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Soft elementary bitopological spaces

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Abstract. In this article a notion of soft quasi-pseudo metric is introduced and its underlying soft bitopology is defined where the relevant soft topologies are elementary soft topologies. Considering the ordering of a pair of soft elements two different notions of pairwise soft separation axioms and weak pairwise soft separation axioms are developed. Some properties of this new soft elementary bitopology regarding pairwise soft separation axioms and weak pairwise soft separation axioms are studied. With some examples and counter examples the relations among them have been justified. Further it is seen that under a given condition the soft quasi-pseudo metric space also satisfies the pairwise soft separation properties.

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1. Introduction and preliminaries

The symmetric requirement of distance function is often too restrictive as the asymmetry appears naturally in many mathematical models. Such asymmetric distance function was first found in quasi-metric space by Wilson [1]. He discussed some properties of quasi-metric space and the relations between metric spaces, quasi-metric spaces and topological spaces. In [2], Kelly showed that a quasi pseudo metric generates base for two different topologies on that set and started a new concept of bitopological space. A bitopological space is a set X equipped with two arbitrary topologies. Later on, Patty [3] and Saegrove [4] have done the conventional investigations on bitopological spaces. In 1972, Reilly [5] studied the separation properties of bitopological spaces.

On the other hand in 1999, Molodtsov [6] introduced soft set as generalization of fuzzy set [7]. Maji et al. [8, 9] successfully applied soft set theory in decision making problems. Also they defined several operations on soft sets to strenghthen soft set theory. Many researchers have contributed in developing mathematical structures in soft set theory. Also, the soft set theory has been associated and applied to decision making, data analysis and various mathematical structures (See [8, 10, 11, 12, 13, 14]). Aktaş and Çağman [15] introduced group structure in soft set, Jun et al. [16, 12] studied soft BCK/BCI-algebras and soft semirings, Majumdar and Samanta [17] introduced notion of soft mapping. Topological structure was introduced in soft setting by Shabir and Naz [18]. From then many authors have taken different approaches to study the topological structure on soft sets (See [19, 20, 21, 22, 23]). Das and Samanta [24, 25] introduced soft metric space and soft normed linear space. In 2014, Soft Bitopological spaces were first studied by Şenel and Çağman [26] and also by B. M. Ittanagi [27] independently. Also Şenel et al. studied soft closed sets on soft bitopological spaces [26], soft topological subspaces [28], Soft ditopological spaces [29], Soft topology generated by L-soft sets [30], etc. Again Chiney and Samanta [19] redefined soft topology (also known as ϵ-soft topology [31]) using elementary union, elementary intersections which are non distributive in nature and the elementary complement which does not follow the excluded middle law. Dutta et al. [32], Roy et al. [33, 34] studied different notions of this type of soft topological spaces.

In this paper, our main goal is to introduce a notion of soft quasi-pseudo metric space using soft elements and to develop a notion of soft bitopology induced by a soft quasi-pseudo metric. Also we investigate the separation properties of a soft quasi-pseudo metric space. In this framework we introduce a notion of soft elementary bitopological space and study some separation axioms on it. A soft bitopological space reduces to a soft topological space if the two soft topologies become identical. Thus the theory of soft bitopological space is more general than the soft topological space and we can consider the soft topological space as a special case of soft bitopological space.

The organization of the paper is as follows : Some preliminary results of soft sets and redefined soft topology related to our work has been discussed. In Section 2, we define soft quasi-pseudo metric and construct soft elementary bitopology. In section 3 and 4, different notions of pairwise soft separation axioms have been introduced and the relation among them are investigated with examples and counter examples. An implication table is given to show the relations among them. Section 5 contains conclusions and future work of the paper.

In order to maintain the length of the paper some preliminary results related to soft set and soft topology are omitted in this paper. Some straightforward proofs are also omitted.

Soft set was introduced by Molodtsov [6] in 1999. Later Ma et al. [35] and Nazmul and Samanta [22] slightly modified his definition and extend it on whole parameter set. In this paper we follow the definition considered by Nazmul and Samanta [22] which is as follows:

Definition 1.1 ([22])**.** Let X be a universal set and A be a set of parameters. Let $\mathcal{P}(X)$ denote the power set of X. A pair $\langle \mathcal{F}, A\rangle$ is called a *soft set* over X, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : A \to \mathcal{P}(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a soft set $\langle \mathcal{F}, A\rangle$ and for each

$\alpha \in A$, $\langle \mathcal{F}, A\rangle(\alpha)$ (simply $\mathcal{F}(\alpha)$) may be considered as the set of α approximate elements of $\langle \mathcal{F}, A\rangle$.

Operations on soft sets such as union, intersection, complement and absolute soft set, null soft set etc. are taken from [22].

Definition 1.2 ([36])**.** Let X be a non-empty set and A be a non-empty parameter set. Then a function $\tilde{x} : A \to X$ is said to be a *soft element* of X. A soft element $\tilde{x}$ of X is said to *belong to* a soft set $\langle \mathcal{F}, A\rangle$ over X, denoted by $\tilde{x}\tilde{\in}\langle \mathcal{F}, A\rangle$, if $\tilde{x}(\lambda) \in \mathcal{F}(\lambda)$ for all $\lambda \in A$.

Let X be an initial universal set and A be a non-empty parameter set. Throughout the paper, we consider the null soft set $\left\langle \tilde{\Phi}, A\right\rangle$ and those soft sets $\langle \mathcal{F}, A\rangle$ over X for which $\mathcal{F}(\alpha) \neq \phi$ for all $\alpha \in A$. We denote this collection by $S(\tilde{X})$ and $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft elements such that $\bar{r}(\lambda) = r$ for all $\lambda \in A$. The singleton soft set will be denoted by $\langle \tilde{x}, A\rangle$. We will denote the soft set generated by a collection $\mathcal{B}$ of soft elements of X and the collection of soft elements of a soft set $\langle \mathcal{F}, A\rangle$ by $SS(\mathcal{B})$ and $SE\langle \mathcal{F}, A\rangle$ respectively.

Definition 1.3 ([24])**.** For any two soft sets $\langle \mathcal{F}, A\rangle$, $\langle \mathcal{G}, A\rangle \in S(\tilde{X})$,

(i) the *elementary union* of $\langle \mathcal{F}, A\rangle$ and $\langle \mathcal{G}, A\rangle$, denoted by $\langle \mathcal{F}, A\rangle \Cup \langle \mathcal{G}, A\rangle$, is defined by $\langle \mathcal{F}, A\rangle \Cup \langle \mathcal{G}, A\rangle = SS(\mathcal{B})$,
where, $\mathcal{B} = \{\tilde{x}\tilde{\in}\left\langle \tilde{X}, A\right\rangle : \tilde{x}\tilde{\in}\langle \mathcal{F}, A\rangle \text{ or } \tilde{x}\tilde{\in}\langle \mathcal{G}, A\rangle\}$, i.e.,

$$\langle \mathcal{F}, A\rangle \Cup \langle \mathcal{G}, A\rangle = SS(SE\langle \mathcal{F}, A\rangle \cup SE\langle \mathcal{G}, A\rangle),$$

(ii) the *elementary intersection* of $\langle \mathcal{F}, A\rangle$ and $\langle \mathcal{G}, A\rangle$, denoted by $\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle$, is defined by $\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle = SS(\mathcal{B})$,
where $\mathcal{B} = \{\tilde{x}\tilde{\in}\left\langle \tilde{X}, A\right\rangle : \tilde{x}\tilde{\in}\langle \mathcal{F}, A\rangle \text{ and } \tilde{x}\tilde{\in}\langle \mathcal{G}, A\rangle\}$, i.e.,

$$\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle = SS(SE\langle \mathcal{F}, A\rangle \cap SE\langle \mathcal{G}, A\rangle).$$

Definition 1.4 ([36])**.** Let $\mathbb{R}$ be the set of real numbers and $\mathfrak{B}(\mathbb{R})$ be the set of all non-empty bounded subset of $\mathbb{R}$. A mapping $\mathcal{F} : A \to \mathfrak{B}(\mathbb{R})$ is said to be a *soft real set*. In particular, if $\mathcal{F}(\lambda)$ is a singleton set for all $\lambda \in A$, then $\langle \mathcal{F}, A\rangle$ is called a *soft real number*. The set of all non-negative soft real numbers will be denoted by $\mathbb{R}(A)^*$.

Proposition 1.5 ([36])**.** *For any two soft real numbers* $\tilde{r}, \tilde{s}$,

(1) $\tilde{r} \tilde{\leq} \tilde{s}$, *if* $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, $\lambda \in A$,
(2) $\tilde{r} \tilde{\geq} \tilde{s}$ *if* $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$, $\lambda \in A$.

Definition 1.6 ([24])**.** A mapping $d : SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(A)^*$ is said to be a *soft metric* on $(\tilde{X}, A)$, if d satisfies the following conditions:

(M1) $d(\tilde{x}, \tilde{y})\tilde{\geq}\bar{0}$ for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$,
(M2) $d(\tilde{x}, \tilde{y}) = \bar{0}$ if and only if $\tilde{x} = \tilde{y}$,
(M3) $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$,
(M4) $d(\tilde{x}, \tilde{z})\tilde{\leq}d(\tilde{x}, \tilde{y}) + d(\tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$.

Definition 1.7 ([19]). Let $\mathcal{T}$ be a collection of soft sets of S$(\tilde{X})$. Then $\mathcal{T}$ is said to be a *soft topology*, if it satisfies the following conditions:

(i) $\left\langle \tilde{\Phi}, A \right\rangle$ and $\left\langle \tilde{X}, A \right\rangle$ belong to $\mathcal{T}$,
(ii) the elementary intersection of any two soft sets of $\mathcal{T}$ belongs to $\mathcal{T}$,
(iii) the elementary union of any number of soft sets of $\mathcal{T}$ belongs to $\mathcal{T}$.

The triplet $\left\langle \tilde{X}, \mathcal{T}, A \right\rangle$ is called a *soft topological space*.

From now on, we shall refer it to an *elementary soft topology*. Each member of $\mathcal{T}$ is called a *soft open set* or $\mathcal{SO}$ *set*. Throughout this paper, the soft topology is taken in the sense of Chiney and Samanta [19]. Definitions of soft closed set or $\mathcal{SC}$ set, soft closure and some soft separation axioms are also taken from [19].

2. Soft quasi-pseudo metric and soft elementary bitopology

Definition 2.1. A mapping $\mathcal{P} : SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(A)^*$ is said to be a *soft quasi-pseudo metric* (SQPM) on $\left\langle \tilde{X}, A \right\rangle$, if $\mathcal{P}$ satisfies the following conditions:

(Q1) $\mathcal{P}(\tilde{x}, \tilde{x}) = \bar{0}$ for all $\tilde{x} \in SE(\tilde{X})$,
(Q2) $\mathcal{P}(\tilde{x}, \tilde{z}) \tilde{\leq} \mathcal{P}(\tilde{x}, \tilde{y}) + \mathcal{P}(\tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$.

In addition, if $\mathcal{P}$ satisfies the condition $\mathcal{P}(\tilde{x}, \tilde{y}) = \bar{0}$ iff $\tilde{x} = \tilde{y}$, then $\mathcal{P}$ is said to be a *soft quasi-metric* (SQM) on $\left\langle \tilde{X}, A \right\rangle$.

The soft set $\left\langle \tilde{X}, A \right\rangle$ with a SQPM, $\mathcal{P}(\ ,\ )$ on it is called a *soft quasi-pseudo metric space* (in short, SQPMS) and is denoted by $\left\langle \tilde{X}, \mathcal{P}, A \right\rangle$. Let $\left\langle \tilde{X}, \mathcal{P}, A \right\rangle$ be a SQPMS and let $\mathcal{Q}(\ ,\ )$ be defined by $\mathcal{Q}(\tilde{x}, \tilde{y}) = \mathcal{P}(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$. Then it can be easily verified that $\mathcal{Q}(\ ,\ )$ is also a SQPM on $\left\langle \tilde{X}, A \right\rangle$ and $\mathcal{Q}(\ ,\ )$ is called the *soft conjugate* of $\mathcal{P}(\ ,\ )$. A SQPMS with SQPM $\mathcal{P}$ and its soft conjugate $\mathcal{Q}$ on $\left\langle \tilde{X}, A \right\rangle$ is denoted by $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$.

Proposition 2.2. (Decomposition Theorem) *If a SQPM* $\mathcal{P}$ *satisfies the condition :*
(Q3) for $(\xi_1, \xi_2) \in X \times X$ *and* $\lambda \in A$, $\{\mathcal{P}(\tilde{x}, \tilde{y})(\lambda) : \tilde{x}(\lambda) = \xi_1, \tilde{y}(\lambda) = \xi_2\}$ *is a singleton set, and if for* $\lambda \in A$, $\mathcal{P}_\lambda : X \times X \to \mathbb{R}^+$ *is defined by* $\mathcal{P}_\lambda(\tilde{x}(\lambda), \tilde{y}(\lambda)) = \mathcal{P}(\tilde{x}, \tilde{y})(\lambda)$ *for all* $\tilde{x}, \tilde{y} \in SE(\tilde{X})$*, then* $\mathcal{P}_\lambda$ *is a quasi-pseudo metric on* X*. Also,* $\mathcal{Q}_\lambda$ *is conjugate to quasi-pseudo metric* $\mathcal{P}_\lambda$ *on* X*.*

Proof. Clearly, $\mathcal{P}_\lambda : X \times X \to \mathbb{R}^+$ is a rule that assigns an ordered pair of X to a non-negative real number for all $\lambda \in A$. Now the well defined property of $\mathcal{P}_\lambda$ for all $\lambda \in A$ follows from the condition ($Q3$) and the soft quasi-pseudo metric axioms gives the pseudo metric conditions of $\mathcal{P}_\lambda$ for all $\lambda \in A$. Then the soft quasi-pseudo metric satisfying ($Q3$) gives a parametrized family of crisp quasi-pseudo metric. Also, by definition of $\mathcal{Q}$, it follows that $\mathcal{Q}_\lambda$ is conjugate to $\mathcal{P}_\lambda$ for all $\lambda \in A$. □

Example 2.3. Consider the set $\mathbb{R}$ of real numbers with usual order and metric. Let d be the metric on $\mathbb{R}$ defined by $d(a, b) = min\{1, |a - b|\} \forall a, b \in \mathbb{R}$.

Now, we define $\mathcal{P} : SE(\tilde{\mathbb{R}}) \times SE(\tilde{\mathbb{R}}) \to \mathbb{R}(A)^*$ by

$$\begin{aligned} \mathcal{P}(\tilde{x}, \tilde{y})(\lambda) &= d(\xi, \eta) \quad if \quad \xi \leq \eta, \\ &= 1 \qquad\quad\;\; if \quad \xi \geq \eta, \end{aligned}$$

where $\tilde{x}(\lambda) = \xi, \tilde{y}(\lambda) = \eta$.
Again, we define $\mathcal{Q} : SE(\tilde{\mathbb{R}}) \times SE(\tilde{\mathbb{R}}) \to \mathbb{R}(A)^*$ by

$$\begin{aligned} \mathcal{Q}(\tilde{x}, \tilde{y})(\lambda) &= d(\xi, \eta) \quad if \quad \xi \geq \eta, \\ &= 1 \qquad\quad\;\; if \quad \xi \leq \eta, \end{aligned}$$

where $\tilde{x}(\lambda) = \xi, \tilde{y}(\lambda) = \eta$.
Then $\mathcal{P}$, $\mathcal{Q}$ are soft quasi-pseudo metrics on $\left\langle \tilde{\mathbb{R}}, A \right\rangle$ and $\mathcal{Q}$ is soft conjugate of $\mathcal{P}$. Also, if for $\lambda \in A$, $\mathcal{P}_\lambda : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ is defined by $\mathcal{P}_\lambda(\tilde{x}(\lambda), \tilde{y}(\lambda)) = \mathcal{P}(\tilde{x}, \tilde{y})(\lambda)$ for all $\tilde{x}, \tilde{y} \in SE(\tilde{\mathbb{R}})$, then $\mathcal{P}_\lambda$ is a quasi-pseudo metric on $\mathbb{R}$.

Definition 2.4. Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ be a SQPMS, $\tilde{r}$ be a positive soft real number and $\tilde{a} \tilde{\in} SE(\tilde{X})$. Then a *soft open ball* $B_{\mathcal{P}}(\tilde{a}, \tilde{r})$ *with centre at* $\tilde{a}$ *and radius* $\tilde{r}$ is a collection of soft elements of $\left\langle \tilde{X}, A \right\rangle$ satisfying $\mathcal{P}(\tilde{x}, \tilde{a}) \tilde{<} \tilde{r}$. Thus

$$B_{\mathcal{P}}(\tilde{a}, \tilde{r}) = \{\tilde{x} \tilde{\in} SE(\tilde{X}) : \mathcal{P}(\tilde{x}, \tilde{a}) \tilde{<} \tilde{r}\}.$$

Definition 2.5. Let $\mathfrak{B}$ be a collection of soft elements of $\left\langle \tilde{X}, A \right\rangle$ in a $SQPMS$ $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$. Then a soft element $\tilde{a}$ is said to be an *interior element* of $\mathfrak{B}$, if there exists a positive soft real number $\tilde{r}$ such that $\tilde{a} \in B_{\mathcal{P}}(\tilde{a}, \tilde{r}) \subset \mathfrak{B}$.

Definition 2.6. Let $\mathfrak{B}$ be a non-null collection of soft elements of $\left\langle \tilde{X}, A \right\rangle$ in a $SQPMS$ $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$. Then $\mathfrak{B}$ is said to be *open in* $\mathcal{P}$, if each elements of $\mathfrak{B}$ is an interior element of $\mathfrak{B}$.

Definition 2.7. Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ be a $SQPMS$. A soft set $\langle \mathcal{F}, A \rangle \in S(\tilde{X})$ is said to be a*soft open* ($\mathcal{SO}$ set) *set* in $\mathcal{P}$, if there is a collection of soft elements $\mathfrak{B}$ of $\left\langle \tilde{X}, A \right\rangle$ which is open in $\mathcal{P}$ and $SS(\mathfrak{B}) = \langle \mathcal{F}, A \rangle$.

Proposition 2.8. (1) *In a $SQPMS$ $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ the null soft set $\left\langle \tilde{\Phi}, A \right\rangle$, the absolute soft set $\left\langle \tilde{X}, A \right\rangle$ and arbitrary elementary union of $\mathcal{SO}$ sets are $\mathcal{SO}$.*

(2) *Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ be a $SQPMS$ satisfying (Q3). Then $\langle \mathcal{G}, A \rangle$ is a $\mathcal{SO}$ set with respect to $\mathcal{P}$ iff $\mathcal{G}(\lambda)$ is open in $(X, \mathcal{P}_\lambda)$ for all $\lambda \in A$.*

(3) *If $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is a $SQPMS$ satisfying (Q3), then elementary intersection of two $\mathcal{SO}$ sets is $\mathcal{SO}$.*

Proof. (1) $\left\langle \tilde{\Phi}, A \right\rangle$ is trivially a $\mathcal{SO}$ set. Obviously, $SE(\tilde{X})$ is the set of all soft elements of $\tilde{X}$ and all of them are interior elements of $SE(\tilde{X})$. Then $SE(\tilde{X})$ is open and $\left\langle \tilde{X}, A \right\rangle = SS(SE(\tilde{X}))$ is soft open in $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$. Again let $\{\langle \mathcal{F}_\alpha, A \rangle\}_{\alpha \in A}$ be an arbitrary family of $\mathcal{SO}$ sets. If $\langle \mathcal{F}_\alpha, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$ $\forall \alpha \in A$, then $\Cup_{\alpha\in A} \langle \mathcal{F}_\alpha, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$, which is a $\mathcal{SO}$ set. Finally, let $\langle \mathcal{F}_\alpha, A\rangle \neq \left\langle \tilde{\Phi}, A\right\rangle$ for at least one $\alpha \in A$. Since $\langle \mathcal{F}_\alpha, A\rangle$ is $\mathcal{SO}$ for each α, there is a collection of $\mathcal{B}_\alpha$ of soft elements of $\langle \mathcal{F}_\alpha, A\rangle$ such that $\mathcal{B}_\alpha$ are open set and $\langle \mathcal{F}_\alpha, A\rangle = SS(\mathcal{B}_\alpha)$. Thus $\Cup_{\alpha\in A} \langle \mathcal{F}_\alpha, A\rangle = SS(\cup_{\alpha\in A}\mathcal{B}_\alpha)$ is $\mathcal{SO}$ set.

(2) Suppose $\langle \mathcal{G}, A\rangle$ is $\mathcal{SO}$. Then there is a collection $\mathcal{B}$ of soft elements of $\langle \mathcal{G}, A\rangle$ such that $\mathcal{B}$ is open and $\langle \mathcal{G}, A\rangle = SS(\mathcal{B})$. Let $x \in SS(\mathcal{B})(\lambda)$. Then there is $\tilde{x}$ in $\mathcal{B}$ such that $\tilde{x}(\lambda) = x$. Since $\mathcal{B}$ is open, there exists $\tilde{r}\tilde{>}\bar{0}$ such that $\tilde{x}\tilde{\in}B_{\mathcal{P}}(\tilde{x},\tilde{r}) \subset \mathcal{B}$, i.e., $x = \tilde{x}(\lambda) \in SS(B_{\mathcal{P}}(\tilde{x},\tilde{r}))(\lambda) \subset SS(\mathcal{B})(\lambda)$ and $SS(B_{\mathcal{P}}(\tilde{x},\tilde{r}))(\lambda)$ is an open ball in $(X, \mathcal{P}_\lambda)$. Thus x is an interior point of $SS(\mathcal{B})(\lambda)$. So $SS(\mathcal{B})(\lambda)$ is open in $(X, \mathcal{P}_\lambda)$. This is true for every $\lambda \in A$.

Conversely, suppose $\langle \mathcal{F}, A\rangle(\lambda) = SS(\mathcal{B})(\lambda)$ is open in $(X, \mathcal{P}_\lambda)$ for every $\lambda \in A$. Let $\tilde{x}\tilde{\in}SS(\mathcal{B})$. Then $\tilde{x}(\lambda) \in SS(\mathcal{B})(\lambda)$ for every $\lambda \in A$. Since $SS(\mathcal{B})(\lambda)$ is open in $(X, \mathcal{P}_\lambda)$, there exists open ball $B_\lambda(\tilde{x}(\lambda), r_\lambda)$ in $(X, \mathcal{P}_\lambda)$ such that $\tilde{x}(\lambda) \in B_\lambda(\tilde{x}(\lambda), r_\lambda) \subset SS(\mathcal{B})(\lambda)$ for every $\lambda \in A$. Consider the soft real number $\tilde{r}$ such that $\tilde{r}(\lambda) = r_\lambda$ $\forall\lambda \in A$. Then $\tilde{r}\tilde{>}\bar{0}$ and $B_{\mathcal{P}}(\tilde{x},\tilde{r})$ is a open ball in $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A\right\rangle$. Thus $\tilde{x} \in B_{\mathcal{P}}(\tilde{x},\tilde{r}) \subset SE(SS(\mathcal{B}))$ and so $SE(SS(\mathcal{B})) = \cup_{\tilde{x}\tilde{\in}SS(\mathcal{B})}B_{\mathcal{P}}(\tilde{x},\tilde{r})$. Since arbitrary union of open set is open, it follows that $SE(SS(\mathcal{B}))$ is open and $SS(\mathcal{B}) = SS(SE(SS(B_{\mathcal{P}}(\tilde{x},\tilde{r}))))$. Hence $SS(\mathcal{B}) = \langle \mathcal{F}, A\rangle$ is $\mathcal{SO}$ in $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A\right\rangle$.

(3) Let $\langle \mathcal{F}, A\rangle$ and $\langle \mathcal{G}, A\rangle$ be two $\mathcal{SO}$. Then there exist open sets $\mathcal{B}_1$, $\mathcal{B}_2$ in $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A\right\rangle$ such that $\langle \mathcal{F}, A\rangle = SS(\mathcal{B}_1)$ and $\langle \mathcal{G}, A\rangle = SS(\mathcal{B}_2)$. If $\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$, then $\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle$ is $\mathcal{SO}$. If $\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle \neq \left\langle \tilde{\Phi}, A\right\rangle$, then $[\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle](\lambda) = \mathcal{F}(\lambda) \cap \mathcal{G}(\lambda)$ $\forall\lambda \in A$. Since $\mathcal{F}(\lambda)$ and $\mathcal{G}(\lambda)$ are open in $(X, \mathcal{P}_\lambda)$ for all $\lambda \in A$, it follows that $[\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle](\lambda)$ is open in $(X, \mathcal{P}_\lambda)$ for all $\lambda \in A$. Thus by (2), $\langle \mathcal{F}, A\rangle \Cap \langle \mathcal{G}, A\rangle$ is $\mathcal{SO}$ set. □

Definition 2.9. Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A\right\rangle$ be a $SQPMS$. A soft set $\langle \mathcal{F}, A\rangle \in S(\tilde{X})$ is said to be a *soft closed* ($\mathcal{SC}$) *set*, if $\langle \mathcal{F}, A\rangle^C \in S(\tilde{X})$ and $\langle \mathcal{F}, A\rangle^C$ is $\mathcal{SO}$ in $\left\langle \tilde{X}, \mathcal{P}, A\right\rangle$.

Example 2.10. Consider the SQPMS as in Example 2.3. Let $\tilde{a}, \tilde{b} \tilde{\in} \left\langle \tilde{\mathbb{R}}, A\right\rangle$. If we define soft set $\langle \mathcal{G}, A\rangle$ by $\mathcal{G}(\lambda) = [\tilde{a}(\lambda), \tilde{b}(\lambda))$ $\forall$ $\lambda \in A$, then $\mathcal{G}(\lambda)$ is open set in $(\mathbb{R}, \mathcal{P}_\lambda)$. Thus $\langle \mathcal{G}, A\rangle$ is soft open set, i.e., $\mathcal{SO}$ w.r.t $\mathcal{P}$. Also, the soft set $\langle \mathcal{F}, A\rangle$ defined by $F(\lambda) = \mathbb{R} \setminus [\tilde{a}(\lambda), \tilde{b}(\lambda))$ $\forall$ $\lambda \in A$. is soft closed set, i.e., $\mathcal{SC}$ w.r.t $\mathcal{P}$, as $\mathcal{F}(\lambda)$ is closed in $(\mathbb{R}, \mathcal{P}_\lambda)$.

Proposition 2.11. *Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A\right\rangle$ be a $SQPMS$ satisfying* $(Q3)$. *Then $\langle \mathcal{F}, A\rangle \in S(\tilde{X})$ is $\mathcal{SC}$ in $\mathcal{P}$ iff $\mathcal{F}(\lambda)$ is proper closed set or X or Φ in $\langle X, \mathcal{P}_\lambda, \mathcal{Q}_\lambda\rangle$ for all $\lambda \in A$.*

Proof. Proof follows from Proposition 2.8 (2) and the definition of soft closed set. □

Proposition 2.12. *Let* $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ *be a SQPMS satisfying (Q3). Let* $\tau_{\mathcal{P}} = \{\langle \mathcal{F}, A\rangle \in S(\tilde{X}) : \tilde{x} \tilde{\in} \langle \mathcal{F}, A\rangle$ *implies there is a soft open set* $\langle \mathcal{G}, A\rangle$ *in* $\mathcal{P}$ *such that* $\tilde{x} \tilde{\in} \langle \mathcal{G}, A\rangle \tilde{\subset} \langle \mathcal{F}, A\rangle\}$. *Then* $\tau_{\mathcal{P}}$ *is an elementary soft topology on* $\left\langle \tilde{X}, A \right\rangle$.

Proof. Clearly $\left\langle \tilde{\Phi}, A \right\rangle$, $\left\langle \tilde{X}, A \right\rangle \in \tau_{\mathcal{P}}$. Let $\langle \mathcal{F}_1, A\rangle$, $\langle \mathcal{F}_2, A\rangle \in \tau_{\mathcal{P}}$. If $\langle \mathcal{F}_1, A\rangle \Cap \langle \mathcal{F}_2, A\rangle = \left\langle \tilde{\Phi}, A \right\rangle$, then $\langle \mathcal{F}_1, A\rangle \Cap \langle \mathcal{F}_2, A\rangle \in \tau_{\mathcal{P}}$. Consider $\langle \mathcal{F}_1, A\rangle \Cap \langle \mathcal{F}_2, A\rangle \neq \left\langle \tilde{\Phi}, A \right\rangle$. Let $\tilde{x} \tilde{\in} \langle \mathcal{F}_1, A\rangle \Cap \langle \mathcal{F}_2, A\rangle$. Then we have

$$\tilde{x} \tilde{\in} \langle \mathcal{F}_1, A\rangle\,,\ \tilde{x} \tilde{\in} \langle \mathcal{F}_2, A\rangle\,.$$

Thus there is soft open set $\langle \mathcal{G}_1, A\rangle$, $\langle \mathcal{G}_2, A\rangle$ in $\mathcal{P}$ such that

$$\tilde{x} \tilde{\in} \langle \mathcal{G}_1, A\rangle \tilde{\subset} \langle \mathcal{F}_1, A\rangle \text{ and } \tilde{x} \tilde{\in} \langle \mathcal{G}_2, A\rangle \tilde{\subset} \langle \mathcal{F}_2, A\rangle\,.$$

So we get

$$\tilde{x} \tilde{\in} \langle \mathcal{G}_1, A\rangle \Cap \langle \mathcal{G}_2, A\rangle \tilde{\subset} \langle \mathcal{F}_1, A\rangle \Cap \langle \mathcal{F}_2, A\rangle\,.$$

Since $\left\langle \tilde{X}, \mathcal{P}, A \right\rangle$ satisfies (Q3), $\langle \mathcal{G}_1, A\rangle \Cap \langle \mathcal{G}_2, A\rangle$ is a soft open set in $\mathcal{P}$. Hence $\langle \mathcal{F}_1, A\rangle \Cap \langle \mathcal{F}_2, A\rangle \in \tau_{\mathcal{P}}$.

Let $\langle \mathcal{F}_i, A\rangle \in \tau_{\mathcal{P}}$ for all $i \in \Delta$ and let $\tilde{x} \tilde{\in} \Cup_{i \in \Delta} \langle \mathcal{F}_i, A\rangle$.

Case **I**: Suppose there exists some $i \in \Delta$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}_i, A\rangle$. Then there exists a soft open set $\langle \mathcal{G}, A\rangle$ in $\mathcal{P}$ such that $\tilde{x} \tilde{\in} \langle \mathcal{G}, A\rangle \tilde{\subset} \langle \mathcal{F}_i, A\rangle \tilde{\subset} \Cup_{i \in \Delta} \langle \mathcal{F}_i, A\rangle$. Thus $\Cup_{i \in \Delta} \langle \mathcal{F}_i, A\rangle \in \tau_{\mathcal{P}}$.

Case **II**: Suppose $\tilde{x} \tilde{\notin} \langle \mathcal{F}_i, A\rangle$ for all $i \in \Delta$ and choose a $\lambda_1 \in \Delta$ such that $\tilde{x}(\alpha_1) \in \langle \mathcal{F}_{\lambda_1}, A\rangle(\alpha_1)$ for some $\alpha_1 \in A$. Choose a soft element $\tilde{y}_{\lambda_1} \tilde{\in} \langle \mathcal{F}_{\lambda_1}, A\rangle$ such that $\tilde{y}_{\lambda_1}(\alpha_1) = \tilde{x}(\alpha_1)$. Then there is a soft open set $\langle \mathcal{G}_{\lambda_1}, A\rangle$ in $\mathcal{P}$ such that $\tilde{y}_{\lambda_1} \tilde{\in} \langle \mathcal{G}_{\lambda_1}, A\rangle \tilde{\subset} \langle \mathcal{F}_{\lambda_1}, A\rangle$. Also choose a $\lambda_2 \in \Delta$ such that $\tilde{x}(\alpha_2) \in \langle \mathcal{F}_{\lambda_2}, A\rangle(\alpha_2)$ for some $\alpha_2 \in A$ and a soft element $\tilde{y}_{\lambda_2} \tilde{\in} \langle \mathcal{F}_{\lambda_2}, A\rangle$ such that $\tilde{y}_{\lambda_2}(\alpha_2) = \tilde{x}(\alpha_2)$. Then there is a soft open set $\langle \mathcal{G}_{\lambda_2}, A\rangle$ in $\mathcal{P}$ such that $\tilde{y}_{\lambda_2} \tilde{\in} \langle \mathcal{G}_{\lambda_2}, A\rangle \tilde{\subset} \langle \mathcal{F}_{\lambda_2}, A\rangle$. Proceeding this way for each $\alpha_i \in A$, we get $\lambda_i \in \Delta$ such that $\tilde{y}_{\lambda_i} \tilde{\in} \langle \mathcal{G}_{\lambda_i}, A\rangle \tilde{\subset} \langle \mathcal{F}_{\lambda_i}, A\rangle$, where $\langle \mathcal{G}_{\lambda_i}, A\rangle$ is a soft open set in $\mathcal{P}$ and $\tilde{y}_{\lambda_i}(\alpha_i) = \tilde{x}(\alpha_i)$. Thus we have

$$\tilde{x} \tilde{\in} \Cup_{\lambda_i \in A} \langle \mathcal{G}_{\lambda_i}, A\rangle \tilde{\subset} \Cup_{i \in \Delta} \langle \mathcal{F}_{\lambda_i}, A\rangle\,.$$

Since $\Cup_{\lambda_i \in A} \langle \mathcal{G}_{\lambda_i}, A\rangle$ is soft open in $\mathcal{P}$, we have $\Cup_{i \in \Delta} \langle \mathcal{F}_{\lambda_i}, A\rangle \in \tau_{\mathcal{P}}$. So $\tau_{\mathcal{P}}$ is an elementary soft topology on $\left\langle \tilde{X}, A \right\rangle$. □

Remark 2.13. Let $\mathcal{Q}(\ ,\ )$ be the soft conjugate of $\mathcal{P}(\ ,\ )$. Let $\tau_{\mathcal{Q}} = \{\langle \mathcal{F}, \mathcal{A}\rangle \in S(\tilde{X})\,;\ \tilde{x} \tilde{\in} \langle \mathcal{F}, A\rangle$ implies there is a soft open set $\langle \mathcal{G}, A\rangle$ in $\mathcal{Q}$ such that $\tilde{x} \tilde{\in} \langle \mathcal{G}, A\rangle \tilde{\subset} \langle \mathcal{F}, A\rangle\}$. Then $\tau_{\mathcal{Q}}$ is an elementary soft topology on $\left\langle \tilde{X}, A \right\rangle$. Thus omitting the condition of symmetry from a soft metric can give rise to two elementary soft topologies on $\left\langle \tilde{X}, A \right\rangle$ in the sense of Chiney and Samanta [19].

Definition 2.14. Let X be a universal set and A be a parameter set. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two arbitrary soft elementary topologies on $\left\langle \tilde{X}, A \right\rangle$ as in Definition 1.7. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is called a *soft elementary bitopological space* ($\mathcal{SEBS}$).

Example 2.15. Let $X = \{x, y, z\},\ A = \{\alpha, \beta\}$ and $\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}, A \rangle\}$, where $\mathcal{F}(\alpha) = \{x, y\},\ \mathcal{F}(\beta) = \{y, z\}$ and $\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{G}, A \rangle\}$, where $\mathcal{G}(\alpha) = \{x, z\}, \mathcal{G}(\beta) = \{x, y, z\}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{SEBS}$.

3. Pairwise soft T_0, T_1 and T_2 spaces

In this section, we discuss about some separation axioms in $\mathcal{SEBS}$ in weak and strong versions which arise for considering the pair of separating soft elements as unordered and ordered pairs. In the rest of the paper $(\tilde{x}, \tilde{y})$ will be considered as an ordered pair of soft elements and $\tilde{x}, \tilde{y}$ as unordered pair of soft elements. In a $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ the soft closure of a soft set $\langle \mathcal{F}, A \rangle$ with respect to the soft topology $\mathcal{T}$ will be denoted by $\overline{\langle \mathcal{F}, A \rangle}^{\mathcal{T}}$.

Definition 3.1. Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{SEBS}$ and $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be *weak pairwise soft* T_0 ($\mathcal{WPST}_0$), if there is $\langle \mathcal{F}, A \rangle \in S(\tilde{X})$ which is either $\mathcal{T}_1$- $\mathcal{SO}$ or $\mathcal{T}_2$- $\mathcal{SO}$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{y}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$ or $\tilde{y} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$.

Definition 3.2. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be *pairwise soft* T_0 ($\mathcal{PST}_0$), if for each pair of soft elements $(\tilde{x}, \tilde{y})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)\ \forall \lambda \in A$, either there is $\langle \mathcal{F}, A \rangle \in \mathcal{T}_1$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{y}(\lambda) \notin \mathcal{F}(\lambda)\ \forall \lambda \in A$ or there is $\langle \mathcal{G}, A \rangle \in \mathcal{T}_2$ such that $\tilde{y} \tilde{\in} \langle \mathcal{G}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{G}(\lambda)$ for all $\lambda \in A$.

Remark 3.3. Clearly a $\mathcal{PST}_0$ space is $\mathcal{WPST}_0$ but not conversely. Also a $\mathcal{PST}T_0$ space is not just a pair of two soft TT_0 topological spaces.

Example 3.4. Let $X = \{a, b\},\ A = \{\alpha, \beta\}$. Let $\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \bar{a}, A \rangle, \langle \bar{b}, A \rangle\}$ and $\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}, A \rangle, \langle \mathcal{G}, A \rangle\}$, where $\mathcal{F}(\alpha) = \{a\}, \mathcal{F}(\beta) = \{b\}; \mathcal{G}(\alpha) = \{b\}, \mathcal{G}(\beta) = \{a\}$ Then clearly $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{PST}_0$ space.

Example 3.5. Let $X = \{a, b\},\ A = \{\alpha, \beta\}$. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be the soft topologies on $\left\langle \tilde{X}, A \right\rangle$ given as follows:

$$\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \bar{a}, A \rangle, \langle \bar{b}, A \rangle, \langle \mathcal{F}_1, A \rangle, \langle \mathcal{F}_2, A \rangle, \langle \mathcal{F}_3, A \rangle\}$$

and

$$\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle\},$$

where $\mathcal{F}_1(\alpha) = \{a\}, \mathcal{F}_1(\beta) = \{b\}; \mathcal{F}_2(\alpha) = \{a\}, \mathcal{F}_2(\beta) = \{a, b\}; \mathcal{F}_3(\alpha) = \{a, b\}$, $\mathcal{F}_3(\beta) = \{b\}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{WPST}_0$ space but not $\mathcal{PST}_0$.

Definition 3.6. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *weak pairwise soft* T_1 *space* ($\mathcal{WPST}_1$), if for any $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$, there is $\langle \mathcal{F}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{G}, A\rangle \in \mathcal{T}_2$ such that either $\tilde{x} \tilde{\in} \langle \mathcal{F}, A\rangle$, $\tilde{y}(\lambda) \notin \mathcal{F}(\lambda)$ and $\tilde{y} \tilde{\in} \langle \mathcal{G}, A\rangle$, $\tilde{x}(\lambda) \notin \mathcal{G}(\lambda)$ for all $\lambda \in A$ or $\tilde{x} \tilde{\in} \langle \mathcal{G}, A\rangle$, $\tilde{y}(\lambda) \notin \mathcal{G}(\lambda)$ and $\tilde{y} \tilde{\in} \langle \mathcal{F}, A\rangle$, $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$.

Definition 3.7. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *pairwise soft* T_1 *space* ($\mathcal{PST}_1$), if for each pair of soft elements $(\tilde{x}, \tilde{y})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$ there is $\langle \mathcal{F}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{G}, A\rangle \in \mathcal{T}_2$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}, A\rangle$, $\tilde{y}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$ and $\tilde{y} \tilde{\in} \langle \mathcal{G}, A\rangle$, $\tilde{x}(\lambda) \notin \mathcal{G}(\lambda)$ for all $\lambda \in A$.

Remark 3.8. It is clear that $\mathcal{PST}_1$ space is $\mathcal{PST}_0$ and $\mathcal{WPST}_1$ space is $\mathcal{WPST}_0$. Also $\mathcal{PST}_1$ space is $\mathcal{WPST}_1$ but not conversely.

Example 3.9. Let $X = \{a, b\}$, $A = \{\alpha, \beta\}$ and let $\mathcal{T}_1$ and $\mathcal{T}_2$ be the soft topologies on $\left\langle \tilde{X}, A \right\rangle$ given as follows:

$$\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}_1, A\rangle, \langle \mathcal{F}_2, A\rangle, \langle \mathcal{F}_3, A\rangle\},$$

$$\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{G}_1, A\rangle, \langle \mathcal{G}_2, A\rangle, \langle \mathcal{G}_3, A\rangle\},$$

where $\mathcal{F}_1(\alpha) = \{a\}, \mathcal{F}_1(\beta) = \{a\}; \mathcal{F}_2(\alpha) = \{b\}, \mathcal{F}_2(\beta) = \{a\}; \mathcal{F}_3(\alpha) = \{a, b\}, \mathcal{F}_3(\beta) = \{a\}; \mathcal{G}_1(\alpha) = \{b\}, \mathcal{G}_1(\beta) = \{b\}; \mathcal{G}_2(\alpha) = \{a\}, \mathcal{G}_2(\beta) = \{b\}; \mathcal{G}_3(\alpha) = \{a, b\}, \mathcal{G}_3(\beta) = \{b\}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is $\mathcal{WPST}_1$ space. Consider the soft elements $\tilde{\xi_1}, \tilde{\xi_2}$, where $\tilde{\xi_1}(\alpha) = a, \tilde{\xi_1}(\beta) = b$ and $\tilde{\xi_2}(\alpha) = b, \tilde{\xi_2}(\beta) = a$. Then for the pair $(\tilde{\xi_1}, \tilde{\xi_2})$, there does not exist any $\mathcal{T}_1$- $\mathcal{SO}$ set containing $\tilde{\xi_1}$ and any $\mathcal{T}_2$- $\mathcal{SO}$ set containing $\tilde{\xi_2}$. Thus $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is not $\mathcal{PST}_1$ space.

Remark 3.10. The following examples show that $\mathcal{WPST}_\circ$ is not equivalent to $\mathcal{WPST}_1$ and $\mathcal{PST}_0$ is not equivalent to $\mathcal{PST}_1$. Also $\mathcal{WPST}_1$ does not imply $\mathcal{PST}_0$.

Example 3.11. Let $X = \{a, b\}$, $A = \{\alpha, \beta\}$ and let $\mathcal{T}_1$ and $\mathcal{T}_2$ be the soft topologies on $\left\langle \tilde{X}, A \right\rangle$ given as follows:

$$\begin{aligned} \mathcal{T}_1 = \{&\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}_1 A\rangle, \langle \mathcal{F}_2, A\rangle, \langle \mathcal{F}_3, A\rangle, \langle \mathcal{F}_4, A\rangle, \langle \mathcal{F}_5, A\rangle, \\ &\langle \mathcal{F}_6, A\rangle, \langle \mathcal{F}_7, A\rangle, \langle \mathcal{F}_8, A\rangle\}, \\ \mathcal{T}_2 = \{&\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle\}, \end{aligned}$$

where $\mathcal{F}_1(\alpha) = \{a\}, \mathcal{F}_1(\beta) = \{a\}$; $\mathcal{F}_2(\alpha) = \{b\}, \mathcal{F}_2(\beta) = \{b\}; \mathcal{F}_3(\alpha) = \{a\}, \mathcal{F}_3(\beta) = \{b\}; \mathcal{F}_4(\alpha) = \{b\}, \mathcal{F}_4(\beta) = \{a\}; \mathcal{F}_5(\alpha) = \{a, b\}, \mathcal{F}_5(\beta) = \{a\}; \mathcal{F}_6(\alpha) = \{a, b\}, \mathcal{F}_6(\beta) = \{b\}; \mathcal{F}_7(\alpha) = \{a\}, \mathcal{F}_7(\beta) = \{a, b\}; \mathcal{F}_8(\alpha) = \{b\}, \mathcal{F}_8(\beta) = \{a, b\}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is $\mathcal{WPST}_0$ but not $\mathcal{WPST}_1$. Consider the $\mathcal{SBS}$ in Example 3.4 which is clearly $\mathcal{PST}_0$. Now for the pair of soft elements $(\bar{a}, \bar{b})$, there does not exist any $\mathcal{T}_2$- $\mathcal{SO}$ set containing the soft element $\bar{b}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is not $\mathcal{PST}_1$.

Example 3.12. Let $X = \{a, b\},\ A = \{\alpha, \beta\}$ and let $\mathcal{T}_1$ and $\mathcal{T}_2$ be the soft topologies on $\left\langle \tilde{X}, A \right\rangle$ given as follows:

$$\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}_1, A \rangle, \langle \mathcal{F}_2, A \rangle, \langle \mathcal{F}_3, A \rangle\},$$

$$\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{G}_1, A \rangle, \langle \mathcal{G}_2, A \rangle, \langle \mathcal{G}_3, A \rangle\},$$

where $\mathcal{F}_1(\alpha) = \{a\}, \mathcal{F}_1(\beta) = \{a\}; \mathcal{F}_2(\alpha) = \{b\}, \mathcal{F}_2(\beta) = \{a\}; \mathcal{F}_3(\alpha) = \{a, b\}, \mathcal{F}_3(\beta) = \{a\}; \mathcal{G}_1(\alpha) = \{b\}, \mathcal{G}_1(\beta) = \{b\}; \mathcal{G}_2(\alpha) = \{a\}, \mathcal{G}_2(\beta) = \{b\}; \mathcal{G}_3(\alpha) = \{a, b\}, \mathcal{G}_3(\beta) = \{b\}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{WPST}_1$ space. Consider the pair of soft element $(\bar{b}, \bar{a})$. Then there does not exist any $\mathcal{T}_1$- soft $\mathcal{SO}$ set containing $\bar{b}$ and any $\mathcal{T}_2$- $\mathcal{SO}$ set containing $\bar{a}$. Thus $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is not $\mathcal{PST}_0$.

Proposition 3.13. *$\mathcal{PST}_1$ is equivalent to soft T_1 in both soft topologies.*

Proof. Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{PST}_1$ space. Let $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then by $\mathcal{PST}_1$, for the pair of soft elements $(\tilde{x}, \tilde{y})$, $\exists\ \langle \mathcal{F}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{G}, A \rangle \in \mathcal{T}_2$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{y}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$ and $\tilde{y} \tilde{\in} \langle \mathcal{G}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{G}(\lambda)$ for all $\lambda \in A$. Again for the pair of soft elements $(\tilde{y}, \tilde{x})$, there exists $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that $\tilde{y} \tilde{\in} \langle \mathcal{U}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{U}(\lambda)\ \forall \lambda \in A$ and $\tilde{x} \tilde{\in} \langle \mathcal{V}, A \rangle,\ \tilde{y}(\lambda) \notin \mathcal{V}(\lambda)\ \forall \lambda \in A$. Thus $\left\langle \tilde{X}, \mathcal{T}_1, A \right\rangle$ and $\left\langle \tilde{X}, \mathcal{T}_2, A \right\rangle$ are soft T_1.

Conversely, let $\left\langle \tilde{X}, \mathcal{T}_1, A \right\rangle$ and $\left\langle \tilde{X}, \mathcal{T}_2, A \right\rangle$ be two soft T_1 topological spaces. Let $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Consider the pair $(\tilde{x}, \tilde{y})$. Since $\left\langle \tilde{X}, \mathcal{T}_1, A \right\rangle$ is soft T_1, there exists $\langle \mathcal{F}, A \rangle,\ \langle \mathcal{G}, A \rangle \in \mathcal{T}_1$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}, \mathcal{A} \rangle$, $\tilde{y}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$ and $\tilde{y} \tilde{\in} \langle \mathcal{G}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{G}(\lambda)$ for all $\lambda \in A$. Again since $\left\langle \tilde{X}, \mathcal{T}_2, A \right\rangle$ is soft T_1, there exists $\langle \mathcal{U}, A \rangle,\ \langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle,\ \tilde{y}(\lambda) \notin \mathcal{U}(\lambda)$ for all $\lambda \in A$ and $\tilde{y} \tilde{\in} \langle \mathcal{V}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{V}(\lambda)$ for all $\lambda \in A$. Thus for the pair $(\tilde{x}, \tilde{y})$, we get a $\mathcal{T}_1$- $\mathcal{SO}$ set $\langle \mathcal{F}, A \rangle$ and a $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{V}, A \rangle$ such that $\tilde{x} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{y}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$ and $\tilde{y} \tilde{\in} \langle \mathcal{V}, A \rangle,\ \tilde{x}(\lambda) \notin \mathcal{V}(\lambda)$ for all $\lambda \in A$. So $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is $\mathcal{PST}_1$ space. □

Definition 3.14. Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{SEBS}$ and $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be *weak pairwise soft* T_2 ($\mathcal{WPST}_2$), if there exist $\langle \mathcal{F}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{G}, A \rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{F}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle =< \tilde{\Phi}, A > \text{ and}$$

$$\text{either } \tilde{x} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{y} \tilde{\in} \langle \mathcal{G}, A \rangle \text{ or } \tilde{y} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{x} \tilde{\in} \langle \mathcal{G}, A \rangle.$$

Definition 3.15. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *pairwise soft T_2 space* ($\mathcal{PST}_2$), if for any pair of soft elements $(\tilde{x}, \tilde{y})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)\ \forall \lambda \in A$, there exists $\langle \mathcal{F}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{G}, A \rangle \in \mathcal{T}_2$ such that $\langle \mathcal{F}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$ and $\tilde{x} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{y} \tilde{\in} \langle \mathcal{G}, A \rangle$.

Remark 3.16. Clearly $\mathcal{WPST}_2$ implies $\mathcal{WPST}_1$ and $\mathcal{PST}_2$ implies $\mathcal{PST}_1$. Also $\mathcal{PST}_2$ implies $\mathcal{WPST}_2$ but not conversely. Consider the $\mathcal{SBS}$ of Example 3.12 which is $\mathcal{WPST}_2$. Now consider the pair of soft elements $(\bar{b}, \bar{a})$. Then there does not exists any $\mathcal{T}_1$- $\mathcal{SO}$ set containing $\bar{b}$ and any $\mathcal{T}_2$- $\mathcal{SO}$ set containing $\bar{a}$. Thus $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is not a $\mathcal{PST}_2$ space. $\mathcal{WPST}_2$ does not imply $\mathcal{PST}_1$. Consider the $\mathcal{SBS}$ of Example 3.12 which is $\mathcal{WPST}_2$ but for the pair of soft elements $(\bar{b}, \bar{a})$. Then there does not exist any $\mathcal{T}_1$- $\mathcal{SO}$ set containing $\bar{b}$ and any $\mathcal{T}_2$- $\mathcal{SO}$ set containing $\bar{a}$. Thus $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is not a $\mathcal{PST}_1$ space. The following example shows that $\mathcal{PST}_1$ is not equivalent to $\mathcal{PST}_2$ and $\mathcal{WPST}_1$ is not equivalent to $\mathcal{WPST}_2$.

Example 3.17. Let X be an uncountable set and A be a parameter set. Let $\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle\} \cup \{\langle \mathcal{F}, A \rangle \tilde{\subseteq} \left\langle \tilde{X}, A \right\rangle : X \backslash \mathcal{F}(\alpha)$ is a finite subset of X for each $\alpha \in A\}$. Let $\tilde{x},\ \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then $\tilde{x} \tilde{\in} \langle \tilde{y}, A \rangle^C,\ \tilde{y} \tilde{\in} \langle \tilde{x}, A \rangle^C$ and $\langle \tilde{y}, A \rangle^C,\ \langle \tilde{x}, A \rangle^C$ are $\mathcal{T}_1$- $\mathcal{SO}$ set. Thus $\left\langle \tilde{X}, \mathcal{T}_1, A \right\rangle$ is a soft T_1 space.

Let $\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle\} \cup \{\langle \mathcal{F}, A \rangle \tilde{\subseteq} \left\langle \tilde{X}, A \right\rangle : X \backslash \mathcal{F}(\alpha)$ is a countable subset of X for each $\alpha \in A\}$. Then similarly, it can be shown that $\left\langle \tilde{X}, \mathcal{T}_2, A \right\rangle$ is a soft T_1 space. Thus $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{PST}_1$ space and so a $\mathcal{WPST}_1$ space. If possible, let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{PST}_2$ space. Then for any $(\tilde{x}, \tilde{y})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$, there exists $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle,\ \tilde{y} \tilde{\in} \langle \mathcal{V}, A \rangle$ and $\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$. Thus $X \backslash \mathcal{U}(\alpha)$ is finite and $X \backslash \mathcal{V}(\alpha)$ is countable for each $\alpha \in A$. Again $\langle \mathcal{U}, A \rangle^C \Cup \langle \mathcal{V}, A \rangle^C = \left\langle \tilde{X}, A \right\rangle$ which implies X is countable which is a contradiction. So $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is neither a $\mathcal{PST}_2$ space nor a $\mathcal{WPST}_2$ space.

4. Pairwise soft regular and pairwise soft normal spaces

Definition 4.1. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be:

(i) $(\mathcal{T}_1)R_0$ *with respect to* $\mathcal{T}_2$, if for any $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and for any soft element $\tilde{x}$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exists a $\mathcal{T}_1$- $\mathcal{SO}$ set $\langle \mathcal{U}, A \rangle$ such that $\tilde{x}(\lambda) \notin \mathcal{U}(\lambda)$ for all $\lambda \in A$ and $\langle \mathcal{F}, A \rangle \tilde{\subseteq} \langle \mathcal{U}, A \rangle$,

(ii) $(\mathcal{T}_2)R_0$ *with respect to* $\mathcal{T}_1$, if for any $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and for any soft element $\tilde{x}$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exists a $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{V}, A \rangle$ such that $\tilde{x}(\lambda) \notin \mathcal{V}(\lambda)$ for all $\lambda \in A$ and $\langle \mathcal{F}, A \rangle \tilde{\subseteq} \langle \mathcal{V}, A \rangle$.

Definition 4.2. A $\mathcal{SBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *pairwise soft* R_0 *space* ($\mathcal{PSR}_\circ$), if it is $(\mathcal{T}_1)R_0$ with respect to $\mathcal{T}_2$ and $(\mathcal{T}_2)R_0$ with respect to $\mathcal{T}_1$.

Proposition 4.3. *$\mathcal{PST}_0$ and $\mathcal{PSR}_0$ together in a $\mathcal{SEBS}$ is equivalent to $\mathcal{PST}_1$.*

Proof. Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{PST}_1$ space. Then it is $\mathcal{PST}_0$. To prove $(\mathcal{T}_2)R_0$, let $\langle \mathcal{F}, A \rangle$ be a $\mathcal{T}_1$- $\mathcal{SC}$ set and $\tilde{x}$ be a soft element with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$. Then for any $\tilde{y} \tilde{\in} \langle \mathcal{F}, A \rangle,\ \tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. By $\mathcal{PST}_1$, for the pair of soft element $(\tilde{x}, \tilde{y})$, there exists a $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{V}_{\tilde{y}}, A \rangle$ containing $\tilde{y}$ and $\tilde{x}(\lambda) \notin \mathcal{V}_{\tilde{y}}(\lambda)$ for all $\lambda \in A$.

Thus $\langle \mathcal{F}, A\rangle \tilde{\subseteq} \underset{\tilde{y}\tilde{\in}\langle \mathcal{F},A\rangle}{\Cup} \langle \mathcal{V}_{\tilde{y}}, A\rangle$. So $\underset{\tilde{y}\tilde{\in}\langle \mathcal{F},A\rangle}{\Cup} \langle \mathcal{V}_{\tilde{y}}, A\rangle \in \mathcal{T}_2$ and $\tilde{x}(\lambda)\notin \underset{\tilde{y}\tilde{\in}\langle \mathcal{F},A\rangle}{\Cup} V_{\tilde{y}}(\lambda)$ for all $\lambda \in A$. Hence $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $(\mathcal{T}_2)R_0$ w.r.t $\mathcal{T}_1$. Similarly, it can be proved that $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $(\tau_1)R_0$ w.r.t τ_2. Therefore $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $\mathcal{PSR}_0$.

Conversely, suppose $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $\mathcal{PST}_0$ and $\mathcal{PSR}_0$. Let $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then by $\mathcal{PST}_0$, for the pair of soft element $(\tilde{x}, \tilde{y})$, two cases may arise.

Case I : Suppose there exists $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ such that $\tilde{x}\tilde{\in}\langle \mathcal{U}, A\rangle$ and $\tilde{y}(\lambda)\notin \mathcal{U}(\lambda)$ for all $\lambda \in A$. Then $\tilde{x}(\lambda)\notin \mathcal{U}^C(\lambda)$ for all $\lambda \in A$, where $\langle \mathcal{U}, A\rangle^C$ is a $\mathcal{T}_1$- $\mathcal{SC}$ set. Then by $\mathcal{PSR}_0$, there is $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that $\tilde{x}(\lambda)\notin \mathcal{V}(\lambda)$ for all $\lambda \in A$ and $\langle \mathcal{U}, A\rangle^C \tilde{\subset} \langle \mathcal{V}, A\rangle$. Thus we have $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that $\tilde{x}\tilde{\in}\langle \mathcal{U}, A\rangle$, $\tilde{y}(\lambda)\notin \mathcal{U}(\lambda)$ for all $\lambda \in A$ and $\tilde{y}\tilde{\in}\langle \mathcal{V}, A\rangle$, $\tilde{x}(\lambda)\notin \mathcal{V}(\lambda)$ for all $\lambda \in A$.

Case II: Suppose for the pair of soft elements $(\tilde{x}, \tilde{y})$, $\exists$ $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that $\tilde{y}\tilde{\in}\langle \mathcal{V}, A\rangle$, $\tilde{x}(\lambda)\notin \langle \mathcal{V}, A\rangle(\lambda)$ for all $\lambda \in A$. Then $\tilde{y}(\lambda)\notin \mathcal{V}^C(\lambda)$ for all $\lambda \in A$, where $\langle \mathcal{V}, A\rangle^C$ is a $\mathcal{T}_2$- $\mathcal{SC}$ set. Thus by $\mathcal{PSR}_0$, there exists $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ such that $\tilde{y}\notin \mathcal{U}(\lambda)$ for all $\lambda \in A$ and $\langle \mathcal{V}, A\rangle^C \tilde{\subset} \langle \mathcal{U}, A\rangle$. So we have $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that $\tilde{x}\tilde{\in}\langle \mathcal{U}, A\rangle$, $\tilde{y}(\lambda)\notin \mathcal{U}(\lambda)$ for all $\lambda \in A$ and $\tilde{y}\tilde{\in}\langle \mathcal{V}, A\rangle$, $\tilde{x}(\lambda)\notin \mathcal{V}(\lambda)$ for all $\lambda \in A$. Hence in both cases, $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $\mathcal{PST}_1$. □

Definition 4.4. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is said to be:

(i) $(\mathcal{T}_1)R_1$ *with respect to* $\mathcal{T}_2$, if for any soft element $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$, $\tilde{x}(\lambda)\notin \overline{\langle \tilde{y}, A\rangle}^{\mathcal{T}_2}(\lambda)$ for all $\lambda \in A$, there exist $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{U}, A\rangle \tilde{\cap} \langle \mathcal{V}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle \text{ and } \tilde{x}\tilde{\in}\langle \mathcal{V}, A\rangle,\ \overline{\langle \tilde{y}, A\rangle}^{\mathcal{T}_2} \tilde{\subseteq} \langle \mathcal{U}, A\rangle,$$

(ii) $(\mathcal{T}_2)R_1$ *with respect to* $\mathcal{T}_1$, if for any soft element $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$, $\tilde{x}(\lambda)\notin \overline{\langle \tilde{y}, A\rangle}^{\mathcal{T}_1}(\lambda)$ for all $\lambda \in A$, then there exist $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{U}, A\rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{U}, A\rangle \tilde{\cap} \langle \mathcal{V}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle \text{ and } \tilde{x}\tilde{\in}\langle \mathcal{U}, A\rangle,\ \overline{\langle \tilde{y}, A\rangle}^{\mathcal{T}_1} \tilde{\subseteq} \langle \mathcal{V}, A\rangle.$$

Definition 4.5. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is said to be a *pairwise soft* R_1 *space* ($\mathcal{PSR}_1$), if it is $(\mathcal{T}_1)R_1$ with respect to $\mathcal{T}_2$ and $(\mathcal{T}_2)R_1$ with respect to $\mathcal{T}_1$.

Proposition 4.6. *$\mathcal{PST}_1$ and $\mathcal{PSR}_1$ together in a $\mathcal{SEBS}$ is equivalent to $\mathcal{PST}_2$.*

Proof. Suppose $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $\mathcal{PST}_2$ space. Then it is $\mathcal{PST}_1$. Thus by Proposition 3.13, $\left\langle \tilde{X}, \mathcal{T}_1, A\right\rangle$ and $\left\langle \tilde{X}, \mathcal{T}_2, A\right\rangle$ are soft T_1 spaces. So $\overline{\langle \tilde{x}, A\rangle}^{\mathcal{T}_1} = \overline{\langle \tilde{x}, A\rangle}^{\mathcal{T}_2} = \langle \tilde{x}, A\rangle$. Since $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $\mathcal{PST}_2$ space, for any soft element $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$, $\tilde{x}(\lambda)\notin \overline{\langle \tilde{y}, A\rangle}^{\mathcal{T}_1}(\lambda)$ for all $\lambda \in A$, there exist $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that $\langle \mathcal{U}, A\rangle \tilde{\cap} \langle \mathcal{V}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$ and $\tilde{x}\tilde{\in}\langle \mathcal{U}, A\rangle$, $\overline{\langle \tilde{y}, A\rangle}^{\mathcal{T}_1} \tilde{\subseteq} \langle \mathcal{V}, A\rangle$. Hence it is $\mathcal{PSR}_1$.

Conversely, let us take $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ as $\mathcal{PST}_1$ and $\mathcal{PSR}_1$ space. Let $(\tilde{x}, \tilde{y})$ be any pair of soft elements with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then $\tilde{x}(\lambda) \notin \overline{\langle \tilde{y}, A \rangle}^{\mathcal{T}_1}(\lambda)$ for all $\lambda \in A$. Thus by $\mathcal{PSR}_1$, there exist $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that $\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$ and $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle$, $\overline{\langle \tilde{y}, A \rangle}^{\mathcal{T}_1} \tilde{\subset} \langle \mathcal{V}, A \rangle$. So $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle$, $\tilde{y} \tilde{\in} \langle \mathcal{V}, A \rangle$ and $\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$. Hence $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is $\mathcal{PST}_2$. □

Definition 4.7. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be:

(i) *soft $\mathcal{T}_1$ regular with respect to $\mathcal{T}_2$*, if for any $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and for any soft element $\tilde{x}$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exist $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle \text{ and } \tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle, \ \langle \mathcal{F}, A \rangle \tilde{\subseteq} \langle \mathcal{V}, A \rangle,$$

(ii) *soft $\mathcal{T}_2$ regular with respect to $\mathcal{T}_1$*, if for any $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and for any soft element $\tilde{x}$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exist $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle \text{ and } \tilde{x} \tilde{\in} \langle \mathcal{V}, A \rangle, \ \langle \mathcal{F}, A \rangle \tilde{\subseteq} \langle \mathcal{U}, A \rangle.$$

Definition 4.8. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *pairwise soft regular space* ($\mathcal{PSR}_2$), if it is soft $\mathcal{T}_1$ regular with respect to $\mathcal{T}_2$ and soft $\mathcal{T}_2$ regular with respect to $\mathcal{T}_1$.

Proposition 4.9. (1) *A $\mathcal{PSR}_2$ space is $\mathcal{PSR}_1$.*
(2) *A $\mathcal{PSR}_1$ space is $\mathcal{PSR}_0$.*

Example 4.10. Let $X = \{x, y, z\}, A = \{\alpha, \beta\}$, $\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}, A \rangle\}$ and $\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{G}, A \rangle\}$, where $\mathcal{F}(\alpha) = \{x, y\}, \mathcal{F}(\beta) = \{z\}$ and $\mathcal{G}(\alpha) = \{z\}, \mathcal{G}(\beta) = \{x, y\}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is soft $\mathcal{T}_1$ regular with respect to $\mathcal{T}_2$ and soft $\mathcal{T}_2$ regular with respect to $\mathcal{T}_1$, i.e., pairwise soft regular space ($\mathcal{PSR}_2$). Thus it is also $\mathcal{PSR}_1$ and $\mathcal{PSR}_0$.

Proposition 4.11. *Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{SEBS}$. Then the following statements are equivalent :*

(1) $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ *is soft $\mathcal{T}_1$ regular with respect to $\mathcal{T}_2$,*
(2) *for any soft element $\tilde{x}$ and $\mathcal{T}_1$- $\mathcal{SO}$ set $\langle \mathcal{U}, A \rangle$ containing $\tilde{x}$ such that $\langle \mathcal{U}, A \rangle^C \neq \left\langle \tilde{\Phi}, A \right\rangle$, there exists $\langle \mathcal{V}, A \rangle \in \mathcal{T}_1$ such that $\tilde{x} \tilde{\in} \langle \mathcal{V}, A \rangle \tilde{\subset} \overline{\langle \mathcal{V}, A \rangle}^{\mathcal{T}_2} \tilde{\subset} \langle \mathcal{U}, A \rangle$,*
(3) *for any soft element $\tilde{x}$ and $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exists $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ such that $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle$ and $\overline{\langle \mathcal{U}, A \rangle}^{\mathcal{T}_2} \tilde{\cap} \langle \mathcal{F}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$.*

Proof. (1) ⇒ (2) : Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be soft $\mathcal{T}_1$ regular with respect to $\mathcal{T}_2$. Let $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle$. Then $\tilde{x}(\lambda) \notin \mathcal{U}^C(\lambda)$ $\forall \lambda \in A$, where $\langle \mathcal{U}, A \rangle^{\mathbb{C}}$ is a $\mathcal{T}_1$- $\mathcal{SC}$

set. Thus by the assumption, there exist $\langle \mathcal{V}, A\rangle \in \mathcal{T}_1$, $\langle \mathcal{F}, A\rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{V}, A\rangle \tilde{\cap} \langle \mathcal{F}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle \text{ and } \tilde{x} \tilde{\in} \langle \mathcal{V}, A\rangle,\ \langle \mathcal{U}, A\rangle^C \tilde{\subset} \langle \mathcal{F}, A\rangle .$$

Since $\langle \mathcal{V}, A\rangle \tilde{\cap} \langle \mathcal{F}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$, $\langle \mathcal{V}, A\rangle \tilde{\subset} \langle \mathcal{F}, A\rangle^{\mathbb{C}}$, where $\langle \mathcal{F}, A\rangle^C$ is a $\mathcal{T}_2$- $\mathcal{SO}$ set. So $\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_2} \tilde{\subset} \langle \mathcal{F}, A\rangle^C$. Again $\langle \mathcal{F}, A\rangle^C \tilde{\subset} \langle \mathcal{U}, A\rangle$. Hence $\tilde{x} \tilde{\in} \langle \mathcal{V}, A\rangle \tilde{\subset} \overline{\langle \mathcal{F}, A\rangle}^{\mathcal{T}_2} \tilde{\subset} \langle \mathcal{U}, A\rangle$.

(2) $\Rightarrow$ (3) : Let $\langle \mathcal{F}, A\rangle$ be any $\mathcal{T}_1$- $\mathcal{SC}$ set and $\tilde{x}$ be any soft element such that $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$. Then $\tilde{x} \tilde{\in} \langle \mathcal{F}, A\rangle^C$, where $\langle \mathcal{F}, A\rangle^{\mathbb{C}}$ is a $\mathcal{T}_1$ - $\mathcal{SO}$ set. Thus by the assumption, there exists $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ such that $\tilde{x} \tilde{\in} \langle \mathcal{U}, A\rangle$ and $\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2} \tilde{\subset} \langle \mathcal{F}, A\rangle^C$. So $\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2} \tilde{\cap} \langle \mathcal{F}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$.

(3) $\Rightarrow$ (1) : Let $\langle \mathcal{F}, A\rangle$ be any $\mathcal{T}_1$- $\mathcal{SC}$ set and $\tilde{x}$ be any soft element such that $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$. Then by the assumption, there exists $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ such that $\tilde{x} \tilde{\in} \langle \mathcal{U}, A\rangle$ and $\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2} \tilde{\cap} \langle \mathcal{F}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$. Thus we have

$$\langle \mathcal{F}, A\rangle \tilde{\subset} (\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2})^C,$$

where $(\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2})^C$ is a $\mathcal{T}_2$- $\mathcal{SO}$ set and $\langle \mathcal{U}, A\rangle \tilde{\cap} (\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2})^C = \left\langle \tilde{\Phi}, A\right\rangle$. So $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is soft $\mathcal{T}_1$ regular with respect to $\mathcal{T}_2$. □

Proposition 4.12. *Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ be a $\mathcal{SEBS}$. Then the following statements are equivalent*

1. $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ *is soft $\mathcal{T}_2$ regular with respect to $\mathcal{T}_1$,*
2. *for any soft element $\tilde{x}$ and $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{V}, A\rangle$ containing $\tilde{x}$ such that $\langle \mathcal{V}, A\rangle^C \neq \left\langle \tilde{\Phi}, A\right\rangle$, there exists $\langle \mathcal{U}, A\rangle \in \mathcal{T}_2$ such that $\tilde{x} \tilde{\in} \langle \mathcal{U}, A\rangle \tilde{\subset} \overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1} \tilde{\subset} \langle \mathcal{V}, A\rangle$,*
3. *for any soft element $\tilde{x}$ and $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{F}, A\rangle$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exists $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that $\tilde{x} \tilde{\in} \langle \mathcal{V}, A\rangle$ and $\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1} \tilde{\cap} \langle \mathcal{F}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$.*

Definition 4.13. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is said to be a *weak pairwise soft T_3 space* ($\mathcal{WPST}_3$), if it is $\mathcal{PSR}_2$ and $\mathcal{WPST}_1$.

Definition 4.14. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is said to be a *pairwise soft T_3 space* ($\mathcal{PST}_3$), if it is $\mathcal{PSR}_2$ and $\mathcal{PST}_1$.

Proposition 4.15. (1) *A $\mathcal{PST}_3$ space is $\mathcal{PST}_2$.*
(2) *A $\mathcal{WPST}_3$ space is $\mathcal{WPST}_2$.*

Example 4.16. Consider the set $\mathbb{R}$ of real numbers and for all $\alpha \in A$ and let $\mathcal{T}_\alpha$ be the usual topology on $\mathbb{R}$. Let $\mathcal{T} = \{\langle \mathcal{F}, A\rangle \in S(\tilde{\mathbb{R}}) : \mathcal{F}(\alpha) \in \mathcal{T}_\alpha\}$ and $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}$. Then $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is pairwise soft T_3 space ($\mathcal{PST}_3$). Thus it is also $\mathcal{PST}_2$.

Remark 4.17. A $\mathcal{WPST}_3$ space is not $\mathcal{PST}_2$. Consider the Example 3.9. Then for the pair of soft elements $(\bar{b}, \bar{a})$, there does not exist any $\mathcal{T}_1$- $\mathcal{SO}$ set $\langle \mathcal{F}, A\rangle$ containing $\bar{b}$ and any $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{G}, A\rangle$ containing $\bar{a}$ such that $\langle \mathcal{F}, A\rangle \tilde{\cap} \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$.

Proposition 4.18. *$\mathcal{PSR}_2$ and $\mathcal{PST}_2$ together in a $\mathcal{SEBS}$ is equivalent to $\mathcal{PST}_3$.*

Proof. Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{PST}_3$ space. Then it is $\mathcal{PSR}_2$. To prove $\mathcal{PST}_2$, consider a pair of soft elements $(\tilde{x}, \tilde{y})$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Since $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is $\mathcal{PST}_1$, $\langle \tilde{x}, A \rangle$, $\langle \tilde{y}, A \rangle$ are $\mathcal{T}_1$ and $\mathcal{T}_2$- $\mathcal{SC}$ set. Then $\tilde{x}(\lambda) \notin \overline{\langle \tilde{y}, A \rangle}^{\mathcal{T}_1}(\lambda)$ for all $\lambda \in A$. Thus there exist $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$, $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that $\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$ and $\tilde{x} \tilde{\in} \langle \mathcal{U}, A \rangle$, $\overline{\langle \tilde{y}, A \rangle}^{\mathcal{T}_1} \tilde{\subset} \langle \mathcal{V}, A \rangle$, i.e., $\tilde{y} \tilde{\in} \langle \mathcal{V}, A \rangle$. So $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{PST}_2$ space.

The converse is straightforward. □

Definition 4.19. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be *pairwise soft normal* ($\mathcal{PSR}_3$), if for any $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and a $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{G}, A \rangle$ with $\langle \mathcal{F}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$, there exist $\langle \mathcal{U}, A \rangle \in \mathcal{T}_1$ and $\langle \mathcal{V}, A \rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{F}, A \rangle \tilde{\subset} \langle \mathcal{V}, A \rangle, \ \langle \mathcal{G}, A \rangle \tilde{\subset} \langle \mathcal{U}, A \rangle \ \text{ and } \ \langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle.$$

Definition 4.20. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *weak pairwise soft T_4 space* ($\mathcal{WPST}_4$), if it is $\mathcal{PSR}_3$ and $\mathcal{WPST}_1$.

Definition 4.21. A $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is said to be a *pairwise soft T_4 space* ($\mathcal{PST}_4$), if it is $\mathcal{PSR}_3$ and $\mathcal{PST}_1$.

Remark 4.22. A $\mathcal{PSR}_3$ space is not $\mathcal{PSR}_2$.

Example 4.23. Let $X = \{a, b\}$, $A = \{\alpha, \beta\}$. Let $\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{F}, A \rangle\}$, $\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A \right\rangle, \left\langle \tilde{X}, A \right\rangle, \langle \mathcal{G}, A \rangle\}$, where $\mathcal{F}(\alpha) = \{b\}, \mathcal{F}(\beta) = \{a\}; \mathcal{G}(\alpha) = \{b\}, \mathcal{G}(\beta) = \{b\}$. Consider the $\mathcal{SEBS}$ $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$. Then it is $\mathcal{PSR}_3$. Since there does not exist any disjoint $\mathcal{SO}$ set of $\mathcal{T}_1$ and $\mathcal{T}_2$, it is not $\mathcal{PSR}_2$.

Proposition 4.24. *Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ be a $\mathcal{SEBS}$. Then the following statements are equivalent :*

(1) $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ *is $\mathcal{PSR}_3$,*

(2) *for any $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and any $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{V}, A \rangle$ [$\langle \mathcal{V}, A \rangle^C \neq \left\langle \tilde{\Phi}, A \right\rangle$] containing $\langle \mathcal{F}, A \rangle$, there exists $\langle \mathcal{U}, A \rangle \in \mathcal{T}_2$ such that $\langle \mathcal{F}, A \rangle \tilde{\subset} \langle \mathcal{U}, A \rangle \tilde{\subset} \overline{\langle \mathcal{U}, A \rangle}^{\mathcal{T}_1} \tilde{\subset} \langle \mathcal{V}, A \rangle$,*

(3) *for any $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and a $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{G}, A \rangle$ with $\langle \mathcal{F}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$, there exists a $\mathcal{T}_2$- $\mathcal{SO}$ set $\langle \mathcal{V}, A \rangle$ containing $\langle \mathcal{F}, A \rangle$ such that $\overline{\langle \mathcal{V}, A \rangle}^{\mathcal{T}_1} \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$.*

Proof. (1) $\Rightarrow$ (2) : Suppose $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is a $\mathcal{PSR}_3$ space. Let $\langle \mathcal{F}, A \rangle$ be any $\mathcal{T}_1$- $\mathcal{SC}$ set and $\langle \mathcal{V}, A \rangle$ be a $\mathcal{T}_2$- $\mathcal{SO}$ set such that $\langle \mathcal{F}, A \rangle \tilde{\subset} \langle \mathcal{V}, A \rangle$. Then $\langle \mathcal{V}, A \rangle^C$

is a $\mathcal{T}_2$- $\mathcal{SC}$ set such that $\langle \mathcal{F}, A\rangle \tilde{\cap} \langle \mathcal{V}, A\rangle^C = \left\langle \tilde{\Phi}, A\right\rangle$. Thus by $\mathcal{PSR}_3$, there exist $\left\langle \mathcal{U}', A\right\rangle \in \mathcal{T}_1$, $\langle \mathcal{U}, A\rangle \in \mathcal{T}$ such that $\langle \mathcal{F}, A\rangle \tilde{\subset} \langle \mathcal{U}, A\rangle$ and $\langle \mathcal{V}, A\rangle^C \tilde{\subset} \left\langle \mathcal{U}', A\right\rangle$. Now $\langle \mathcal{U}, A\rangle \tilde{\subset} \left\langle \mathcal{U}', A\right\rangle^C$, where $\left\langle \mathcal{U}', A\right\rangle^C$ is a $\mathcal{T}_1$- $\mathcal{SC}$ set. So $\overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_1} \tilde{\subset} \left\langle \mathcal{U}', A\right\rangle^C$. Again $\left\langle \mathcal{U}', A\right\rangle^C \tilde{\subset} \langle \mathcal{V}, A\rangle$. Hence $\langle \mathcal{F}, A\rangle \tilde{\subset} \langle \mathcal{U}, A\rangle \tilde{\subset} \overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_1} \tilde{\subset} \langle \mathcal{V}, A\rangle$.

(2) $\Rightarrow$ (3) : Suppose the condition (2) holds. Let $\langle \mathcal{F}, A\rangle$ be any $\mathcal{T}_1$- $\mathcal{SC}$ set and $\langle \mathcal{G}, A\rangle$ be any $\mathcal{T}_2$- $\mathcal{SC}$ set with $\langle \mathcal{F}, A\rangle \tilde{\cap} \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$. Then $\langle \mathcal{F}, A\rangle \tilde{\subset} \langle \mathcal{G}, A\rangle^C$, where $\langle \mathcal{G}, A\rangle^C$ is a $\mathcal{T}_2$- $\mathcal{SO}$ set. Thus by the condition (2), $\exists$ $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ such that

$$\langle \mathcal{F}, A\rangle \tilde{\subset} \langle \mathcal{V}, A\rangle \text{ and } \overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1} \tilde{\subset} \langle \mathcal{G}, A\rangle^C.$$

So $\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1} \tilde{\cap} \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$.

(3) $\Rightarrow$ (1) : Suppose the condition (3) holds. Let $\langle \mathcal{F}, A\rangle$ be any $\mathcal{T}_1$- $\mathcal{SC}$ set and $\langle \mathcal{G}, A\rangle$ be any $\mathcal{T}_2$- $\mathcal{SC}$ set with $\langle \mathcal{F}, A\rangle \tilde{\cap} \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$. Then by the condition (3), $\exists$ $\langle \mathcal{V}, A\rangle \in \mathcal{T}_2$ with $\langle \mathcal{F}, A\rangle \tilde{\subset} \langle \mathcal{V}, A\rangle$ and $\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1} \tilde{\cap} \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$. Thus $\langle \mathcal{G}, A\rangle \tilde{\subset} (\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1})^C$, where $(\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1})^{\mathbb{C}} \in \mathcal{T}_1$ and $\langle \mathcal{V}, A\rangle \tilde{\cap} (\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_1})^C = \left\langle \tilde{\Phi}, A\right\rangle$. So $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ is $\mathcal{PSR}_3$. □

Proposition 4.25. *Let $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ be a $\mathcal{SEBS}$. Then the following statements are equivalent :*

(1) $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A\right\rangle$ *is $\mathcal{PSR}_3$,*

(2) *for any $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{F}, A\rangle$ and any $\mathcal{T}_1$- $\mathcal{SO}$ set $\langle \mathcal{V}, A\rangle$ $[\langle \mathcal{V}, A\rangle^C \neq \left\langle \tilde{\Phi}, A\right\rangle]$ containing $\langle \mathcal{F}, A\rangle$, there exists $\langle \mathcal{U}, A\rangle \in \mathcal{T}_1$ such that* $\langle \mathcal{F}, A\rangle \tilde{\subset} \langle \mathcal{U}, A\rangle \tilde{\subset} \overline{\langle \mathcal{U}, A\rangle}^{\mathcal{T}_2} \tilde{\subset} \langle \mathcal{V}, A\rangle$,

(3) *for any $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{F}, A\rangle$ and a $\mathcal{T}_2$- $\mathcal{SC}$ set $\langle \mathcal{G}, A\rangle$ with $\langle \mathcal{F}, A\rangle \tilde{\cap} \langle \mathcal{G}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$, there exists a $\mathcal{T}_1$- $\mathcal{SO}$ set $\langle \mathcal{V}, A\rangle$ containing $\langle \mathcal{G}, A\rangle$ such that* $\overline{\langle \mathcal{V}, A\rangle}^{\mathcal{T}_2} \tilde{\cap} \langle \mathcal{F}, A\rangle = \left\langle \tilde{\Phi}, A\right\rangle$.

Proof. The proofs are similar to Proposition 4.24. □

Remark 4.26. A $\mathcal{PST}_4$ space is $\mathcal{PST}_3$.

Example 4.27. Let $X = \{a, b\}$, $A = \{\alpha, \beta\}$ and let $\mathcal{T}_1$ and $\mathcal{T}_2$ be the soft topologies on $\left\langle \tilde{X}, A\right\rangle$ given as follows:

$$\mathcal{T}_1 = \{\left\langle \tilde{\Phi}, A\right\rangle, \left\langle \tilde{X}, A\right\rangle, \langle \mathcal{F}_1, A\rangle, \langle \mathcal{F}_2, A\rangle, \langle \mathcal{F}_3, A\rangle, \langle \mathcal{F}_4, A\rangle, \langle \mathcal{F}_5, A\rangle\},$$

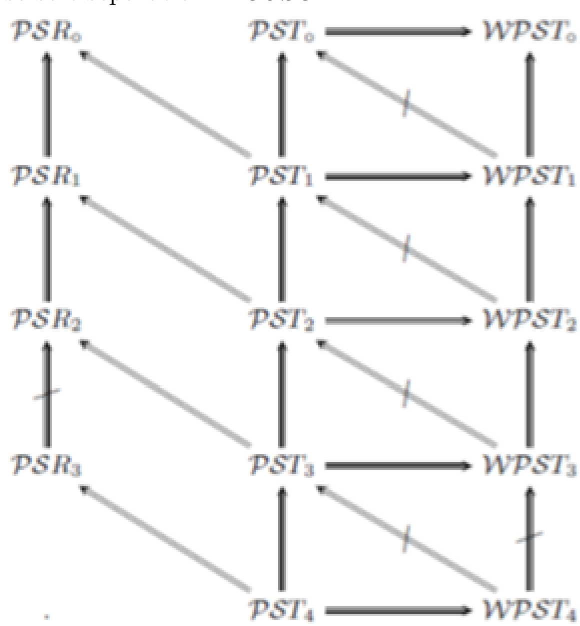
$$\mathcal{T}_2 = \{\left\langle \tilde{\Phi}, A\right\rangle, \left\langle \tilde{X}, A\right\rangle, \langle \mathcal{G}_1, A\rangle, \langle \mathcal{G}_2, A\rangle, \langle \mathcal{G}_3, A\rangle, \langle \mathcal{G}_4, A\rangle, \langle \mathcal{G}_5, A\rangle\},$$

where $\mathcal{F}_1(\alpha) = \{a\}, \mathcal{F}_1(\beta) = \{a\}; \mathcal{F}_2(\alpha) = \{b\}, \mathcal{F}_2(\beta) = \{b\}; \mathcal{F}_3(\alpha) = \{a\}, \mathcal{F}_3(\beta) = \{b\}; \mathcal{F}_4(\alpha) = \{a\}, \mathcal{F}_4(\beta) = \{a, b\}; \mathcal{F}_5(\alpha) = \{a, b\}, \mathcal{F}_5(\beta) = \{b\}; \mathcal{G}_1(\alpha) = \{a\}, \mathcal{G}_1(\beta) =$

$\{b\}; \mathcal{G}_2(\alpha) = \{b\}, \mathcal{G}_2(\beta) = \{a\}; \mathcal{G}_3(\alpha) = \{b\}, \mathcal{G}_3(\beta) = \{b\}; \mathcal{G}_4(\alpha) = \{a, b\}, \mathcal{G}_4(\beta) = \{b\}; \mathcal{G}_5(\alpha) = \{b\}, \mathcal{G}_5(\beta) = \{a, b\}$. Then the $\mathcal{SEBS} < \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A >$ is $\mathcal{WPST}_4$. Now consider the $\mathcal{T}_1$- $\mathcal{SC}$ set $\langle \mathcal{H}, A \rangle$, where $\mathcal{H}(\alpha) = \{a\}, \mathcal{H}(\beta) = \{a\}$. Then $\bar{b} \tilde{\notin} \langle \mathcal{H}, A \rangle$, but there does not exist a $\mathcal{T}_2$- $\mathcal{SO}$ set containing $\langle \mathcal{H}, A \rangle$ other than $\left\langle \tilde{X}, A \right\rangle$. Thus $\left\langle \tilde{X}, \mathcal{T}_1, \mathcal{T}_2, A \right\rangle$ is not $\mathcal{WPST}_3$ space.

Remark 4.28. A $\mathcal{WPST}_4$ space is not $\mathcal{PST}_3$. Consider the Example 3.9 which is clearly a $\mathcal{WPST}_4$ space but not $\mathcal{PST}_1$. Then it is not $\mathcal{PST}_3$.

The following Figure indicates some implications proven in above section concerning pairwise soft separation in $\mathcal{SEBS}$.



Definition 4.29. A $SQPMS$ $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is said to be *pairwise soft Hausdorff*, if for any two soft elements $\tilde{x}, \tilde{y}$ with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$, there exist a $\mathcal{P}$- $\mathcal{SO}$ set $\langle \mathcal{F}, A \rangle$ containing $\tilde{x}$ and a $\mathcal{Q}$- $\mathcal{SO}$ set $\langle \mathcal{G}, A \rangle$ containing $\tilde{y}$ such that $\langle \mathcal{F}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$.

Proposition 4.30. *A $SQPMS$ satisfying (Q3) is pairwise soft Hausdorff.*

Proof. Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ be a $SQPMS$ with a soft conjugate $\mathcal{Q}$ satisfying (Q3). Let $\tilde{x}, \tilde{y}$ be two soft elements with $\tilde{x}(\lambda) \neq \tilde{y}(\lambda)$ for all $\lambda \in A$. Then $\mathcal{P}_\lambda(\tilde{x}(\lambda), \tilde{y}(\lambda)) > 0$ for all $\lambda \in A$. Thus $\mathcal{P}(\tilde{x}, \tilde{y}) \tilde{>} \bar{0}$. Let $\tilde{r}$ be a soft real number such that $0 < \tilde{r}(\lambda) < \frac{1}{2}\mathcal{P}(\tilde{x}, \tilde{y})(\lambda)$ for all $\lambda \in A$. Consider the $\mathcal{P}$- $\mathcal{SO}$ set $\mathfrak{F}_1 = B_{\mathcal{P}}(\tilde{x}, \tilde{r})$ and the $\mathcal{Q}$- $\mathcal{SO}$ set

$\mathcal{F}_2 = B_{\mathcal{Q}}(\tilde{y}, \tilde{r})$. Then $\tilde{x} \in \mathcal{F}_1$ and $\tilde{y} \in \mathcal{F}_2$ and $\mathcal{F}_1 \cap \mathcal{F}_2 = \Phi$. Thus $SS(\mathcal{F}_1) \tilde{\cap} SS(\mathcal{F}_2) = \left\langle \tilde{\Phi}, A \right\rangle$. □

Definition 4.31. A $SQPMS$ $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is said to be:

(i) *$\mathcal{P}$ soft regular with respect to $\mathcal{Q}$*, if for any $\mathcal{P}$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and for any soft element $\tilde{x}$ with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$, there exist a $\mathcal{P}$- $\mathcal{SO}$ set $\langle \mathcal{U}, A \rangle$ and a $\mathcal{Q}$- $\mathcal{SC}$ set $\langle \mathcal{V}, A \rangle$ such that $\tilde{x} \tilde{\in} \langle \mathcal{V}, A \rangle$, $\langle \mathcal{F}, A \rangle \tilde{\subset} \langle \mathcal{V}, A \rangle$ and $\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$.

(ii) *pairwise soft regular*, if it is $\mathcal{P}$ soft regular w.r.t $\mathcal{Q}$ and $\mathcal{Q}$ soft regular w.r.t $\mathcal{P}$.

Definition 4.32. A $SQPMS$ $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is said to be *pairwise normal*, if for any $\mathcal{P}$- $\mathcal{SC}$ set $\langle \mathcal{F}, A \rangle$ and for any $\mathcal{Q}$- $\mathcal{SC}$ set $\langle \mathcal{G}, A \rangle$ with $\langle \mathcal{F}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$, there exist $\mathcal{P}$- $\mathcal{SO}$ set $\langle \mathcal{U}, A \rangle$ containing $\langle \mathcal{G}, A \rangle$ and $\mathcal{Q}$- $\mathcal{SO}$ set $\langle \mathcal{V}, A \rangle$ containing $\langle \mathcal{F}, A \rangle$ such that $\langle \mathcal{U}, A \rangle \tilde{\cap} \langle \mathcal{V}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$.

Proposition 4.33. *A $SQPMS$ satisfying ($Q3$) is pairwise soft regular.*

Proof. Let $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ be a $SQPMS$ satisfying ($Q3$). Let $\langle \mathcal{F}, A \rangle$ be a $\mathcal{P}$- $\mathcal{SC}$ set and $\tilde{x}$ be a soft element with $\tilde{x}(\lambda) \notin \mathcal{F}(\lambda)$ for all $\lambda \in A$. Since $\mathcal{F}(\lambda)$ is closed in $\langle X, \mathcal{P}_\lambda \rangle$, there exist open sets $\mathcal{H}_\lambda$ in $\langle X, \mathcal{P}_\lambda \rangle$ and $\mathcal{G}_\lambda$ in $\langle X, \mathcal{Q}_\lambda \rangle$ such that $\tilde{x}(\lambda) \in \mathcal{H}_\lambda$ and $\mathcal{F}(\lambda) \subset \mathcal{G}_\lambda$ for all $\lambda \in A$. Consider the $\mathcal{P}$- $\mathcal{SO}$ set $\langle \mathcal{H}, A \rangle$ and $\mathcal{Q}$- $\mathcal{SO}$ set $\langle \mathcal{G}, A \rangle$, where $\mathcal{H}(\lambda) = H_\lambda$ and $\mathcal{G}(\lambda) = \mathcal{G}_\lambda$ for all $\lambda \in A$. Then $\tilde{x} \tilde{\in} \langle \mathcal{H}, A \rangle$, $\langle \mathcal{F}, A \rangle \tilde{\subset} \langle \mathcal{G}, A \rangle$ and $\langle \mathcal{H}, A \rangle \tilde{\cap} \langle \mathcal{G}, A \rangle = \left\langle \tilde{\Phi}, A \right\rangle$. Thus $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is $\mathcal{P}$ soft regular with respect to $\mathcal{Q}$. Similarly, it can be shown that $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is $\mathcal{Q}$ soft regular with respect to $\mathcal{P}$. So $\left\langle \tilde{X}, \mathcal{P}, \mathcal{Q}, A \right\rangle$ is pairwise soft regular. □

Remark 4.34. A $SQPMS$ satisfying ($Q3$) is pairwise soft normal.

Example 4.35. Let us consider the $SQPMS$ as in Example 2.3. Then it is pairwise soft Hausdorff, pairwise soft regular and pairwise soft normal as it satisfies ($Q3$).

5. Conclusion and future work

In this study, we introduce the notions of soft quasi-pseudo metric and soft elementary bitopological spaces using redefined soft topology. Two different types of pairwise separation axioms are introduced and their properties are discussed. Still there is a scope to study important properties like bicompactness, connectedness, countability axioms, metrizability etc. in soft elementary bitopological spaces. It will be necessary to carry out more theoretical research to establish a general framework for the topological applications. This is just a beginning of study and much further studies are required to develop this field.

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