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Application of soft sets theory to multilattices

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Abstract. Multilattices generalize lattices by replacing the axiom of existence of a least upper bound for two elements by the set of minimal upper bounds and dually. In this work, we use multilattice as the universe set in soft sets theory. The notions of *f-soft multilattice, r-soft multilattice, soft ideal, soft filter, idealistic multilattice* are defined and several related properties are investigated.

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1. Introduction

Nearly all the economic and environmental policies are likely to be evaluate unfolding against a backdrop of great uncertainty. These uncertainties may be related to price or quantity, socio-economic or political context, state of the environment etc. Uncertainties arise in various other fields such as engineering, education and medical science. Theories such as probability, fuzzy sets [1], vague sets [2] and rough sets [3] are consider as mathematical tools for dealing with uncertainties and can be applied in these earlier theories. As pointed out in [4], all these theories have their inherent difficulties, possibly related to the inadequacy of the parameterization tools. For this reason, Molodtsov [4] proposed in 1999 *soft set theory* as a new mathematical tool for modeling vagueness and uncertainty. Since then, soft sets theory has been rapidly developed, with several applications. In 2003, Maji et al. [5] studied soft set theory and defined its basic operators. In the following years, Ali et al. [6] contributed to operational research by developing additional operators on soft sets. A survey on some of the main developments of applications of soft set theory in decision making problems is given in [7]. Numerous studies have been carried out on soft algebraic structures such as soft groups [8], soft semirings [9], soft rings [10], soft lattices [11, 12], soft BCK/BCI-algebras [13], soft BL-algebras [14].

Dymek and Walendziak have generalized many of the results from these algebras to universal algebras in [15].

On the other hand, one of the most widely used algebraic theories, whose importance no longer needs to be demonstrated is lattice theory. This theory mirrors the partially ordered sets theory but does not exhaust it as mentioned Benado in [16]. That is why, he generalizes the notion of structure to that of hyperstructure, which extends lattices to multilattices by dropping the uniqueness condition regarding upper and lower bounds and replacing it by the existence of minimal upper bounds and maximal lower bounds [17]. Multilattices are better suited to deal with situations where the partially ordered sets studied are hyperstructures with respect to the required order. Applications of multilattices theory in fields such as Formal Concept Analysis and fuzzy set theory can be found in [18, 19].

Our aim in this paper is to link multilattices to soft sets theory. Specifically, we defined the notions of f-soft multilattice, r-soft multilattice, soft ideal, soft filter, idealistic soft multilattice and studied their related properties. The structure of the paper is the following: we first recalled some basic notions related to soft sets theory and multilattices in section 2. In section 3, we defined the notion f-soft multilattice and r-soft multilattice and study their properties related to operations on soft sets. We focus on soft ideal and soft filter of an f-soft multilattice in section 4 while section 5 deal with idealistic soft multilattice. Finally, we draw some conclusions and prospects for future work in section 6.

2. Soft sets and multilattices

In this first section, we recall some basic notions related to soft sets theory and multilattices.

2.1. **Soft sets.** The notion of soft set was introduced by Molodtsov [4] as a generalization of Zadeh's fuzzy set [1], in order to better manage uncertainties. Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U.

Definition 2.1 ([4])**.** A *soft set* over U is a pair (f, E) such that $f : E \to P(U)$ is a map.

Let (f, E) be a soft set over a universe U and $A = \{x \in E : f(x) \neq \varnothing\}$. The set A is called the *support* of the soft set (f, E). A soft set is said to be *non-null*, if its support is not empty. The *null soft set* is a soft set with an empty support.

Throughout this study, we identify a soft set (f, E) with support A to the function f_A, where $f_A : E \to P(U)$ such that $f_A(x) = f(x)$ if $x \in A$ and $f_A(x) = \varnothing$ if $x \notin A$.

Example 2.2. Let us consider a soft set (f, E) which describes the attractiveness of hotels that Mr. X would like to visit for a stay.

U is the set of hotels under consideration. Assume there are six hotels, and then $U = \{h1, h2, h3, h4, h5, h6\}$. $E = \{e, u, w, r, l\}$ is the set of decision parameters, with e : expensive, u : beautiful, w : wooden, r : in good repair and l : good location.
Thus $f(w)$ means "wooden hotels" and its functional value is the set of wooden hotels of the consider universe. Suppose that $f(e) = \{h2, h4\}$, $f(u) = \{h1, h3\}$, $f(w) = \varnothing$, $f(r) = \{h1, h3, h5\}$ and $f(l) = \{h1\}$. Then (f, E) is a soft set over U.

Its support is $A = \{e, u, r, l\}$. We can identify (f, E) to
$f_A = \{(e, \{h2, h4\}), (u, \{h1, h3\}), (r, \{h1, h3, h5\}), (l, \{h1\})\}$ as mentioned above.

If $f(x) = \varnothing$ for all $x \in E$, then $(f, E) = f_\varnothing$ is the null soft set. In the following, the null soft set will be denoted by Φ.

Definition 2.3 ([5])**.** For two soft sets f_A and g_B over a universe U, f_A is called a *soft subset* of g_B, if

(i) $A \subseteq B$,
(ii) $f(x) \subseteq g(x)$ for all $x \in A$.

This relationship is denoted $f_A \tilde{\subseteq} g_B$. If f_A is a soft subset of g_B, then g_B is said to be a *soft super set* of f_A.

Definition 2.4 ([5])**.** Two soft sets f_A and g_B are said to be *soft equals*, if f_A is a soft subset and a soft super set of g_B.

From [5], we set the intersection and the union to be as follows.

Definition 2.5 ([5])**.** Let f_A and g_B be two non-null soft sets over a common universe U.

(i) The *intersection* of f_A and g_B, denoted by $f_A \tilde{\cap} g_B$, is the soft set over U defined as follows:

$$f_A \tilde{\cap} g_B = \Phi, \text{ if } C = A \cap B = \varnothing,$$

$$f_A \tilde{\cap} g_B = h_C, \text{ if } C \neq \varnothing,$$

where $h(x) = f(x) \cap g(x)$ for all $x \in C$.

(ii) The *union* of f_A and g_B, denoted by $f_A \tilde{\cup} g_B$, is the soft set over U defined as follows:

$$f_A \tilde{\cup} g_B = h_{A \cup B},$$

where $h(x) = f(x) \cup g(x)$ for all $x \in A \cup B$.

Then we have $\Phi \tilde{\cap} \Phi = \Phi \tilde{\cup} \Phi = \Phi \tilde{\cap} f_A = \Phi$ and $\Phi \tilde{\cup} f_A = f_A$, for all non-null soft set f_A.

Definition 2.6 ([5])**.** Let f_A and g_B be two non-null soft sets over a common universe U.

(i) "f_A AND g_B", denoted by $f_A \tilde{\wedge} g_B$, is the soft set over U defined as follows:

$$f_A \tilde{\wedge} g_B = \Phi, \text{ if } C = \{(x, y) \in A \times B : f(x) \cap g(y) \neq \varnothing\} = \varnothing,$$

$$f_A \tilde{\wedge} g_B = h_C, \text{ if } C \neq \varnothing,$$

where $h(x, y) = f(x) \cap g(y)$ for all $(x, y) \in C$.

(ii) "f_A OR g_B", denoted by $f_A \tilde{\vee} g_B$, is the soft set over U defined as follows:

$$f_A \tilde{\vee} g_B = h_{A \times B},$$

where $h(x, y) = f(x) \cup g(y)$ for all $(x, y) \in A \times B$.

Consequently, $\Phi \tilde{\wedge} \Phi = \Phi \tilde{\vee} \Phi = \Phi \tilde{\wedge} f_A = \Phi \tilde{\vee} f_A = \Phi$ for all non-null soft set f_A.

The intersection can be extended and the union restricted as follows.

Definition 2.7 ([6]). Let f_A and g_B be two non-null soft sets over a common universe U. Then the *restricted union* of f_A and g_B, denoted by $f_A \cup_{\mathfrak{r}} g_B$, is the soft set over U defined as follows:

$$f_A \cup_{\mathfrak{r}} g_B = \Phi, \text{ if } A \cap B = \varnothing,$$
$$f_A \cup_{\mathfrak{r}} g_B = h_{A\cap B}, \text{ if } A \cap B \neq \varnothing,$$

where $h(x) = f(x) \cup g(x)$ for all $x \in A \cap B$.

Definition 2.8 ([6]). Let f_A and g_B be two non-null soft sets over a common universe U. Then the *extended intersection* of f_A and g_B, denoted by $f_A \cap_{\mathfrak{g}} g_B$, is the soft set over U defined as follows:

$$f_A \cap_{\mathfrak{g}} g_B = h_C,$$

where $C = (A \cup B) \setminus \{x \in A \cap B : f(x) \cap g(x) = \varnothing\}$ and for all $x \in C$,

$$h(x) = \begin{cases} f(x) & \text{if } x \in A \setminus B \\ g(x) & \text{if } x \in B \setminus A \\ f(x) \cap g(x) & \text{else.} \end{cases}$$

Example 2.9. Let us consider our previous universe $U = \{h1, h2, h3, h4, h5, h6\}$ and the set of parameters $E = \{e, u, w, r, l\}$ of Example 2.2. We defined the soft sets:

$$f_A = \{(e, \{h1, h3\})\}, \ g_B = \{(e, \{h1, h2\}), (u, \{h2, h3, h5\})\},$$
$$h_C = \{(w, \{h1, h2\}), (l, \{h4, h5\})\}, \ t_D = \{(e, \{h3, h6\}), (u, \{h1, h4, h6\}), (w, \{h5\})\}.$$

Then we have

$$f_A \tilde{\cap} g_B = \{(e, \{h1\})\}, \ f_A \tilde{\cup} g_B = \{(e, \{h1, h2, h3\}), (u, \{h2, h3, h5\})\},$$
$$g_B \tilde{\cap} h_C = \Phi, \ g_B \tilde{\cap} t_D = \Phi,$$
$$f_A \tilde{\wedge} g_B = \{((e, e), \{h1\})\}, \ f_A \tilde{\wedge} t_D = \{((e, e), \{h3\}), ((e, u), \{h1\})\},$$
$$g_B \tilde{\wedge} t_D = \{((e, u), \{h1\}), ((u, e), \{h3\}), ((u, w), \{h5\})\},$$
$$f_A \tilde{\vee} g_B = \{((e, e), \{h1, h2, h3\}), ((e, u), \{h1, h2, h3, h5\})\},$$
$$f_A \cup_{\mathfrak{r}} g_B = \{(e, \{h1, h2, h3\})\},$$
$$f_A \cap_{\mathfrak{g}} g_B = \{(e, \{h1\}), (u, \{h2, h3, h5\})\}.$$

Remark 2.10. The difference and the symmetric difference of two sets can be extended in the framework of soft sets.

Definition 2.11. Let f_A and g_B be two soft sets over a common universe U. Then the *difference* of f_A to g_B, denoted by $g_A \smile f_A$ is the soft set over U denoted as follows:

$$g_B \smile f_A = h_B,$$

where $h(x) = g(x) \setminus f(x)$ for all $x \in B$.

Consequently, $\Phi \smile \Phi = \Phi \smile f_A = f_A \smile f_A = \Phi$, $f_A \smile \Phi = f_A$ for all soft set f_A.

The symmetric difference can now be easily defined.

Definition 2.12. Let f_A and g_B be two soft sets over a common universe U. Then the *symmetric difference* between f_A and g_B, denoted by $f_A \tilde{\Delta} g_B$, is the soft set over U defined as follows:

$$f_A \tilde{\Delta} g_B = (f_A \smile g_B) \tilde{\cup} (g_B \smile f_A).$$

One can observe that $f_A\tilde{\Delta}g_B = (f_A\tilde{\cup}g_B) \smile (f_A\tilde{\cap}g_B)$. Of cause, the properties of the symmetric difference in set theory can be easily verified in soft sets theory.

In [6], the notion of *restricted difference* is defined as $g_B \smile_{\mathfrak{r}} f_A = h_{A\cap B}$, where $h(x) = g(x) \setminus f(x)$ for all $x \in A \cap B$. Using this difference, we can get the *restricted symmetric difference* as $f_A\tilde{\Delta}_{\mathfrak{r}}g_B = (f_A \smile_{\mathfrak{r}} g_B)\tilde{\cup}(g_B \smile_{\mathfrak{r}} f_A)$.

2.2. **Multilattices.** Given a partially ordered set $(M, \leq)$ and $X \subseteq M$ a subset of M, a *multi-supremum* of X is a minimal element of the set of upper bounds of X and a *multi-infimum* of X is a maximal element of the set of lower bounds of X. By **multisup(X)** (resp. **multiinf(X)**), we denoted the set of multi-suprema (resp. multi-infima) of X.

Definition 2.13 ([20])**.** A partially ordered set (poset) $(M, \leq)$ is an *ordered multilattice*, if for all $a,\ b,\ x \in M$ with $a \leq x$ and $b \leq x$, there exists $z \in \mathbf{multisup(\{a,b\})}$ such that $z \leq x$ and for all $a,\ b,\ x \in M$ with $x \leq a$ and $x \leq b$, there exists $z \in \mathbf{multiinf(\{a,b\})}$ such that $x \leq z$.

Remark 2.14. Definition 2.13 is consistent with the existence of two incompatible elements without any multi-supremum or infimum. That is, we can have

$$\mathbf{multisup(\{a,b\})} = \varnothing \text{ or } \mathbf{multiinf(\{a,b\})} = \varnothing \text{ for some } a,\ b \in M.$$

Definition 2.15. [21]A multilattice $(M, \leq)$ is said to be *full*, if for all $a, b \in M$, $\mathbf{multisup(\{a,b\})} \neq \varnothing$ and $\mathbf{multiinf(\{a,b\})} \neq \varnothing$.

We denote $a \sqcap b = \mathbf{multisup(\{a,b\})}$, $a \sqcup b = \mathbf{multiinf(\{a,b\})}$ for all $a, b \in M$.

Definition 2.16 ([20])**.** Let $(M, \leq)$ be a multilattice and $X \subseteq M$ be a non emppty subset of M.

(i) X is called a *full-submultilattice* (briefly, f-submultilattice) of M, if

$$a \sqcup b \subseteq X \text{ and } a \sqcap b \subseteq X \text{ for all } a,\ b \in X.$$

(ii) X is called a *restricted submultilattice* (briefly, r-submultilattice) of M, if

$$(a \sqcup b) \cap X \neq \varnothing \text{ and } (a \sqcap b) \cap X \neq \varnothing \text{ for all } a,\ b \in X.$$

One might think as stated in [20] that every f-submultilattice of a multilattice M is an r-submultilattice of M. It is not hold in general. Indeed, we can consider $M = \{0, a, b\}$ with $0 \leq a$ and $0 \leq b$. $(M, \leq)$ is a multilattice which is naturally an f-submultilattice of M but is not an r-submultilattice of M. However, if M is a full multilattice, then every f-submultilattice of a multilattice M is an r-submultilattice of M as well.

Proposition 2.17. *Let $(M, \leq)$ be a multilattice.*

(1) *For all $a \in M$, $\{a\}$ is an f-submultilattice and an r-submultilattice of M.*

(2) *For all $a, b \in M$ with $a \leq b$, $\{a, b\}$ is an f-submultilattice and an r-submultilattice of M. More generally, every finite chain of a multilattice is an f-submultilattice and an r-submultilattice.*

Proof. Immediately follows from Definition 2.16. □

Example 2.18. On Figure 1, we have a full multilattice and a multilattice which is not full.

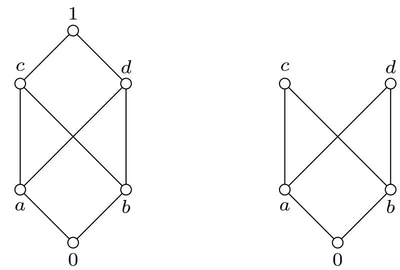

FIGURE 1. Left : a full multilattice. Right : a multilattice which is not full.

Remark 2.19. Let $(M, \leq)$ be a multilattice and $X, Y \subseteq M$ be two subsets of M. If X and Y are f-submultilattices of M with $X \cap Y \neq \varnothing$, then $X \cap Y$ is an f-submultilattice of M.

The intersection of r-submultilattices is not an r-submultilattice in general. Indeed, let consider the multilattice in the left side of Figure 1. $X = \{0, a, b, c\}$ and $Y = \{a, b, d, 1\}$ are r-submultilattices of M but $X \cap Y = \{a, b\}$ is not an r-submultilattice of M since $a \sqcap b = \{c, d\}$ and $\{c, d\} \cap \{a, b\} = \varnothing$.

3. f-soft multilattice and r-soft multilattice

In the following, we will take a multilattice as our universe set with a set E of parameters. M in what follows will be the support or the multilattice structure.

Definition 3.1. Let $(M, \leq)$ be a multilattice and f_A be a non-null soft set over M.

(i) f_A is called a *full soft multilattice* (briefly, f-soft multilattice) over M, if $f(x)$ is an f-submultilattice of M for all $x \in A$.

(ii) f_A is called a *restricted soft multilattice* (briefly, r-soft multilattice) over M, if $f(x)$ is an r-submultilattice of M for all $x \in A$.

Example 3.2. Let consider the full multilattice M in the left side of Figure 1 and $E = \{e, u, w\}$. Let f_A and g_B be the soft sets over M defined by :

$$f_A = \{(e, \{0, a\}), (u, \{b, d\})\} \text{ and } g_B = \{(e, \{a, b, c\}), (u, \{a, c\})\}.$$

Then we have $f(e) = \{0, a\}$, $f(u) = \{b, d\}$. As $0 \leq a$ and $b \leq d$, by Proposition 2.17 (ii), $f(e)$ and $f(u)$ are f-submultilattices of M. Thus f_A is an f-soft multlattice over M. However, $g(e) = \{a, b, c\}$ is not an f-submultilattice of M, as $a \sqcup b = \{c, d\}$ and we don't have $\{c, d\} \subseteq \{a, b, c\}$. So g_B is not an f-soft multilattice over M. Moreover, $a \sqcap b = \{0\}$ and hence $(a \sqcap b) \cap g(e) = \varnothing$. Therefore g_B is not an r-soft multilattice over M.

Assume that $h_C = \{(e, \{0, a, b, c\}), (u, \{0\}), (w, \{b, c, d, 1\})\}$. Then it is an r-soft multilattice over M which is not an f-soft multilattice.

Remark 3.3. Let $(M, \leq)$ be a multilattice and f_A be an f-soft multilattice (resp. r-soft multilattice) over M. Then for all non empty subset B of A, f_B is an f-soft multilattice (resp. r-soft multilattice) over M.

Theorem 3.4. *Let f_A and g_B be two non-null f-soft multilattices over a multilattice M as universe. Then either $f_A \tilde{\cap} g_B = \Phi$ or $f_A \tilde{\cap} g_B$ is an f-soft multilattice over M.*

Proof. Suppose that f_A and g_B are f-soft multilattices over M and $f_A \tilde{\cap} g_B \neq \Phi$. By Definition 2.5 (i), $f_A \tilde{\cap} g_B = h_C$ with $C = \{x \in A \cap B : f(x) \cap g(x) \neq \varnothing\}$ and $h(x) = f(x) \cap g(x)$ for all $x \in C$.

For all $x \in C$, $f(x)$ and $g(x)$ are f-submultilattices of M, as f_A and g_B are f-soft multilattices over M. By Remark 2.19, $f(x) \cap g(x)$ is an f-submultilattice of M. It is follows that $h(x)$ is an f-submultilattice of M for all $x \in C$. Finaly, $h_C = f_A \tilde{\cap} g_B$ is an f-soft multilattice over M. □

Let consider the full multilattice M of Figure 1 and two soft sets f_A and g_B over M defined by $f_A = \{(e, \{0, a, b, c\})\}$, $g_B = \{(e, \{a, b, d, 1\})\}$. f_A and g_B are both r-soft multilattices over M but $f_A \tilde{\cap} g_B = \{(e, \{a, b\})\}$ is not an r-soft multilattice over M. Thus the intersection of two r-soft multilattices is not always an r-soft multilattices.

Theorem 3.5. *Let f_A and g_B be two non-null f-soft multilattices over a common multilattice M as universe. If $f_A \tilde{\wedge} g_B \neq \Phi$, then $f_A \tilde{\wedge} g_B$ is an f-soft multilattice over M.*

Proof. Assume that $f_A \tilde{\wedge} g_B \neq \Phi$. Then $f_A \tilde{\wedge} g_B = h_C$ with $\varnothing \neq C = \{(x, y) \in A \times B : f(x) \cap g(y) \neq \varnothing\}$ and $h(x, y) = f(x) \cap g(y)$ for all $(x, y) \in C$. As f_A and g_B are f-soft multilattices of M, $f(x)$ and $g(y)$ are f-submultilattices of M. By Remark 2.19, $f(x) \cap g(y)$ is an f-submultilattice of M. Thus $h(x, y) = f(x) \cap g(y)$ is an f-submultilattice of M. It follows that $h_C = f_A \tilde{\wedge} g_B$ is an f-soft multilattice over M. □

From the full multilattice M of Figure 1, $f_A = \{(e, \{0, a, b, c\})\}$ and $g_B = \{(e, \{a, b, d, 1\})\}$ are r-soft multilattices and $f_A \tilde{\wedge} g_B = \{((e, e), \{a, b\})\}$ is not an r-soft multilattice over M.

Theorem 3.6. *Let f_A and g_B be two non-null f-soft multilattices* (resp. r-soft multilattices) *over a common universe M. If $A \cap B = \varnothing$, then $f_A \tilde{\cup} g_B$ is an f-soft multilattice* (resp. r-soft multilattices) *over M.*

Proof. By Definition 2.5 (ii), $f_A \tilde{\cup} g_B = h_{A \cup B}$ with $h(x) = f(x) \cup g(x)$ for all $x \in A \cup B$. As $A \cap B = \varnothing$, if $x \in A \cup B$, then $x \in A \setminus B$ or $x \in B \setminus A$. If $x \in A \setminus B$, then $h(x) = f(x)$. As f_A is an f-soft multilattice over M, $f(x)$ is an f-submultilattice of M and thus $h(x)$ is an f-submultilattice of M. We can achieve the conclusion with $x \in B \setminus A$ similarly. This imply that $h_{A \cup B}$ is an f-soft multilattice over M.

The proof for r-soft multilattice is similar. □

If f_A and g_B are two non-null f-soft multilattices over M, $f_A \tilde{\vee} g_B$ may not be an f-soft multilattice even if $A \cap B = \varnothing$. Indeed, from the full multilattice M of Figure 1, let consider $f_A = \{(e, \{0, a\})\}$ and $g_B = \{(u, \{0, b\})\}$. They are two f-soft multilattices over M. However, $f_A \tilde{\vee} g_B = \{((e, u), \{0, a, b\})\}$ is not an f-soft multilattice over M because $\{0, a, b\}$ is not an f-submultilattice of M. One can also observe that f_A and g_B are r-soft multlattices but $f_A \tilde{\vee} g_B$ is not.

Theorem 3.7. *Let f_A and g_B be two non-null f-soft multilattices* (resp. r-soft multilattices) *over a common universe M such that for all $x \in A \cup B$, $f(x) \subseteq g(x)$ or $g(x) \subseteq f(x)$. Then $f_A \tilde{\cup} g_B$ is an f-soft multilattice* (resp. r-soft multilattice) *over M.*

Theorem 3.8. *Let f_A and g_B be two non-null f-soft multilattices over a common universe multilattice M. Then $f_A \cap_{\mathfrak{g}} g_B$ is an f-soft multilattice over M if it is non-null.*

Proof. By Definition 2.8, if $f_A \cap_{\mathfrak{g}} g_B$ is non-null, then $f_A \cap_{\mathfrak{g}} g_B = h_C$,
where $C = (A \cup B) \setminus \{x \in A \cap B\} : f(x) \cap g(x) = \varnothing\}$ and for all $x \in C$,

$$h(x) = \begin{cases} f(x) & \text{if } x \in A \setminus B \\ g(x) & \text{if } x \in B \setminus A \\ f(x) \cap g(x) & \text{else.} \end{cases}$$

As f_A, g_B are f-soft multilattices and the fact that the non empty intersection of two f-submultilattices is an f-submultilattice (Remark 2.19), it follows that for all $x \in C$, $h(x)$ is an f-submultilattice of M. That is $f_A \cap_{\mathfrak{g}} g_B$ is an f-soft multilattice over M as required. □

4. Soft ideal and soft filter of an f- soft multilattice

Ideals and filters are some kind of substructures that are useful in various algebraic structure (groups, rings, lattices, $\cdots$). In what follows, we introduce the notion of soft ideal and soft filter of an f-soft multilattice. We first recalled the notions of ideal and filter of a multilattice.

Definition 4.1 ([22])**.** A non empty subset I of a multilattice M is called an it ideal of M, if the following conditions are satisfied:

(IM1) for all $a,\ b \in I$, $a \sqcup b \subseteq I$,
(IM2) for all $b \in M$ and $a \in I$, $a \sqcap b \subseteq I$,
(IM3) for all $a,\ b \in M$ such that $(a \sqcap b) \cap I \neq \varnothing$, $a \sqcap b \subseteq I$.

The dual notion of ideal is that of filter.

Definition 4.2 ([22])**.** A non empty subset F of a multilattice M is called a *filter* of M, if the following conditions are satisfied:

(IM1) for all $a,\ b \in F$, $a \sqcap b \subseteq F$,
(IM2) for all $b \in M$ and $a \in F$, $a \sqcup b \subseteq F$,
(IM3) for all $a,\ b \in M$ such that $(a \sqcup b) \cap F \neq \varnothing$, $a \sqcup b \subseteq F$.

Remark 4.3. Let $(M, \leq)$ be a multilattice, I an ideal of M and F a filter of M. One can observe that:

(1) I and F are f-submultilattices of M (and r-submultilattices of M if M is full),
(2) M is an ideal and a filter of M,
(3) if M has a bottom element 0, then $\{0\}$ is an ideal of M,
(4) if M has a top element 1, then $\{1\}$ is a filter of M.

Proposition 4.4. *Let $(M, \leq)$ be a multilattice, I, J be two ideals of M, and F, G two filters of M. Then*

(1) *$I \cap J$ is empty or an ideal of M,*
(2) *$F \cap G$ is empty or a filter of M.*

Proof. Assume that I and J are ideals of M. If $a, b \in I \cap J$, then $a \sqcup b \subseteq I$ and $a \sqcup b \subseteq J$ which implies that $a \sqcup b \subseteq I \cap J$. If $a \in I \cap J$ and $b \in M$, then $a \sqcap b \subseteq I$ and $a \sqcap b \subseteq J$, i.e., $a \sqcap b \subseteq I \cap J$. Finally, if $a, b \in M$ and $(a \sqcap b) \cap (I \cap J) \neq \varnothing$, then $((a \sqcap b) \cap I) \cap J \neq \varnothing$, which implies that $(a \sqcap b) \cap I \neq \varnothing$, and that $a \sqcap b \subseteq I$, as I is an ideal of M. Similarly, we have $a \sqcap b \subseteq J$. Thus $a \sqcap b \subseteq I \cap J$.
We conclude by Definition 4.1 that $I \cap J$ is an ideal of M.

The proof of (2) is similar. □

From Definition 2.16, it is natural to see that if X is an f-submultilattice of a multilattice M, then X is a multilattice on its own right with the restriction of the partial order of M. As pointed out in [19], this is not true for an r-submultilattice in general. We can then understand the notions of soft ideal and soft filter of an f-soft multilattice as follows.

Definition 4.5. Let f_A be an f-soft multilattice and g_I be a non-null soft set over a multilattice M. g_I is called a *soft ideal* of f_A, if it satisfies the following conditions:
(i) $I \subseteq A$,
(ii) for all $x \in I$, $g(x)$ is an ideal of $f(x)$.

Definition 4.6. Let f_A be an f-soft multilattice and g_F be a non-null soft set over a multilattice M. g_F is called a *soft filter* of f_A, if it satisfies the following conditions:
(i) $F \subseteq A$,
(ii) for all $x \in F$, $g(x)$ is a filter of $f(x)$.

Using Remark 4.3, one can observe that any soft ideal (resp. soft filter) is an f-soft multilattice over M.

Since filters and ideals are dual, the properties of filters can be obtained from those of ideals by duality. In the following, we present only some of the properties of soft ideals.

Theorem 4.7. *Let f_A be an f-soft multilattice over a multilattice M. Let g_I and h_J be two soft ideals of f_A. Then $g_I \tilde{\cap} h_J$ is a soft ideal of f_A if it is non-null.*

Proof. Assume that $g_I \tilde{\cap} h_J$ is non-null. Then $g_I \tilde{\cap} h_J = t_K$ with $K = I \cap J$ and $t(x) = g(x) \cap h(x)$ for all $x \in K$. As g_I and h_J are soft ideals of f_A, $I \subseteq A$ and $J \subseteq A$, that is $K = I \cap J \subseteq A$. For all $x \in K$, $x \in I$, $x \in J$ and thus $g(x)$ and $h(x)$ are ideals of $f(x)$. It follows that $t(x) = g(x) \cap h(x)$ is an ideal of $f(x)$. □

Example 4.8. Consider our multilattice P in Figure 2. Define:

$$f_A = \{(e, \{0, b_0, b_1, \cdots, b_{12}, a_5\}), (u, \{0, a_0, \cdots, a_5\}), (w, \{c_0, \cdots, c_9, a_5\})\},$$
$$h_I = \{(e, \{0, b_0, b_1, \cdots, b_6\}), (u, \{0, a_0, \cdots, a_5\}), (w, \{c_0, c_1, c_2, c_3\})\},$$
$$t_F = \{(e, \{b_6, b_7, \cdots, b_{12}, a_5\}), (u, \{0, a_0, \cdots, a_5\}), (w, \{c_3, c_4, \cdots, c_9, a_5\})\}.$$

Then f_A is an f-soft multilattices over P, h_I is a soft ideal and t_F a soft filter of f_A.

The union of two soft ideals of an f-soft multilattice is not always a soft ideal of this f-soft multilattice. Indeed, let f_A be as in Example 4.8. Consider the following two non-null soft sets h_I and g_J over the multilattice P given by:

$$h_I = \{(e, \{0, b_0\}), (u, \{0, a_0, \cdots, a_5\}), (w, \{c_0, c_1, c_2, c_3\})\},$$

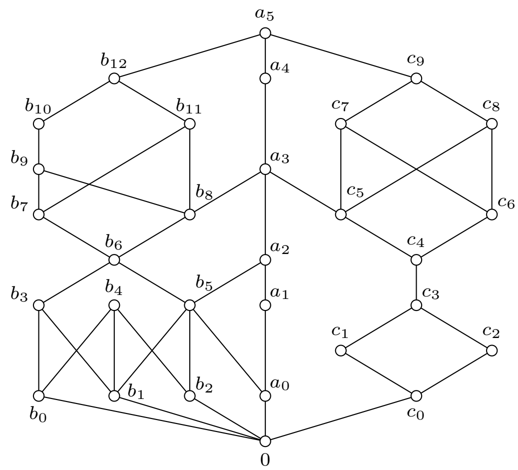

FIGURE 2. Multilattice with bottom element which is not full.

$$g_J = \{(e, \{0, b_1\}), (u, \{0, a_0, \cdots, a_5\}), (w, \{c_0, c_1, c_2, c_3\})\}.$$

Then clearly, h_I and g_J are soft ideals of f_A. However,

$$t_{I\cup J} = h_I \tilde{\cup} g_J = \{(e, \{0, b_0, b_1\}), (u, \{0, a_0, \cdots, a_5\}), (w, \{c_0, c_1, c_2, c_3\})\}$$

is not a soft ideal of f_A, because $t(e) = h(e) \cup g(e) = \{0, b_0, b_1\}$ is not an ideal of $f(e) = \{0, b_0, b_1, \cdots, b_{12}, a_5\}$, as $b_0 \sqcup b_1 = \{b_3, b_4\}$ is not a subset of $t(e)$.

The following result gives a sufficient condition for the union of two soft ideals of an f-soft multilattice to be an ideal of this f-soft multilattice.

Theorem 4.9. *Let f_A be a soft multilattice over a multilattice M. Let g_I and h_J be two soft ideals of f_A. If $I \cap J = \varnothing$, then $g_I \tilde{\cup} h_J$ is a soft ideal of f_A.*

Proof. By Definition 2.5, $g_I \tilde{\cup} h_J = t_{I\cup J}$, where $t(x) = g(x) \cup h(x)$ for all $x \in I \cup J$. As g_I and h_J are soft ideals of f_A, we have $I \subseteq A$ and $J \subseteq A$, which implies that $I \cup J \subseteq A$. If $x \in I \cup J$, then either $x \in I \setminus J$ or $x \in J \setminus I$. That is, $t(x) = g(x)$ or $t(x) = h(x)$. In both cases, $t(x)$ is an ideal of $f(x)$ as g_I and h_J are soft ideals of f_A. We can conclude that $g_I \tilde{\cup} h_J$ is a soft ideal of f_A. □

Theorem 4.10. *Let f_A be an f-soft multilattice over a multilattice M. Let g_I and h_J be two soft ideals of f_A. Then $g_I \cap_{\mathfrak{g}} h_J$ is a soft ideal of f_A if it is non-null.*

Proof. Assume that $g_I \cap_{\mathfrak{g}} h_J$ is non-null. Then $g_I \cap_{\mathfrak{g}} h_J = t_K$, with $K = (I \cup J) \setminus \{x \in I \cap J : g(x) \cap h(x) = \varnothing\}$ and for all $x \in K$, $t(x) = g(x)$ if $x \in I \setminus J$, $t(x) = h(x)$ if $x \in J \setminus I$ and $t(x) = g(x) \cap h(x)$ if $x \in I \cap J$. As g_I and h_J are soft ideals of f_A, $I \subseteq A$ and $J \subseteq A$. That is, $K \subseteq A$. Knowing that the intersection of two ideals of a multilattice is also an ideal (Proposition 4.4), $t(x) = g(x) \cap h(x)$ is an ideal of $f(x)$, if $x \in I \cap J$. For the others two cases, $t(x)$ is evidently an ideal of $f(x)$, as g_I and h_J are soft ideals of f_A. It follows that $g_I \cap_{\mathfrak{g}} h_J$ is a soft ideal of f_A. □

5. Idealistic soft multilattices

In section 4, the notions of soft ideal and soft filter have been presented, but they are related to an f-soft multilattice. In this section, we present the notion of idealistic soft multilattice which is not related to an f-soft multilattice but to the universe, hence to the multilattice.

Definition 5.1. Let f_A be a non-null soft set over a multilattice M. Then f_A is called an *idealistic soft multilattice* over M ,if $f(x)$ is an ideal of M for all $x \in A$.

Example 5.2. Consider the full multilattice of Figure 1 and define

$$f_A = \{(e, \{0, a\}), (u, \{0, b\}), (w, \{0\})\}.$$

Then f_A is an idealistic soft multilattice over the multilattice M parameterized by $E = \{e, u, w\}$.

Remark 5.3. A soft ideal of an f-soft multilattice over a multilattice M is not always an idealistic soft multilattice over M. Indeed, h_I in Example 4.8 is not an idealistic soft multilattice because $h(w) = \{c_0, c_1, c_2, c_3\}$ is not an ideal of M.

Remark 5.4. A soft set whose support is a non empty subset of the support of an idealistic soft multilattice over a multilattice M is also an idealistic soft multilattice over M.

Proposition 5.5. *Let f_A and g_B be two idealistic soft multilattices over a multilattice M.*

(1) *$f_A \tilde{\cap} g_B$ is an idealistic soft multilattice over M if it is non-null.*
(2) *If $A \cap B = \varnothing$, then $f_A \tilde{\cup} g_B$ is an idealistic soft multilattice over M.*
(3) *$f_A \tilde{\wedge} g_B$ is an idealistic soft multilattice over M if it is non-null.*
(4) *$f_A \cap_{\mathfrak{g}} g_B$ is an idealistic soft multilattice over M if it is non-null.*

Proof. (1) The proof is similar to Theorem 4.7.
(2) The proof is similar to Theorem 4.9.
(3) The proof naturally follows from Definition 2.6 (i) and Proposition 4.4 (1).
(4) The proof is similar to Theorem 4.10. □

Definition 5.6. Let M be a multilattice with a bottom element "0" and let f_A be an idealistic soft multilattice over M. Then f_A is said to be:
(i) *trivial*, if $f(x) = \{0\}$ for all $x \in A$,
(ii) *whole*, if $f(x) = M$ for all $x \in A$.

The most used definition of homomorphism of hyperstructures is that introduced originally in [16] and used by Cabrera et al. [21] in the framework of multilattices.

Definition 5.7. [21]A map $f : M \to N$ between multilattices is said to be a *homomorphism*, if for all $x,\ y \in M$,

$$f(x \sqcap y) \subseteq f(x) \sqcap f(y) \text{ and } f(x \sqcup y) \subseteq f(x) \sqcup f(y).$$

When the initial multilattice is full, the notion of homomorphism can be characterized in terms of equalities as follows.

Theorem 5.8 ([21]). *Let $f : M \to N$ be a map between multilattices where M is full. Then f is a homomorphism if and only if for all $x,\ y \in M$,*

$$f(x \sqcap y) = (f(x) \sqcap f(y)) \cap f(M) \text{ and } f(x \sqcup y) = (f(x) \sqcup f(y)) \cap f(M).$$

Corollary 5.9. *Let $f : M \to N$ be a surjective map between multilattices where M is full. Then f is a homomorphism if and only if for all $x,\ y \in M$,*

$$f(x \sqcap y) = f(x) \sqcap f(y) \text{ and } f(x \sqcup y) = f(x) \sqcup f(y).$$

Proposition 5.10. *Let $f : M \to N$ be a homomorphism of multilattices, I be an ideal of M and J be an ideal of N.*

(1) *$f^{-1}(J)$ is an ideal of M.*
(2) *If M is full and f bijective, then $f(I)$ is an ideal of N.*

Proof. (1) Firstly, let $a,\ b \in f^{-1}(J)$. Then $f(a), f(b) \in J$. As J is an ideal of N, $f(a) \sqcup f(b) \subseteq J$. As f is a homomorphism, $f(a \sqcup b) \subseteq f(a) \sqcup f(b)$. Thus $f(a \sqcup b) \subseteq J$. So $a \sqcup b \subseteq f^{-1}(J)$.

Secondly, let $a \in M$ and $b \in f^{-1}(J)$. Then $f(b) \in J$ and $f(a) \in N$. As J is an ideal of N, $f(a) \sqcap f(b) \subseteq J$. As f is a homomorphism, $f(a \sqcap b) \subseteq f(a) \sqcap f(b)$. Thus $f(a \sqcap b) \subseteq J$. So $a \sqcap b \subseteq f^{-1}(J)$.

Finally, let $a,\ b \in f^{-1}(J)$ such that $(a \sqcap b) \cap f^{-1}(J) \neq \varnothing$. Let $c \in (a \sqcap b) \cap f^{-1}(J)$. Then $f(c) \in J$ and $f(c) \in f(a \sqcap b)$, that is, $f(a \sqcap b) \cap J \neq \varnothing$. As f is a homomorphism, $f(a \sqcap b) \subseteq f(a) \sqcap f(b)$, which implies that $(f(a) \sqcap f(b)) \cap J \neq \varnothing$. Since J is an ideal of N, $f(a) \sqcap f(b) \subseteq J$. It follows that $f(a \sqcap b) \subseteq J$. Thus $a \sqcap b \subseteq f^{-1}(J)$.

(2) With M full and f surjective, by Corollary 5.9, we have that for all $x, y \in M$,

$$f(x \sqcap y) = f(x) \sqcap f(y) \text{ and } f(x \sqcup y) = f(x) \sqcup f(y).$$

Firstly, let $a, b \in f(I)$. Then there exit $x,\ y \in I$ such that $f(x) = a$ and $f(y) = b$. Thus $a \sqcup b = f(x) \sqcup f(y) = f(x \sqcup y)$. As I is an ideal of M, $x \sqcup y \subseteq I$, which implies that $f(x \sqcup y) \subseteq f(I)$. It follows that $a \sqcup b \subseteq f(I)$.

Secondly, let $a \in N$ and $b \in f(I)$. Then there exist $y \in I$ such that $b = f(y)$. By the surjectivity of f, there exists an element $x \in M$ such that $a = f(x)$. Thus $a \sqcap b = f(x) \sqcap f(y) = f(x \sqcap y)$. As I is an ideal of M, $x \sqcap y \subseteq I$, which implies that $f(x \sqcap y) \subseteq f(I)$. It follows that $a \sqcap b \subseteq f(I)$.

Finally, let $a,\ b \in N$ such that $(a \sqcap b) \cap f(I) \neq \varnothing$. As f is surjective, there are $x,\ y \in M$ such that $a = f(x)$ and $b = f(y)$. Then $(a \sqcap b) \cap f(I) = (f(x) \sqcap f(y)) \cap f(I) = f(x \sqcap y) \cap f(I) = f((x \sqcap y) \cap I)$, as f injective. Since $(a \sqcap b) \cap f(I) \neq \varnothing$, we have $(x \sqcap y) \cap I \neq \varnothing$. As I is an ideal of M, $x \sqcap y \subseteq I$. That is, $f(x \sqcap y) \subseteq f(I)$. It follows that $a \sqcap b = f(x) \sqcap f(y) = f(x \sqcap y) \subseteq f(I)$. □

Let f_A be a soft set over a multilattice M and $\varphi : M \to N$ be a map between multilattices. That is f is a mapping from E to $P(M)$. Let consider the mapping $\tilde{\varphi} : P(M) \to P(N)$ such that $\tilde{\varphi}(X) = \varphi(X)$ (the direct image), for all $X \in P(M)$. We can see that $(\tilde{\varphi} \circ f, E)$ is a soft set over N. If $x \in A$, then $f(x) \neq \varnothing$, i.e., $\varphi(f(x)) \neq \varnothing$. If rather $f(x) = \varnothing$ $(x \notin A)$, then naturally, $\varphi(f(x)) = \varnothing$. Thus f and $(\tilde{\varphi} \circ f, E)$ have the same support A; we will denoted $(\tilde{\varphi} \circ f, E)$ by φf_A in the sequel.

Proposition 5.11. *Let M be a full multilattice, $\varphi : M \to N$ be a surjective homomorphism of multilattices. If f_A is an f-soft multilattice over M, then φf_A is an f-soft multilattice over N.*

Proof. Assume that f_A is an f-soft multilattice over M. Let $x \in A$ and $a, b \in \varphi(f(x))$. Then there exist $c,\ d \in f(x)$ such that $a = \varphi(c)$ and $b = \varphi(d)$. We have to show that $a \sqcup b \subseteq \varphi(f(x))$ and $a \sqcap b \subseteq \varphi(f(x))$. As f_A is an f-soft multilattice over M, $f(x)$ is an f-submultilattice of M. Thus $c \sqcup d \subseteq f(x)$ and $c \sqcap d \subseteq f(x)$ which implies that $\varphi(c \sqcup d) \subseteq \varphi(f(x))$ and $\varphi(c \sqcap d) \subseteq \varphi(f(x))$. As M is full and φ is surjective, it follows from Corollary 5.9 that $\varphi(c \sqcup d) = \varphi(c) \sqcup \varphi(d)$ and $\varphi(c \sqcap d) = \varphi(c) \sqcap \varphi(d)$. Finally, we have $\varphi(c) \sqcup \varphi(d) \subseteq \varphi(f(x))$ and $\varphi(c) \sqcap \varphi(d) \subseteq \varphi(f(x))$ which implies that $a \sqcup b \subseteq \varphi(f(x))$ and $a \sqcup b \subseteq \varphi(f(x))$ as required. □

Proposition 5.12. *Let M be a full multilattice, $\varphi : M \to N$ be a bijective homomorphism of multilattices and f_A an f-soft multilattice over M. If h_I is a soft ideal of f_A, then φh_I is a soft ideal of φf_A.*

Proof. Proposition 5.11 already reassures that φf_A is an f-soft multilattice over N. $I \subseteq A$ stems from the fact that h_I is a soft ideal of f_A. It remains to show that $\varphi(h(x))$ is an ideal of $\varphi(f(x))$ for all $x \in I$. In fact, if $x \in I$, then $h(x)$ is an ideal of $f(x)$. Note that $f(x)$ is a full multilattice on it own right. Thus applying Proposition 5.10, $\varphi(h(x))$ is an ideal of $\varphi(f(x))$ as required. □

Proposition 5.13. *Let $\varphi : M \to N$ be a bijective homomorphism of multilattices with M a full multilattice. If f_A is an idealistic soft multilattice over M, then φf_A is an idealistic soft multilattice over N.*

Proof. Assume that φ is a bijective homomorphism and f_A an idealistic soft multilattice. Then f_A is non-null, which implies that φf_A is a non-null soft set over N. For all $x \in A$, $f(x)$ is an ideal of M, as f_A is an idealistic soft multilattice. As M is full and φ is bijective, we deduce from Proposition 5.10 (2) that $\varphi(f(x))$ is an ideal of N. It follows that φf_A is an idealistic soft multilattice over N. □

Let $\varphi : M \to N$ be a homomorphism of multilattices such that N has a bottom element "0_N". Then the *kernel* of φ is the set $Ker(\varphi) = \{x \in M : \varphi(x) = 0_N\}$.

Proposition 5.14. *Let f_A be an idealistic soft multilattice over a multilattice M and $\varphi : M \to N$ be a bijective homomorphism of multilattices.*

(1) *If N has bottom element and $f(x) = Ker(\varphi)$ for all $x \in A$, then φf_A is the trivial idealistic soft multilattice over N.*

(2) *If f_A is whole, then φf_A is the whole idealistic soft multilattice over N.*

Proof. (1) Suppose $f(x) = Ker(\varphi)$ for all $x \in A$. Then $\varphi f(x) = \varphi(f(x)) = \varphi(Ker(\varphi)) = \{0_N\}$ for all $x \in A$. Thus φf_A is the trivial idealistic soft multilattice over N by Definition 5.6 (1) and Proposition 5.13.

(2) Suppose f_A is whole. Then $f(x) = M$ for all $x \in A$. Thus $\varphi f(x) = \varphi(f(x)) = \varphi(M)$ for all $x \in A$. As φ is surjective, $\varphi(M) = N$. Finally, $\varphi f(x) = N$ for all $x \in A$. It follows from Definition 5.6 (2) and Proposition 5.13 that φf_A is the whole idealistic soft multilattice over N. □

6. Conclusion

In this paper, we have considered multilattices as universe set in the framework of soft sets theory. The notions of f-soft multilattice and r-soft multilattice have been introduced and some of their properties have been inventigated. We have introduced some related notions such as soft ideals, soft filters, and idalistic soft multilattices with illustrating examples.We have investigated some basic properties of these concepts by using soft sets theory. One may further consider some extensions of multilattices such as residuated multilattices in the framework of soft sets.

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