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On deferred Cesàro summability and statistical convergence for the sets of triple sequences

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Abstract. In this paper, we investigate the concept of deferred Cesàro mean in the Wijsman sense for triple sequences. We present the concepts of strongly deferred Cesàro summability and deferred statistical convergence in the Wijsman sense for triple sequences of sets. Also, we examine the relationships between these notions and then we prove various theorems associated with the deferred statistical convergence in the Wijsman sense for triple sequences of sets.

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1. Introduction

The concept of statistical convergence was investigated under the name almost convergence by Zygmund [1]. It was formally introduced by Fast [2]. Later the idea was associated with summability theory by Fridy [3], and many others (See [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]). The studies of triple sequences have seen rapid growth. The initial work on the statistical convergence of triple sequences was established by Şahiner et al. [15] and the other researches was continued by [16, 17, 18, 19]. Lacunary statistical convergence was given by Fridy and Orhan [20]. After that Patterson and Savaş [21] and Esi and Savaş [22] studied lacunary statistically convergent sequences.

Over the years, on the some convergence concepts for sequences of sets have been worked by several authors. One of them, which will be examined in this study, is the Wijsman sense [23, 24, 25]. The concepts of convergence and Cesàro summability in the Wijsman sense for double sequences of sets were given by Nuray et al. [26]. Some remarkable results on this topic can be reviewed in [27, 28, 29, 30, 31].

The concept of deferred Cesàro mean for real (or complex) sequences was presented by Agnew [32]. Küçükaslan and Yılmaztürk [33] put forward the deferred statistical convergence for single sequences. Deferred Cesàro summability and deferred statistical convergence for double sequences were investigated by Dağadur and Sezgek [34]. The notions of strongly deferred Cesàro summability and deferred statistical convergence for sequences of sets were examined by Altınok et al. [35]. See [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46] for more details.

2. Preliminary

In this section, we recall some preliminary definitions and results as ready references for the purpose.

For a metric space (Y,ρ), $d(y,T)$ indicate the distance from y to T where

$$d(y,T) := d_y(T) = \inf_{t\in T} \rho(y,t)$$

for any $t \in T$ and any non-empty $T \subseteq Y$.

For a non-empty set Y, let a function $f : \mathbb{N} \to 2^Y$ be determined by $f(i) = T_i \in 2^Y$ for all $i \in \mathbb{N}$. Then, the sequence $\{T_i\} = \{T_1, T_2, ...\}$, which is the codomain elements of f, is named sequences of sets.

Throughout the study, (Y,ρ) will be considered as metric space and T, T_{ijk} will be considered as any non-empty closed subsets of Y.

A triple sequence of sets $\{T_{ijk}\}$ is called to be *convergent* in Wijsman sense to the set T, provided that for each $y \in Y$,

$$\lim_{i,j,k\to\infty} d_y(T_{ijk}) = d_y(T)$$

and it is demonstrated by $T_{ijk} \overset{W_3}{\to} T$ ([29]).

A triple sequence of sets $\{T_{ijk}\}$ is called to be *Cesàro summable* in the Wijsman sense to the set T ([29]), provided that for each $y \in Y$,

$$\lim_{m,n,o\to\infty} \frac{1}{mno} \sum_{i,j,k}^{m,n,o} d_y(T_{ijk}) = d_y(T).$$

A triple sequence of sets $\{T_{ijk}\}$ is called to be statistically convergent in the Wijsman sense to the set T provided that for each $\gamma > 0$ and all $y \in Y$,

$$\lim_{m,n,o\to\infty} \frac{1}{mno} |\{(i,j,k) : i \le m, j \le n, k \le o, |d_y(T_{ijk}) - d_y(T)| \ge \gamma\}| = 0$$

The set of all statistical convergence triple sequences of sets in the Wijsman sense is indicated by $\{W_3S\}$ ([29]).

A sequence of sets $\{T_i\}$ is called to be *strongly deferred Cesàro summable* in the Wijsman sense to the set T, provided that for each $y \in Y$,

$$\lim_{m\to\infty} \frac{1}{u(m) - p(m)} \sum_{i=p(m)+1}^{u(m)} |d_y(T_i) - d_y(T)| = 0,$$

where $\{p(m)\}$ and $\{u(m)\}$ are sequences of non-negative integers satisfying

$$p(m) < u(m) \text{ and } \lim_{m\to\infty} u(m) = \infty$$

and it is showed by $T_i \overset{[WD]}{\to} T$ ([37]).

A sequence of sets $\{T_i\}$ is called to be *deferred statistically convergent* in the Wijsman sense to the set T, provided that for each $\gamma > 0$ and all $y \in Y$,

$$\lim_{m\to\infty} \frac{1}{u(m) - p(m)} \left|\{p(m) < i \le u(m) : |d_y(T_i) - d_y(T)| \ge \gamma\}\right| = 0$$

and it is demonstrated by $T_i \overset{WDS}{\to} T$ ([37]).

A triple sequence $\theta_{m,n,o} = \{(k_m, l_n, s_o)\}$ is called to be a *triple lacunary sequence* ([29]), if there are three increasing sequences (k_m), (l_n) and (s_o) of integers so that

$$\begin{array}{c} k_0 = 0,\ h_m = k_m - k_{m-1} \to \infty \text{ as } m \to \infty \\ l_0 = 0,\ h_n = l_n - l_{n-1} \to \infty \text{ as } n \to \infty \\ s_o = 0,\ h_o = s_o - s_{o-1} \to \infty \text{ as } o \to \infty \end{array}$$

3. Main results

In this section, we present the notion of deferred Cesàro mean in the Wijsman sense for triple sequences of sets and then introduce the concepts of strongly deferred Cesàro summability and deferred statistical convergence in the Wijsman sense for triple sequences of sets.

Definition 3.1. The *deferred Cesàro mean* $D_{\psi,\varkappa,\phi}$ in the Wijsman sense of a triple sequence of sets $\mathcal{T} = \{T_{ijk}\}$ is determined by

$$\begin{aligned} (D_{\psi,\varkappa,\phi}\mathcal{T})_{mno} &= \frac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{i=p(m)+1}^{u(m)} \sum_{j=q(n)+1}^{v(n)} \sum_{k=r(o)+1}^{w(o)} d_y(T_{ijk}) \\ &:= \frac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1 \\ j=q(n)+1 \\ k=r(o)+1}}^{u(m),v(n),w(o)} d_y(T_{ijk}) \end{aligned}$$

for each $y \in Y$, where $\{p(m)\}$, $\{u(m)\}$, $\{q(n)\}$, $\{v(n)\}$, $\{r(o)\}$ and $\{w(o)\}$ are sequences of non-negative integers satisfying subsequent conditions:

$$\begin{array}{l} p(m) < u(m),\ \lim_{m\to\infty} u(m) = \infty, \\ q(n) < v(n),\ \lim_{n\to\infty} v(n) = \infty, \\ r(o) < w(o),\ \lim_{o\to\infty} w(o) = \infty \end{array} \tag{3.1}$$

and

$$u(m) - p(m) = \psi(m),\ v(n) - q(n) = \varkappa(n),\ w(o) - r(o) = \phi(o). \tag{3.2}$$

Throughout the paper, unless otherwise specified $\{p(m)\}$, $\{u(m)\}$, $\{q(n)\}$, $\{v(n)\}$, $\{r(o)\}$ and $\{w(o)\}$ are contemplated as sequences of non-negative integers satisfying (3.1) and (3.2).

Definition 3.2. The triple sequence of sets $\{T_{ijk}\}$ is called to be *Wijsman deferred Cesàro summable* to a set T, provided that for every $y \in Y$,

$$\lim_{m,n,o\to\infty} \frac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1 \\ j=q(n)+1 \\ k=r(o)+1}}^{u(m),v(n),w(o)} d_y(T_{ijk}) = d_y(T).$$

In this case, we write $T_{ijk} \overset{W_3 D}{\to} T$.

Definition 3.3. The triple sequence of sets $\{T_{ijk}\}$ is called to be *Wijsman strongly deferred Cesàro summable* to a set T, provided that for every $y \in Y$,

$$\lim_{m,n,o\to\infty} \frac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1}}^{u(m),v(n),w(o)} |d_y(T_{ijk}) - d_y(T)| = 0.$$

In this case, we write $T_{ijk} \overset{[W_3 D]}{\to} T$.
The set of all strongly deferred Cesàro summable triple sequences of sets in the Wijsman sense is indicated by $\{[W_3 D]\}$.

Remark 3.4. (1) For $p(m) = 0$, $u(m) = m$, $q(n) = 0$, $v(n) = n$, $r(o) = 0$ and $w(o) = o$, the concept of Wijsman strongly deferred Cesàro summability coincides with the concept of Wijsman strongly Cesàro summability for triple sequences of sets.

(2) For $p(m) = k_{m-1}$, $u(m) = k_m$, $q(n) = l_{n-1}$, $v(n) = l_n$, $r(o) = s_{o-1}$ and $w(o) = s_o$ where $\{(k_m, l_n, s_o)\}$ is a triple lacunary sequence, the concept of Wijsman strongly deferred Cesàro summability coincides with the concept of Wijsman strongly lacunary summability for triple sequences of sets.

Definition 3.5. The triple sequence of sets $\{T_{ijk}\}$ is called to be *Wijsman deferred statistical convergent* to the set T, provided that for each $\gamma > 0$ and every $y \in Y$,

$$\lim_{m,n,o\to\infty} \tfrac{1}{\psi(m)\varkappa(n)\phi(o)} |\{(i,j,k) : p(m) < i \leq u(m), q(n) < j \leq v(n) \\ r(o) < k \leq w(o), |d_y(T_{ijk}) - d_y(T)| \geq \gamma\}| = 0.$$

In this case, we write $T_{ijk} \overset{W_3 DS}{\to} T$.
The set of all Wijsman deferred statistical convergent triple sequences of sets is indicated by $\{[W_3 SD]\}$.

Remark 3.6. (1) For $p(m) = 0$, $u(m) = m$, $q(n) = 0$, $v(n) = n$, $r(o) = 0$ and $w(o) = o$, the concept of Wijsman deferred statistical convergence coincides with the concept of Wijsman statistical convergence for triple sequences of sets.

(2) For $p(m) = k_{m-1}$, $u(m) = k_m$, $q(n) = l_{n-1}$, $v(n) = l_n$, $r(o) = s_{o-1}$ and $w(o) = s_o$ where $\{(k_m, l_n, s_o)\}$ is a triple lacunary sequence, the concept of Wijsman deferred statistical convergence coincides with the concept of Wijsman lacunary statistical convergence for triple sequences of sets.

Now, we examine the relationships between the notions given in this section and then we prove various theorems associated with the Wijsman deferred statistical convergence for triple sequences of sets.

Theorem 3.7. *If a triple sequence of sets $\{T_{ijk}\}$ is Wijsman strongly deferred Cesàro summable to a set T, then this sequence is Wijsman deferred statistical convergent to the set T.*

Proof. Let $T_{ijk} \overset{[W_3 D]}{\rightarrow} T$. Then for each $\gamma > 0$ and all $y \in Y$, we get

$$\begin{aligned}\sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1}}^{u(m),v(n),w(o)} \left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right| &\geq \sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1\\ \left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right|\geq\gamma}}^{u(m),v(n),w(o)} \left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right| \\ &\geq \gamma\left|\left\{(i,j,k): p\left(m\right)<i\leq u\left(m\right), q\left(n\right)<j\leq v\left(n\right)\right.\right. \\ &\qquad \left.\left. r\left(o\right)<k\leq w\left(o\right), \left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right|\geq\gamma\right\}\right|\end{aligned}$$

and thus

$$\begin{aligned}&\frac{1}{\gamma}\frac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1}}^{u(m),v(n),w(o)} \left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right| \\ &\quad\geq \frac{1}{\psi(m)\varkappa(n)\phi(o)}\left|\left\{(i,j,k): p\left(m\right)<i\leq u\left(m\right), q\left(n\right)<j\leq v\left(n\right)\right.\right. \\ &\qquad\qquad \left.\left. r\left(o\right)<k\leq w\left(o\right), \left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right|\geq\gamma\right\}\right|.\end{aligned}$$

For $m,n,o \to \infty$, according to our supposition, the statement on the left side of the above inequality convergent to 0. As a result, we acquire $T_{ijk} \overset{W_3 DS}{\rightarrow} T$. □

Corollary 3.8. *The converse of the Theorem 3.7 is not true in general. We can contemplate the subsequent example to demonstrate this situation.*

Example 3.9. Let $X = \mathbb{R}^3$ and a triple sequence of sets $\{T_{ijk}\}$ be determined as following:

$$T_{ijk} = \begin{cases} \{ijk\}; & \begin{array}{l} p\left(m\right)<i\leq p\left(m\right)+\left[\left|\sqrt{\psi\left(m\right)}\right|\right] \\ q\left(n\right)<j\leq q\left(n\right)+\left[\left|\sqrt{\varkappa\left(n\right)}\right|\right] \\ r\left(o\right)<k\leq r\left(o\right)+\left[\left|\sqrt{\phi\left(o\right)}\right|\right], \\ (i,j,k)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} \end{array} \\ \{0\}; & \text{otherwise.} \end{cases}$$

This sequence is not bounded. Also, this sequences is Wijsman deferred statistical convergent to the set $T = \{0\}$, but is not Wijsman strongly deferred Cesàro summable.

A triple sequence of sets $\{T_{ijk}\}$ is called to be bounded when $\sup d_y\left(T_{ijk}\right) < \infty$ for all $y \in Y$. At the same time, L^3_∞ indicates the set of all bounded triple sequences of sets.

Theorem 3.10. *If a triple sequence of sets $\{T_{ijk}\}$ is bounded and Wijsman deferred statistical convergent to a set T, then this sequence is Wijsman strongly deferred Cesàro summable to the set T.*

Proof. Let $\{T_{ijk}\}$ is bounded and $T_{ijk} \overset{W_3 DS}{\rightarrow} T$. Since $\{T_{ijk}\} \in L^3_\infty$, there exists a $H > 0$ such that

$$\left|d_y\left(T_{ijk}\right)-d_y\left(T\right)\right| \leq H$$

for all i, j, k and $y \in Y$. Then for every $\gamma > 0$ and each $y \in Y$, we get

$$\begin{aligned}
&\tfrac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1}}^{u(m),v(n),w(o)} \left| d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \\
&= \tfrac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1\\ |d_y(T_{ijk})-d_y(T)|\geq\gamma}}^{u(m),v(n),w(o)} \left| d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \\
&+ \tfrac{1}{\psi(m)\varkappa(n)\phi(o)} \sum_{\substack{i=p(m)+1\\ j=q(n)+1\\ k=r(o)+1\\ |d_y(T_{ijk})-d_y(T)|<\gamma}}^{u(m),v(n),w(o)} \left| d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \\
&\leq \tfrac{H}{\psi(m)\varkappa(n)\phi(o)} \left|\{(i,j,k) : p\left(m\right) < i \leq u\left(m\right)\right. \\
&\quad \left. q\left(n\right) < j \leq v\left(n\right), r\left(o\right) < k \leq w\left(o\right), \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \geq \gamma\}\right| + \gamma.
\end{aligned}$$

For m, n, $o \to \infty$, according to our supposition, the statement on the right side of the above inequality convergent to 0. As a result, we acquire $T_{ijk} \overset{[W_3 D]}{\to} T$. □

Corollary 3.11. *Assume that L^3_∞ be a set of bounded triple sequences of sets. The family of Wijsman strongly deferred Cesàro summable triple sequences is the same as the collection of Wijsman deferred statistical convergent triple sequences. Then, we have*

$$\{[W_3 D]\} \cap L^3_\infty = \{W_3 DS\} \cap L^3_\infty.$$

Theorem 3.12. *Assume that $\{T_{ijk}\}$, $\{U_{ijk}\}$, $\{V_{ijk}\}$ be three triple sequences of sets such that $T_{ijk} \subset U_{ijk} \subset V_{ijk}$ for all $(i,j,k) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}$. Then*

$$T_{ijk} \overset{W_3 DS}{\to} U \text{ and } V_{ijk} \overset{W_3 DS}{\to} U \Rightarrow U_{ijk} \overset{W_3 DS}{\to} U.$$

Proof. Presume that $T_{ijk} \overset{W_3 DS}{\to} U$, $V_{ijk} \overset{W_3 DS}{\to} U$ and $T_{ijk} \subset U_{ijk} \subset V_{ijk}$ for all $(i,j,k) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}$ and all $y \in Y$,

$$T_{ijk} \subset U_{ijk} \subset V_{ijk} \Rightarrow d_y\left(V_{ijk}\right) \leq d_y\left(U_{ijk}\right) \leq d_y\left(T_{ijk}\right)$$

supplies. Then for each $\gamma > 0$, we obtain

$$\begin{aligned}
&\{(i,j,k) : p\left(m\right) < i \leq u\left(m\right), q\left(n\right) < j \leq v\left(n\right) \\
&\quad r\left(o\right) < k \leq w\left(o\right), \left|d_y\left(U_{ijk}\right) - d_y\left(U\right)\right| \geq \gamma\}|\} \\
&\quad = \{(i,j,k) : p\left(m\right) < i \leq u\left(m\right), q\left(n\right) < j \leq v\left(n\right) \\
&\quad r\left(o\right) < k \leq w\left(o\right), d_y\left(U_{ijk}\right) \geq d_y\left(U\right) + \gamma\}|\} \\
&\quad \cup \{(i,j,k) : p\left(m\right) < i \leq u\left(m\right), q\left(n\right) < j \leq v\left(n\right) \\
&\quad r\left(o\right) < k \leq w\left(o\right), d_y\left(U_{ijk}\right) \leq d_y\left(U\right) - \gamma\}|\} \\
&\quad \subset \{(i,j,k) : p\left(m\right) < i \leq u\left(m\right), q\left(n\right) < j \leq v\left(n\right) \\
&\quad r\left(o\right) < k \leq w\left(o\right), d_y\left(T_{ijk}\right) \geq d_y\left(U\right) + \gamma\}|\} \\
&\quad \cup \{(i,j,k) : p\left(m\right) < i \leq u\left(m\right), q\left(n\right) < j \leq v\left(n\right) \\
&\quad r\left(o\right) < k \leq w\left(o\right), d_y\left(V_{ijk}\right) < d_y\left(U\right) - \gamma\}|\}
\end{aligned}$$

and thus

$$\begin{aligned}&\tfrac{1}{\psi(m)\varkappa(n)\phi(o)}\left|\{(i,j,k):p(m)<i\leq u(m),q(n)<j\leq v(n)\right.\\&\qquad\left. r(o)<k\leq w(o),|d_y(U_{ijk})-d_y(U)|\geq\gamma\}|\}\right|\\&<\tfrac{1}{\psi(m)\varkappa(n)\phi(o)}\left|\{(i,j,k):p(m)<i\leq u(m),q(n)<j\leq v(n)\right.\\&\qquad\left. r(o)<k\leq w(o),|d_y(T_{ijk})-d_y(U)|\geq\gamma\}|\}\right|\\&+\tfrac{1}{\psi(m)\varkappa(n)\phi(o)}\left|\{(i,j,k):p(m)<i\leq u(m),q(n)<j\leq v(n)\right.\\&\qquad\left. r(o)<k\leq w(o),|d_y(V_{ijk})-d_y(U)|\geq\gamma\}|\}\right|.\end{aligned}$$

For $m,n,o\to\infty$, by our supposition, the statement on the right side of the above inequality convergent to 0. As a result, we acquire $U_{ijk}\overset{W_3DS}{\to}U$. □

Theorem 3.13. *Assume that* $\left\{\frac{p(m)}{\psi(m)}\right\}$, $\left\{\frac{q(n)}{\varkappa(n)}\right\}$, $\left\{\frac{r(o)}{\phi(o)}\right\}$ *be bounded. Then*

$$T_{ijk}\overset{W_3S}{\to}T\Rightarrow T_{ijk}\overset{W_3DS}{\to}T.$$

Proof. Assume that $T_{ijk}\overset{W_3S}{\to}T$. Then for each $\gamma>0$ and all $y\in Y$, we acquire

$$\begin{aligned}&\lim_{m,n,o\to\infty}\tfrac{1}{mno}\left|\{(i,j,k):i\leq m,j<n,\right.\\&\qquad\qquad\left. k<o,|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|=0.\end{aligned}$$

Thus it is well-known fact that

$$\begin{aligned}&\lim_{m,n,o\to\infty}\tfrac{1}{u(m)v(n)w(o)}\left|\{(i,j,k):i\leq u(m),j<v(n),\right.\\&\qquad\qquad\left. k<w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|=0.\end{aligned}$$

Also, since

$$\begin{aligned}&\{(i,j,k):p(m)<i\leq u(m),q(n)<j\leq v(n),\\&\qquad r(o)<k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}|\}\\&\subseteq\{(i,j,k):i\leq u(m),j\leq v(n)\\&\qquad k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}|\},\end{aligned}$$

we obtain

$$\begin{aligned}&\tfrac{1}{\psi(m)\varkappa(n)\phi(o)}\left|\{(i,j,k):p(m)<i\leq u(m),q(n)<j\leq v(n),\right.\\&\qquad\left. r(o)<k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\}\\&\leq\tfrac{u(m)v(n)w(o)}{\psi(m)\varkappa(n)\phi(o)}\tfrac{1}{u(m)v(n)w(o)}\left|\{(i,j,k):i\leq u(m),j\leq v(n),\right.\\&\qquad\left. k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\}\\&=\left(1+\tfrac{2p(m)}{\psi(m)}\right)\left(1+\tfrac{2q(n)}{\varkappa(n)}\right)\left(1+\tfrac{2r(o)}{\phi(o)}\right)\tfrac{1}{u(m)v(n)w(o)}.\\&\qquad\left|\{(i,j,k):i\leq u(m),j\leq v(n),\right.\\&\qquad\left. k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\}.\end{aligned}$$

When $\left\{\frac{p(m)}{\psi(m)}\right\}$, $\left\{\frac{q(n)}{\varkappa(n)}\right\}$, $\left\{\frac{r(o)}{\phi(o)}\right\}$ are bounded in above inequality, by the supposition, we have $T_{ijk}\overset{W_3DS}{\to}T$ for $m,n,o\to\infty$. □

Theorem 3.14. *Let* $u(m)=m$, $v(n)=n$ *and* $w(o)=o$ *for all* $m,n,o\in\mathbb{N}$. *Then*

$$T_{ijk}\overset{W_3DS}{\to}T\Rightarrow T_{ijk}\overset{W_3S}{\to}T.$$

Proof. Let $u(m) = m$, $v(n) = n$ and $w(o) = o$ for all $m, n, o \in \mathbb{N}$. Also, we assume that $T_{ijk} \overset{W_3DS}{\to} T$. Utilizing the technique given by Agnew [32], we can consider the subsequent sequences:

$$\begin{aligned}
&p(m) = m^{(1)} > p\left(m^{(1)}\right) = m^{(2)} > p\left(m^{(2)}\right) = m^{(3)} > ...\\
&q(n) = n^{(1)} > q\left(n^{(1)}\right) = n^{(2)} > q\left(n^{(2)}\right) = n^{(3)} > ...\\
&r(o) = o^{(1)} > r\left(o^{(1)}\right) = o^{(2)} > r\left(o^{(2)}\right) = o^{(3)} > ...
\end{aligned}$$

for all $m, n, o \in \mathbb{N}$. Then for each $\gamma > 0$ and all $y \in Y$, we can write

$$\begin{aligned}
&\{(i,j,k) : i \leq m, j \leq n, k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\}\\
&\quad = \left\{(i,j,k) : i \leq m^{(1)}, j \leq n^{(1)}, k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(1)} < i \leq m, j \leq n^{(1)}, k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(1)}, n^{(1)} < j \leq n, o^{(1)} < k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(1)}, j \leq n^{(1)}, o^{(1)} < k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(1)}, n^{(1)} < j \leq n, k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(1)} < i \leq m, j \leq n^{(1)}, o^{(1)} < k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(1)} < i \leq m, n^{(1)} < j \leq n, k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(1)} < i \leq m, n^{(1)} < j \leq n, o^{(1)} < k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}.
\end{aligned}$$

Here, we can rewrite some of the above sets, respectively, as follows:

$$\begin{aligned}
&\left\{(i,j,k) : i \leq m^{(1)}, j \leq n^{(1)}, k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad = \left\{(i,j,k) : i \leq m^{(2)}, j \leq n^{(2)}, k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(2)} < i \leq m^{(1)}, j \leq n^{(2)}, k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(2)}, n^{(2)} < j \leq n, o^{(2)} < k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(2)}, j \leq n^{(2)}, o^{(2)} < k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(2)}, n^{(2)} < j \leq n^{(1)}, k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(2)} < i \leq m^{(1)}, j \leq n^{(2)}, o^{(2)} < k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(2)} < i \leq m^{(1)}, n^{(2)} < j \leq n^{(1)}, k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(2)} < i \leq m^{(1)}, n^{(2)} < j \leq n^{(1)}, o^{(2)} < k \leq o^{(1)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}
\end{aligned}$$

and

$$\begin{aligned}
&\left\{(i,j,k) : i \leq m^{(1)}, n^{(1)} < j \leq n, o^{(1)} < k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&= \left\{(i,j,k) : i \leq m^{(2)}, n^{(1)} < j \leq n, o^{(1)} < k \leq o, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\cup \left\{(i,j,k) : m^{(1)} < i \leq m^{(2)}, n^{(1)} < j \leq n^{(2)}, o^{(1)} < k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}.
\end{aligned}$$

Similarly, we can write

$$\begin{aligned}
&\left\{(i,j,k) : i \leq m^{(2)}, j \leq n^{(2)}, k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad = \left\{(i,j,k) : i \leq m^{(3)}, j \leq n^{(3)}, k \leq o^{(3)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(3)} < i \leq m^{(2)}, j \leq n^{(3)}, k \leq o^{(3)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(3)}, n^{(3)} < j \leq n^{(2)}, o^{(3)} < k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(3)}, j \leq n^{(3)}, o^{(3)} < k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : i \leq m^{(3)}, n^{(3)} < j \leq n^{(2)}, k \leq o^{(3)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(3)} < i \leq m^{(2)}, j \leq n^{(3)}, o^{(3)} < k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(3)} < i \leq m^{(2)}, n^{(3)} < j \leq n^{(2)}, k \leq o^{(3)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}\\
&\quad \cup \left\{(i,j,k) : m^{(3)} < i \leq m^{(2)}, n^{(3)} < j \leq n^{(2)}, o^{(3)} < k \leq o^{(2)}, |d_y(T_{ijk}) - d_y(T)| \geq \gamma\right\}.
\end{aligned}$$

When this process is continued similarly, we get

$$\begin{aligned}
&\left\{(i,j,k): i \le m^{(t_1-1)}, j \le n^{(t_2-1)}, k \le o^{(t_3-1)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&= \left\{(i,j,k): i \le m^{(t_1)}, j \le n^{(t_2)}, k \le o^{(t_3)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): m^{(t_1)} < i \le m^{(t_1-1)}, j \le n^{(t_2)}, k \le o^{(t_3)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): i \le m^{(t_1)}, n^{(t_2)} < j \le n^{(t_2-1)}, o^{(t_3)} < k \le o^{(t_3-1)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): i \le m^{(t_1)}, j \le n^{(t_2-1)}, o^{(t_3)} < k \le o^{(t_3-1)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): i \le m^{(t_1)}, n^{(t_2)} < j \le n^{(t_2-1)}, k \le o^{(t_3)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): m^{(t_1)} < i \le m^{(t_1-1)}, j \le n^{(t_2-1)}, o^{(t_3)} < k \le o^{(t_3-1)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): m^{(t_1)} < i \le m^{(t_1-1)}, n^{(t_2-1)} < j \le n^{(t_2)}, k \le o^{(t_3)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\} \\
&\cup \left\{(i,j,k): m^{(t_1)} < i \le m^{(t_1-1)}, n^{(t_2)} < j \le n^{(t_2-1)}, o^{(t_3)} < k \le o^{(t_3-1)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\}.
\end{aligned}$$

where t_1, t_2, t_3 are fixed positive integers so that $m^{(t_1)} \ge 1$, $m^{(t_1+1)} = 0$, $n^{(t_2)} \ge 1$, $n^{(t_2+1)} = 0$ and $o^{(t_3)} \ge 1$, $o^{(t_3+1)} = 0$. From this all process, for whole $m, n, o \in \mathbb{N}$ and all $y \in Y$, we get

$$\begin{aligned}
&\tfrac{1}{mno}\left|\left\{(i,j,k): i \le m, j < n, k < o, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\}\right| \\
&\quad = \sum_{(k,l,s)=(0,0,0)}^{(t_1+1,t_2+1,t_3+1)} \frac{\left(m^{(k)}-m^{(k+1)}\right)\left(n^{(l)}-n^{(l+1)}\right)\left(o^{(s)}-o^{(s+1)}\right)}{mno} R_{mno}
\end{aligned}$$

where

$$\begin{aligned}
R_{mno} := {}& \frac{1}{\left(m^{(k)}-m^{(k+1)}\right)\left(n^{(l)}-n^{(l+1)}\right)\left(o^{(s)}-o^{(s+1)}\right)} \cdot \\
& \left|\left\{(i,j,k): m^{(k+1)} < i \le m^{(k)}, n^{(l+1)} < j \le n^{(l)}, \right.\right. \\
& \left.\left. o^{(s+1)} < k \le o^{(s)}, \left|d_y\left(T_{ijk}\right) - d_y\left(T\right)\right| \ge \gamma\right\}\right|.
\end{aligned}$$

Contemplating the subsequent matrix $\mathcal{J}_{mno}^{kls}$,

$$\mathcal{J}_{mno}^{kls} := \begin{cases} \frac{\left(m^{(k)}-m^{(k+1)}\right)\left(n^{(l)}-n^{(l+1)}\right)\left(o^{(s)}-o^{(s+1)}\right)}{mno}, & \begin{array}{r} k = 0,1,2,...,t_1 \\ l = 0,1,2,...,t_2 \\ s = 0,1,2,...,t_3 \end{array} \\ 0 & \text{otherwise,} \end{cases}$$

where $m^{(0)} = m$, $n^{(0)} = n$ and $o^{(0)} = o$, it is obvious that the Wijsman statistical convergence of the triple sequence of sets $\{T_{ijk}\}$ is equivalent to the convergence of transform under the matrix $\mathcal{J}_{mno}^{kls}$ of the sequence R_{mno}. Since the matrix $\mathcal{J}_{mno}^{kls}$ is regular, by the supposition, we obtain $T_{ijk} \overset{W_3S}{\to} T$ for $m, n, o \to \infty$. □

Corollary 3.15. *Assume that* $\left\{\frac{p(m)}{m-p(m)}\right\}$, $\left\{\frac{q(n)}{n-q(n)}\right\}$, $\left\{\frac{r(o)}{o-r(o)}\right\}$ *are bounded. Then,*

$$T_{ijk} \overset{W_3S}{\to} T \Leftrightarrow T_{ijk} \overset{W_3DS}{\to} T.$$

The subsequent theorems will be contemplated under the restrictions:

$$\begin{gathered}
p\left(m\right) < p'\left(m\right) < u'\left(m\right) < u\left(m\right), \\
q\left(n\right) < q'\left(n\right) < v'\left(n\right) < v\left(n\right) \text{ and} \\
r\left(o\right) < r'\left(o\right) < w'\left(o\right) < w\left(o\right)
\end{gathered}$$

for all $m, n, o \in \mathbb{N}$ where all of these are sequences of non-negative integers.

Theorem 3.16. *If* $\left\{\frac{\psi(m)\varkappa(n)\phi(o)}{\psi'(m)\varkappa'(n)\phi'(o)}\right\}$ *is bounded, then*

$$\{W_3DS\}_{[\psi,\varkappa,\phi]} \subset \{W_3DS\}_{[\psi',\varkappa',\phi']}.$$

Proof. Take $T_{ijk} \to T(\{W_3DS\}_{[\psi,\varkappa,\phi]})$. For each $\gamma > 0$ and all $y \in Y$, since

$$\begin{aligned} &\{(i,j,k) : p'(m) < i \le u'(m), q'(n) < j \le v'(n) \\ &\quad r'(o) < k \le w'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\subset \{(i,j,k) : p(m) < i \le u(m), q(n) < j \le v(n) \\ &\quad r(o) < k \le w(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\}, \end{aligned}$$

we get

$$\begin{aligned} &\frac{1}{\psi'(m)\varkappa'(n)\phi'(o)} |\{(i,j,k) : p'(m) < i \le u'(m), q'(n) < j \le v'(n) \\ &\quad r'(o) < k \le w'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\}| \\ &\le \frac{\psi(m)\varkappa(n)\phi(o)}{\psi'(m)\varkappa'(n)\phi'(o)} \left(\frac{1}{\psi(m)\varkappa(n)\phi(o)} |\{(i,j,k) : p(m) < i \le u(m), \right. \\ &\quad \left. q(n) < j \le v(n), r(o) < k \le w(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\}| \right). \end{aligned}$$

When $\left\{ \frac{\psi(m)\varkappa(n)\phi(o)}{\psi'(m)\varkappa'(n)\phi'(o)} \right\}$ is bounded in above inequality, by the supposition, we acquire $T_{ijk} \to T(\{W_3DS\}_{[\psi',\varkappa',\phi']})$ for $m, n, o \to \infty$. □

Theorem 3.17. *If the sets* $\{i : p(m) < i \le p'(m)\}$, $\{i : u'(m) < i \le u(m)\}$, $\{j : q(n) < i \le q'(n)\}$, $\{j : v'(n) < i \le v(n)\}$, $\{k : r(o) < k \le r'(o)\}$ *and* $\{k : w'(o) < k \le w(o)\}$ *are finite for all* $m, n, o \in \mathbb{N}$, *then*

$$\{W_3DS\}_{[\psi',\varkappa',\phi']} \subset \{W_3DS\}_{[\psi,\varkappa,\phi]}.$$

Proof. Let $T_{ijk} \to T(\{W_3DS\}_{[\psi',\varkappa',\phi']})$. For each $\gamma > 0$ and $y \in Y$, since

$$\begin{aligned} &\{(i,j,k) : p(m) < i \le u(m), q(n) < j \le v(n) \\ &\quad r(o) < k \le w(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &= \{(i,j,k) : p(m) < i \le p'(m), q(n) < j \le q'(n) \\ &\quad r(o) < k \le r'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : p(m) < i \le p'(m), q'(n) < j \le v'(n) \\ &\quad r'(o) < k \le w'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : p(m) < i \le p'(m), v'(n) < j \le v(n) \\ &\quad w'(o) < k \le w(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : p'(m) < i \le u'(m), q(n) < j \le q'(n) \\ &\quad r(o) < k \le r'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : p'(m) < i \le u'(m), q'(n) < j \le v'(n) \\ &\quad r'(o) < k \le w'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : p'(m) < i \le u'(m), v'(n) < j \le v(n) \\ &\quad w'(o) < k \le w(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : u'(m) < i \le u(m), q(n) < j \le q'(n) \\ &\quad r(o) < k \le r'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : u'(m) < i \le u(m), q'(n) < j \le v'(n) \\ &\quad r'(o) < k \le w'(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\} \\ &\cup \{(i,j,k) : u'(m) < i \le u(m), v'(n) < j \le v(n) \\ &\quad w'(o) < k \le w(o), |d_y(T_{ijk}) - d_y(T)| \ge \gamma\}, \end{aligned}$$

we obtain

$$\begin{aligned}
&\tfrac{1}{\psi(m)\varkappa(n)\phi(o)}\left|\{(i,j,k):p(m)<i\leq u(m),q(n)<j\leq v(n)\right.\\
&\quad\left. r(o)<k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad\leq\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):p(m)<i\leq p'(m),q(n)<j\leq q'(n)\right.\\
&\quad\left. r(o)<k\leq r'(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):p(m)<i\leq p'(m),q'(n)<j\leq v'(n)\right.\\
&\quad\left. r'(o)<k\leq w'(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):p(m)<i\leq p'(m),v'(n)<j\leq v(n)\right.\\
&\quad\left. w'(o)<k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):p'(m)<i\leq u'(m),q(n)<j\leq q'(n)\right.\\
&\quad\left. r(o)<k\leq r'(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):p'(m)<i\leq u'(m),q'(n)<j\leq v'(n)\right.\\
&\quad\left. r'(o)<k\leq w'(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):p'(m)<i\leq u'(m),v'(n)<j\leq v(n)\right.\\
&\quad\left. w'(o)<k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):u'(m)<i\leq u(m),q(n)<j\leq q'(n)\right.\\
&\quad\left. r(o)<k\leq r'(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):u'(m)<i\leq u(m),q'(n)<j\leq v'(n)\right.\\
&\quad\left. r'(o)<k\leq w'(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|\\
&\quad+\tfrac{1}{\psi'(m)\varkappa'(n)\phi'(o)}\left|\{(i,j,k):u'(m)<i\leq u(m),v'(n)<j\leq v(n)\right.\\
&\quad\left. w'(o)<k\leq w(o),|d_y(T_{ijk})-d_y(T)|\geq\gamma\}\right|.
\end{aligned}$$

If the sets $\{i:p(m)<i\leq p'(m)\}$, $\{i:u'(m)<i\leq u(m)\}$, $\{j:q(n)<i\leq q'(n)\}$, $\{j:v'(n)<i\leq v(n)\}$, $\{k:r(o)<k\leq r'(o)\}$ and $\{k:w'(o)<k\leq w(o)\}$ are finite for all $m,n,o\in\mathbb{N}$ in the above inequality, by the assumption, we obtain $T_{ijk}\to T(\{W_3DS\}_{[\psi,\varkappa,\phi]})$ for $m,n,o\to\infty$. □

4. Conclusion

In this study, we investigated deferred Cesàro mean in the Wijsman sense for triple sequences of sets and then put forward to the concepts of Wijsman strongly deferred Cesàro summability and Wijsman deferred statistical convergence for triple sequences of sets. In addition, investigated the relationships between these concepts and then to prove some theorems associated with the concepts of Wijsman deferred statistical convergence for triple sequences of sets was purposed.

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