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On fuzzy dot hyper K-ideal of a hyper K-algebra

D. Tefera Gerima

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Abstract. In this paper, the methods introduced in fuzzy dot d-subalgebras and fuzzy dot BF-subalgebras are applied on fuzzy hyper K-ideals of a hyper K-algebra. The concepts of a fuzzy dot hyper K-subalgebra, a fuzzy dot hyper K-ideal, a fuzzy dot weak hyper K-ideal and the Cartesian product in a fuzzy dot hyper K-algebra are discussed. Furthermore, homomorphism in fuzzy dot hyper K-algebras is investigated.

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Corresponding Author: D. Gerima Tefera (gerima233@gmail.com)

1. Introduction

Now a days hyper structures are widely used in both pure and applied mathematics. During the exploration of properties of set difference,the idea of fuzzy (weak, strong) hyper p-ideals presented and characterization of these ideals was conferred using different concepts like that of level subsets, hyper homomorphic preimage etc. The connections between fuzzy (weak, strong) hyper p-ideals were discussed, and the strongest fuzzy relation on a hyper BCK-algebra was introduced [1]and the idea of sub implicative algebra in hom-set,1-commutative algebra of hom-set was introduced by Gerima [2], and the concepts of ideals and filters on Hilbert implication algebras was discussed by Gerima [3].

Zadeh [4] introduced the notion of fuzzy set, and Berhanu et al. [5] explored the idea of P_0-almost distributive fuzzy lattice, and furthermore, the concept of hyper K-algebras develop by Borzooei and Mehdizahedi [6] which lead to the introduction of fuzzy hyper K-algebra. He and Xin [7] introduced the idea of fuzzy hyper lattices, the relations between the weak homomorphism of fuzzy hyper lattices and the weak homomorphism of corresponding hyper lattices. The notion of fuzzy sets is applied to investigate the relations between fuzzy distributive hyper BCI-ideals and distributive hyper BCI-Ideals of a hyper BCI-algebra elaborated by Nisaret et al. [8].

Radfar et al. [9] investigated some types of hyper filters in hyper BE-algebras and the relationship between hyper filters. The notion of fuzzy hyper K-sub algebra and fuzzy(weak) hyper K-ideals of a hyper K-algebra and also shown that fuzzy hyper K-ideal is a fuzzy weak hyper K-ideal initiated by Borzooei and Mehdizahedi [10] and the notions of fuzzy hyper UP-sub algebra and fuzzy hyper UP-filter and established some of their properties introduced by Amairanto and Isla [11].

Gerima [12] investigated on fuzzy dot d-sub algebras and fuzzy dot d-ideals of a d-algebra. The idea of fuzzy dot BF-sub algebras, fuzzy dot Cartesian product of BF-sub algebras, and fuzzy strong relations in fuzzy dot BF-sub algebras introduced by Gerima and Fantahun Tigist [13].

Throughout this paper, H represents a hyper K-algebra unless otherwise mentioned.

2. Preliminaries

Definition 2.1 ([6])**.** Let H be a nonempty set and $\circ : H \times H \to P^*(H) = P(H) - \{\varnothing\}$. Then "$\circ$" is the hyper operation on H, and $(H, \circ, 0)$ is said to be a *hyper K-algebra*, if it satisfies the following axioms for all $x, y, z \in H$:

(1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(3) $x < x$,
(4) $x < y,\ y < x \Rightarrow x = y$,
(5) $0 < x$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H,\ A < B$ is defined by there exist $a \in A,\ b \in B$ such that $a < b$.

Remark 2.2 ([9])**.** If $A, B \subseteq H$, then $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$ of H, where $a \circ b$ may be written as $\{a\} \circ b, a \circ \{b\}, \{a\} \circ \{b\}$.

Definition 2.3 ([6])**.** Let S be a nonempty subset of H. Then S is a hyper K-su balgebra of H if and only if $x \circ y \subseteq S$ for all $x,\ y \in H$.

Definition 2.4 ([10])**.** Let $(H, \circ_1, 0_1)$ and $(H, \circ_2, 0_2$ be two hyper K-algebras. Then $f : H_1 \to H_2$ defined by $f(x \circ_1 y) = f(x) \circ_2 f(y)$ for all $x,\ y \in H_1$ is called a *homomorphism* in hyper K-algebras.

Definition 2.5 ([6])**.** Let I be a nonempty subset of H.

(1) I is called a *weak hyper K-ideal of H*, if it satisfies the following conditions:
(i) $0 \in I$,
(ii) $x \circ y \subseteq I$ and $y \in I$ imply that $x \in I$ for all $x,\ y \in H$.

(2) I is called a hyper K-ideal of H, if it satisfies the following conditions:
(i) $0 \in I$,
(ii)$x \circ y < I$, $y \in I \Rightarrow x \in I$ for all $x,\ y \in H$.

For a nonempty set X, a mapping $\mu : X \to [0, 1]$ is called a fuzzy subset of X and $[0, 1]^X$ denotes the set of all fuzzy subsets of X. Let $a,\ b \in [0, 1]$ and let

$\{a_j \in [0,1] : j \in J\}$, where J is a index set. Then we write

$$min\{a,b\} = a \wedge b,\ max\{a,b\} = a \vee b,\ inf_{j\in J}a_j = \bigwedge_{j\in J} a_j \text{ and } sup_{j\in J}a_j = \bigvee_{j\in J} a_j.$$

Definition 2.6 ([6]). Let $\mu,\ \lambda \in [0,1]^H$ and let $\{\mu_j \in [0,1]^H : j \in J\}$ be a family of fuzzy subsets of H. Then we define the followings: for all $x \in H$,

(1) $(\mu \cap \lambda)(x) = \mu(x) \wedge \lambda(x)$,
(2) $(\bigcap_{j\in J} \mu_j)(x) = \bigwedge_{j\in J} \mu_j(x)$,
(3) $(\mu \cup \lambda)(x) = \mu(x) \vee \lambda(x)$,
(4) $(\bigcup_{j\in J} \mu_j)(x) = \bigvee_{j\in J} \mu_j(x)$.

Definition 2.7 ([10]). Let μ be a fuzzy subset of H. Then μ is called a fuzzy hyper K-sub algebra of H, if for all $x, y \in H$, we have $\mu(t) \geq \mu(x) \wedge \mu(y)\}$ for all $t \in x \circ y$.

Definition 2.8 ([10]). Let μ be a fuzzy subset of H. Then μ is said to be a *fuzzy hyper K ideal* of H, if it satisfies the following conditions: for all $x,\ y \in H$,

(1) $\mu(0) \geq \mu(x)$,
(2) $\mu(x) \geq \bigvee_{t\in x\circ y} \mu(t) \wedge \mu(y)$.

Definition 2.9 ([7]). Let μ be a fuzzy subset of H. Then μ is said to be a fuzzy weak hyper k- ideal of H if for all $x, y \in H$.

(1) $\mu(0) \geq \mu(x)$.
(2) $\mu(x) \geq min\{inf_{t\in x\circ y}\mu(t), \mu(y)\}$.

3. Fuzzy dot hyper K-subalgebra of a hyper K- algebra H

Definition 3.1. Let μ be a fuzzy hyper K-sub algebra of H. Then μ is called a *fuzzy dot hyper K-subalgebra* of H, if for any $x,\ y \in H$, we have

$$\mu(a) \geq \mu(x).\mu(y) \text{ for each } a \in x \circ y.$$

Example 3.2. Let $H = \{0, 1, 2\}$. Then define "∘" by the table below:

∘	0	1	2
0	{0}	{0}	{0}
1	{1}	{0, 1}	{0, 1}
2	{2}	{1,2}	{0,1,2}

Then $(H, \circ, 0)$ is a hyper K-algebra. Define a fuzzy subset μ by

$$\mu(0) = 0.8,\ \mu(1) = 0.6,\ \mu(2) = 0.7.$$

Then $\mu(a) \geq \mu(1).\mu(2) = 0.6.0.7 = 0.42$, $a \in 1 \circ 2$. Thus $\mu(a) \geq 0.42$. So μ is a fuzzy dot hyper K-sub algebra.

Lemma 3.3. *Let μ be a fuzzy dot hyper K-sub algebra of H. Then we have*

$$\mu(0) \geq (\mu(x))^2 \text{ for any } x \in H.$$

Definition 3.4. Let $\{\mu_j : j \in J\}$ be a family of fuzzy dot hyper K-sub algebra of H. Then $\bigcap_{j\in J} \mu_j = inf_{j\in J}\mu_j$.

Proposition 3.5. *If $\{\mu_i : j \in J\}$ is a family of fuzzy dot hyper K-sub algebra of H, then $\bigcap_{j\in J}\mu_j$ is a fuzzy dot hyper K-sub algebra of H.*

Proof. Let $x,\ y \in H$ and let $a \in x \circ y$. Then

$$\begin{aligned}(\textstyle\bigcap_{j\in J}\mu_j)(a) &= \textstyle\bigwedge_{j\in J}\mu_j(a)\\ &\geq \textstyle\bigwedge_{j\in J}[\mu_j(x).\mu_j(y)]\\ &= (\textstyle\bigwedge_{j\in I}\mu_j(x)).(\textstyle\bigwedge_{j\in J}\mu_j(y))\\ &= (\textstyle\bigcap_{j\in J}\mu_j)(x).(\textstyle\bigcap_{j\in J}\mu_j)(y).\end{aligned}$$

Thus $\bigcap_{j\in J}\mu_j$ is a fuzzy dot hyper K-sub algebra of H. □

Definition 3.6. Let $\mu \in [0,1]^H$, let $x,\ y \in H$ and let $\alpha \in [0,1]$. Then $\mu_\alpha = \{a \in x \circ y : \mu(a) \geq \alpha\}$ is called an *upper level subset* of μ.

Theorem 3.7. *Let μ be a fuzzy sub algebra of H. Then μ is a fuzzy dot hyper K-sub algebra of H if and only if for each $\alpha \in [0,1]$, μ_α is a hyper K-sub algebra of H.*

Proof. Suppose μ be a fuzzy dot hyper K-sub algebra of H and let $\alpha \in [0,1]$. It is clear that $0 \in H$ and $0 \in x \circ x$ for any $x \in H$. Let $\mu(x) = \sqrt{\alpha}$. Then we have $\mu(0) \geq \mu(x).\mu(x) = \alpha$. Thus $0 \in \mu_\alpha$.

Now let $x,\ y \in \mu_\alpha$, let $\mu(x) = \sqrt{\alpha},\ \mu(y) = \sqrt{\alpha}$ and let $a \in x \circ y$. Then

$$\mu(a) \geq \mu(x).\mu(y) = \sqrt{\alpha}.\sqrt{\alpha} = \alpha.$$

Thus $a \in \mu_\alpha$. So $x \circ y \subseteq \mu_\alpha$. Hence μ_α is a hyper K-sub algebra of H.

Conversely, suppose μ_α is a hyper K-subalgebra of H for each $\alpha \in [0,1]$ and let $x,\ y \in \mu_\alpha$. Then $\mu(x) \geq \alpha$ and $\mu(y) \geq \alpha$. Put $\mu(x) = \alpha$ and $\mu(y) = \alpha$. Let $a \in x \circ y$. Then $\mu(a) \geq \alpha \geq \alpha^2 = \alpha.\alpha = \mu(x).\mu(y)$ for all $a \in x \circ y$. Thus we get

$$\mu(a) \geq \mu(x).\mu(y) \text{ for all } x,\ y \in \mu_\alpha,\ x,\ y \in H.$$

So μ is a fuzzy dot hyper K-sub algebra. □

Theorem 3.8. *Let $(H_1, \circ_1, 0_1)$, let $(H_2, \circ_2, 0_2)$ be two hyper K-algebras and let $f : H_1 \to H_2$ be a homomorphism.*

(1) *If μ is a fuzzy dot hyper K-subalgebra of H_2, then $f^{-1}(\mu)$ is a fuzzy dot hyper K-subalgebra of H_1.*

(2) *If f is onto and λ is a fuzzy dot hyper K-sub algebra of H_1, then $f(\lambda)$ is a fuzzy dot hyper K-sub algebra of H_2.*

Proof. (1) Suppose μ is a fuzzy dot hyper K-sub algebra of H_2, let $x,\ y \in H_1$ and let $a \in x \circ_1 y$. Since f is a homomorphism, $f(a) \in f(x \circ_1 y) = f(x) \circ_2 f(y)$. Then we have

$$\begin{aligned}f^{-1}(\mu)(a) &= \mu(f(a))\\ &\geq \mu(f(x)) \wedge \mu(f(y)) \text{ [Since } f(a) \in f(x) \circ_2 f(y)]\\ &\geq \mu(f(x)).\mu(f(y))\\ &= f^{-1}(\mu(x)).f^{-1}(\mu(y)).\end{aligned}$$

Thus $f^{-1}(\mu)(a) \geq f^{-1}(\mu)(x).f^{-1}(\mu)(y)$. So $f^{-1}(\mu)$ is a fuzzy dot hyper K-sub algebra of H_1.

(2) Suppose f is onto and λ is a fuzzy dot hyper K-sub algebra of H_1, let $x,\ y \in H_2$ and let $b \in x \circ_2 y$. It is obvious that $f^{-1}(x),\ f^{-1}(y)$ and $f^{-1}(b)$ are nonempty sets. Let $a = f^{-1}(x)$ and $c = f^{-1}(y)$. Then $b \in x \circ_2 y = f(a) \circ_2 f(c) = f(a \circ_1 c)$. Thus

there is $h_{ac} \in a \circ_1 c$ such that$f(h_{ac}) = b$. Since λ is a fuzzy dot hyper K-sub algebra of H_1, we get

$$\lambda(h_{ac}) \geq \lambda(a).\lambda(c). \tag{3.1}$$

Thus we have

$$\begin{aligned} f(\lambda)(b) &= \bigvee_{h \in f^{-1}(b)} \lambda(h) \\ &\geq \bigvee_{h_{ac} \in a \circ_1 c} \lambda(h_{ac}) \\ &\geq \bigvee_{a \in f^{-1}(x),\ c \in f^{-1}(y)} [\lambda(a).\lambda(c)] \text{ [By (3.1)]} \\ &\geq (\bigvee_{a \in f^{-1}(x)} \lambda(a)).(\bigvee_{c \in f^{-1}(y)} \lambda(c)) \\ &= f(\lambda)(x).f(\lambda)(y). \end{aligned}$$

So $f(\lambda)(b) \geq f(\lambda)(x).f(\lambda)(y)$. Hence $f(\lambda)$ is a fuzzy dot hyper K-sub algebra of H_2. □

4. Fuzzy dot hyper K-ideals of a hyper K-algebra

Definition 4.1. Let μ be a fuzzy hyper K-ideal of H. Then μ is said to be a *fuzzy dot hyper K-ideal* of H, if it satisfies the following conditions: for all $x,\ y \in H$,

(i) $\mu(0) \geq (\mu(x))^2$,

(ii) $\mu(x) \geq \bigvee_{a \in x \circ y} \mu(a).\mu(y)$.

Example 4.2. Let $H = \{0, a, b\}$ be a set with the hyper operation $\circ$ on H defined by the table below:

∘	0	a	b
0	{0}	{0}	{0}
a	{a}	{0, a}	{0,a}
b	{b}	{a,b}	{0,a,b}

Then $(H, \circ, 0)$ is a hyper K-algebra. Let $I = \{0, a\}$ be the hyper K-ideal of H and let μ be the fuzzy ideal of I defined by $\mu(0) = 0.8$ and $\mu(a) = 0.6$. Then we have

(i) $\mu(0) \geq (\mu(x))^2$,

(ii) $\mu(x) \geq \bigvee_{a \in x \circ y} \mu(a) \wedge \mu(y) \geq \bigvee_{a \in x \circ y} \mu(a).\mu(y)$.

Thus μ is a fuzzy dot hyper K-ideal of H.

Definition 4.3. Let $\{\mu_j\},\ j \in J$ be a family of fuzzy hyper K-ideal of a hyper K-algebra H. Then

(1) $\bigcap_{j \in J} \mu_i = inf_{j \in J} \mu_j$,

(2) $\bigcup_{j \in J} \mu_j = sup_{j \in J} \mu_j$.

Proposition 4.4. *If $\{\mu_j : j \in J\}$ is a family of fuzzy dot hyper K-ideal of H, then $\bigcap_{j \in J} \mu_j$ is a fuzzy dot hyper K-ideal of H.*

Proof. Suppose $\{\mu_j\},\ j \in J$ is a family of fuzzy dot hyper K-ideal of H. Then for any $x \in H$, we have $0 \in x \circ x$.

(i)
$$\begin{aligned} (\bigcap_{j \in J} \mu_j)(0) &= \bigwedge_{j \in J} \mu_j(0) \\ &\geq \bigwedge_{j \in J} [\mu_j(x) \wedge \mu_j(x)] \\ &\geq \bigwedge_{j \in J} [\mu_j(x).\mu_j(x)] \\ &= \bigwedge_{j \in J} (\mu_j(x))^2 \\ &= (\bigcap_{j \in J} \mu_j^2)(x). \end{aligned}$$

Then $(\bigcap_{j \in J} \mu_j(0)) \geq (\bigcap_{j \in J} \mu_j^2)(x))$ for all $x \in H$.

(ii) Since $\mu_j,\ j \in J$ is a fuzzy dot hyper K-ideal of H, we get

$$\begin{aligned}(\bigcap_{j\in J}\mu_j)(x) &= \bigwedge_{j\in J}\mu_j(x)\\ &\geq \bigwedge_{j\in J}[\bigvee_{a\in x\circ y}\mu_j(a)\wedge\mu_j(y)]\\ &\geq \bigwedge_{j\in J}[\bigvee_{a\in x\circ y}\mu_j(a).\mu_j(y)]\\ &\geq [(\bigvee_{a\in x\circ y}\bigwedge_{j\in J}\mu_j(a)).\bigwedge_{j\in J}\mu_j(y)]\\ &= \bigvee_{a\in x\circ y}(\bigcap_{j\in J}\mu_j)(a).(\bigcap_{j\in J}\mu_j)(y).\end{aligned}$$

Then $(\bigcap_{j\in J}\mu_j(x)) \geq \bigvee_{a\in x\circ y}(\bigcap_{j\in J}\mu_j)(a).(\bigcap_{j\in J}\mu_j)(y)$.
Thus $\bigcap_{j\in J}\mu_j$ is a fuzzy dot hyper K -ideal of H. □

Proposition 4.5. *If $\{\mu_j : j \in J\}$ is a chain of family of fuzzy dot hyper K-ideal of H, then $\bigcup_{j\in J}\mu_j$ is a fuzzy dot hyper K-ideal of H.*

Proof. The proof is similar to Proposition 3.4 with simple modification. □

Definition 4.6. Let μ be a fuzzy ideal of H and let $\alpha \in [0,1]$. Then μ_α is called a *hyper K-ideal* of H.

Theorem 4.7. *Let μ be a fuzzy hyper K-ideal of H and let $\alpha \in [0,1]$. Then μ is a fuzzy dot hyper K-ideal of H if and only if μ_α is a hyper K-ideal of H.*

Proof. Suppose μ is a fuzzy dot hyper K-ideal of H and let $\alpha \in [0,1]$.

(i) It is clear that $\mu(0) \geq (\mu(x))^2$ for all $x \in H$.

(ii) It is obvious that $\mu(x) \geq \bigvee_{a\in x\circ y}\mu(a).\mu(y)$ for all $x,\ y \in X$. Put $\mu(x) = \sqrt{\alpha}$. Then $\mu(0) \geq (\mu(x))^2 = (\sqrt{\alpha})^2 = \alpha$.
Thus $0 \in \mu_\alpha$.

(iii) Let $x,\ y \in H$ such that $x \circ y < \mu_\alpha$ and $y \in \mu_\alpha$. Then for all $a \in x \circ y$, there exist $b \in \mu_\alpha$ such that $a < b$ implies $\mu(a) \geq \mu(b) \geq \alpha$. Thus $\bigvee_{a\in x\circ y}\mu(a) \geq \alpha$. By definition of fuzzy hyper K-ideal of H, we have

$$\mu(x) \geq \bigvee_{a\in x\circ y}[\mu(a)\wedge mu(y)] \geq \alpha.$$

So $\mu(x) \geq \alpha$. Hence $x \in \mu_\alpha$. Therefore μ_α is a hyper K-ideal of H.

Conversely, suppose $\mu_\alpha,\ \alpha \in [0,1]$ be a hyper K-ideal of H. For any $x \in H$, setting $\mu(x) = \alpha$. Then $x \in \mu_\alpha$. Since $0 \in \mu_\alpha$, we get

$$\mu(0) \geq \alpha = \mu(x) = \mu(x)\wedge\mu(x) \geq \mu(x).\mu(x) = (\mu(x))^2.$$

. Thus $\mu(0) \geq (\mu(x))^2$ for all $x \in H$.

Now Let $x,\ y \in H$ be arbitrary and let $\alpha = \bigvee_{a\in x\circ y}\mu(a)\wedge\mu(y)$. Then for $y \in \mu_\alpha$ and $u \in x \circ y$, we have

$$\mu(u) \geq \bigvee_{a\in x\circ y}\mu(a) \geq \bigvee_{a\in x\circ y}\mu(a)\wedge\mu(y) = \alpha.$$

Thus $\mu(u) \geq \alpha$, i.e., $u \in \mu_\alpha$. Since $x \circ y < \mu_\alpha$ and $y \in \mu_\alpha$, $x \in \mu_\alpha$. So we get

$$\mu(x) \geq \alpha = \bigvee_{a\in x\circ y}\mu(a)\wedge\mu(y) \geq \bigvee_{a\in x\circ y}\mu(a).\mu(y).$$

Hence μ is a fuzzy dot hyper K-ideal of H. □

Proposition 4.8. *Let μ be a fuzzy dot hyper K-ideal of H. Then $x \circ y < z$ implies $\mu(x) \geq \mu(z).\mu(y)$ for any $x,\ y,\ z \in H$.*

Proof. Let μ be a fuzzy dot hyper K-ideal of X and let $x,\ y,\ z \in H$. Then

$$\begin{aligned}\mu(x) &\geq \bigvee_{a\in x\circ y} \mu(a) \wedge \mu(y)\\ &\geq \mu(z) \wedge \mu(y)\\ &\geq \mu(z).\mu(y).\end{aligned}$$

Thus $\mu(x) \geq \mu(z).\mu(y)$. □

5. Fuzzy dot weak hyper K-Ideal of a hyper K-algebra

Definition 5.1. Let I be a weak hyper K-ideal of H, let μ be a fuzzy subset of I and let μ be a fuzzy weak hyper K-ideal of H if for all $x, y \in I \subseteq H$. Then μ is called a *fuzzy dot weak hyper K-ideal* of H, if it satisfies the following conditions:
(i) $\mu(0) \geq (\mu(x))^2$ for all $x \in I$,
(ii) $\mu(x) \geq \bigwedge_{z\in x\circ y} \mu(z).\mu(y)$ for all $x \circ y \subseteq I$ and $y \in I, x \in H$.

Example 5.2. Let $H = \{0, 1, 2\}$ be a set and let $\circ$ is defined by the table below:

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0, 1}	{0,1}
2	{2}	{2}	{0,1}

Then $(H, \circ, 0)$ is a hyper K-algebra. Let $I = \{0, 1\}$ be the weak hyper K-ideal of a hyper K-algebra and let μ be the fuzzy subset of I defined by

$$\mu(0) = t_0 \text{ and } \mu(1) = t_1 \text{ for } t_0,\ t_1 \in [0, 1] \text{ with } t_0 > t_1 > t_2.$$

Then we have
(i) $\mu(0) \geq (\mu(1))^2$, since $t_0 > t_1$ implies that $(t_0)^2 \geq (t_1)^2$,
(ii) $\mu(x) \geq \bigwedge_{z\in x\circ 1} \mu(z).\mu(1)$ for $x \in H$.
Thus μ is a fuzzy weak hyper K-ideal of H.

The following is an immediate consequence of Definition 5.1.

Proposition 5.3. *Let μ be a fuzzy subset of H and let μ be a fuzzy weak hyper K-ideal of H. If μ is a fuzzy dot weak hyper K-ideal of H, then $0 \in \mu_t$ with $t \in [0, 1]$ and $\mu_t \neq \varnothing$.*

Proposition 5.4. *Let μ be a fuzzy dot hyper K-ideal of H. Then μ is a fuzzy weak hyper K-ideal of H.*

Proof. Let μ be a fuzzy dot hyper K-ideal of H. Then clearly, $\mu(0) \geq (\mu(x))^2$ for all $x \in H$.

Now Let $x,\ y \in H$. Then we have

$$\begin{aligned}\mu(x) &\geq \bigvee_{a\in x\circ y} \mu(a) \wedge \mu(y)\\ &\geq \bigwedge_{a\in x\circ y} \mu(a) \wedge \mu(y)\\ &\geq \bigwedge_{a\in x\circ y} \mu(a).\mu(y).\end{aligned}$$

Thus μ is a fuzzy dot weak hyper K-ideal of H. □

Proposition 5.5. *If $\{\mu_i : j \in J\}$ is a family of fuzzy dot weak hyper K-ideal of H, then $\bigcap_{j\in J} \mu_j$ is a fuzzy dot weak hyper K-ideal of H.*

Proof. Suppose $\{\mu_j : j \in J\}$ is a fuzzy dot weak hyper K-ideal of H and let $x \in H$. Then we get

$$\begin{aligned}(\bigcap_{j\in J}\mu_j)(0) &= \bigwedge_{j\in J}\mu_j(0)\\ &= \bigwedge_{j\in J}(\mu_j(x))^2\\ &= (\bigcap_{j\in J}\mu_j^2)(x)).\end{aligned}$$

Also, we have

$$\begin{aligned}(\bigcap_{j\in J}\mu_j)(x) &= \bigwedge_{j\in J}[\bigwedge_{a\in x\circ y}\mu_j(a)\wedge\mu_j(y)]\\ &\geq \bigwedge_{j\in J}[\bigwedge_{a\in x\circ y}\mu_j(a).\mu_j(y)]\\ &= [\bigwedge_{a\in x\circ y}\bigwedge_{j\in J}\mu_j(a)].\bigwedge_{j\in J}\mu_i(y)\\ &= \bigwedge_{a\in x\circ y}(\bigcap_{j\in J}\mu_j(a)).(\bigcap_{j\in J}\mu_j)(y).\end{aligned}$$

Then $\bigcap_{j\in J}\mu_i$ is a fuzzy dot weak hyper K-ideal of H. □

6. The Cartesian product in fuzzy dot hyper K-ideal of a hyper K-algebra

Definition 6.1. Let $(H, \circ_1, 0_1)$, $(K, \circ_2, 0_2)$ be hyper K-algebras and let $H \times K$ be the Cartesian product of H and K. Define the hyper operation " $\circ$ " on $H \times K$ as follows: for any $(a, b),\ (c, d) \in H \times K$,

$$(a, b) \circ (c, d) = (a \circ_1 c, b \circ_2 d).$$

Then $(H \times K, \circ, (0_1, 0_2))$ is called the *hyper product of hyper K-algebra*.

Definition 6.2. Let μ, and λ be a fuzzy dot hyper K-ideal of H. Then the product of μ and λ , denoted by $\mu \times \lambda$, is a fuzzy subset of $H \times H$ defined as follows: for each $(x, y) \in H \times H$,

$$(\mu \times \lambda)(x, y) = \mu(x).\lambda(y).$$

Proposition 6.3. *Let μ and λ be a fuzzy dot hyper K-ideal of H. Then $\mu \times \lambda$ is a fuzzy dot hyper K-ideal of $H \times H$.*

Proof. Let μ and λ be a fuzzy dot hyper K-ideal of H and let $(x, y) \in H \times H$. Then we have

$$\begin{aligned}(\mu\times\lambda)(0,0) &= \mu(0).\lambda(0)\\ &\geq (\mu(x))^2.(\lambda(y))^2\\ &= (\mu(x).\mu(x)).(\lambda(y).\lambda(y))\\ &= (\mu(x).\lambda(y)).(\mu(x).\lambda(x))\\ &= (\mu\times\lambda)(x,y).(\mu\times\lambda)(x,y)\\ &= ((\mu\times\lambda)(x,y))^2.\end{aligned}$$

Also, we get

$$\begin{aligned}(\mu\times\lambda)(x,x) &= \mu(x).\lambda(x)\ \text{[By definition 6.2]}\\ &\geq (\bigvee_{a\in x\circ y}\mu(a)).\mu(y)).(\bigvee_{b\in x\circ y}\lambda(b).\lambda(y))\\ &= (\bigvee_{a\in x\circ y}\mu(a).\bigvee_{b\in x\circ y}\lambda(b)).(\mu(y).\lambda(y))\\ &= \bigvee_{(a,b)_{(a,b)\in(x\circ y,x\circ y)}}(\mu\times\lambda)(a,b).(\mu\times\lambda)(y,y).\end{aligned}$$

Thus $\mu \times \lambda$ is a fuzzy dot hyper K-ideals of a hyper K-algebra $H \times H$. □

Proposition 6.4. *Let μ, and λ be a fuzzy dot hyper K-ideal of H. Then if $\mu \times \lambda$ is a fuzzy dot hyper K-ideal of $H \times H$, then $\mu \times \lambda$ is a fuzzy dot weak hyper K-ideal of $H \times H$.*

Proof. Suppose $\mu \times \lambda$ be a fuzzy dot hyper K-ideal of a hyper algebra $H \times H$ and let $(x, y) \in H \times H$. Then we get

$$\begin{aligned}(\mu \times \lambda)(0,0) &= \mu(0).\lambda(0)\\ &\geq (\mu(x))^2.(\lambda(y))^2\\ &= (\mu(x).\lambda(y)).(\mu(x).\lambda(y))\\ &= (\mu \times \lambda)(x,y).(\mu \times \lambda)(x,y)\\ &= (\mu \times \lambda)^2(x,y).\end{aligned}$$

Also, we have

$$\begin{aligned}(\mu \times \lambda)(x,x) &= \mu(x).\lambda(x)\\ &\geq (\textstyle\bigvee_{a\in x\circ y} \mu(a).\mu(y)).(\bigvee_{b\in x\circ y} \lambda(b).\lambda(y))\\ &= \textstyle\bigvee_{(a,b)\in(x\circ y, x\circ y)} (\mu(a).\lambda(b)).(\mu(y).\lambda(y))\\ &\geq \textstyle\bigwedge_{(a,b)\in(x\circ y, x\circ y)} (\mu(a).\lambda(b)).(\mu(y).\lambda(y))\\ &= \textstyle\bigwedge_{(a,b)\in(x\circ y, x\circ y)} (\mu \times \lambda)(a,b).(\mu \times \lambda)(y,y).\end{aligned}$$

Thus $\mu \times \lambda$ is a fuzzy dot weak hyper K-ideal of $H \times H$. □

Proposition 6.5. *Let μ, and λ be a fuzzy dot weak hyper K-ideal of H. Then $\mu \times \lambda$ is a fuzzy dot weak hyper K-ideal of $H \times H$.*

Proof. Let μ and λ be a fuzzy dot weak hyper K-ideals of H and let $?(x, y) \in H \times H$. Then we have

$$\begin{aligned}(\mu \times \lambda)(0,0) &= \mu(0).\lambda(0)\\ &\geq (\mu(x))^2.(\lambda(x))^2\\ &= (\mu \times \lambda)(x,x).\end{aligned}$$

Also, we get

$$\begin{aligned}(\mu \times \lambda)(x,x) &= \mu(x).\lambda(x)\\ &\geq \textstyle\bigwedge_{a\in x\circ y} \mu(a).\mu(y). \wedge_{b\in x\circ y} \lambda(b).\lambda(y)\\ &= \textstyle\bigwedge_{(a,b)\in(x\circ y, x\circ y)} (\mu \times \lambda)(a,b).(\mu \times \lambda)(y,y).\end{aligned}$$

Thus the result holds.

□

7. Conclusions

In this paper, the idea of fuzzy dot hyper K-subalgebras, fuzzy dot hyper K-ideals, fuzzy dot weak hyper K-ideals. Family of fuzzy dot hyper K-subalgebras, characterization theorems of upper level subset sand chain of family of fuzzy dot hyper K-ideals are introduced. Furthermore, the concepts of homomorphism in fuzzy dot hyper K-algebra, the Cartesian product in fuzzy dot hyper K-algebra with different properties are investigated. As the future work the author will work the applications and hyper implication algebra.

8. Data availability and conflict of interest

Data availability

The data used to support the findings of this study are included by citation with in the study of the article. The author allow this manuscript to be available as open access for readers and researchers. No figures, photo and pictures in main manuscript and no separate tables.

CONFLICT OF INTEREST

There is no conflict of interest between authors.

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D. TEFERA GERIMA (gerima233@gmail.com)
Department of Mathematics, College of Natural science, Wollo University, Ethiopia