Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Operations on temporal intuitionistic fuzzy sets

R. Parvathi, C. Yuvapriya

Received 3 December 2021; Revised 5 January 2022; Accepted 12 June 2022

Abstract. In this paper, theory of temporal intuitionistic fuzzy sets (TIFSs) is developed. Geometric interpretation of TIFSs is given. A few basic operations and normalization on TIFSs are defined and a few of their properties are analysed. Also, the five types of cartesian products of TIFSs are defined and the properties of TIFS based on these definitions are discussed.

2020 AMS Classification: 03E72

Keywords: Temporal intuitionistic fuzzy sets, Operations an temporal intuitionistic fuzzy sets

Corresponding Author: C. Yuvapriya (yuvapriya.c@gmail.com)

1. Introduction

Fuzzy sets (FSs) introduced by Zadeh in 1965 [1] are generalization of crisp sets. Atanassov introduced the concept of intuitionistic fuzzy sets (IFSs) in 1983 [2] as an extension of FSs. These sets include not only the membership of the element in the set but also the non-membership of the element along with degree of hesitancy. Atanassov also extended the concept of IFSs into temporal intuitionistic fuzzy sets (TIFSs) [3]. To handle the uncertainties in real life, fuzzy set theory helps a lot in different aspects. Intuitionistic fuzzy set theory gives better result than fuzzy set theory in handling the uncertainties and vagueness.

Soft set theory was introduced by Molodtsov [4] as a general mathematical tool for dealing with problems that contain uncertainty. New soft topologies using restricted and extended intersections on L-soft sets are introduced and the differences of these soft topologies are studied in [5]. The concept of soft bitopological Hausdorff space (SBT Hausdorff space) is presented in [6]. The properties of soft topology and soft subtopology are discussed in [7]. In [8], soft closed sets, soft α-closed, soft semi-closed, soft pre-closed, regular soft closed, soft g-closed and soft sg-closed are defined on soft bitopological space and related properties of these soft sets are given and compared their properties with each other. In [9], in order to apply the concept of octahedron sets to multi-criteria group decision-making problems, several similarity and distance measures for octahedron sets are defined and a multi-criteria group decision-making method with linguistic variables in octahedron set environment is presented.

Time is an important parameter in real life. Many mathematical solutions will get modification based on the parameter '*time*'. So, the mathematical model of the real life applications need one more parameter that is '*time*'. In intuitionistic fuzzy set theory, the parameter '*time*' is included and the new theory called, temporal intuitionistic fuzzy set theory is developed. In fixed time moment, the temporal intuitionistic fuzzy set (TIFS) is a standard IFS. But TIFS is used to represent the moving objects in different time moments.

The objective of this paper is to introduce a few interesting operations on TIFSs and study their basic properties. Also, it is very important to note that these operations are useful in defining contrast intensification operator and morphological operators. Hence, the ultimate aim of the authors is to apply these operators in video processing, that is, to contribute TIFSs tools in video processing. Further, it is planned to design and develop a video processing algorithm using TIFSs.

This paper is organised as follows: In Section 2, the basic definitions of FSs, IFSs and TIFSs and the related concepts are given. In Section 3, Geometric interpretation of TIFSs is given. In Section 4, some relations and operations on TIFSs are defined and discussed. Section 5 concludes the paper.

2. Preliminaries

In this section, the basic definitions related to fuzzy sets, intuitionistic fuzzy sets and temporal intuitionistic fuzzy sets are presented.

Definition 2.1 ([1])**.** Let X be a non-empty set. A *fuzzy set* (FS) A drawn from X is defined as $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0, 1]$ denotes the membership of the element x in the fuzzy set A.

Definition 2.2 ([2])**.** Let$X$ be a non-empty set. An *intuitionistic fuzzy set* (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ with $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$ represents the degrees of membership and non-membership of the element x to the IFS A. For each IFS, the *intuitionistic index or hesitancy degree* of x in X to the IFS A is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Definition 2.3 ([3, 10])**.** Let E be the universe and T be a non-empty set of time moments. Then, a *temporal intuitionistic fuzzy set* (TIFS) is an object having the form

$$A(T) = \{\langle x, \mu_A(x, t), \nu_A(x, t)\rangle : (x, t) \in E \times T\},$$

where

(i) $A \subset E$ is a fixed set.

(ii) $\mu_A(x, t)$ and $\nu_A(x, t)$ denote the degrees of membership and non-membership respectively of the element (x, t) such that $0 \le \mu_A(x, t) + \nu_A(x, t) \le 1$ for all $(x, t) \in E \times T$

Definition 2.4 ([3, 10])**.** For every two TIFSs,

$$A(T') = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle : \langle x,t\rangle \in E \times T'\}$$

$$B(T'') = \{\langle x, \mu_B(x,t), \nu_B(x,t)\rangle : \langle x,t\rangle \in E \times T''\},$$

where T' and T'' have finite number of distinct time-elements or they are time-intervals. The union operator is defined as:

$$A(T') \cup B(T'') = \left\{\left\langle x, \mu_{A(T')\cup B(T'')}(x,t), \nu_{A(T')\cup B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E \times (T^{'} \cup T^{''})\right\},$$

where $\left\langle x, \mu_{A(T')\cup B(T'')}(x,t), \nu_{A(T')\cup B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \langle x, \mu_A(x,t'), \nu_A(x,t')\rangle, & \text{if } t = t' \in T' - T'' \\ \langle x, \mu_B(x,t''), \nu_B(x,t'')\rangle, & \text{if } t = t'' \in T'' - T' \\ \langle x, max(\mu_A(x,t'), \mu_B(x,t'')), min(\nu_A(x,t'), \nu_B(x,t''))\rangle, & \text{if } t = t' = t'' \in T' \cap T'' \\ \langle x, 0, 1\rangle, & \text{otherwise} \end{cases}$$

The following two operators C^* and I^* over a TIFS A are defined as

$$C^*(A(T)) = \left\{\left\langle x, sup_{t\in T}\mu_{A(T)}(x,t), inf_{t\in T}\nu_{A(T)}(x,t)\right\rangle | x \in E\right\},$$

$$I^*(A(T)) = \left\{\left\langle x, inf_{t\in T}\mu_{A(T)}(x,t), sup_{t\in T}\nu_{A(T)}(x,t)\right\rangle | x \in E\right\}.$$

Definition 2.5 ([3, 10])**.** For every two TIFSs,

$$A(T') = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle : \langle x,t\rangle \in E \times T'\}$$

$$B(T'') = \{\langle x, \mu_B(x,t), \nu_B(x,t)\rangle : \langle x,t\rangle \in E \times T''\},$$

where T' and T'' have finite number of distinct time-elements or they are time-intervals. The intersection operator is defined as:

$$A(T') \cap B(T'') \left\{\left\langle x, \mu_{A(T')\cap B(T'')}(x,t), \nu_{A(T')\cap B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E \times (T^{'} \cup T^{''})\right\}$$

where $\left\langle x, \mu_{A(T')\cap B(T'')}(x,t), \nu_{A(T')\cap B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \langle x, \mu_A(x,t'), \nu_A(x,t')\rangle, & \text{if } t = t' \in T' - T'' \\ \langle x, \mu_B(x,t''), \nu_B(x,t'')\rangle, & \text{if } t = t'' \in T'' - T' \\ \langle x, min(\mu_A(x,t'), \mu_B(x,t'')), max(\nu_A(x,t'), \nu_B(x,t''))\rangle, & \text{if } t = t' = t'' \in T' \cap T'' \\ \langle x, 0, 1\rangle, & \text{otherwise.} \end{cases}$$

3. Geometric interpretation of temporal intuitionistic fuzzy sets

In this section, the geometric interpretation of TIFS is given. Let E be the universe, T be a non-empty set of time moments and $E \times T$ is the cartesian product of the sets E and T. Then, a geometric interpretation of TIFS is given in the following figure. This figure is the combination of the geometric interpretation of intuitionistic fuzzy sets and the temporal scale in [10].

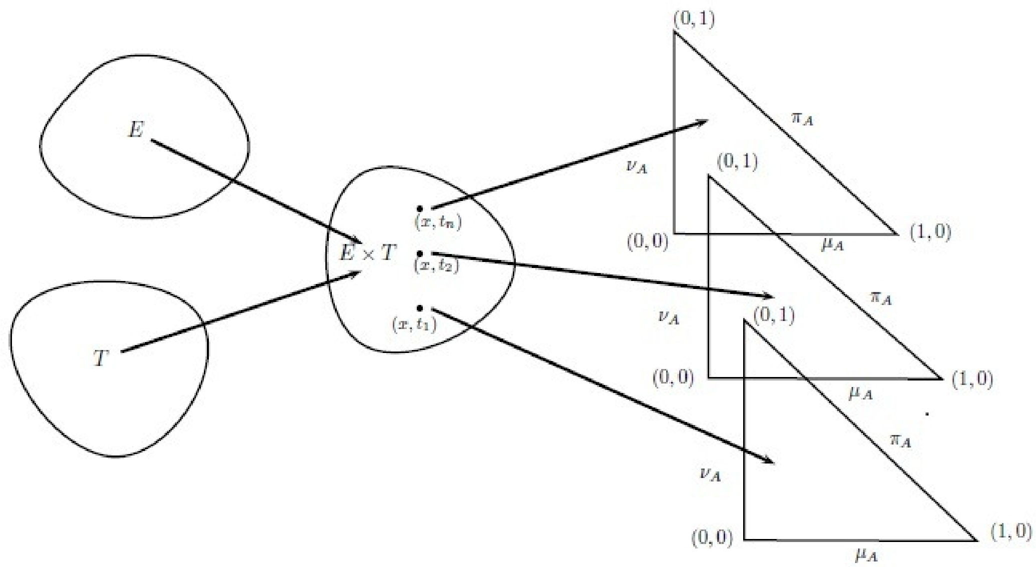

FIGURE 1. Geometric interpretation of temporal intuitionistic fuzzy sets [10]

4. Some relations and operations on temporal intuitionistic fuzzy sets

In this section, relations and operations over IFSs in [3, 10] are extended to TIFSs. Though these relations and operations are similar to intuitionistic fuzzy sets, it is necessary to extend them to TIFSs for designing and developing video processing algorithm, which involves 'time' parameter.

Definition 4.1 (Extended from [3, 10])**.** For every two TIFSs,

$$A(T') = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle : \langle x, t\rangle \in E \times T'\}$$

and

$$B(T'') = \{\langle x, \mu_B(x,t), \nu_B(x,t)\rangle : \langle x, t\rangle \in E \times T''\},$$

the following relations and operations are defined:

(i) $A(T') \subseteq B(T'')$ if and only if for every $\langle x, t\rangle \in E \times (T' \cup T'')$ and $t' \leq t''$,

$$\mu_A(x,t') \leq \mu_B(x,t'') \text{ and } \nu_A(x,t') \geq \nu_B(x,t''),$$

(ii) $A(T') \supseteq B(T'')$ if and only if for every $\langle x, t\rangle \in E \times (T' \cup T'')$ and $t' \geq t''$,

$$\mu_A(x,t') \geq \mu_B(x,t'') \text{ and } \nu_A(x,t') \leq \nu_B(x,t''),$$

(iii) $A(T') = B(T'')$ if and only if for every $\langle x, t\rangle \in E \times (T' \cup T'')$ and $t' = t''$,

$$\mu_A(x,t' = \mu_B(x,t'') \text{ and } \nu_A(x,t') = \nu_B(x,t''),$$

(iv) $\overline{A(T')} = \{\langle x, \nu_A(x,t), \mu_A(x,t)\rangle : \langle x, t\rangle \in E \times T'\},$

(v) $A(T')+B(T'') = \left\{\left\langle x, \mu_{A(T')+B(T'')}(x,t), \nu_{A(T')+B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E\times(T^{'}\cup T^{''})\right\}$,
where $\left\langle x, \mu_{A(T')+B(T'')}(x,t), \nu_{A(T')+B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \langle x, \mu_A(x,t'), \nu_A(x,t')\rangle, & \text{if } t=t'\in T'-T'' \\ \langle x, \mu_B(x,t''), \nu_B(x,t'')\rangle, & \text{if } t=t''\in T''-T' \\ \langle x, \mu_A(x,t')+\mu_B(x,t'')-\mu_A(x,t').\mu_B(x,t''), \nu_A(x,t').\nu_B(x,t'')\rangle, & \text{if } t=t'=t''\in T'\cap T'' \\ \langle x,0,1\rangle, & \text{otherwise,} \end{cases}$$

(vi) $A(T').B(T'') = \left\{\left\langle x, \mu_{A(T').B(T'')}(x,t), \nu_{A(T').B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E\times(T^{'}\cup T^{''})\right\}$,
where $\left\langle x, \mu_{A(T').B(T'')}(x,t), \nu_{A(T').B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \langle x,0,1\rangle, & \text{if } t=t'\in T'-T'' \\ \langle x,0,1\rangle, & \text{if } t=t''\in T''-T' \\ \langle x, \mu_A(x,t').\mu_B(x,t''), \nu_A(x,t')+\nu_B(x,t'')-\nu_A(x,t').\nu_B(x,t'')\rangle, & \text{if } t=t'=t''\in T'\cap T'' \\ \langle x,0,1\rangle, & \text{otherwise,} \end{cases}$$

(vii) $A(T')@B(T'') = \left\{\left\langle x, \mu_{A(T')@B(T'')}(x,t), \nu_{A(T')@B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E\times(T^{'}\cup T^{''})\right\}$,
where $\left\langle x, \mu_{A(T')@B(T'')}(x,t), \nu_{A(T')@B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \left\langle x, \frac{\mu_A(x,t')}{2}, \frac{\nu_A(x,t')+1}{2}\right\rangle, & \text{if } t=t'\in T'-T'' \\ \left\langle x, \frac{\mu_B(x,t'')}{2}, \frac{1+\nu_B(x,t'')}{2}\right\rangle, & \text{if } t=t''\in T''-T' \\ \left\langle x, \frac{\mu_A(x,t')+\mu_B(x,t'')}{2}, \frac{\nu_A(x,t')+\nu_B(x,t'')}{2}\right\rangle, & \text{if } t=t'=t''\in T'\cap T'' \\ \langle x,0,1\rangle, & \text{otherwise,} \end{cases}$$

(viii) $A(T')\$B(T'') = \left\{\left\langle x, \mu_{A(T')\$B(T'')}(x,t), \nu_{A(T')\$B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E\times(T^{'}\cup T^{''})\right\}$,
where $\left\langle x, \mu_{A(T')\$B(T'')}(x,t), \nu_{A(T')\$B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \left\langle x, 0, \sqrt{\nu_A(x,t')}\right\rangle, & \text{if } t=t'\in T'-T'' \\ \left\langle x, 0, \sqrt{\nu_A(x,t'')}\right\rangle, & \text{if } t=t''\in T''-T' \\ \left\langle x, \sqrt{\mu_A(x,t').\mu_B(x,t'')}, \sqrt{\nu_A(x,t').\nu_B(x,t'')}\right\rangle, & \text{if } t=t'=t''\in T'\cap T'' \\ \langle x,0,1\rangle, & \text{otherwise,} \end{cases}$$

(xi) $A(T')*B(T'') = \left\{\left\langle x, \mu_{A(T')*B(T'')}(x,t), \nu_{A(T')*B(T'')}(x,t)\right\rangle \mid \langle x,t\rangle \in E\times(T^{'}\cup T^{''})\right\}$,
where $\left\langle x, \mu_{A(T')*B(T'')}(x,t), \nu_{A(T')*B(T'')}(x,t)\right\rangle =$

$$\begin{cases} \left\langle x, \frac{\mu_A(x,t')}{2}, \frac{1}{2}\right\rangle, & \text{if } t=t'\in T'-T'' \\ \left\langle x, \frac{\mu_B(x,t'')}{2}, \frac{1}{2}\right\rangle, & \text{if } t=t''\in T''-T' \\ \left\langle x, \frac{\mu_A(x,t')+\mu_B(x,t'')}{2.(\mu_A(x,t').\mu_B(x,t'')+1)}, \frac{\nu_A(x,t')+\nu_B(x,t'')}{2.(\nu_A(x,t').\nu_B(x,t'')+1)}\right\rangle, & \text{if } t=t'=t''\in T'\cap T'' \\ \langle x,0,1\rangle, & \text{otherwise,} \end{cases}$$

(x) $A(T') \bowtie B(T'') = \left\{ \left\langle x, \mu_{A(T') \bowtie B(T'')}(x,t), \nu_{A(T') \bowtie B(T'')}(x,t) \right\rangle \mid \langle x,t \rangle \in E \times (T^{'} \cup T^{''}) \right\}$,
where $\left\langle x, \mu_{A(T') \bowtie B(T'')}(x,t), \nu_{A(T') \bowtie B(T'')}(x,t) \right\rangle =$

$$\begin{cases} \left\langle x, 0, 2.\frac{\nu_A(x,t'}{\nu_A(x,t'+1} \right\rangle, & \text{if } t = t' \in T' - T'' \\ \left\langle x, 0, 2.\frac{\nu_B(x,t''}{\nu_B(x,t''+1} \right\rangle, & \text{if } t = t'' \in T'' - T' \\ \left\langle x, 2.\frac{(\mu_A(x,t').\mu_B(x,t''))}{(\mu_A(x,t')+\mu_B(x,t''))}, 2.\frac{(\nu_A(x,t').\nu_B(x,t''))}{(\nu_A(x,t')+\nu_B(x,t''))} \right\rangle, & \text{if } t = t' = t'' \in T' \cap T'' \\ \langle x, 0, 1 \rangle, & \text{otherwise.} \end{cases}$$

Definition 4.2 (Extended from [3])**.** Let E be a non-empty set and let $A(T)$ be a temporal intuitionistic fuzzy set. Then

$$(A(T))^n = \{\langle x, \mu_A(x,t)^n, (1 - \nu_A(x,t))^n \rangle : \langle x,t \rangle \in E \times T\},$$

$$nA(T) = \{\langle x, (1 - (1 - \mu_A(x,t))^n, \nu_A(x,t))^n \rangle : \langle x,t \rangle \in E \times T\}.$$

Definition 4.3 (Extended from [3])**.** Let $A(T)$ be TIFS. Then, the necessity operator is defined as

$$\Box A(T) = \{\langle x, \mu_A(x,t), 1 - \mu_A(x,t) \rangle : \langle x,t \rangle \in (E \times T)\}.$$

Also, the possibility operator is defined as

$$\Diamond A(T) = \{\langle x, 1 - \nu_A(x,t), \nu_A(x,t) \rangle : \langle x,t \rangle \in (E \times T)\}.$$

Proposition 4.4 (Extended from [3])**.** *Let $A(T)$ be TIFS. Then the following equalities are hold:*

$$\begin{aligned} \overline{\Box \overline{A(T)}} &= \Diamond A(T), \\ \overline{\Diamond \overline{A(T)}} &= \Box A(T), \\ \Box A(T) \subset A(T) &\subset \Diamond A(T), \\ \Box\Box A(T) &= \Box A(T), \\ \Box\Diamond A(T) &= \Diamond A(T), \\ \Diamond\Box A(T) &= \Box A(T), \\ \Diamond\Diamond A(T) &= \Diamond A(T). \end{aligned}$$

Proof. $\overline{\Box \overline{A(T)}} = \overline{\Box \{\langle x, \nu_A(x,t), \mu_A(x,t) \rangle : \langle x,t \rangle \in E \times T\}}$
$= \overline{\{\langle x, \nu_A(x,t), 1 - \nu_A(x,t) \rangle : \langle x,t \rangle \in E \times T\}}$
$= \{\langle x, 1 - \nu_A(x,t), \nu_A(x,t) \rangle : \langle x,t \rangle \in E \times T\}$
$= \Diamond A(T)$. □

Theorem 4.5. *Let $A(T')$ and $B(T'')$ be two TIFSs. Then the following results hold:*

$$\begin{aligned}
\Box(A(T) \cap B(T'')) &= \Box A(T') \cap \Box B(T''),\\
\Box(A(T) \cup B(T'')) &= \Box A(T') \cup \Box B(T''),\\
\overline{\Box((\overline{A(T')} + \overline{B(T'')})} &= \Diamond A(T') \cdot \Diamond B(T''),\\
\overline{\Box((\overline{A(T')} \cdot \overline{B(T'')})} &= \Diamond A(T') + \Diamond B(T''),\\
\Box(A(T) @ B(T'')) &= \Box A(T') @ \Box B(T''),\\
\Box(A(T) \$ B(T'')) &\supset \Box A(T') \$ \Box B(T''),\\
\Box(A(T) \bowtie B(T'')) &\supset \Box A(T') \bowtie \Box B(T''),\\
\Diamond(A(T) \cap B(T'')) &= \Diamond A(T') \cap \Diamond B(T''),\\
\Diamond(A(T) \cup B(T'')) &= \Diamond A(T') \cup \Diamond B(T''),\\
\overline{\Diamond((\overline{A(T')} + \overline{B(T'')})} &= \Box A(T') \cdot \Box B(T''),\\
\overline{\Diamond((\overline{A(T')} \cdot \overline{B(T'')})} &= \Box A(T') + \Box B(T''),\\
\Diamond(A(T) @ B(T'')) &= \Diamond A(T') @ \Diamond B(T''),\\
\Diamond(A(T) \$ B(T'')) &\subset \Diamond A(T') \$ \Diamond B(T''),\\
\Diamond(A(T) \bowtie B(T'')) &\subset \Diamond A(T') \bowtie \Diamond B(T'').
\end{aligned}$$

Proof. $\Box(A(T) \cap B(T''))$

$= \Box \{\langle x, min(\overline{\mu}_A(x,t), \overline{\mu}_B(x,t)), max(\overline{\nu}_A(x,t), \overline{\nu}_B(x,t))\rangle : \langle x,t\rangle \in E \times T' \cup T''\}$

$= \{\langle x, min(\overline{\mu}_A(x,t), \overline{\mu}_B(x,t)), 1 - min(\overline{\mu}_A(x,t), \overline{\mu}_B(x,t))\rangle : \langle x,t\rangle \in E \times T' \cup T''\}$

$= \{\langle x, min(\overline{\mu}_A(x,t), \overline{\mu}_B(x,t)), max(1 - \overline{\mu}_A(x,t), 1 - \overline{\mu}_B(x,t))\rangle : \langle x,t\rangle \in E \times T' \cup T''\}$

$= \{\langle x, \mu_A(x,t), 1 - \mu_A(x,t)\rangle : \langle x,t\rangle \in (E \times T)\} \cap \{\langle x, \mu_B(x,t), 1 - \mu_B(x,t)\rangle : \langle x,t\rangle \in (E \times T)\}$

$= \Box A(T') \cap \Box B(T'')$,

where

$$\overline{\mu}_A(x,t) = \begin{cases} \mu_A(x,t), & t \in T^{'} \\ 0, & t \in T^{''} - T^{'} \end{cases}$$

$$\overline{\mu}_B(y,t) = \begin{cases} \mu_B(y,t), & t \in T^{''} \\ 0, & t \in T^{'} - T^{''} \end{cases}$$

$$\overline{\nu}_A(x,t) = \begin{cases} \nu_A(x,t), & t \in T^{'} \\ 1, & t \in T^{''} - T^{'} \end{cases}$$

$$\overline{\nu}_B(y,t) = \begin{cases} \nu_B(y,t), & t \in T^{''} \\ 1, & t \in T^{'} - T^{''}. \end{cases}$$

□

Definition 4.6. The *normalization* of a TIFS $A(T)$ of the universe E, denoted by $NORM(A(T))$, is defined as

$$NORM(A(T)) = \{\langle x, \mu_{NORM}(x,t), \nu_{NORM}(x,t)\rangle : \langle x,t\rangle \in E \times T\},$$

where

$$\mu_{NORM}(x,t) = \frac{\mu_A(x,t)}{\text{Sup}(\mu_A(x,t))}$$

and

$$\nu_{NORM}(x,t) = \frac{\nu_A(x,t) - \inf\,(\nu_A(x,t))}{1 - \inf\,(\nu_A(x,t))}.$$

Definition 4.7 (Extended from [3])**.** Let E_1 and E_2 be two different universes and let

$$A(T') = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle : \langle x,t\rangle \in E_1 \times T'\}$$
$$B(T'') = \{\langle x, \mu_B(y,t), \nu_B(y,t)\rangle : \langle y,t\rangle \in E_2 \times T''\}$$

be two TIFSs. Then the *Cartesian product* of two TIFSs $A(T')$ and $B(T'')$ are defined as follows:

- the Cartesian product " $\times_1$ "
$A \times_1 B = \{\langle\langle x,y\rangle, \overline{\mu}_A(x,t)\cdot\overline{\mu}_B(y,t), \overline{\nu}_A(x,t)\cdot\overline{\nu}_B(y,t)\rangle :$
$\langle x,t\rangle \in E_1 \times T'$ and $\langle y,t\rangle \in E_2 \times T''\}$,
- the Cartesian product " $\times_2$ "
$A \times_2 B = \{\langle\langle x,y\rangle, \overline{\mu}_A(x,t)+\overline{\mu}_B(y,t)-\overline{\mu}_A(x,t)\cdot\overline{\mu}_B(y,t), \overline{\nu}_A(x,t)\cdot\overline{\nu}_B(y,t)\rangle :$
$\langle x,t\rangle \in E_1 \times T'$ and $\langle y,t\rangle \in E_2 \times T''\}$,
- the Cartesian product " $\times_3$ "
$A \times_3 B = \{\langle\langle x,y\rangle, \overline{\mu}_A(x,t)\cdot\overline{\mu}_B(y,t), \overline{\nu}_A(x,t)+\overline{\nu}_B(y,t)-\overline{\nu}_A(x,t)\cdot\overline{\nu}_B(y,t)\rangle :$
$\langle x,t\rangle \in E_1 \times T'$ and $\langle y,t\rangle \in E_2 \times T''\}$,
- the Cartesian product " $\times_4$ "
$A \times_4 B = \{\langle\langle x,y\rangle, min(\overline{\mu}_A(x,t),\overline{\mu}_B(y,t)), max(\overline{\nu}_A(x,t),\overline{\nu}_B(y,t))\rangle :$
$\langle x,t\rangle \in E_1 \times T'$ and $\langle y,t\rangle \in E_2 \times T''\}$,
- the Cartesian product " $\times_5$ "
$A \times_5 B = \{\langle\langle x,y\rangle, max(\overline{\mu}_A(x,t),\overline{\mu}_B(y,t)), min(\overline{\nu}_A(x,t),\overline{\nu}_B(y,t))\rangle :$
$\langle x,t\rangle \in E_1 \times T'$ and $\langle y,t\rangle \in E_2 \times T''\}$,

where

$$\overline{\mu}_A(x,t) = \begin{cases} \mu_A(x,t), & t \in T^{'} \\ 0, & t \in T^{''} - T^{'} \end{cases}$$

$$\overline{\mu}_B(y,t) = \begin{cases} \mu_B(y,t), & t \in T^{''} \\ 0, & t \in T^{'} - T^{''} \end{cases}$$

$$\overline{\nu}_A(x,t) = \begin{cases} \nu_A(x,t), & t \in T^{'} \\ 1, & t \in T^{''} - T^{'} \end{cases}$$

$$\overline{\nu}_B(y,t) = \begin{cases} \nu_B(y,t), & t \in T^{''} \\ 1, & t \in T^{'} - T^{''}. \end{cases}$$

Proposition 4.8. *Let E_1, E_2, E_3 be three universes and T be time moments. Consider four TIFSs $A(T'), B(T')$ (over $E_1 \times T'$), $C(T'')$ (over $E_2 \times T''$) and $D(T''')$*

(over $E_3 \times T'''$). Then the following relations are hold:

$$(A(T) \times C(T'')) \times D(T''') = A(T) \times (C(T'') \times D(T'''),$$
$$(A(T') \cup B(T') \times C(T'') = (A(T') \times C(T'') \cup (B(T') \times C(T'')),$$
$$(A(T') \cap B(T')) \times C(T''') = (A(T') \times C(T'')) \cap (B(T') \times C(T'')),$$
$$C(T'') \times (A(T') \cup B(T')) = (C(T'') \times A(T') \cup (C(T'') \times B(T')),$$
$$C(T'') \times (A(T') \cap B(T')) = (C(T'') \times A(T')) \cap (C(T'') \times B(T')),$$

where $\times \in \{\times_1, \times_2, \times_3, \times_4, \times_5\}$.

Proof. $(A(T') \times_1 C(T'')) \times_1 D(T''')$
$= \{\langle\langle x, y\rangle, \overline{\mu}_A(x,t) \cdot \overline{\mu}_C(y,t), \overline{\nu}_A(x,t) \cdot \overline{\nu}_C(y,t)\rangle : \langle x, t\rangle \in E_1 \times T'$
and $\langle y, t\rangle \in E_2 \times T''\} \times_1 D(T''')$
$= \{\left\langle \langle x, y\rangle, \overline{\overline{\mu}_A(x,t) \cdot \overline{\mu}_C(y,t)} \cdot \overline{\mu}_D(z,t), \overline{\overline{\nu}_A(x,t) \cdot \overline{\nu}_C(y,t)} \cdot \overline{\nu}_D(z,t)\right\rangle :$
$\langle x, t\rangle \in E_1 \times T', \langle y, t\rangle \in E_2 \times T'' \& \langle z, t\rangle \in E_3 \times T'''\}$
$= \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle : \langle x, t\rangle \in E_1 \times T'\}$
$\times_1\{\langle\langle y, z\rangle, \overline{\mu}_C(y,t) \cdot \overline{\mu}_D(z,t), \overline{\nu}_C(y,t) \cdot \overline{\nu}_D(z,t)\rangle :$
$\langle y, t\rangle \in E_2 \times T'' \& \langle z, t\rangle \in E_3 \times T'''\}$
$= A(T') \times_1 (C(T'') \times_1 D(T''')$.
The validity of $(A(T') \times C(T'')) \times D(T''') = A(T') \times (C(T'') \times D(T'''))$ is proved with respect to the Cartesian product " $\times_1$ ". Other results can also be proved in a similar way. □

Proposition 4.9. *Let E_1, E_2, E_3 be three universes and T be time moments. Consider four TIFSs $A(T'), B(T')$ (over $E_1 \times T'$), $C(T'')$ (over $E_2 \times T''$) and $D(T''')$ (over $E_3 \times T'''$). Then the following relations are hold:*

$$(A(T') + B(T')) \times C(T'') \subset (A(T') \times C(T'')) + (B(T') \times C(T'')),$$
$$(A(T').B(T')) \times C(T'') \supset (A(T') \times C(T'')).(B(T') \times C(T'')),$$
$$(A(T')@B(T')) \times C(T'') = (A(T') \times C(T''))@(B(T') \times C(T'')),$$
$$C(T'') \times (A(T') + B(T')) \subset (C(T'') \times A(T')) + (C(T'') \times B(T')),$$
$$C(T'') \times (A(T').B(T')) \supset (C(T'') \times A(T')).(C(T'') \times B(T')),$$
$$C(T'') \times (A(T')@B(T')) = (C(T'') \times A(T'))@(C(T'') \times B(T')),$$

where $\times \in \{\times_1, \times_2, \times_3\}$.

Proof. $(A(T') + B(T')) \times_1 C(T'')$
$= \{\langle x, \overline{\mu}_A(x,t) + \overline{\mu}_B(x,t) - \overline{\mu}_A(x,t).\overline{\mu}_B(x,t), \overline{\nu}_A(x,t).\overline{\nu}_B(x,t)\rangle : \langle x, t\rangle \in E_1 \times (T' \cup T'')\}$
$\times_1 \{\langle y, \mu_C(y,t), \nu_C(y,t)\rangle : \langle y, t\rangle \in E_2 \times T'''\}$
$= \{\left\langle \langle x, y\rangle, \overline{\overline{\mu}_A(x,t) + \overline{\mu}_B(x,t) - \overline{\mu}_A(x,t).\overline{\mu}_B(x,t)} \cdot \overline{\mu}_C(y,t), \overline{\overline{\nu}_A(x,t) \cdot \overline{\nu}_B(x,t)} \cdot \overline{\nu}_C(y,t)\right\rangle :$
$\langle x, t\rangle \in E_1 \times (T' \cup T'') \& \langle y, t\rangle \in E_2 \times T'''\}$
$\subset \{\langle\langle x, y\rangle, \overline{\overline{\mu}_A(x,t) \cdot \overline{\mu}_C(y,t) + \overline{\mu}_B(x,t) \cdot \overline{\mu}_C(y,t) - \overline{\mu}_A(x,t).\overline{\mu}_B(x,t) \cdot \overline{\mu}_C(y,t)}, \overline{\overline{\nu}_A(x,t) \cdot \overline{\nu}_B(x,t)}$
$\cdot \overline{\nu}_C(y,t)\rangle : \langle x, t\rangle \in E_1 \times (T' \cup T'') \& \langle y, t\rangle \in E_2 \times T'''\}$

$= \{\langle\langle x,y\rangle, \overline{\mu}_A(x,t)\cdot\overline{\mu}_C(y,t), \overline{\nu}_A(x,t)\cdot\overline{\nu}_C(y,t)\rangle : \langle x,t\rangle \in E_1\times T' \& \langle y,t\rangle \in E_2\times T''\}+$
$\{\langle\langle x,y\rangle, \overline{\mu}_B(x,t)\cdot\overline{\mu}_C(y,t), \overline{\nu}_B(x,t)\cdot\overline{\nu}_C(y,t)\rangle : \langle x,t\rangle \in E_1\times T' \& \langle y,t\rangle \in E_2\times T'''\}$
$= (A(T')\times_1 C(T'')) + (B(T')\times_1 C(T''))$.
The validity of $(A(T') + B(T'))\times C(T'') \subset (A(T')\times C(T'')) + (B(T')\times C(T''))$ is proved with respect to the Cartesian product " $\times_1$ ". Other results can also be proved in a similar way. □

5. Conclusion

In this work, theory of temporal intuitionistic fuzzy sets (TIFSs) has been developed. Operations like normalization and five types of cartesian products on TIFSs have been defined and a few of their properties are also analysed. Also, the authors proposed to work on morphological operators of TIFS and their applications in video processing in their future work.

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[2] K. T. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia 1983.
[3] K. T. Atanassov, Intuitionistic fuzzy sets: Theory and Applications, Physica-Verlag, Heidelberg 1999.
[4] D. Molodtsov, Soft set theory:First results, Comput. Math. Appl. 37 (4-5) (1999) 19–31.
[5] G. Şenel, Soft Topology Generated by L-Soft Sets, Journal of New Theory 4 (24) (2018) 88–100.
[6] G. Şenel, A New Approach to Hausdorff Space Theory via the Soft Sets, Mathematical Problems in Engineering 9 (2016) 1–6.
[7] G. Şenel and N. Çağman, Soft topological subspaces, Ann. Fuzzy Math. Inform. 10 (4) (2015) 525–535.
[8] G. Şenel and N. Çağman, Soft Closed Sets on Soft Bitopological Space, Journal of New Results in Science 3 (5) (2014) 57–66.
[9] G. Şenel, J. G. Lee and K. Hur, Distance and Similarity Measures for Octahedron Sets and Their Application to MCGDM Problems, Mathematics 8 (2020) 1690.
[10] K. T. Atanassov, On Intuitionistic fuzzy sets theory, Springer, Berlin, Heidelberg 2012.

R.Parvathi (paarvathis@rediffmail.com)
Associate Professor & Head, Department of Mathematics, Vellalar College for Women, Erode - 638012, Tamilnadu, India

C.Yuvapriya (yuvapriya.c@gmail.com)
Research Scholar, Department of Mathematics, Vellalar College for Women, Erode - 638012, Tamilnadu, India