Theorem 2.36 ([24]). *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal space, then every pairwise fuzzy residual set is a pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Theorem 2.37 ([24]). *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, then (X, T_1, T_2) is a pairwise fuzzy submaximal space.*

Theorem 2.38 ([24]). *If every pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire space.*

Theorem 2.39 ([24]). *If every pairwise fuzzy G_δ-set is a fuzzy pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire space.*

Theorem 2.40 ([20]). *If λ is a pairwise fuzzy first category set in a pairwise fuzzy globally disconnected space (X, T_1, T_2), then λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2).*

Theorem 2.41 ([16]). *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then (X, T_1, T_2) is a pairwise fuzzy second category space.*

Theorem 2.42 ([25]). *If λ is a pairwise fuzzy σ-first category set in (X, T_1, T_2), then there is a pairwise fuzzy F_σ-set δ in (X, T_1, T_2) such that $\lambda \leq \delta$.*

Theorem 2.43 ([25]). *If $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where ($\lambda_k$'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy σ-second category space.*

Theorem 2.44 ([26]). *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy σ-second category space, then (X, T_1, T_2) is a pairwise fuzzy weakly Volterra space.*

3. Pairwise fuzzy almost P-spaces

Definition 3.1. A fuzzy bitopological space (X, T_1, T_2) is said to be a *pairwise fuzzy almost P-space*, if for each non-zero pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). That is, (X, T_1, T_2) is a pairwise fuzzy almost P-space, if $int_{T_1} int_{T_2}(\lambda) \neq 0$ and $int_{T_2} int_{T_1}(\lambda) \neq 0$ for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2).

Example 3.2. Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ, γ, β and α defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.6$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$;
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4$, $\mu(b) = 0.7$, $\mu(c) = 0.6$;
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.5$, $\gamma(b) = 0.3$, $\gamma(c) = 0.7$;
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.5$, $\beta(b) = 0.2$, $\beta(c) = 0.7$;
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.5$, $\alpha(b) = 0.4$, $\alpha(c) = 0.6$.
Then $T_1 = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, \gamma \wedge (\lambda \vee \mu), \mu \wedge (\lambda \vee \gamma), \lambda \wedge (\mu \vee \gamma), \gamma \vee (\lambda \wedge \mu), \mu \vee (\lambda \wedge \gamma), \lambda \vee (\mu \wedge \gamma), \lambda \wedge \mu \wedge \gamma, \lambda \vee \mu \vee \gamma, 1\}$ and $T_2 = \{0, \lambda, \mu, \beta, \lambda \vee \mu, \lambda \vee \beta, \mu \vee \beta, \lambda \wedge \mu, \lambda \wedge \beta, \mu \wedge \beta, \beta \wedge (\lambda \vee \mu), \mu \wedge (\lambda \vee \beta), \lambda \wedge (\mu \vee \beta), \beta \vee (\lambda \wedge \mu), \mu \vee (\lambda \wedge \beta), \lambda \vee (\mu \wedge \beta), \lambda \wedge \mu \wedge \beta, \lambda \vee \mu \vee \beta, 1\}$ are fuzzy topologies on X. By computation, λ, μ, $\lambda \vee \mu$, $\lambda \vee \gamma$, $\mu \vee \gamma$, $\lambda \wedge \mu$, $\mu \wedge (\lambda \vee \gamma)$, $\lambda \wedge (\mu \vee \gamma)$, $\gamma \vee (\lambda \wedge \mu)$, $\mu \vee (\lambda \wedge \gamma)$, $\lambda \vee (\mu \wedge \gamma)$, $\lambda \vee \mu \vee \gamma$, $\lambda \vee \beta$, $\mu \vee \beta$, $\mu \wedge (\lambda \vee \beta)$, $\lambda \wedge (\mu \vee \beta)$, $\beta \vee (\lambda \wedge \mu)$, $\mu \vee (\lambda \wedge \beta)$, $\lambda \vee (\mu \wedge \beta)$, $\lambda \vee \mu \vee \beta$ are pairwise fuzzy open sets in (X, T_1, T_2). By computation $\alpha = [\lambda \vee (\mu \wedge \gamma)] \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)] \wedge [\lambda \vee \mu \vee \beta] \wedge [\lambda \vee \mu] \wedge [\lambda \vee \beta]$ and $\lambda \wedge \mu = [\lambda \vee \mu] \wedge [\lambda \vee \beta] \wedge [\mu \vee \gamma] \wedge [\lambda \wedge (\mu \vee \gamma)] \wedge [\mu \wedge (\lambda \vee \beta)]$. Then α and $\lambda \wedge \mu$ are pairwise fuzzy G_δ-sets in (X, T_1, T_2). Also,
$int_{T_2} int_{T_1}(\alpha) = int_{T_2}([\lambda \wedge (\mu \vee \gamma)] = \lambda \wedge (\mu \vee \beta) \neq 0$,
$int_{T_1} int_{T_2}(\alpha) = int_{T_1}([\lambda \wedge (\mu \vee \beta)] = \lambda \wedge (\mu \vee \gamma) \neq 0$,
$int_{T_2} int_{T_1}(\lambda \wedge \mu) = int_{T_2}(\lambda \wedge \mu) = \lambda \wedge \mu \neq 0$,
$int_{T_1} int_{T_2}(\lambda \wedge \mu) = int_{T_1}(\lambda \wedge \mu) = \lambda \wedge \mu \neq 0$.
Thus for the pairwise fuzzy G_δ-sets α and $\lambda \wedge \mu$ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\alpha) \neq 0$ and $int_{T_i} int_{T_j}(\lambda \wedge \mu) \neq 0$ ($i \neq j$ and $i, j = 1, 2$) implies that (X, T_1, T_2) is a pairwise fuzzy almost P-space.

Remark 3.3. Clearly, each pairwise fuzzy P-space is a pairwise fuzzy almost P-space. For, if λ is a non-zero pairwise fuzzy G_δ-set in a pairwise fuzzy P-space (X, T_1, T_2), then λ is a pairwise fuzzy open set in (X, T_1, T_2) and $int_{T_i}(\lambda) = \lambda$, ($i = 1, 2$) and hence $int_{T_i} int_{T_j}(\lambda) = \lambda \neq 0$ ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). But the converse need not be true. That is, pairwise fuzzy almost P-spaces need not be pairwise fuzzy P-spaces. For, in Example 3.2, α is a pairwise fuzzy G_δ-set in

(X, T_1, T_2). But α is not a pairwise fuzzy open set in (X, T_1, T_2) and hence (X, T_1, T_2) is not a pairwise fuzzy P-space.

Proposition 3.4. *If μ is a pairwise fuzzy F_σ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), then μ is not a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Let μ be a pairwise fuzzy F_σ-set in (X, T_1, T_2). Then $1 - \mu$ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set $1-\mu$ in (X, T_1, T_2), $int_{T_i} int_{T_j}(1-\mu) \neq 0$ ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). This implies that $1 - cl_{T_i} cl_{T_j}(\mu) \neq 0$ in (X, T_1, T_2). Thus $cl_{T_i} cl_{T_j}(\mu) \neq 1$ in (X, T_1, T_2). So μ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

Proposition 3.5. *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy almost P-space, then there is no pairwise fuzzy nowhere dense and pairwise fuzzy G_δ-set in (X, T_1, T_2).*

Proof. Suppose that there exists a pairwise fuzzy nowhere dense and pairwise fuzzy G_δ-set λ in (X, T_1, T_2). Since λ is a pairwise fuzzy nowhere dense set, $int_{T_i} cl_{T_j}(\lambda) = 0$, ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). But $int_{T_i} int_{T_j}(\lambda) \leq int_{T_i} cl_{T_j}(\lambda)$, implies that $int_{T_i} int_{T_j}(\lambda) = 0$, a contradiction to (X, T_1, T_2) being the pairwise fuzzy almost P-space in which for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, in (X, T_1, T_2). Then there is no pairwise fuzzy nowhere dense and pairwise fuzzy G_δ-set in a pairwise fuzzy almost P-space (X, T_1, T_2). □

Remark 3.6. In view of Proposition 3.5, one will have the following result: "In pairwise fuzzy almost P-spaces, the pairwise fuzzy G_δ-sets are not pairwise fuzzy nowhere dense sets".

Proposition 3.7. *If λ is a pairwise fuzzy regular G_δ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), then $int_{T_i} int_{T_j}(\lambda) \neq 0$, ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2).*

Proof. Suppose λ is a pairwise fuzzy regular G_δ-set in (X, T_1, T_2). By Theorem 2.28, the pairwise fuzzy regular G_δ-set λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). □

Proposition 3.8. *If μ is a pairwise fuzzy regular F_σ-set in (X, T_1, T_2), then μ is not a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Suppose μ is a pairwise fuzzy regular F_σ-set in (X, T_1, T_2). By Theorem 2.29, the pairwise fuzzy regular F_σ-set μ is a pairwise fuzzy F_σ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space and by Proposition 3.4, for a pairwise fuzzy F_σ-set μ in (X, T_1, T_2), μ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

Proposition 3.9. *If λ is a pairwise fuzzy G_δ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), then λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2).*

Proof. Suppose λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). Now $int_{T_i} int_{T_j}(\lambda) \leq int_{T_i} cl_{T_j}(\lambda)$, implies that $int_{T_i} cl_{T_j}(\lambda) \neq 0$. Then λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2). □

Proposition 3.10. *If λ is a pairwise fuzzy G_δ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), then $cl_{T_i} int_{T_j}(1-\lambda) \neq 1$, $(i \neq j$ and $i, j = 1, 2)$, in (X, T_1, T_2).*

Proof. Suppose λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, by Proposition 3.9, the pairwise fuzzy G_δ-set λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2). Then $int_{T_i} cl_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Thus $1 - int_{T_i} cl_{T_j}(\lambda) \neq 1$. So $cl_{T_i} int_{T_j}(1-\lambda) \neq 1$ in (X, T_1, T_2). □

Proposition 3.11. *If λ is a non-zero pairwise fuzzy dense and pairwise fuzzy G_δ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), then $1-\lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Suppose λ bis a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Then by Theorem 2.30, $1-\lambda$ is a pairwise fuzzy σ-nowhere dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Now $cl_{T_i} cl_{T_j}(1-\lambda) = 1 - int_{T_i} int_{T_j}(\lambda) \neq 1$. Thus $1-\lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

Proposition 3.12. *If μ is a pairwise fuzzy F_σ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), then λ is a pairwise fuzzy cs dense set in (X, T_1, T_2).*

Proof. Suppose μisbe a pairwise fuzzy F_σ-set in (X, T_1, T_2). Then $1-\mu$ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space,and by Proposition 3.9, $1-\mu$ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2). Thus μ is a pairwise fuzzy cs dense set in (X, T_1, T_2). □

Proposition 3.13. *If (λ_k)'s $(k = 1$ to $\infty)$ are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in a pairwise fuzzy almost P-space (X, T_1, T_2), then $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, in (X, T_1, T_2).*

Proof. Suppose (λ_k)'s $(k = 1$ to $\infty)$ is a pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Then by Theorem 2.30, $(1-\lambda_k)$'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Thus $\vee_{k=1}^{\infty}(1-\lambda_k)$ is a pairwise fuzzy σ-first category set in (X, T_1, T_2). By Theorem 2.42, there is a pairwise fuzzy F_σ-set δ in (X, T_1, T_2) such that $\vee_{k=1}^{\infty}(1-\lambda_k) \leq \delta$. This implies that $1 - \wedge_{k=1}^{\infty}(\lambda_k) \leq \delta$ and $(1-\delta) \leq \wedge_{k=1}^{\infty}(\lambda_k)$. So $int_{T_i}(1-\delta) \leq int_{T_i}[\wedge_{k=1}^{\infty}(\lambda_k)]$. Since $1-\delta$ is a pairwise fuzzy G_δ-set in a pairwise fuzzy almost P-space (X, T_1, T_2), $int_{T_i} int_{T_j}(1-\delta) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Hence $int_{T_i}\ int_{T_j}(1-\delta) \leq int_{T_i}(1-\delta)$. This implies that $int_{T_i}(1-\delta) \neq 0$. Therefore $int_{T_i}[\wedge_{k=1}^{\infty}(\lambda_k)] \neq 0$ in (X, T_1, T_2). Now $int_{T_i}[\wedge_{k=1}^{\infty}(\lambda_k)] \leq \wedge_{k=1}^{\infty}(\lambda_k)$, implies that $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$ in (X, T_1, T_2). □

4. Pairwise fuzzy almost P-spaces and other fuzzy bitopological Spaces

The following propositions give the conditions for the pairwise fuzzy submaximal spaces to become pairwise fuzzy almost P-spaces.

Proposition 4.1. *If the pairwise fuzzy G_δ-sets are pairwise fuzzy dense sets in a pairwise fuzzy submaximal space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy almost P-space.*

Proof. Suppose λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Then λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ is a pairwise fuzzy open set in (X, T_1, T_2) and $int_{T_i}(\lambda) = \lambda,\ i = 1, 2$. Thus $int_{T_i} int_{T_j}(\lambda) = int_{T_i}(\lambda) = \lambda \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). So (X, T_1, T_2) is a pairwise fuzzy almost P-space. □

Proposition 4.2. *If the pairwise fuzzy F_σ-sets are pairwise fuzzy nowhere dense sets in a pairwise fuzzy submaximal space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy almost P-space.*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). Then $1-\lambda$ is a pairwise fuzzy F_σ-set in (X, T_1, T_2). By hypothesis, $1 - \lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Thus by Theorem 2.31, $1 - (1 - \lambda)$ is a pairwise fuzzy dense set in (X, T_1, T_2). That is, λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ is a pairwise fuzzy open set in (X, T_1, T_2) and $int_{T_i}(\lambda) = \lambda,\ i = 1, 2$. So $int_{T_i} int_{T_j}(\lambda) = int_{T_i}(\lambda) = \lambda \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy almost P-space. □

Proposition 4.3. *If λ is a pairwise fuzzy residual set in a pairwise fuzzy globally disconnected and pairwise fuzzy almost P-space (X, T_1, T_2), then λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy globally disconnected space, by Theorem 2.32, λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Now $int_{T_i} int_{T_j}(\lambda) \leq int_{T_i} cl_{T_j}(\lambda)$, implies that $int_{T_i} cl_{T_j} \neq 0$. Then λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2). □

Proposition 4.4. *If λ is a pairwise fuzzy first category set in a pairwise fuzzy globally disconnected and pairwise fuzzy almost P-space (X, T_1, T_2), then λ is a pairwise fuzzy cs dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then $1 - \lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy globally disconnected and pairwise fuzzy almost P-space, by Proposition 4.3, $1 - \lambda$ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2). Then λ is a pairwise fuzzy cs dense set in (X, T_1, T_2). □

Proposition 4.5. *If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-space (X, T_1, T_2), then $1 - \lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, by Theorem 2.34, $1 - \lambda$ is a pairwise fuzzy first category set in (X, T_1, T_2). Also, since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Now $cl_{T_i} cl_{T_j}(1 - \lambda) = 1 - int_{T_i} int_{T_j}(\lambda) \neq 1$. Then $1 - \lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

Proposition 4.6. *If λ is a pairwise fuzzy G_δ-set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-space (X, T_1, T_2), then $1 - \lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, by Proposition 3.10, $cl_{T_i} int_{T_j}(1 - \lambda) \neq 1$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Also, since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space,and by Theorem 2.33, $cl_{T_i} cl_{T_j}(1 - \lambda) \neq 1$. Then $1 - \lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

Proposition 4.7. *If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-space (X, T_1, T_2), then λ is a pairwise fuzzy residual set in (X, T_1, T_2) such that $int_{T_i} int_{T_j}(\lambda) \neq 0$, in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, by Theorem 2.34, $1 - \lambda$ is a pairwise fuzzy first category set in (X, T_1, T_2). Then λ is a pairwise fuzzy residual set in (X, T_1, T_2). Also, since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Thus λ is a pairwise fuzzy residual set in (X, T_1, T_2) such that $int_{T_i} int_{T_j}(\lambda) \neq 0$ in (X, T_1, T_2). □

The following proposition shows that all pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-spaces are pairwise fuzzy irresolvable spaces.

Proposition 4.8. *If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-space, then (X, T_1, T_2) is a pairwise fuzzy irresolvable space.*

Proof. Let λ be a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-space, by Proposition 4.5, $1 - \lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2). Then for a pairwise fuzzy dense set λ in (X, T_1, T_2), $1 - \lambda$ is not a pairwise fuzzy dense set in (X, T_1, T_2). Thus (X, T_1, T_2) is a pairwise fuzzy irresolvable space. □

Proposition 4.9. *If λ is a pairwise fuzzy first category set in a pairwise fuzzy submaximal and pairwise fuzzy almost P-space (X, T_1, T_2), then λ is not a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, by Theorem 2.35, $1 - \lambda$ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Also, since (X, T_1, T_2) is a pairwise fuzzy almost P-space, for a pairwise fuzzy G_δ-set $1 - \lambda$, $int_{T_i} int_{T_j}(1 - \lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Then $1 - cl_{T_i} cl_{T_j}(\lambda) \neq 0$ in (X, T_1, T_2). So hence $cl_{T_i} cl_{T_j}(\lambda) \neq 1$. Hence λ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

Proposition 4.10. *If λ is a pairwise fuzzy residual set in a pairwise fuzzy submaximal and pairwise fuzzy almost P-space (X, T_1, T_2), then $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$, in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, by Theorem 2.36, λ is a pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). □

The following propositions give the conditions for all pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec spaces to become pairwise fuzzy almost P-spaces.

Proposition 4.11. *If pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy almost P-space.*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). By hypothesis, λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, by Theorem 2.37, (X, T_1, T_2) is a pairwise fuzzy submaximal space. Then the pairwise fuzzy G_δ-sets are pairwise fuzzy dense sets in a pairwise fuzzy submaximal space (X, T_1, T_2). Thus by Proposition 4.1, (X, T_1, T_2) is a pairwise fuzzy almost P-space. □

Proposition 4.12. *If pairwise fuzzy F_σ-set is a pairwise fuzzy nowhere dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy almost P-space.*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). Then $1-\lambda$ is a pairwise fuzzy F_σ-set in (X, T_1, T_2). By hypothesis, $1 - \lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Thus by Theorem 2.31, $1 - (1 - \lambda)$ is a pairwise fuzzy dense set in (X, T_1, T_2). That is, λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, by Theorem 2.37, (X, T_1, T_2) is a pairwise fuzzy submaximal space. So all pairwise fuzzy G_δ-sets are pairwise fuzzy dense sets in a pairwise fuzzy submaximal space (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ is a pairwise fuzzy open set in (X, T_1, T_2). Hence by Proposition 4.1, (X, T_1, T_2) is a pairwise fuzzy almost P-space. □

Proposition 4.13. *If each pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire and pairwise fuzzy almost P-space.*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). Then by hypothesis, λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy submaximal space, by Theorem 2.38, (X, T_1, T_2) is a pairwise fuzzy Baire space. Also, since (X, T_1, T_2) is a pairwise fuzzy submaximal space, by Proposition 4.1, (X, T_1, T_2) is a pairwise fuzzy almost P-space. Thus (X, T_1, T_2) is a pairwise fuzzy Baire and pairwise fuzzy almost P-space. □

Proposition 4.14. *If each pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire and pairwise fuzzy almost P-space.*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). Then by hypothesis, λ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, by Theorem 2.39, (X, T_1, T_2) is a pairwise fuzzy Baire space. Also by Proposition 4.11, (X, T_1, T_2) is a pairwise fuzzy almost P-space. Thus (X, T_1, T_2) is a pairwise fuzzy Baire and pairwise fuzzy almost P-space. □

Proposition 4.15. *If each pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy second category and pairwise fuzzy almost P-space.*

Proof. The proof follows from Proposition 4.13 and Theorem 2.41. □

Proposition 4.16. *If each pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy second category and pairwise fuzzy almost P-space.*

Proof. The proof follows from Proposition 4.14 and Theorem 2.41. □

Proposition 4.17. *If λ is a pairwise fuzzy first category set in a pairwise fuzzy globally disconnected and pairwise fuzzy almost P-space, then λ is a not pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Supposeet λ is a pairwise fuzzy first category set in a pairwise fuzzy globally disconnected space (X, T_1, T_2). Then by Theorem 2.40, λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, by Proposition 3.4, λ is not a pairwise fuzzy dense set in (X, T_1, T_2). □

The following proposition gives the condition for the pairwise fuzzy almost P-spaces to become pairwise fuzzy σ-second category spaces.

Proposition 4.18. *If there are pairwise fuzzy dense and pairwise fuzzy G_δ-sets (λ_k)'s $(k = 1 \text{ to } \infty)$ in a pairwise fuzzy almost P-space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy σ-second category space.*

Proof. Suppose (λ_k)'s $(k = 1 \text{ to } \infty)$ are the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy almost P-space, by Proposition 3.13, $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, in (X, T_1, T_2). Then by Theorem 2.43, (X, T_1, T_2) is a pairwise fuzzy σ-second category space. □

The following proposition gives the condition for the pairwise fuzzy open hereditarily irresolvable spaces to become pairwise fuzzy almost P-spaces.

Proposition 4.19. *If a pairwise fuzzy G_δ-set is a pairwise fuzzy somewhere dense set in a pairwise fuzzy open hereditarily irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy almost P-space.*

Proof. Let λ be a pairwise fuzzy G_δ-set in (X, T_1, T_2). Then by hypothesis, λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2). Thus $int_{T_1} cl_{T_2}(\lambda) \neq 0$ and $int_{T_2} cl_{T_1}(\lambda) \neq 0$, in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy open hereditarily irresolvable space, $int_{T_1} int_{T_2}(\lambda) \neq 0$ and $int_{T_2} int_{T_1}(\lambda) \neq 0$, for the fuzzy set λ in (X, T_1, T_2). So (X, T_1, T_2) is a pairwise fuzzy almost P-space. □

The following proposition gives the condition for the pairwise fuzzy almost P-spaces to become pairwise fuzzy weakly Volterra spaces.

Proposition 4.20. *If there are pairwise fuzzy dense and pairwise fuzzy G_δ-sets (λ_k)'s ($k = 1$ to ∞) in a pairwise fuzzy almost P-space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy weakly Volterra space.*

Proof. The proof follows from Proposition 4.18 and Theorem 2.44. □

5. Conclusion

In this paper, the concept of pairwise fuzzy almost P-space is introduced by means of pairwise fuzzy G_δ-set. It is established that pairwise fuzzy G_δ-sets are not pairwise fuzzy nowhere dense sets in pairwise fuzzy almost P-spaces. The conditions for all pairwise fuzzy submaximal spaces and all pairwise fuzzy open hereditarily irresolvable spaces to become pairwise fuzzy almost P-spaces are established. Also, the conditions for pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec spaces to become pairwise fuzzy almost P-spaces are established. It is shown that all pairwise fuzzy strongly irresolvable and pairwise fuzzy almost P-spaces are pairwise fuzzy irresolvable spaces. Moreover, the condition for all pairwise fuzzy almost P-spaces to become pairwise fuzzy σ-second category spaces and pairwise fuzzy weakly Volterra spaces are also established. This work may be extended to study fuzzy Lindeloffness of pairwise fuzzy Baire spaces and study fuzzy Volterraness, fuzzy basically disconnectedness of fuzzy bitopological spaces and fuzzy soft bitopological spaces.

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.
[3] A. I. Veksler, P'-points, P'-sets, P'-spaces, A new class of order-continuous measure and functional, Soviet Math. Dokl. 4 (5) (1973) 1445–1450.
[4] R. Levy, Almost P-spaces, Canad. J. Math. XXIX (2) (1977) 284–288.
[5] Chang Il Kim, Almost P-spaces, Commun. Korean Math. Soc. 18 (4) (2003) 695–701.
[6] G. Şenel, Lee Jeong Gon and Kul Hur, Distance and similarity measures for octahedron sets and their application to MCGDM problems, Mathematics 8 (2020) 1690.
[7] G. Şenel, Soft topology generated by L-Soft sets, Journal of New Theory 4 (24) (2018) 88–100.
[8] G. Şenel, A new approach to Hausdorff space theory via the soft sets, Mathematical Problems in Engineering 2016 (9) (2016) 1-6. Doi:10.1155/2016/2196743.
[9] G. Şenel and Cagman Naim, Soft topological subspaces, Ann. Fuzzy Math. Inform. 10 (4) (2015) 525–535.
[10] G. Şenel and Cagman Naim, Soft closed sets on soft bitopological space, Journal of New Results in Science 3 (5) (2014) 57–66.
[11] A. Kandil and M. E. El-Shafee, Biproximities and fuzzy bitopological spaces, Simen Stevin 63 (1989) 45–66.
[12] G. Thangaraj and V. Chandiran, On pairwise fuzzy Volterra spaces, Ann. Fuzzy Math. Inform. 7(6) (2014) 1005–1012.
[13] G. Thangaraj and E. Roseline Gladis, On pairwise fuzzy P-spaces, Adv. Fuzzy Math. 12(5) (2017), 1075–1085.
[14] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981) 14–32.
[15] G. Thangaraj, On pairwise fuzzy resolvable and fuzzy irresolvable spaces, Bull. Cal. Math. Soc. 102 (1) (2010) 59–68.

[16] G. Thangaraj and S. Sethuraman, On pairwise fuzzy Baire bitopological spaces, Gen. Math. Notes. 20 (2) (2014) 12–21.
[17] G. Thangaraj and V. Chandiran, On pairwise fuzzy regular Volterra spaces, J. Appl .Math. & Informatics 33 (5-6) (2015) 503–515.
[18] G. Thangaraj and S. Sethuraman, A note on pairwise fuzzy Baire spaces, Ann. Fuzzy Math. Inform. 8(5) (2014), 729–737.
[19] G. Thangaraj and A. Vinothkumar, On pairwise fuzzy σ-Baire spaces, Ann. Fuzzy Math. Inform. 9 (4) (2015) 529–536.
[20] G. Thangaraj and A. Vinothkumar, Pairwise fuzzy globally disconnected spaces and σ-Baire spaces, Global J. Math. Sci. 9 (2) (2017) 117–125.
[21] G. Thangaraj and P. Vivakanandan, A short note on pairwise fuzzy irresolvable spaces and pairwise fuzzy open hereditarily irresolvable spaces, J. Comput. Mathematica 4 (1) (2020) 26–47.
[22] G. Thangaraj and V. Chandiran, A note on pairwise fuzzy Volterra spaces, Ann. Fuzzy Math. Inform. 9 (3) (2015) 365–372.
[23] G. Thangaraj and P. Vivakanandan, On pairwise fuzzy strongly irresolvable spaces, Inter. J. Innovative Sci. Engin & Tech. 2 (10) (2015) 94–111.
[24] G. Thangaraj and S. Sethuraman, Some remarks on pairwise fuzzy Baire Spaces, Ann. Fuzzy Math. Inform. 9 (5) (2014) 683–691.
[25] G. Thangaraj and A. Vinothkumar, A note on pairwise fuzzy σ-Baire bitopological spaces, Ann. Fuzzy Math. Inform. 11 (5) (2016) 729–736.
[26] G. Thangaraj and V. Chandiran, Some remarks on pairwise fuzzy weakly Volterra spaces, Inter. J. Fuzzy Math. and Sys. 4 (2) (2014) 227–235.

G. Thangaraj (g.thangaraj@rediffmail.com)
Department of Mathematics, Thiruvalluvar University, Vellore-632115, Tamil Nadu, India

E. Roseline Gladis (roselinegladis@gmail.com)
Department of Mathematics, Voorhees College, Vellore-632001, Tamil Nadu, India