

## On fuzzy Moscow spaces

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**ABSTRACT.** In this paper, the notion of fuzzy Moscow space is introduced and studied. It is established that fuzzy extremally disconnected spaces are fuzzy Moscow spaces and fuzzy Moscow and fuzzy P-spaces are fuzzy extremally disconnected spaces. It is obtained that fuzzy Moscow spaces are not fuzzy hyperconnected spaces and not fuzzy superconnected spaces. A condition under which fuzzy Moscow spaces become fuzzy hyperconnected spaces is obtained. It is also established that fuzzy semi-open sets are fuzzy pre-open sets in fuzzy Moscow and fuzzy P-spaces.

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**Keywords:** Fuzzy  $G_\delta$ -set, Fuzzy  $F_\sigma$ -set, Fuzzy semi-open set, Fuzzy residual set, Fuzzy P-space, Fuzzy Baire space, Fuzzy globally disconnected space, Fuzzy extremally disconnected space, Fuzzy hyperconnected space.

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### 1. INTRODUCTION

The concept of fuzzy set as a new approach for modelling uncertainties was introduced by Zadeh [1] in the year 1965. The concept of fuzzy topological spaces was introduced by Chang [2] in 1968. Chang's works paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of Moscow space in classical topology was introduced by Arhangel'skii [3] and the notion of Moscow space can be considered as a straightforward generalization of extremal disconnectedness. The notion of a Moscow space generalizes the notion of a perfectly  $\kappa$ -normal space introduced independently by Blair [4]. The notion of a Moscow space plays a crucial role in the theory of C-embeddings.

The concept of fuzzy extremally disconnected spaces was introduced and studied by Ghosh [5]. The notion of fuzzy P-spaces was introduced and studied by Thangaraj and Balasubramanian [17]. In this paper, the notion of fuzzy Moscow space is introduced by means of fuzzy  $G_\delta$ -sets. Several characterizations of fuzzy Moscow spaces are established. It is established that fuzzy extremally disconnected spaces are fuzzy

Moscow spaces and fuzzy Moscow and fuzzy P-spaces are fuzzy extremally disconnected spaces and fuzzy basically disconnected spaces. It is obtained that fuzzy Moscow spaces, are not fuzzy hyperconnected spaces and not fuzzy superconnected spaces. The condition under which fuzzy Moscow spaces become fuzzy hyperconnected spaces is obtained. It is also established that fuzzy regular closed sets are fuzzy open sets and fuzzy semi-open sets are fuzzy pre-open sets in fuzzy Moscow and fuzzy P-spaces. Also it is established that fuzzy closure of fuzzy  $\beta$ -open sets are fuzzy open sets in fuzzy Moscow and fuzzy P-spaces. A condition under which fuzzy basically disconnected spaces becomes fuzzy Moscow spaces, is also obtained in this paper. In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [6, 7, 8, 9]. Recently, Şenel et al. [10] defined an octahedron set composed of an interval-valued fuzzy set, an intuitionistic set and a fuzzy set that will provide nice information about uncertainty and vagueness and applied the concept of octahedron sets to multi-criteria group decision-making problems.

## 2. PRELIMINARIES

Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by  $(X, T)$  or simply by  $X$ , we will denote a fuzzy topological space due to Chang (1968). Let  $X$  be a non-empty set and  $I$  the unit interval  $[0, 1]$ . A fuzzy set in  $X$  is a mapping from  $X$  into  $I$ . The fuzzy set  $0_X$  is defined as  $0_X(x) = 0$ , for all  $x \in X$  and the fuzzy set  $1_X$  is defined as  $1_X(x) = 1$ , for all  $x \in X$ .

**Definition 2.1** ([2]). Let  $(X, T)$  be a fuzzy topological space and  $\lambda$  be any fuzzy set in  $(X, T)$ . The *interior*, the *closure* and the *complement* of  $\lambda$  are defined respectively as follows:

- (i)  $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda, \mu \in T\}$ ,
- (ii)  $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$ ,
- (iii)  $\lambda'(x) = 1 - \lambda(x)$  for all  $x \in X$ .

For a family  $\{\lambda_i \mid i \in I\}$  of fuzzy sets in  $(X, T)$ , the union  $\psi = \vee_i(\lambda_i)$  and intersection  $\delta = \wedge_i(\lambda_i)$  are defined respectively as:

- (iv)  $\psi(x) = \sup_i\{\lambda_i(x) \mid x \in X\}$ ,
- (v)  $\delta(x) = \inf_i\{\lambda_i(x) \mid x \in X\}$ .

**Lemma 2.2** ([11]). For a fuzzy set  $\lambda$  of a fuzzy topological space  $X$ ,

- (1)  $1 - int(\lambda) = cl(1 - \lambda)$ ,
- (2)  $1 - cl(\lambda) = int(1 - \lambda)$ .

**Definition 2.3.** A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called a

- (i) *fuzzy regular-open*, if  $\lambda = intcl(\lambda)$  and *fuzzy regular-closed*, if  $\lambda = clint(\lambda)$  [11],
- (ii) *fuzzy pre-open*, if  $\lambda \leq intcl(\lambda)$  and *fuzzy pre-closed*, if  $clint(\lambda) \leq \lambda$  [12],
- (iii) *fuzzy semi-open*, if  $\lambda \leq clint(\lambda)$  and *fuzzy semi-closed*, if  $intcl(\lambda) \leq \lambda$  [11],
- (iv) *fuzzy  $\beta$ -open*, if  $\lambda \leq clintcl(\lambda)$  and *fuzzy  $\beta$ -closed*, if  $intclint(\lambda) \leq \lambda$  [13],
- (v) *fuzzy  $G_\delta$ -set*, if  $\lambda = \wedge_{i=1}^\infty(\lambda_i)$ , where  $\lambda_i \in T$  for  $i \in I$  [14],
- (vi) *fuzzy  $F_\sigma$ -set* if  $\lambda = \vee_{i=1}^\infty(\lambda_i)$ , where  $1 - \lambda_i \in T$  for  $i \in I$  [14].

**Definition 2.4.** The fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$ , is called a

- (i) *fuzzy dense set*, if there exists no fuzzy closed set  $\mu$  in  $(X, T)$  such that  $\lambda < \mu < 1$ , i.e.,  $cl(\lambda) = 1$  in  $(X, T)$  [16],
- (ii) *fuzzy nowhere dense set*, if there exists no non-zero fuzzy open set  $\mu$  in  $(X, T)$  such that  $\mu < cl(\lambda)$ , i.e.,  $intcl(\lambda) = 0$  in  $(X, T)$  [16],
- (iii) *fuzzy first category set*, if  $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy nowhere dense sets in  $(X, T)$ . Any other fuzzy set in  $(X, T)$  is said to be of fuzzy second category [16],
- (iv) *fuzzy somewhere dense set*, if there exists a non-zero fuzzy open set  $\mu$  in  $(X, T)$  such that  $\mu < cl(\lambda)$ , i.e.,  $intcl(\lambda) \neq 0$ , in  $(X, T)$  [19],
- (v) *fuzzy residual set*, if  $1 - \lambda$  is a fuzzy first category set in  $(X, T)$  [20],
- (vi) *fuzzy  $\sigma$ -boundary set*, if  $\lambda = \bigvee_{i=1}^{\infty} (\mu_i)$ , where  $\mu_i = cl(\lambda_i) \wedge (1 - \lambda_i)$  and  $(\lambda_i)$ 's are fuzzy regular open sets in  $(X, T)$  [21],
- (vii) *fuzzy co- $\sigma$ -boundary set*, if  $1 - \lambda$  is a fuzzy  $\sigma$ -boundary set in  $(X, T)$  [21],
- (viii) *fuzzy  $\sigma$ -nowhere dense set*, if is a fuzzy  $F_{\sigma}$ -set with  $int(\lambda) = 0$  in  $(X, T)$  [23].

**Definition 2.5.** A fuzzy topological space  $(X, T)$  is called a

- (i) *fuzzy P-space* if each fuzzy  $G_{\delta}$ -set in  $(X, T)$  is fuzzy open in  $(X, T)$  [17],
- (ii) *fuzzy almost P-space*, if for every non-zero fuzzy  $G_{\delta}$ -set  $\lambda$  in  $(X, T)$ ,  $int(\lambda) \neq 0$  in  $(X, T)$  [24],
- (iii) *fuzzy globally disconnected space*, if each fuzzy semi-open set is fuzzy open in  $(X, T)$  [26],
- (iv) *fuzzy hyperconnected space*, if every non-null fuzzy open subset of  $(X, T)$  is fuzzy dense in  $(X, T)$  [29],
- (v) *fuzzy perfectly disconnected space*, if for any two non-zero fuzzy sets  $\lambda$  and  $\mu$  defined on  $X$  with  $\lambda \leq (1 - \mu)$ ,  $cl(\lambda) \leq 1 - cl(\mu)$  in  $(X, T)$  [27],
- (vi) *fuzzy submaximal space*, if for each fuzzy set  $\lambda$  in  $(X, T)$  such that  $cl(\lambda) = 1$ ,  $\lambda \in T$  [14],
- (vii) *fuzzy Baire space*, if  $int(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy nowhere dense sets in  $(X, T)$  [20],
- (viii) *fuzzy superconnected space*, if it has no proper fuzzy regular open set [30],
- (ix) *fuzzy  $DG_{\delta}$ -space*, if each fuzzy dense (but not fuzzy open) set in  $(X, T)$  is a fuzzy  $G_{\delta}$ -set in  $(X, T)$  [28],
- (x) *fuzzy extremally disconnected space*, if the closure of every fuzzy open set of  $(X, T)$  is fuzzy open in  $(X, T)$  [5],
- (xi) *fuzzy basically disconnected space*, if the closure of every fuzzy open  $F_{\sigma}$ -set of  $(X, T)$  is fuzzy open in  $(X, T)$  [17],
- (xii) *fuzzy nodef space*, if each fuzzy nowhere dense set is a fuzzy  $F_{\sigma}$ -set in  $(X, T)$  [28],
- (xiii) *fuzzy open hereditarily irresolvable space*, if  $intcl(\lambda) \neq 0$ , for any non-zero fuzzy set  $\lambda$  defined on  $X$ ,  $int(\lambda) \neq 0$  in  $(X, T)$  [18].

**Definition 2.6** ([31]). Let  $(X, T)$  be any fuzzy topological space and  $\lambda$  be any fuzzy set in  $(X, T)$ . The fuzzy semi-closure and the fuzzy semi-interior of  $\lambda$  are defined as follows:

- (i)  $s-cl(\lambda) = \bigwedge \{ \mu \mid \lambda \leq \mu, \mu \text{ is fuzzy semi-closed in } (X, T) \}$ ,
- (ii)  $s-int(\lambda) = \bigvee \{ \mu \mid \mu \leq \lambda, \mu \text{ is fuzzy semi-open in } (X, T) \}$ .

**Theorem 2.7** ([11]). *In a fuzzy space  $X$ ,*

- (1) *the closure of a fuzzy open set is a fuzzy regular closed set,*
- (2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

**Theorem 2.8** ([20]). *If  $\lambda$  is a fuzzy residual set in a fuzzy topological space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\mu$  in  $(X, T)$  such that  $\mu \leq \lambda$ .*

**Lemma 2.9** ([31]). *Let  $A$  be a fuzzy set of a fuzzy topological space  $X$ . Then  $\text{int}(A) \leq s\text{-int}(A) \leq A \leq s\text{-cl}(A) \leq \text{cl}(A)$ .*

**Theorem 2.10** ([15]). *For any fuzzy topological space  $(X, T)$ , the following are equivalent:*

- (1)  *$X$  is fuzzy extremally disconnected space,*
- (2) *for each fuzzy closed set  $\lambda$ ,  $\text{int}(\lambda)$  is fuzzy closed,*
- (3) *for each fuzzy open set  $\lambda$ ,  $\text{cl}(\lambda) + \text{cl}[1 - \text{cl}(\lambda)] = 1$ ,*
- (4) *for every pair of fuzzy open sets  $\lambda$  and  $\mu$  in  $X$  with  $\text{cl}(\lambda) + \mu = 1$ ,*

$$\text{cl}(\lambda) + \text{cl}(\mu) = 1.$$

**Theorem 2.11** ([21]). *If  $\lambda$  is a fuzzy  $\sigma$ -boundary set in a fuzzy topological space  $(X, T)$ , then  $\lambda$  is a fuzzy  $F_\sigma$ -set in  $(X, T)$ .*

**Theorem 2.12** ([26]). *If  $\lambda$  is a fuzzy residual set in the fuzzy globally disconnected space  $(X, T)$ , then  $\lambda$  is a fuzzy  $G_\delta$ -set in  $(X, T)$ .*

**Theorem 2.13** ([20]). *If  $\gamma$  is a fuzzy residual set in a fuzzy almost  $P$ -space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\mu$  in  $(X, T)$  such that  $\mu \leq \gamma$  in  $(X, T)$ .*

**Theorem 2.14** ([22]). *Let  $(X, T)$  be a fuzzy topological space. Then, the following are equivalent:*

- (1)  *$(X, T)$  is a fuzzy Baire space,*
- (2) *each non-zero fuzzy open set is a fuzzy second category set in  $(X, T)$ .*

**Theorem 2.15** ([17]). *For any fuzzy topological space  $(X, T)$ , the following are equivalent :*

- (1)  *$X$  is fuzzy basically disconnected,*
- (2) *for each fuzzy closed  $G_\delta$ -set  $\lambda$ ,  $\text{int}(\lambda)$  is fuzzy closed,*
- (3) *for each fuzzy open  $F_\sigma$ -set  $\lambda$ ,  $\text{cl}(\lambda) + \text{cl}[1 - \text{cl}(\lambda)] = 1$ .*

**Theorem 2.16** ([25]). *If  $\lambda$  is a fuzzy residual set in the fuzzy submaximal space  $(X, T)$ , then  $\lambda$  is a fuzzy  $G_\delta$ -set in  $(X, T)$ .*

**Theorem 2.17** ([21]). *If  $\lambda$  is a fuzzy co- $\sigma$ -boundary set in a fuzzy perfectly disconnected space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\delta$  in  $(X, T)$  such that  $\lambda \leq \delta$ .*

**Theorem 2.18** ([5]). *A fuzzy topological space  $(X, T)$  is fuzzy extremally disconnected if and only if  $\text{FSO}(X) \subset \text{FPO}(X)$ .*

**Theorem 2.19** ([5]). *The following are equivalent for a fuzzy topological space  $(X, T)$ :*

- (1)  *$(X, T)$  is fuzzy extremally disconnected,*
- (2) *the closure of every fuzzy semi-pre-open set in  $X$  is fuzzy open.*

(3) the closure of every fuzzy pre-open set in  $X$  is fuzzy open.

**Theorem 2.20** ([23]). If  $\lambda$  is a fuzzy  $\sigma$ -nowhere dense set in the fuzzy topological space  $(X, T)$ , then  $\lambda$  is a fuzzy first category set in  $(X, T)$ .

**Theorem 2.21** ([5]). In a fuzzy extremally disconnected space  $(X, T)$ ,  $cl(\mu) = s-cl(\mu)$  for any fuzzy semi-open set  $\mu$  in  $(X, T)$ .

**Theorem 2.22** ([28]). If  $\lambda$  is a fuzzy dense (but not fuzzy open) set in a fuzzy  $DG_\delta$ -space  $(X, T)$ , then  $\lambda$  is a fuzzy residual set in  $(X, T)$ .

**Theorem 2.23** ([28]). If  $\lambda$  is a fuzzy dense ( but not fuzzy open ) set in a fuzzy  $DG_\delta$ -space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\eta$  in  $(X, T)$  such that  $\eta \leq \lambda$ .

**Theorem 2.24** ([28]). If  $\gamma$  is a fuzzy dense set in the fuzzy open hereditarily and fuzzy nodef space  $(X, T)$ , then  $\lambda$  is a fuzzy  $G_\delta$ -set such that  $int(\lambda) \neq 0$ , in  $(X, T)$ .

**Theorem 2.25** ([25]). If a fuzzy topological space  $(X, T)$  is a fuzzy hyperconnected and fuzzy  $P$ -space, then  $(X, T)$  is a fuzzy Baire space.

### 3. FUZZY MOSCOW SPACES

In this section, we introduce the concept of fuzzy Moscow spaces and study various properties related to it.

**Definition 3.1.** A fuzzy topological space  $(X, T)$  is called a *fuzzy Moscow space*, if for each fuzzy open set  $\lambda$  in  $(X, T)$ ,  $cl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .

**Example 3.2.** Let  $X = \{a, b, c\}$ . Let  $I = [0, 1]$ . The fuzzy sets  $\alpha$ ,  $\beta$  and  $\gamma$  are defined on  $X$  as follows:

$$\begin{aligned} \alpha : X \rightarrow I \text{ is defined by } & \alpha(a) = 0.6; & \alpha(b) = 0.4; & \alpha(c) = 0.6, \\ \beta : X \rightarrow I \text{ is defined by } & \beta(a) = 0.5; & \beta(b) = 0.5; & \beta(c) = 0.5, \\ \gamma : X \rightarrow I \text{ is defined by } & \gamma(a) = 0.4; & \gamma(b) = 0.6; & \gamma(c) = 0.4. \end{aligned}$$

Then  $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \wedge \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \vee \gamma, 1\}$  is a fuzzy topology on  $X$ . Moreover, we can easily check that the following equalities hold:

$$\alpha = \alpha \wedge (\alpha \vee \beta) \wedge (\alpha \vee \gamma), \quad \alpha \wedge \beta = \alpha \wedge (\beta \vee \gamma) \wedge (\alpha \vee \beta),$$

$$\alpha \wedge \gamma = \alpha \wedge \beta \wedge (\beta \wedge \gamma), \quad \beta \wedge \gamma = \beta \wedge \gamma \wedge (\alpha \vee \beta), \quad \gamma = (\alpha \vee \gamma) \wedge (\beta \vee \gamma) \wedge \gamma.$$

Thus  $\alpha$ ,  $\gamma$ ,  $\alpha \wedge \beta$ ,  $\alpha \wedge \gamma$  and  $\beta \wedge \gamma$  are fuzzy  $G_\delta$ -sets in  $(X, T)$ . By computation, one can find that

$$\begin{aligned} cl(\alpha) &= 1 - \gamma = \alpha \vee (\alpha \wedge \beta) \vee (\alpha \wedge \gamma), \\ cl(\beta) &= 1 - \beta = (\alpha \wedge \beta) \vee (\beta \wedge \gamma), \\ cl(\gamma) &= 1 - \alpha = (\beta \wedge \gamma) \vee \gamma \vee (\alpha \wedge \gamma), \\ cl(\alpha \vee \beta) &= 1 - (\beta \wedge \gamma) = \alpha \vee (\beta \wedge \gamma), \\ cl(\alpha \vee \gamma) &= 1 - (\alpha \wedge \gamma) = \alpha \vee \gamma, \\ cl(\beta \vee \gamma) &= 1 - (\alpha \wedge \beta) = (\alpha \wedge \beta) \vee (\beta \wedge \gamma) \vee \gamma, \\ cl(\alpha \wedge \beta) &= 1 - (\beta \vee \gamma) = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma), \\ cl(\alpha \wedge \gamma) &= 1 - (\alpha \vee \gamma) = (\alpha \wedge \gamma), \\ cl(\beta \wedge \gamma) &= 1 - (\alpha \vee \beta) = (\beta \wedge \gamma) \vee (\alpha \wedge \gamma). \end{aligned}$$

So for each fuzzy open set  $\lambda (= \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma)$

in  $(X, T)$ ,  $cl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$  where  $(\delta_i)$ 's ( $=\alpha, \gamma, \alpha \wedge \beta, \alpha \wedge \gamma$  and  $\beta \wedge \gamma$ ) are fuzzy  $G_\delta$ -sets in  $(X, T)$  shows that  $(X, T)$  is a fuzzy Moscow space.

**Example 3.3.** Let  $X = \{a, b, c\}$ . Let  $I = [0, 1]$ . The fuzzy sets  $\lambda, \mu$  and  $\gamma$  are defined on  $X$  as follows:

$$\begin{aligned} \alpha : X \rightarrow I \text{ is defined by } & \lambda(a) = 0.8; & \lambda(b) = 0.6; & \lambda(c) = 0.7, \\ \mu : X \rightarrow I \text{ is defined by } & \mu(a) = 0.6; & \beta(b) = 0.9; & \beta(c) = 0.8, \\ \gamma : X \rightarrow I \text{ is defined by } & \gamma(a) = 0.7; & \gamma(b) = 0.5; & \gamma(c) = 0.9, \\ \eta : X \rightarrow I \text{ is defined by } & \eta(a) = 0.7; & \eta(b) = 0.6; & \eta(c) = 0.8. \end{aligned}$$

Then  $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \wedge \gamma), \gamma \vee (\lambda \wedge \mu), \lambda \wedge (\mu \vee \gamma), \mu \wedge (\lambda \vee \gamma), \gamma \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \gamma, \lambda \wedge \mu \wedge \gamma, 1\}$  is a fuzzy topology on  $X$ . By computation, one can find that

$$\begin{aligned} \lambda &= \lambda \wedge (\lambda \vee \mu) \wedge (\lambda \vee \gamma) \wedge [\lambda \vee (\mu \wedge \gamma)] \wedge (\lambda \vee \mu \vee \gamma), \\ \lambda \wedge \gamma &= \gamma \wedge (\mu \vee \gamma) \wedge [\lambda \wedge (\mu \vee \gamma)] \wedge [\gamma \wedge (\lambda \vee \mu)] \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)], \\ \lambda \wedge \mu \wedge \gamma &= \mu \wedge (\lambda \wedge \mu) \wedge (\mu \wedge \gamma) \wedge [\mu \wedge (\lambda \vee \gamma)], \end{aligned}$$

and

$$\eta = (\mu \vee \gamma) \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)].$$

Thus  $\lambda, \lambda \wedge \gamma, \lambda \wedge \mu \wedge \gamma$  and  $\eta$  are fuzzy  $G_\delta$ -sets in  $(X, T)$ . By computation, one can find that  $cl(\lambda) = 1 \neq \lambda \vee (\lambda \wedge \gamma) \vee [\lambda \wedge \mu \wedge \gamma] \vee \eta$ . This shows that  $(X, T)$  is not a fuzzy Moscow space.

**Proposition 3.4.** *If  $\lambda$  is a fuzzy open set in the fuzzy Moscow space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\mu$  in  $(X, T)$  such that  $\mu \leq cl(\lambda)$ .*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space,  $cl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Clearly,  $\delta_i \leq \bigvee_{i=1}^{\infty} (\delta_i)$ . Let  $\mu = \delta_i$ . Then  $\mu \leq \bigvee_{i=1}^{\infty} (\delta_i) = cl(\lambda)$ . Thus for the fuzzy open set  $\lambda$  in  $(X, T)$ , there exists a fuzzy  $G_\delta$ -set  $\mu$  in  $(X, T)$  such that  $\mu \leq cl(\lambda)$ .  $\square$

**Proposition 3.5.** *If  $\eta$  is a fuzzy closed set in the fuzzy Moscow space  $(X, T)$ , then there exists a fuzzy  $F_\sigma$ -set  $\delta$  in  $(X, T)$  such that  $int(\eta) \leq \delta$ .*

*Proof.* Let  $\eta$  be a fuzzy closed set in  $(X, T)$ . Then  $1 - \eta$  is the fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is the fuzzy Moscow space and by Proposition 3.4, there exists an fuzzy  $G_\delta$ -set  $\mu$  in  $(X, T)$  such that  $\mu \leq cl(1 - \eta)$ . Thus  $\mu \leq 1 - int(\eta)$ . This implies that  $int(\eta) \leq 1 - \mu$ . Let  $\delta = 1 - \mu$ . Then  $\delta$  is a fuzzy  $F_\sigma$ -set in  $(X, T)$ . Thus for the fuzzy closed set  $\eta$  in  $(X, T)$ , there exists a fuzzy  $F_\sigma$ -set  $\delta$  in  $(X, T)$  such that  $int(\eta) \leq \delta$ .  $\square$

**Proposition 3.6.** *If  $\lambda$  is a fuzzy regular closed set in the fuzzy Moscow space  $(X, T)$ , then  $\lambda = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy regular closed set in  $(X, T)$ . Then  $clint(\lambda) = \lambda$  in  $(X, T)$ . Now  $int(\lambda) \neq 0$  [otherwise,  $int(\lambda) = 0$ , will imply that  $clint(\lambda) = cl[0] = 0$ , a contradiction]. Since the fuzzy topological space  $(X, T)$  is the fuzzy Moscow space, for the fuzzy open set  $int(\lambda)$  in  $(X, T)$ ,  $cl[int(\lambda)] = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Thus  $\lambda = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .  $\square$

**Proposition 3.7.** *If  $\mu$  is a fuzzy regular open set in the fuzzy Moscow space  $(X, T)$ , then  $\mu = \bigwedge_{i=1}^{\infty}(\eta_i)$ , where  $(\eta_i)$ 's are fuzzy  $F_{\sigma}$ -sets in  $(X, T)$ .*

*Proof.* Let  $\mu$  be a fuzzy regular open set in  $(X, T)$ . Then  $1 - \mu$  is a fuzzy regular closed set in  $(X, T)$ . Since  $(X, T)$  is the fuzzy Moscow space, by Proposition 3.6,  $1 - \mu = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_{\delta}$ -sets in  $(X, T)$ . Thus  $\mu = 1 - \bigvee_{i=1}^{\infty}(\delta_i) = \bigwedge_{i=1}^{\infty}(1 - \delta_i)$ . Let  $\eta_i = 1 - \delta_i$ . Then  $(\eta_i)$ 's are fuzzy  $F_{\sigma}$ -sets in  $(X, T)$ . Thus  $\mu = \bigwedge_{i=1}^{\infty}(\eta_i)$ , where  $(\eta_i)$ 's are fuzzy  $F_{\sigma}$ -sets in  $(X, T)$ .  $\square$

**Proposition 3.8.** *If each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy residual set in the fuzzy Moscow space  $(X, T)$ , then there exists a fuzzy regular closed set  $\gamma$  in  $(X, T)$  such that  $\gamma \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ .*

*Proof.* Let each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy residual set in  $(X, T)$ . Then by Theorem 2.8, there exist fuzzy  $G_{\delta}$ -sets  $\eta_i$  in  $(X, T)$  such that  $\eta_i \leq \lambda_i$ . This implies that  $\bigvee_{i=1}^{\infty}(\eta_i) \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\delta$  in  $(X, T)$ ,  $cl(\delta) = \bigvee_{i=1}^{\infty}(\eta_i)$ . Thus  $cl(\delta) \leq \bigvee_{i=1}^{\infty}(\lambda_i)$  in  $(X, T)$ . Since  $\delta$  is the fuzzy open set in  $(X, T)$ , by Theorem 2.7 (1),  $cl(\delta)$  is the fuzzy regular closed set in  $(X, T)$ . Let  $\gamma = cl(\delta)$ . Thus, if  $(\lambda_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy residual sets in  $(X, T)$ , then there exists a fuzzy regular closed set  $\gamma$  in  $(X, T)$  such that  $\gamma \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ .  $\square$

**Proposition 3.9.** *If each  $\mu_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the fuzzy Moscow space  $(X, T)$ , then there exists a fuzzy regular open set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty}(\mu_i) \leq \delta$ .*

*Proof.* Let each  $\mu_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy first category set in  $(X, T)$ . Then  $(1 - \mu_i)$ 's are fuzzy residual sets in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, by Proposition 3.8, there exists a fuzzy regular closed set  $\gamma$  in  $(X, T)$  such that  $\gamma \leq \bigvee_{i=1}^{\infty}(1 - \mu_i)$ . This implies that  $\gamma \leq [1 - \bigwedge_{i=1}^{\infty}(\mu_i)]$ . Thus  $\bigwedge_{i=1}^{\infty}(\mu_i) \leq (1 - \gamma)$ . Let  $\delta = 1 - \gamma$ . Then  $\delta$  is the fuzzy regular open set in  $(X, T)$ . Thus for the fuzzy first category sets  $(\mu_i)$ 's in  $(X, T)$ , there exists a fuzzy regular open set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty}(\mu_i) \leq \delta$ .  $\square$

**Proposition 3.10.** *If each  $\mu_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the fuzzy Moscow space  $(X, T)$ , then there exists a fuzzy open and fuzzy semi-closed set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty}(\mu_i) \leq \delta$ .*

*Proof.* The Proof follows from Proposition 3.9 and from the fact that fuzzy regular open sets in fuzzy topological spaces  $(X, T)$  are both fuzzy open and fuzzy semi-closed.  $\square$

**Proposition 3.11.** *If each  $\mu_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the Moscow space  $(X, T)$ , then  $s-cl[\bigwedge_{i=1}^{\infty}(\mu_i)] \neq 1$  in  $(X, T)$ .*

*Proof.* Let each  $\mu_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy first category set in  $(X, T)$ . Then by Proposition 3.9, there exists a fuzzy regular open set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty}(\mu_i) \leq \delta$ . Since fuzzy regular open sets in fuzzy topological spaces are both fuzzy open and fuzzy semi-closed,  $\delta$  is a fuzzy open and fuzzy semi-closed set in  $(X, T)$ . Thus  $\bigwedge_{i=1}^{\infty}(\mu_i) \leq \delta$  implies that  $s-cl[\bigwedge_{i=1}^{\infty}(\mu_i)] \neq 1$  in  $(X, T)$ .  $\square$



**Remark 3.12.** From the above Proposition, one will have the following result: “If  $(\mu_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy first category sets in the Moscow space  $(X, T)$ , then  $\bigwedge_{i=1}^{\infty}(\mu_i)$  is not fuzzy semi-dense in  $(X, T)$ ”.

**Proposition 3.13.** *If each  $\delta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy  $G_\delta$ -set in the fuzzy Moscow space  $(X, T)$ , then  $s\text{-int}[\bigvee_{i=1}^{\infty}(\delta_i)] \neq 0$  in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy semi-open set in  $(X, T)$ . Then  $\lambda \leq clint(\lambda)$  in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, for the fuzzy open set  $int(\lambda)$  in  $(X, T)$ ,  $cl[int(\lambda)] = \bigvee_{i=1}^{\infty}(\eta_i)$ , where  $(\eta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$  [ $(\eta_i)$ 's are considered from fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's]. Thus  $\lambda \leq \bigvee_{i=1}^{\infty}(\eta_i)$  in  $(X, T)$ . This implies that  $\lambda \leq \bigvee_{i=1}^{\infty}(\eta_i) \leq \bigvee_{i=1}^{\infty}(\delta_i)$ . So  $s\text{-int}[\bigvee_{i=1}^{\infty}(\delta_i)] \neq 0$  in  $(X, T)$ .  $\square$

**Proposition 3.14.** *If each  $\gamma_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy  $F_\sigma$ -set in the fuzzy Moscow space  $(X, T)$ , then  $s\text{-cl}[\bigwedge_{i=1}^{\infty}(\gamma_i)] \neq 1$  in  $(X, T)$ .*

*Proof.* Let each  $\gamma_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy  $F_\sigma$ -set in  $(X, T)$ . Then  $(1 - \gamma_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, by Proposition 3.13,  $s\text{-int}[\bigvee_{i=1}^{\infty}(1 - \gamma_i)] \neq 0$  in  $(X, T)$ . Now  $s\text{-int}[\bigvee_{i=1}^{\infty}(1 - \gamma_i)] = s\text{-int}[1 - \bigwedge_{i=1}^{\infty}(\gamma_i)] = 1 - s\text{-cl}[\bigwedge_{i=1}^{\infty}(\gamma_i)]$ , implies that  $1 - s\text{-cl}[\bigwedge_{i=1}^{\infty}(\gamma_i)] \neq 0$  in  $(X, T)$ . Thus  $s\text{-cl}[\bigwedge_{i=1}^{\infty}(\gamma_i)] \neq 1$  in  $(X, T)$ .  $\square$

**Remark 3.15.** From the above Proposition, one will have the following result: “If  $(\mu_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy  $F_\sigma$ -sets in the Moscow space  $(X, T)$ , then  $\bigwedge_{i=1}^{\infty}(\mu_i)$  is not fuzzy semi-dense in  $(X, T)$ ”.

**Proposition 3.16.** *If each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy  $\sigma$ -boundary set in the fuzzy Moscow space  $(X, T)$ , then  $\bigwedge_{i=1}^{\infty}(\lambda_i)$  is a fuzzy open set in  $(X, T)$ .*

*Proof.* Let each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy  $\sigma$ -boundary set in  $(X, T)$ . Then by Theorem 2.11, the fuzzy  $\sigma$ -boundary sets  $(\lambda_i)$ 's are fuzzy  $F_\sigma$ -sets in  $(X, T)$ . Thus  $(1 - \lambda_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\lambda$  in  $(X, T)$ ,  $cl(\lambda) = \bigvee_{i=1}^{\infty}(1 - \lambda_i)$ , where  $(1 - \lambda_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . This implies that  $cl(\lambda) = 1 - \bigwedge_{i=1}^{\infty}(\lambda_i)$ . So  $\bigwedge_{i=1}^{\infty}(\lambda_i) = 1 - cl(\lambda)$ . Since  $1 - cl(\lambda)$  is a fuzzy open set in  $(X, T)$ ,  $\bigwedge_{i=1}^{\infty}(\lambda_i)$  is a fuzzy open set in  $(X, T)$ .  $\square$

**Proposition 3.17.** *If each  $\mu_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy co- $\sigma$ -boundary set in the fuzzy Moscow space  $(X, T)$ , then  $\bigvee_{i=1}^{\infty}(\mu_i)$  is a fuzzy closed set in  $(X, T)$ .*

*Proof.* Let each  $\mu_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy co- $\sigma$ -boundary set in  $(X, T)$ . Then  $(1 - \mu_i)$ 's are fuzzy  $\sigma$ -boundary in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, by Proposition 3.16,  $\bigwedge_{i=1}^{\infty}(1 - \mu_i)$  is a fuzzy open set in  $(X, T)$ . Now  $\bigwedge_{i=1}^{\infty}(1 - \mu_i) = 1 - \bigvee_{i=1}^{\infty}(\mu_i)$ , implies that  $1 - \bigvee_{i=1}^{\infty}(\mu_i)$  is the fuzzy open set. Thus  $\bigvee_{i=1}^{\infty}(\mu_i)$  is a fuzzy closed set in  $(X, T)$ .  $\square$

**Proposition 3.18.** *If  $\lambda$  is a fuzzy semi-open set in the fuzzy Moscow space  $(X, T)$ , then  $\lambda \leq \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy semi-open set in  $(X, T)$ . Then  $\lambda \leq clint(\lambda)$  in  $(X, T)$ . Now  $int(\lambda) \neq 0$  [otherwise,  $int(\lambda) = 0$ , will imply that  $clint(\lambda) = cl[0] = 0$  and this will imply that  $\lambda = 0$ , a contradiction]. Since the fuzzy topological space  $(X, T)$  is a



fuzzy Moscow space, for the fuzzy open set  $int(\lambda)$  in  $(X, T)$ ,  $cl[int(\lambda)] = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Thus  $\lambda \leq \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .  $\square$

**Proposition 3.19.** *If  $\lambda$  is a fuzzy pre-closed set in the fuzzy Moscow space  $(X, T)$ , then  $\bigvee_{i=1}^{\infty} (\delta_i) \leq \lambda$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy pre-closed set in  $(X, T)$ . Then  $clint(\lambda) \leq \lambda$ , in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, for the fuzzy open set  $int(\lambda)$  in  $(X, T)$ ,  $cl[int(\lambda)] = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Thus  $\bigvee_{i=1}^{\infty} (\delta_i) \leq \lambda$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .  $\square$

**Proposition 3.20.** *If  $\lambda$  is a fuzzy  $\beta$ -open set in the fuzzy Moscow space  $(X, T)$ , then  $\lambda \leq \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy  $\beta$ -open set in  $(X, T)$ . Then  $\lambda \leq clintcl(\lambda)$ , in  $(X, T)$ . Now  $intcl(\lambda) \neq 0$ . Since the fuzzy topological space  $(X, T)$  is the fuzzy Moscow space, for the fuzzy open set  $intcl(\lambda)$  in  $(X, T)$ ,  $cl[intcl(\lambda)] = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Thus  $\lambda \leq \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .  $\square$

**Proposition 3.21.** *If  $\lambda$  is a fuzzy somewhere dense set in the fuzzy Moscow space  $(X, T)$ , then*

- (1)  $clintcl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ ,
- (2)  $int(\lambda) \leq \bigvee_{i=1}^{\infty} (\delta_i)$  in  $(X, T)$ .

*Proof.* (1) Let  $\lambda$  be a fuzzy somewhere dense set in  $(X, T)$ . Then  $intcl(\lambda) \neq 0$ , in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space, for the fuzzy open set  $intcl(\lambda)$  in  $(X, T)$ ,  $cl[intcl(\lambda)] = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Thus  $clintcl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ .

(2). By (1),  $clintcl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Then  $intcl(\lambda) \leq \bigvee_{i=1}^{\infty} (\delta_i)$  and  $int(\lambda) \leq intcl(\lambda)$ , implies that  $int(\lambda) \leq \bigvee_{i=1}^{\infty} (\delta_i)$  in  $(X, T)$ .  $\square$

**Proposition 3.22.** *If each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy  $\sigma$ -nowhere dense set in the fuzzy Moscow space  $(X, T)$ , then there exists a fuzzy regular open set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty} (\lambda_i) \leq \delta$ .*

*Proof.* Let each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy  $\sigma$ -nowhere dense set in  $(X, T)$ . Then by Theorem 2.20,  $(\lambda_i)$ 's are fuzzy first category sets in  $(X, T)$ . Thus  $(1 - \lambda_i)$ 's are fuzzy residual sets in  $(X, T)$ . So by Theorem 2.8, there exist fuzzy  $G_\delta$ -sets  $\eta_i$  in  $(X, T)$  such that  $\eta_i \leq (1 - \lambda_i)$ . This implies that  $\bigvee_{i=1}^{\infty} (\eta_i) \leq \bigvee_{i=1}^{\infty} (1 - \lambda_i)$ . Since  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\mu$  in  $(X, T)$ ,  $cl(\mu) = \bigvee_{i=1}^{\infty} (\eta_i)$ . Hence  $cl(\mu) \leq \bigvee_{i=1}^{\infty} (1 - \lambda_i)$  and  $cl(\mu) \leq 1 - \bigwedge_{i=1}^{\infty} (\lambda_i)$  in  $(X, T)$ . This implies that  $\bigwedge_{i=1}^{\infty} (\lambda_i) \leq 1 - cl(\mu)$ . By Theorem 2.7 (1),  $cl(\mu)$  is a fuzzy regular closed. Therefore  $1 - cl(\mu)$  is a fuzzy regular open set in  $(X, T)$ . Let  $\delta = 1 - cl(\mu)$ . Then there exists a fuzzy regular open set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty} (\lambda_i) \leq \delta$ .  $\square$

#### 4. FUZZY MOSCOW SPACES AND OTHER FUZZY TOPOLOGICAL SPACES

In this section, we discuss some relationships among fuzzy Moscow spaces and other fuzzy spaces.

**Proposition 4.1.** *If the fuzzy topological space  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, then  $(X, T)$  is a fuzzy extremally disconnected space.*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space,  $cl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is the fuzzy P-space, the fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's are fuzzy open sets in  $(X, T)$ . Then  $\bigvee_{i=1}^{\infty} (\delta_i)$  is a fuzzy open set in  $(X, T)$ . Thus  $cl(\lambda)$  is a fuzzy open set in  $(X, T)$ . So for the fuzzy open set  $\lambda$ ,  $cl(\lambda)$  is a fuzzy open set in  $(X, T)$  implies that  $(X, T)$  is a fuzzy extremally disconnected space.  $\square$

**Proposition 4.2.** *If  $\mu$  is a fuzzy closed set in the fuzzy Moscow and fuzzy P-space, then there exists a fuzzy  $F_\sigma$ -set  $\delta$  in  $(X, T)$  such that  $clint(\mu) \leq \delta$ .*

*Proof.* Let  $\mu$  be a fuzzy closed set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1,  $(X, T)$  is a fuzzy extremally disconnected space. Then by Theorem 2.10, for the fuzzy closed set  $\mu$ ,  $int(\mu)$  is fuzzy closed in  $(X, T)$  and thus  $cl[int(\mu)] = int(\mu)$ . Thus by Proposition 3.5, there exists a fuzzy  $F_\sigma$ -set  $\delta$  in  $(X, T)$  such that  $int(\mu) \leq \lambda$ . So for the fuzzy closed set  $\mu$  in  $(X, T)$ , there exists a fuzzy  $F_\sigma$ -set  $\delta$  in  $(X, T)$  such that  $clint(\mu) \leq \delta$ .  $\square$

**Proposition 4.3.** *If the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, then  $(X, T)$  is not a fuzzy superconnected space.*

*Proof.* Let the fuzzy topological space  $(X, T)$  be a fuzzy Moscow space and  $(\mu_i)$ 's ( $i = 1$  to  $\infty$ ) be fuzzy first category sets in  $(X, T)$ . Then by Proposition 3.9, there exists a fuzzy regular open set  $\delta$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty} (\mu_i) \leq \delta$ . Thus  $(X, T)$  is not a fuzzy superconnected space.  $\square$

**Proposition 4.4.** *If the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, then  $(X, T)$  is not a fuzzy hyperconnected space.*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space,  $cl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Then by Theorem 2.7 (1),  $cl(\lambda)$  being the closure of a fuzzy open set  $\lambda$  is a fuzzy regular closed set and thus a closed set in  $(X, T)$ . So  $cl(\lambda) \neq 1$ , in  $(X, T)$ . Hence for the fuzzy open set  $\lambda$ ,  $cl(\lambda) \neq 1$ , in  $(X, T)$  implies that  $(X, T)$  is not a fuzzy hyperconnected space.  $\square$

The following Proposition gives a condition for the fuzzy Moscow spaces to become fuzzy hyperconnected spaces.

**Proposition 4.5.** *If each fuzzy  $G_\delta$ -set is a fuzzy dense set in the fuzzy Moscow space  $(X, T)$ , then  $(X, T)$  is a fuzzy hyperconnected space.*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is an fuzzy Moscow space,  $cl(\lambda) = \bigvee_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Then by the hypothesis, the fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's are fuzzy dense sets in  $(X, T)$ . Now  $cl[cl(\lambda)] = cl[\bigvee_{i=1}^{\infty} (\delta_i)] \geq \bigvee_{i=1}^{\infty} [cl(\delta_i)] = \bigvee_{i=1}^{\infty} (1) = 1$ , implies that  $cl[cl(\lambda)] = 1$ . Since  $cl[cl(\lambda)] = cl(\lambda)$ ,  $cl(\lambda) = 1$ . Thus the fuzzy open set  $\lambda$  is a fuzzy dense set in  $(X, T)$ . So  $(X, T)$  is the fuzzy hyperconnected space.  $\square$

The following Proposition shows that fuzzy extremally disconnected spaces are fuzzy Moscow spaces.

**Proposition 4.6.** *If the fuzzy topological space  $(X, T)$  is a fuzzy extremally disconnected space, then  $(X, T)$  is a fuzzy Moscow space.*

*Proof.* Let  $\lambda$  be the fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is the fuzzy extremally disconnected space,  $cl(\lambda)$  is the fuzzy open set in  $(X, T)$ . Since  $\lambda \leq cl(\lambda)$ ,  $\lambda = \lambda \wedge cl(\lambda)$ . Now  $\lambda = \lambda \wedge cl(\lambda)$ , where  $\lambda \in T$ . Then  $cl(\lambda) \in T$  implies that  $\lambda$  is a fuzzy  $G_\delta$ -set in  $(X, T)$ . Since  $cl(\lambda)$  is the fuzzy open set in  $(X, T)$  and  $(X, T)$  is the fuzzy extremally disconnected space,  $cl[cl(\lambda)]$  is a fuzzy open set in  $(X, T)$ . Now  $cl(\lambda) \leq cl[cl(\lambda)]$  and  $cl(\lambda) = cl(\lambda) \wedge cl[cl(\lambda)]$ . Thus  $cl(\lambda)$  is the fuzzy  $G_\delta$ -set in  $(X, T)$ . So  $cl(\lambda) = \lambda \vee cl(\lambda)$ , where  $\lambda$  and  $cl(\lambda)$  are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Hence  $(X, T)$  is a fuzzy Moscow space.  $\square$

The following Proposition gives a condition for fuzzy basically disconnected spaces to become fuzzy Moscow spaces.

**Proposition 4.7.** *If a fuzzy topological space  $(X, T)$  is an fuzzy basically disconnected space in which fuzzy open sets are fuzzy  $F_\sigma$ -sets, then  $(X, T)$  is an fuzzy Moscow space.*

*Proof.* Let  $\lambda$  be an fuzzy open set in  $(X, T)$ . Then by the hypothesis,  $\lambda$  is an fuzzy  $F_\sigma$ -set in  $(X, T)$ . Since  $(X, T)$  is an fuzzy basically disconnected space, by Theorem 2.15, for the fuzzy open  $F_\sigma$ -set  $\lambda$ ,  $cl(\lambda) + cl[1 - cl(\lambda)] = 1$ . Thus  $cl(\lambda) + 1 - intcl(\lambda) = 1$ . So  $cl(\lambda) = intcl(\lambda)$ . This implies that  $cl(\lambda)$  is an fuzzy open set in  $(X, T)$ . Hence  $(X, T)$  is a fuzzy extremally disconnected space. Therefore by Proposition 4.6,  $(X, T)$  is a fuzzy Moscow space.  $\square$

**Proposition 4.8.** *If each  $\delta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy residual set in the fuzzy Moscow and fuzzy globally disconnected space  $(X, T)$ , then  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy semi-open set in  $(X, T)$ .*

*Proof.* Let each  $\delta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy residual set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy globally disconnected space, by Theorem 2.12,  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Now consider  $\bigvee_{i=1}^{\infty}(\delta_i)$  in  $(X, T)$  and  $(X, T)$  being the fuzzy Moscow space, for some fuzzy open set  $\lambda$  in  $(X, T)$ ,  $cl(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since  $\lambda$  is a fuzzy open set in  $(X, T)$ ,  $\lambda \leq intcl(\lambda)$ . Then  $cl(\lambda) \leq clintcl(\lambda)$ . Thus  $cl(\lambda)$  is a fuzzy semi-open set in  $(X, T)$ . So  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy semi-open set in  $(X, T)$ .  $\square$

**Proposition 4.9.** *If each  $\eta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the fuzzy Moscow and fuzzy globally disconnected space  $(X, T)$ , then  $\bigwedge_{i=1}^{\infty}(\eta_i)$  is a fuzzy semi-closed set in  $(X, T)$ .*

*Proof.* Let each  $\eta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy first category set in  $(X, T)$ . Then  $(1 - \eta_i)$ 's are fuzzy residual sets in  $(X, T)$ . Since  $(X, T)$  is the fuzzy Moscow and fuzzy globally disconnected space, by Proposition 4.8,  $\bigvee_{i=1}^{\infty}(1 - \eta_i)$  is a fuzzy semi-open set in  $(X, T)$ . Now  $\bigvee_{i=1}^{\infty}(1 - \eta_i) = 1 - \bigwedge_{i=1}^{\infty}(\eta_i)$  and this implies that  $1 - \bigwedge_{i=1}^{\infty}(\eta_i)$  is a fuzzy semi-open set in  $(X, T)$ . Thus  $\bigwedge_{i=1}^{\infty}(\eta_i)$  is a fuzzy semi-closed set in  $(X, T)$ .  $\square$

**Proposition 4.10.** *If the fuzzy topological space  $(X, T)$  is a fuzzy Moscow and fuzzy  $P$ -space, then  $(X, T)$  is a fuzzy basically disconnected space.*

*Proof.* Let  $(X, T)$  be a fuzzy Moscow and fuzzy P-space. Then by Proposition 4.1,  $(X, T)$  is a fuzzy extremally disconnected space. Since fuzzy extremally disconnected spaces are fuzzy basically disconnected spaces,  $(X, T)$  is a fuzzy basically disconnected space.  $\square$

**Proposition 4.11.** *If each  $\gamma_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy residual set in the fuzzy Moscow and fuzzy almost P-space  $(X, T)$ , then there exists a fuzzy open set in  $(X, T)$  such that  $cl(\lambda) \leq \bigvee_{i=1}^{\infty}(\gamma_i)$ .*

*Proof.* Let each  $\gamma_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy residual set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy almost P-space, by Theorem 2.13, there exists fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's in  $(X, T)$  such that  $\delta_i \leq \gamma_i$ , in  $(X, T)$ . Then  $\bigvee_{i=1}^{\infty}(\delta_i) \leq \bigvee_{i=1}^{\infty}(\gamma_i)$  in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\lambda$  in  $(X, T)$ ,  $cl(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Thus  $cl(\lambda) \leq \bigvee_{i=1}^{\infty}(\gamma_i)$  in  $(X, T)$ . So, if  $(\delta_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy residual sets in the fuzzy Moscow and fuzzy almost P-space  $(X, T)$ , then there exists a fuzzy open set  $\lambda$  in  $(X, T)$  such that  $cl(\lambda) \leq \bigvee_{i=1}^{\infty}(\gamma_i)$ .  $\square$

**Proposition 4.12.** *If each  $\eta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the fuzzy Moscow and fuzzy almost P-space  $(X, T)$ , then there exists a fuzzy open set  $\alpha$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty}(\eta_i) \leq \alpha$ .*

*Proof.* Let each  $\eta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy first category set in  $(X, T)$ . Then  $(1 - \eta_i)$ 's are fuzzy residual sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy almost P-space, by Proposition 4.11, for the fuzzy residual sets  $(1 - \eta_i)$ 's in  $(X, T)$ , there exists a fuzzy open set  $\lambda$  in  $(X, T)$  such that  $cl(\lambda) \leq \bigvee_{i=1}^{\infty}(1 - \eta_i)$ . Now  $\bigvee_{i=1}^{\infty}(1 - \eta_i) = 1 - \bigwedge_{i=1}^{\infty}(\eta_i)$ . Thus  $cl(\lambda) = 1 - \bigwedge_{i=1}^{\infty}(\eta_i)$ . This implies that  $\bigwedge_{i=1}^{\infty}(\eta_i) = 1 - cl(\lambda)$ . Let  $\alpha = 1 - cl(\lambda)$ . Then  $\alpha$  is a fuzzy open set in  $(X, T)$ . Thus, if  $(\eta_i)$ 's are fuzzy first category sets in the fuzzy Moscow and fuzzy almost P-space  $(X, T)$ , then there exists a fuzzy open set  $\alpha$  in  $(X, T)$  such that  $\bigwedge_{i=1}^{\infty}(\eta_i) \leq \alpha$  in  $(X, T)$ .  $\square$

**Proposition 4.13.** *If each  $\gamma_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy residual set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then  $int(\bigvee_{i=1}^{\infty}(\gamma_i)) \neq 0$ , in  $(X, T)$ .*

*Proof.* Let each  $\gamma_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy residual set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1,  $(X, T)$  is a fuzzy extremally disconnected space. Since  $(X, T)$  is a fuzzy P-space,  $(X, T)$  is a fuzzy almost P-space [since for every non-zero fuzzy  $G_\delta$ -set  $\lambda$  in  $(X, T)$ ,  $int(\lambda) = \lambda \neq 0$ ]. Then  $(X, T)$  is a fuzzy Moscow and fuzzy almost P-space. Then by Proposition 4.11, there exists a fuzzy open set  $\lambda$  in  $(X, T)$  such that  $cl(\lambda) \leq \bigvee_{i=1}^{\infty}(\gamma_i)$ . Since  $(X, T)$  is a fuzzy extremally disconnected space,  $cl(\lambda)$  is a fuzzy open set in  $(X, T)$ . Thus  $cl(\lambda) \leq \bigvee_{i=1}^{\infty}(\gamma_i)$  and  $cl(\lambda) \in T$  implies that  $int(\bigvee_{i=1}^{\infty}(\gamma_i)) \neq 0$  in  $(X, T)$ .  $\square$

**Proposition 4.14.** *If each  $\eta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then  $cl(\bigvee_{i=1}^{\infty}(\gamma_i)) \neq 1$ , in  $(X, T)$ .*

*Proof.* Let each  $\eta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy first category set in  $(X, T)$ . Then  $(1 - \eta_i)$ 's are fuzzy residual sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, for the fuzzy residual sets  $(1 - \eta_i)$ 's in  $(X, T)$ , by Proposition 4.13,  $int(\bigvee_{i=1}^{\infty}(1 - \eta_i)) \neq 0$  in  $(X, T)$ . Thus  $int(1 - \bigwedge_{i=1}^{\infty}(\eta_i)) \neq 0$  in  $(X, T)$ . So by

Lemma 2.2,  $\text{int}(1 - \bigwedge_{i=1}^{\infty}(\eta_i)) = 1 - \text{cl}(\bigwedge_{i=1}^{\infty}(\eta_i))$ . Hence  $1 - \text{cl}(\bigwedge_{i=1}^{\infty}(\eta_i)) \neq 0$ , implies that  $\text{cl}(\bigwedge_{i=1}^{\infty}(\eta_i)) \neq 1$  in  $(X, T)$ .  $\square$

**Remark 4.15.** From the above Proposition, one will have the following result: “If  $(\eta_i)$ ’s ( $i = 1$  to  $\infty$ ) are fuzzy first category sets in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then  $\bigwedge_{i=1}^{\infty}(\eta_i)$  is not fuzzy dense in  $(X, T)$ ”.

**Proposition 4.16.** *If  $\lambda$  is a fuzzy open set in the fuzzy Baire, fuzzy Moscow and fuzzy P-space  $(X, T)$ , then  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy second category sets in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space,  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy  $G_\delta$ -sets in  $(X, T)$  and  $(X, T)$  being the fuzzy P-space, the fuzzy  $G_\delta$ -sets  $(\delta_i)$ ’s are fuzzy open sets in  $(X, T)$ . In the fuzzy Baire space  $(X, T)$ , by Theorem 2.14, the fuzzy open sets are  $(\delta_i)$ ’s are fuzzy second category sets in  $(X, T)$ . Then for the fuzzy open set in  $(X, T)$ ,  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy second category sets in  $(X, T)$ .  $\square$

**Corollary 4.17.** *If the fuzzy topological space  $(X, T)$  is a fuzzy Baire, fuzzy Moscow and fuzzy P-space, then for the fuzzy second category sets  $(\delta_i)$ ’s,  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy regular closed set in  $(X, T)$ .*

*Proof.* The Proof follows from Proposition 4.15 and Theorem 2.7 (1).  $\square$

**Proposition 4.18.** *If each fuzzy  $G_\delta$ -set is a fuzzy dense set in a fuzzy Moscow and fuzzy P-space  $(X, T)$ , then for a fuzzy open set in  $(X, T)$ ,  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy second category sets in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space,  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since the fuzzy  $G_\delta$ -sets are fuzzy dense sets in the fuzzy Moscow space  $(X, T)$ , by Proposition 4.5,  $(X, T)$  is an fuzzy hyperconnected space. Then  $(X, T)$  is a fuzzy hyperconnected and fuzzy P-space. Thus by Theorem 2.25,  $(X, T)$  is a fuzzy Baire space. So by Proposition 4.16, for the fuzzy open set  $\lambda$  in the fuzzy Baire, fuzzy Moscow and fuzzy P-space  $(X, T)$ ,  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy second category sets in  $(X, T)$ .  $\square$

**Proposition 4.19.** *If each  $\delta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy  $G_\delta$ -set in a fuzzy Moscow and fuzzy extremally disconnected space  $(X, T)$ , then  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy open set in  $(X, T)$ .*

*Proof.* Let each  $\delta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy  $G_\delta$ -set in  $(X, T)$ . Since the fuzzy topological space  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\lambda$  in  $(X, T)$ ,  $\text{cl}(\lambda) = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ ’s are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy extremally disconnected space, for the fuzzy open set  $\lambda$  in  $(X, T)$ ,  $\text{cl}(\lambda)$  is a fuzzy open set in  $(X, T)$ . Then  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy open set in  $(X, T)$ .  $\square$

**Proposition 4.20.** *If each  $\delta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy residual set in the fuzzy submaximal and fuzzy Moscow space  $(X, T)$ , then  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy regular closed set in  $(X, T)$ .*

*Proof.* Let each  $\delta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy residual set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy submaximal space, by Theorem 2.16, the fuzzy residual sets  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$  and  $(X, T)$  being the fuzzy Moscow space, for some fuzzy open set  $\delta$  in  $(X, T)$ ,  $cl(\delta) = \bigvee_{i=1}^{\infty}(\delta_i)$ . Since  $\delta$  is a fuzzy open set in  $(X, T)$ . Then by Theorem 2.7 (1),  $cl(\delta)$  is a fuzzy regular closed set in  $(X, T)$ . Thus  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy regular closed set in  $(X, T)$ .  $\square$

**Corollary 4.21.** *If each  $\eta_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy first category set in the fuzzy submaximal and fuzzy Moscow space  $(X, T)$ , then  $\bigwedge_{i=1}^{\infty}(\eta_i)$  is a fuzzy regular open set in  $(X, T)$ .*

*Proof.* Let each  $\eta_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy first category set in  $(X, T)$ . Then  $(1 - \eta_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy residual sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy submaximal and fuzzy Moscow space, by Proposition 4.20,  $\bigvee_{i=1}^{\infty}(1 - \eta_i)$  is an fuzzy regular closed set in  $(X, T)$ . Now  $\bigvee_{i=1}^{\infty}(1 - \eta_i) = 1 - \bigwedge_{i=1}^{\infty}(\eta_i)$  implies that  $1 - \bigwedge_{i=1}^{\infty}(\eta_i)$  is an fuzzy regular closed set in  $(X, T)$ . Thus  $\bigwedge_{i=1}^{\infty}(\eta_i)$  is an fuzzy regular open set in  $(X, T)$ .  $\square$

**Proposition 4.22.** *If each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy co- $\sigma$ -boundary set in the fuzzy Moscow and fuzzy perfectly disconnected space  $(X, T)$ , then there exists a fuzzy regular closed set  $\delta$  in  $(X, T)$  such that  $\bigvee_{i=1}^{\infty}(\lambda_i) \leq \delta$ .*

*Proof.* Let each  $\lambda_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy co- $\sigma$ -boundary set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy perfectly disconnected space, by Theorem 2.17, there exist fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's in  $(X, T)$  such that  $\lambda_i \leq \delta_i$ . This implies that  $\bigvee_{i=1}^{\infty}(\lambda_i) \leq \bigvee_{i=1}^{\infty}(\delta_i)$ . Since  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\eta$  in  $(X, T)$ ,  $cl(\eta) = \bigvee_{i=1}^{\infty}(\delta_i)$ . Then by Theorem 2.7 (1),  $cl(\eta)$  is a fuzzy regular closed set in  $(X, T)$ . Let  $\delta = cl(\eta)$ . Then there exists a fuzzy regular closed set  $\delta$  in  $(X, T)$  such that  $\bigvee_{i=1}^{\infty}(\lambda_i) \leq \delta$ .  $\square$

**Corollary 4.23.** *If each  $\mu_i$  ( $i = 1$  to  $\infty$ ) is a fuzzy  $\sigma$ -boundary set in the fuzzy Moscow and fuzzy perfectly disconnected space  $(X, T)$ , then there exists a fuzzy regular open set  $\eta$  in  $(X, T)$  such that  $\eta \leq \bigwedge_{i=1}^{\infty}(\mu_i)$ .*

*Proof.* Let each  $\mu_i$  ( $i = 1$  to  $\infty$ ) be a fuzzy  $\sigma$ -boundary set in  $(X, T)$ . Then  $(1 - \mu_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy co- $\sigma$ -boundary sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy perfectly disconnected space, by Proposition 4.22, there exists a fuzzy regular closed set  $\delta$  in  $(X, T)$  such that  $\bigvee_{i=1}^{\infty}(1 - \mu_i) \leq \delta$ . This implies that  $1 - \bigwedge_{i=1}^{\infty}(\mu_i) \leq \delta$  and  $1 - \delta \leq \bigwedge_{i=1}^{\infty}(\mu_i)$ . Let  $\eta = 1 - \delta$ . Then  $\eta$  is a fuzzy regular open set in  $(X, T)$ . Thus there exists a fuzzy regular open set  $\eta$  in  $(X, T)$  such that  $\eta \leq \bigwedge_{i=1}^{\infty}(\mu_i)$ .  $\square$

**Proposition 4.24.** *If  $\lambda$  is a fuzzy regular closed set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then  $\lambda$  is a fuzzy open set in  $(X, T)$ .*

*Proof.* Let  $\lambda$  be a fuzzy regular closed set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space, by Proposition 3.6,  $\lambda = \bigvee_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy P-space, the fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's are fuzzy open sets in  $(X, T)$ . Then  $\bigvee_{i=1}^{\infty}(\delta_i)$  is a fuzzy open set in  $(X, T)$ . Thus  $\lambda$  is a fuzzy open set in  $(X, T)$ .  $\square$

**Proposition 4.25.** *If the fuzzy topological space  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, then each fuzzy semi-open set in  $(X, T)$  is a fuzzy pre-open set in  $(X, T)$ .*



*Proof.* Let  $\lambda$  be a fuzzy semi-open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1,  $(X, T)$  is a fuzzy extremally disconnected space. Then by Theorem 2.18,  $\lambda$  is a fuzzy pre-open set in  $(X, T)$ .  $\square$

**Proposition 4.26.** *Let  $(X, T)$  be a fuzzy Moscow and fuzzy P-space, then*

- (1) *if  $\lambda$  is a fuzzy  $\beta$ -open (semi pre-open) set, then  $cl(\lambda)$  is a fuzzy open set in  $(X, T)$ ,*
- (2) *if  $\lambda$  is a fuzzy pre-open set, then  $cl(\lambda)$  is a fuzzy open set in  $(X, T)$ .*

*Proof.* (1) Suppose  $\lambda$  is a fuzzy  $\beta$ -open (semi pre-open) set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1,  $(X, T)$  is a fuzzy extremally disconnected space. Then, by Theorem 2.19 (2),  $cl(\lambda)$  is a fuzzy open set in  $(X, T)$ .

(2) Suppose  $\lambda$  is a fuzzy pre-open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1,  $(X, T)$  is a fuzzy extremally disconnected space. Then by Theorem 2.19 (3),  $cl(\lambda)$  is the fuzzy open set in  $(X, T)$ .  $\square$

**Proposition 4.27.** *If  $\lambda$  is a fuzzy  $G_\delta$ -set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\eta$  in  $(X, T)$  such that  $cl(\lambda) \leq \eta$ .*

*Proof.* Let  $\lambda$  be a fuzzy  $G_\delta$ -set in  $(X, T)$ . Then  $\lambda = \bigwedge_{i=1}^\infty (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy open sets in  $(X, T)$ . Now  $cl(\lambda) = cl(\bigwedge_{i=1}^\infty (\lambda_i)) \leq \bigwedge_{i=1}^\infty (cl(\lambda_i))$ . Since  $(X, T)$  is the fuzzy Moscow and fuzzy P-space, by Proposition 4.1,  $(X, T)$  is the fuzzy extremally disconnected space. Thus  $(cl(\lambda_i))$ 's are fuzzy open sets in  $(X, T)$ . So  $\bigwedge_{i=1}^\infty (cl(\lambda_i))$  is a fuzzy  $G_\delta$ -set in  $(X, T)$ . Let  $\eta = \bigwedge_{i=1}^\infty (cl(\lambda_i))$ . Then  $\eta$  is a fuzzy  $G_\delta$ -set in  $(X, T)$ . Thus for the fuzzy  $G_\delta$ -set  $\lambda$ , there exists a fuzzy  $G_\delta$ -set  $\eta$  in  $(X, T)$  such that  $cl(\lambda) \leq \eta$ .  $\square$

**Proposition 4.28.** *If  $\lambda$  is a fuzzy open set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then there exists a fuzzy open set  $\mu$  in  $(X, T)$  such that  $\lambda \leq cl(\mu)$ .*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow space,  $cl(\lambda) = \bigvee_{i=1}^\infty (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy  $G_\delta$ -sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space, by Proposition 4.27, for the fuzzy  $G_\delta$ -sets  $(\delta_i)$ 's in  $(X, T)$ , there exist fuzzy  $G_\delta$ -sets  $(\eta_i)$ 's in  $(X, T)$  such that  $cl(\delta_i) \leq \eta_i$ . Then we get

$$\bigvee_{i=1}^\infty (\delta_i) \leq \bigvee_{i=1}^\infty (cl(\delta_i)) \leq \bigvee_{i=1}^\infty (\eta_i).$$

This implies that  $cl(\lambda) \leq \bigvee_{i=1}^\infty (\eta_i)$ . Since  $(X, T)$  is a fuzzy Moscow space, for some fuzzy open set  $\mu$  in  $(X, T)$ ,  $cl(\mu) = \bigvee_{i=1}^\infty (\eta_i)$ . Thus  $\lambda \leq cl(\lambda) \leq cl(\mu)$ . So there exists a fuzzy open set  $\mu$  in  $(X, T)$  such that  $\lambda \leq cl(\mu)$ .  $\square$

**Corollary 4.29.** *If  $\lambda$  is the fuzzy open set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then there exists a fuzzy open set  $\eta$  in  $(X, T)$  such that  $\lambda \leq \eta$ .*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy Moscow and fuzzy P-space  $(X, T)$ , by Proposition 4.28, there exists a fuzzy open set  $\mu$  in  $(X, T)$  such that  $\lambda \leq cl(\mu)$ . Then by Theorem 2.7 (1),  $cl(\mu)$  is a fuzzy regular closed set in  $(X, T)$ . Thus by Proposition 4.24, the fuzzy regular closed set  $cl(\mu)$  is a fuzzy open set in  $(X, T)$ . Let  $cl(\mu) = \eta$ . Then for the fuzzy open set  $\lambda$ , there exists a fuzzy open set  $\eta$  in  $(X, T)$  such that  $\lambda \leq \eta$ .  $\square$



**Proposition 4.30.** *If  $\lambda$  is a fuzzy semi-open set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then  $cl(\lambda) = s-cl(\lambda)$ , in  $(X, T)$ .*

*Proof.* The Proof follows from Theorem 2.21 and Proposition 4.1. □

**Proposition 4.31.** *If  $(\lambda_i)$ 's  $(i = 1$  to  $\infty)$  are fuzzy dense ( but not fuzzy open ) sets in the fuzzy Moscow and fuzzy  $DG_\delta$ -space  $(X, T)$ , then there exists a fuzzy regular closed set  $\gamma$  in  $(X, T)$  such that  $\gamma \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ .*

*Proof.* The Proof follows from Theorem 2.22 and Proposition 3.8. □

**Proposition 4.32.** *If each  $\lambda_i$   $(i = 1$  to  $\infty)$  is a fuzzy dense (but not fuzzy open) set in the fuzzy Moscow and fuzzy  $DG_\delta$ -space  $(X, T)$ , then there exists a fuzzy open set  $\eta$  in  $(X, T)$  such that  $cl(\eta) \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ .*

*Proof.* Let each  $\lambda_i$   $(i = 1$  to  $\infty)$  be a fuzzy dense (but not fuzzy open) set in  $(X, T)$ . Since  $(X, T)$  is the fuzzy  $DG_\delta$ -space, by Theorem 2.23, there exist fuzzy  $G_\delta$ -sets  $(\eta_i)$ 's in  $(X, T)$  such that  $\eta_i \leq \lambda_i$ . Then  $\bigvee_{i=1}^{\infty}(\eta_i) \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ . Since  $(X, T)$  is the fuzzy Moscow space, for some fuzzy open set  $\eta$  in  $(X, T)$ ,  $cl(\eta) \leq \bigvee_{i=1}^{\infty}(\lambda_i)$ . Thus  $cl(\eta) \leq \bigvee_{i=1}^{\infty}(\lambda_i)$  in  $(X, T)$ . □

**Proposition 4.33.** *If each  $\lambda_i$   $(i = 1$  to  $\infty)$  is a fuzzy dense set in the fuzzy open hereditarily irresolvable, fuzzy Moscow and fuzzy nodef space  $(X, T)$ , then there exists a fuzzy open set  $\mu$  in  $(X, T)$  such that  $cl(\mu) = \bigvee_{i=1}^{\infty}(\lambda_i)$ , where  $int(\lambda_i) \neq 0$  in  $(X, T)$ .*

*Proof.* Let each  $\lambda_i$   $(i = 1$  to  $\infty)$  be a fuzzy dense set in  $(X, T)$ . Since  $(X, T)$  is a fuzzy open hereditarily irresolvable and fuzzy nodef space, by Theorem 2.24,  $(\lambda_i)$ 's are fuzzy  $G_\delta$ -sets such that  $int(\lambda_i) \neq 0$  in  $(X, T)$ . Now consider  $\bigvee_{i=1}^{\infty}(\lambda_i)$  in  $(X, T)$ . Since  $(X, T)$  is the fuzzy Moscow space, for some fuzzy open set  $\mu$  in  $(X, T)$  such that  $cl(\mu) = \bigvee_{i=1}^{\infty}(\lambda_i)$  in  $(X, T)$ . Then  $cl(\mu) = \bigvee_{i=1}^{\infty}(\lambda_i)$ , where  $int(\lambda_i) \neq 0$  in  $(X, T)$ . □

**Proposition 4.34.** *If  $\lambda$  is a fuzzy open set in the fuzzy Moscow and fuzzy P-space  $(X, T)$ , then there exists a fuzzy  $G_\delta$ -set  $\eta$  in  $(X, T)$  such that  $\eta \leq intcl(\lambda)$ .*

*Proof.* Let  $\lambda$  be a fuzzy open set in  $(X, T)$ . Then  $1 - \lambda$  is a fuzzy closed set in  $(X, T)$ . Since  $(X, T)$  is the fuzzy Moscow and fuzzy P-space, by Proposition 4.2, there exists a fuzzy  $F_\sigma$ -set  $\delta$  in  $(X, T)$  such that  $clint(1 - \lambda) \leq \delta$ . Then  $1 - intcl(\lambda) \leq \delta$ . Thus  $1 - \delta \leq intcl(\lambda)$ . Let  $\eta = 1 - \delta$ . Then  $\eta$  is a fuzzy  $G_\delta$ -set in  $(X, T)$ . Thus there exists a fuzzy  $G_\delta$ -set  $\eta$  in  $(X, T)$  such that  $\eta \leq intcl(\lambda)$ . □

## 5. CONCLUSIONS

In this paper, the notion of fuzzy Moscow space is introduced by means of fuzzy  $G_\delta$ -sets. It is established that fuzzy extremally disconnected spaces are fuzzy Moscow spaces and fuzzy Moscow and fuzzy P-spaces, are fuzzy extremally disconnected spaces and fuzzy basically disconnected spaces. It is obtained the fuzzy Moscow spaces, are not fuzzy hyperconnected spaces and not fuzzy superconnected spaces. The condition under which fuzzy Moscow spaces, become fuzzy hyperconnected spaces, is obtained. Fuzzy closure of an fuzzy open set in fuzzy Baire, fuzzy Moscow and fuzzy P-space, is obtained by means of fuzzy second category sets . It is also established that fuzzy regular closed sets are fuzzy open sets and fuzzy semi-open

sets are fuzzy pre-open sets in fuzzy Moscow and fuzzy P-spaces. A condition under which fuzzy basically disconnected spaces becomes fuzzy Moscow spaces, is also obtained in this paper.

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