Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (x 2023), pp. 1–x
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI

http://ribs.wonkwang.ac.kr

On some strong irresolute functions defined by betaopen sets

Ali H. Kocaman

Received 10 October 2023; Revised 30 December 2023; Accepted 19 January 2024

Abstract. This paper is to introduce and investigate new classes of generalizations of non-continuous functions, to obtain some of their properties and to hold decompositions of strong $\alpha lc\beta$-irresoluteness and $s\beta lc$-irresolute in topological spaces.

2020 AMS Classification: 03E72, 54A40

Keywords: β-open set, locally closed set, α-open set, preopen set, strongly $\alpha lc\beta$-irresolute, $s\beta lc$-irresolute function.

Corresponding Author: Ali H. Kocaman (akocaman@ohu.edu.tr)

1. Introduction

In 1989, Ganster and Reilly [1] introduced and studied the notion of LC-continuous functions. Recall the concepts of α-open [2] (resp. locally closed [3], semi-open [4], preopen [5], g-closed [6], rg-closed [7], αlc-[8]) sets and strongly α-irresolute [9] (strongly α-continuous [10] functions in topological spaces. In 1996, Dontchev [11] introduced a stronger form of LC-continuity called contra-continuity. Recently, continuity and irresoluteness of functions in topological spaces have been researched by many mathematicians (See [12, 13]. The aim of this paper is to define and investigate the notions of new classes of functions, namely strongly $\alpha lc\beta$-irresolute, strongly $slc\beta$-irresolute, strongly $plc\beta$-irresolute, strongly $\beta lc\beta$-irresolute, strongly $\alpha glc\beta$-irresolute, strongly $sglc\beta$-irresolute, strongly $pglc\beta$-irresolute, strongly $\beta glc\beta$-irresolute, strongly $\alpha rglc\beta$-irresolute, strongly $srglc\beta$-irresolute, strongly $prglc\beta$-irresolute, strongly $\beta rglc\beta$-irresolute, $\alpha\beta lc$-irresolute, $s\beta lc$-irresolute, $p\beta lc$-irresolute, $\alpha g\beta lc$-irresolute, $sg\beta lc$-irresolute, $pg\beta lc$-irresolute, $\alpha rg\beta lc$-irresolute, $srg\beta lc$-irresolute, $prg\beta lc$-irresolute and to obtain some properties of these functions in topological spaces.

2. Preliminaries

Throughout this paper, spaces always mean topological spaces and $f : X \to Y$ denotes a single valued function of a space (X, τ) into a space (Y, v). Let S be a subset of a space (X, τ). The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively.

Here we recall the following known definitions and properties.

Definition 2.1. A subset S of a space (X, τ) is said to be *α-open* [2] (resp. *semi-open* [4], *preopen* [5], *β-open* [14] or *semi-preopen* [15]), if S ⊂Int(Cl(Int(S))) (resp. S ⊂ Cl(Int(S)), S ⊂ Int(Cl(S)), S ⊂ Cl(Int(Cl(S)))).

Definition 2.2 ([3])**.** A subset A of a space (X, τ) is called a *locally closed* (briefly, *LC*) *set*, if A = S ∩ F, where S is open and F is closed.

The family of all α-open (resp. semi-open, preopen, β-open) sets in a space (X, τ) is denoted by $\tau^\alpha = \alpha$(X) (resp. SO(X), PO(X), βO(X) = SPO(X)). It is shown in [2] that τ^α is a topology on X. Moreover, $\tau \subset \tau^\alpha$= PO(X) ∩ SO(X) ⊂ βO(X).

Definition 2.3. A subset A of a space (X, τ) is called a:

(i) *generalized closed set* (briefly, g-closed) [6], if Cl(A) ⊂ U, whenever A ⊂ U and U is open,

(ii) *regular generalized closed set* (for short, rg-closed) [7], if Cl(A) ⊂ U, whenever A ⊂ U and U is regular open.

Remark 2.4 ([16])**.** Closed → g-closed → rg-closed. In general, none of the implications is reversible.

Definition 2.5 ([8, 17])**.** A subset A of a space (X, τ) is called:

(i) an *αlc-set*, if A = S ∩ F, where S is α-open and F is closed,
(ii) an *slc-set*, if A = S ∩ F, where S is semi-open and F is closed.
(iii) a *plc-set*, if A = S ∩ F, where S is preopen and F is closed,
(iv) a *βlc-set*, if A = S ∩ F, where S is β-open and F is closed,
(v) an *αglc-set*, if A = S ∩ F, where S is α-open and F is g-closed,
(vi) an *sglc-set*, if A = S ∩ F, where S is semi-open and F is g-closed,
(vii) a *pglc-set*, if A = S ∩ F, where S is preopen and F is g-closed.
(viii) a *βglc-set*, if A = S ∩ F, where S is β-open and F is g-closed,
(ix) an *αrglc-set*, if A = S ∩ F, where S is α-open and F is rg-closed,
(x) an *srglc-set*, if A = S ∩ F, where S is semi-open and F is rg-closed,
(xi) a *prglc-set*, if A = S ∩ F, where S is preopen and F is rg-closed,
(xii) a *βrglc-set*, if A = S ∩ F, where S is β-open and F is rg-closed.

The family of all αlc-sets (resp. slc-sets, plc-sets, βlc-sets, αglc-sets, sglc-sets, pglc-sets, βglc-sets, αrglc-sets, srglc-sets, prglc-sets, βrglc-sets) in a space (X, τ) is denoted by αLC(X) (resp. SLC(X), PLC(X), βLC(X), αGLC(X), SGLC(X), PGLC(X), βGLC(X), αRGLC(X), SRGLC(X), PRGLC(X), βRGLC(X)). Moreover, α(X) ⊂ αLC(X) ⊂ PLC(X) and PO(X) ⊂ PLC(X) [17].

Lemma 2.6 ([18])**.** *Let (X, τ) be a topological space. Then we have*

(1) *αLC(X) ⊂ αGLC(X) ⊂ αRGLC(X),*
(2) *PLC(X) ⊂ PGLC(X) ⊂ PRGLC(X),*

(3) *SLC(X)* $\subset$ *SGLC(X)* $\subset$ *SRGLC(X)*,
(4) β*LC(X)* $\subset$ β*GLC(X)* $\subset$ β*RGLC(X)*.

Proof. This observes from Definition 2.5. □

Definition 2.7. A topological space (X, τ) is called a $T_{1/2}$*-space* [6] (resp. T_{rg}*-space* [19]), if every g-closed (resp. rg-closed) subset of X is closed (resp. g-closed).

Theorem 2.8 ([17])**.** *Let* (X, τ) *be a* $T_{1/2}$*-space. Then we have*
(1) α*GLC(X)* = α*LC(X)*,
(2) *SGLC(X)* = *SLC(X)*,
(3) *PGLC(X)* = *PLC(X)*,
(4) β*GLC(X)* = β*LC(X)*.

Theorem 2.9 ([17])**.** *Let* (X, τ) *be a* T_{rg}*-space. Then we have*
(1) α*RGLC(X)* = α*GLC(X)*,
(2) *SRGLC(X)* = *SGLC(X)*,
(3) *PRGLC(X)* = *PGLC(X)*,
(4) β*RGLC(X)* = β*GLC(X)*.

Corollary 2.10 ([20])**.** *Let* (X, τ) *be a* $T_{1/2}$*-space and* T_{rg}*-space. Then we have*
(1) α*RGLC(X)* = α*GLC(X)* =α*LC(X)*,
(2) *SRGLC(X)* = *SGLC(X)* = *SLC(X)*,
(3) *PRGLC(X)* = *PGLC(X)* = *PLC(X)*,
(4) β*RGLC(X)* = β*GLC(X)* = β*LC(X)*.

Lemma 2.11 ([17])**.** *Let A and B be subsets of a topological space* (X, τ)*. Then we have*
(1) *if* $A \in PO(X)$ *and* $B \in \alpha LC(X)$*, then* $A \cap B \in \alpha LC(A)$,
(2) *if* $A \in PO(X)$ *and* $B \in SLC(X)$*, then* $A \cap B \in SLC(A)$,
(3) *if* $A \in SO(X)$ *and* $B \in PLC(X)$*, then* $A \cap B \in PLC(A)$,
(4) *if* $A \in \alpha(X)$ *and* $B \in \beta LC(X)$*, then* $A \cap B \in \beta LC(A)$.

Lemma 2.12. *Let* (X, τ) *be a topological space. Then we have*
(1) $\alpha(X) = PO(X) \cap \alpha LC(X)$ [21],
(2) $SO(X) = SPO(X) \cap \alpha LC(X)$ [8].

Definition 2.13 ([3])**.** A topological space (X, τ) is called a *submaximal space*, if every dense subset of X is open in X.

Definition 2.14 ([2])**.** A topological space (X, τ) is called an *extremally disconnected space*, if the closure of each open subset of X is open in X.

The following theorem follows from the fact that if (X, τ) is a submaximal and extremally disconnected space, then $\tau = \tau^{\alpha}$ = SO(X) = PO(X) = βO(X) (See [22, 23]).

Theorem 2.15 ([17])**.** *Let* (X, τ) *be a submaximal and extremally disconnected space. Then we have*
(1) α*lc-set* $\Longleftrightarrow$ *slc-set* $\Longleftrightarrow$ *plc-set* $\Longleftrightarrow$ β*lc-set*,
(2) α*glc-set* $\Longleftrightarrow$ *sglc-set* $\Longleftrightarrow$ *pglc-set* $\Longleftrightarrow$ β*glc-set*,
(3) α*rglc-set* $\Longleftrightarrow$ *srglc-set* $\Longleftrightarrow$ *prglc-set* $\Longleftrightarrow$ β*rglc-set*.

3. Generalizations of some types strong functions

Definition 3.1 ([15])**.** A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *α-precontiuous*, if $f^{-1}(V)$ is preopen set in X for every α-open subset V of Y.

Definition 3.2. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *irresolute* [24] (resp. *semi αlc-continuous* [18]), if $f^{-1}(V)$ is semi-open set(resp. αlc-set) in X for every semi-open subset V of Y.

Definition 3.3 ([17])**.** A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *αlc-irresolute*, if $f^{-1}(V)$ is αlc-set in X for every αlc-set in V of Y.

Definition 3.4. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *β-irresolute* [25] (resp. *strongly Semi β-irresolute* [26]), if $f^{-1}(V)$ is β-open set (resp. semi-open) in X for every β-open subset V of Y.

Definition 3.5. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *strongly αlcβ-irresolute* (resp. *strongly slcβ-irresolute*, *strongly plcβ-irresolute*, *strongly βlcβ-irresolute*), if $f^{-1}(V)$ is β-open set in X for every αlc-set (resp. slc-set, plc-set, βlc-set) V of Y.

Definition 3.6. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *strongly αglcβ-irresolute* (resp. *strongly sglcβ-irresolute*, *strongly pglcβ-irresolute*, *strongly βglcβ-irresolute*), if $f^{-1}(V)$ is β-open set in X for every αglc-set (resp. sglc-set, pglc-set, βglc-set) V of Y.

Definition 3.7. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *strongly αrglcβ-irresolute* (resp. *strongly srglcβ-irresolute*, *strongly prglcβ-irresolute*, *strongly βrglcβ-irresolute*), if $f^{-1}(V)$ is β-open set in X for every αrglc-set (resp. srglc-set, prglc-set, βrglc-set) V of Y.

Definition 3.8. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *$\alpha\beta$lc-irresolute* (resp. *sβlc-irresolute*, *pβlc-irresolute*, *βlc-irresolute* [17]), if $f^{-1}(V)$ is βlc-set in X for every αlc-set (resp. slc-set, plc-set, βlc-set) V of Y.

Definition 3.9. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *$\alpha g\beta$lc-irresolute* (resp. *sgβlc-irresolute*, *pgβlc-irresolute*, *$\beta g\beta$lc-irresolute*), if $f^{-1}(V)$ is βlc-set in X for every αglc-set (resp. sglc-set, pglc-set, βglc-set) V of Y.

Definition 3.10. A function $f : (X, \tau) \to (Y, \upsilon)$ is said to be *$\alpha rg\beta$lc-irresolute* (resp. *srgβlc-irresolute*, *prgβlc-irresolute*, *$\beta rg\beta$lc-irresolute*), if $f^{-1}(V)$ is βlc-set in X for every αrglc-set (resp. srglc-set, prglc-set, βrglc-set) V of Y.

From the definitions, we have the following relationships:

$$\begin{array}{ccc} \text{semi } \alpha\text{-irresoluteness} & \longrightarrow & \text{semi } \alpha\text{lc-continuity} \\ \downarrow & & \downarrow \\ \text{strong } \alpha\text{lc}\beta\text{-irresoluteness} & \longrightarrow & \text{s}\beta\text{lc-irresoluteness} \end{array}$$

Figure 1

However, the converses of the above implications are not true in general by the following examples.

Example 3.11. Let X = {a, b, c} and let a function $f : (X, \tau) \to (Y, \upsilon)$ be the identity. If τ = {X , ϕ, {a}} and υ = {X, ϕ, {b,c}} are two topologies on X, then f is semi αlc-continuity and sβlc-irresolute but it is not strongly αlcβ-irresolute and Semi α-irresolute.

Example 3.12. Let X = {a, b, c} and let a function $f : (X, \tau) \to (Y, \upsilon)$ be the identity. If τ = {X , ϕ, {a}, {b, c}} and υ = {X , ϕ, {a,b}} are two topologies on X, then f is strongly αlcβ-irresolute and sβlc-irresolute but it is not semi αlc-continuity Semi α-irresolute.

Theorem 3.13. *Let X be a topological space and let $(Y_\lambda)_\Lambda \in \Lambda$ be a family of topological spaces. If a function $f : X \to \Pi_{\lambda\in\Lambda} Y_\lambda$ is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute), then $P_\lambda \circ f : X \to Y_\lambda$ is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute) for each $\lambda\epsilon\Lambda$, where P_λ is the projection of $\Pi_{\lambda\in\Lambda} Y_\lambda$ onto Y_λ.*

Proof. Suppose f is strongly αlcβ-irresolute and let V_λ be any αlc-set of Y_λ for each $\lambda \in \Lambda$. Since P_λ is continuous and open, it is αlc-irresolute [17]. Then $P_\lambda^{-1}(V_\lambda)$ is αlc-set in $\Pi_{\lambda\in\Lambda} Y_\lambda$. Since f is strongly αlcβ-irresolute, $f^{-1}(P_\lambda^{-1}(V_\lambda)) = (P_\lambda \circ f)^{-1}(V_\lambda)$ is an β-open set in X. Thus $P_\lambda \circ f$ is strongly αlcβ-irresolute. Similarly, the other assertions are proved. □

Theorem 3.14. *If $f : (X, \tau) \to (Y, \upsilon)$ is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute) and A is α-open subsets of X, then the restriction $f/_A : A \to Y$ is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute).*

Proof. Suppose f is strongly αlcβ-irresolute and let V be any αlc-set of Y for each $\lambda \in \Lambda$. Since f is strongly αlcβ-irresolute, $f^{-1}(V)$ is a β-open in X. Since A is α-open in X, $(f/_A)^{-1}(V) = A \cap f^{-1}(V)$ is a β-open in A by Lemma 2.2 (4) in [17]. Then $f/_A$ is strongly αlcβ-irresolute. The other assertions are similarly proved. □

Theorem 3.15. *Let $f : X \to Y$ be a function and $g : Y \to Z$ be strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute) function. If f is β-irresolute, then the composition $g \circ f : X \to Z$ is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute,* *strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute).*

Proof. Let g be strongly αlcβ-irresolute. Suppose f is β-irresolute and let W be any αlc-set subset of Z. Since g is strongly αlcβ-irresolute $g^{-1}(W)$ is β-open in Y. Since f is β-irresolute, $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is β-open in X. Then $g \circ f$ strongly αlcβ-irresolute. The other assertions are similarly proved. □

Theorem 3.16. *Let $f : X \to Y$ be a function and $g : Y \to Z$ be αlc-irresolute (resp. slc-irresolute, plc-irresolute, βlc-irresolute, αglc-irresolute, sglc-irresolute, pglc-irresolute, βglc-irresolute, αrglc-irresolute, srglc-irresolute, prglc-irresolute, βrglc-irresolute) function. If f is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute), then the composition $g \circ f : X \to Z$ is strongly αlcβ-irresolute (resp. strongly slcβ-irresolute, strongly plcβ-irresolute, strongly βlcβ-irresolute, strongly αglcβ-irresolute, strongly sglcβ-irresolute, strongly pglcβ-irresolute, strongly βglcβ-irresolute, strongly αrglcβ-irresolute, strongly srglcβ-irresolute, strongly prglcβ-irresolute, strongly βrglcβ-irresolute).*

Proof. Let g be αlc-irresolute. Suppose f is strongly αlcβ-irresolute and let W be any αlc-set subset of Z. Since g is αlc-irresolute, $g^{-1}(W)$ is αlc-set in Y. Since f is strongly αlcβ-irresolute, $(gof)^{-1}(W)$=$f^{-1}(g^{-1}(W))$ is β-open in X. Then $g \circ f : X \to Z$ is strongly αlcβ-irresolute. The other assertions are similarly proved. □

Theorem 3.17. *Let $f : X \to Y$ be a function and $g : Y \to Z$ be strongly αlc-irresolute (resp. Strongly slc-irresolute, strongly plc-irresolute, strongly βlc-irresolute, strongly αglc-irresolute, strongly sglc-irresolute, strongly pglc-irresolute, strongly βglc-irresolute, strongly αrglc-irresolute, strongly srglc-irresolute, strongly prglc-irresolute, strongly βrglc-irresolute) function. If f is α-precontinuous, then the composition $g \circ f : X \to Z$ is strongly αlc-preirresolute (resp. strongly slc-preirresolute, strongly plc-preirresolute, strongly βlc-preirresolute, strongly αglc-preirresolute, strongly sglc-preirresolute, strongly pglc-preirresolute, strongly βglc-preirresolute, strongly αrglc-preirresolute, strongly srglc-preirresolute, strongly prglc-preirresolute, strongly βrglc-preirresolute).*

Proof. Let g be strongly αlc-irresolute. Suppose f is α-precontinuous and let W be any αlc-set subset of Z. Since g is strongly αlc-irresolute, $g^{-1}(W)$ is α-open in Y. Since f is α-precontinuity, $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is preopen in X. Then $g \circ f$ strongly αlc-preirresolute. The other assertions are similarly proved. □

Theorem 3.18. *Let (X, τ) be a $T_{1/2}$-space and let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*

(1) *strongly αglcβ-irresolute $\iff$ strongly αlcβ-irresolute,*
(2) *strongly sglcβ-irresolute $\iff$ strongly slcβ-irresolute,*
(3) *strongly pglcβ-irresolute $\iff$ strongly plcβ-irresolute,*
(4) *strongly βglcβ-irresolute $\iff$ strongly βlcβ-irresolute.*

Proof. It is obvious from Theorem 2.8. □

Theorem 3.19. *Let (X, τ) be a $T_{1/2}$-space and let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*
(1) *$\alpha g\beta lc$-irresolute $\iff$ $\alpha\beta lc$-irresolute,*
(2) *$sg\beta lc$-irresolute $\iff$ $s\beta lc$-irresolute,*
(3) *$pg\beta lc$-irresolute $\iff$ $p\beta lc$-irresolute,*
(4) *$\beta g\beta lc$-irresolute $\iff$ βlc-irresolute.*

Proof. It is obvious from Theorem 2.8 □

Theorem 3.20. *Let (X, τ) be a T_{rg}-space and let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*
(1) *strongly $\alpha rglc\beta$-irresolute $\iff$ strongly $\alpha glc\beta$-irresolute.*
(2) *strongly $srglc\beta$-irresolute $\iff$ strongly $sglc\beta$-irresolute,*
(3) *strongly $prglc\beta$-irresolute $\iff$ strongly $pglc\beta$-irresolute,*
(4) *strongly $\beta rglc\beta$-irresolute $\iff$ strongly $\beta glc\beta$-irresolute.*

Proof. It is obvious from Theorem Theorem 2.9 □

Theorem 3.21. *Let (X, τ) be a T_{rg}-space and let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*
(1) *$\alpha rg\beta lc$-irresolute $\iff$ $\alpha g\beta lc$-irresolute,*
(2) *$srg\beta lc$-irresolute $\iff$ $sg\beta lc$-irresolute,*
(3) *$prg\beta lc$-irresolute $\iff$ $pg\beta lc$-irresolute,*
(4) *$\beta rg\beta lc$-irresolute $\iff$ $\beta g\beta lc$-irresolute.*

Proof. It is obvious from Theorem Theorem 2.9 □

Corollary 3.22. *Let (X, τ) be a $T_{1/2}$-space and T_{rg}-space. Let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*
(1) *strongly $\alpha rglc\beta$-irresolute $\iff$ strongly $\alpha glc\beta$-irresolute $\iff$ strongly $\alpha lc\beta$-irresolute,*
(2) *strongly $srglc\beta$-irresolute $\iff$ strongly $sglc\beta$-irresolute $\iff$ strongly $slc\beta$-irresolute,*
(3) *strongly $prglc\beta$-irresolute $\iff$ strongly $pglc\beta$-irresolute $\iff$ strongly $plc\beta$-irresolute,*
(4) *strongly $\beta rglc\beta$-irresolute $\iff$ strongly $\beta glc\beta$-irresolute $\iff$ strongly $\beta lc\beta$-irresolute.*

Proof. It is obvious from Corollary 2.10. □

Corollary 3.23. *Let (X, τ) be a $T_{1/2}$-space and T_{rg}-space. Let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*
(1) *$\alpha rg\beta lc$-irresolute $\iff$ $\alpha g\beta lc$-irresolute $\iff$ $\alpha\beta lc$-irresolute,*
(2) *$srg\beta lc$-irresolute $\iff$ $sg\beta lc$-irresolute $\iff$ $s\beta lc$-irresolute,*
(3) *$prg\beta lc$-irresolute $\iff$ $pg\beta lc$-irresolute $\iff$ $p\beta lc$-irresolute,*
(4) *$\beta rg\beta lc$-irresolute $\iff$ $\beta g\beta lc$-irresolute $\iff$ βlc-irresolute.*

Proof. It is obvious from Corollary 2.10. □

Theorem 3.24. *Let (X, τ) be a submaximal and extremally disconnected space and let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*

(1) *strongly $\alpha lc\beta$-irresolute $\Longleftrightarrow$ strongly $slc\beta$-irresolute $\Longleftrightarrow$ strongly $plc\beta$-irresolute $\Longleftrightarrow$ strongly $\beta lc\beta$-irresolute,*

(2) *strongly $\alpha glc\beta$-irresolute $\Longleftrightarrow$ strongly $sglc\beta$-irresolute $\Longleftrightarrow$ strongly $pglc\beta$-irresolute $\Longleftrightarrow$ strongly $\beta glc\beta$-irresolute,*

(3) *strongly $\alpha rglc\beta$-irresolute $\Longleftrightarrow$ strongly $srglc\beta$-irresolute $\Longleftrightarrow$ strongly $prglc\beta$-irresolute $\Longleftrightarrow$ strongly $\beta rglc\beta$-irresolute.*

Proof. It is obvious from Theorem 2.15. □

Theorem 3.25. *Let (X, τ) be a submaximal and extremally disconnected space and let $f : (X, \tau) \to (Y, \upsilon)$ be a function. Then we have*

(1) *$\alpha\beta lc$-irresolute $\Longleftrightarrow$ $s\beta lc$-irresolute $\Longleftrightarrow$ $p\beta lc$-irresolute $\Longleftrightarrow$ βlc-irresolute,*

(2) *$\alpha g\beta lc$-irresolute $\Longleftrightarrow$ $sg\beta lc$-irresolute $\Longleftrightarrow$ $pg\beta lc$-irresolute $\Longleftrightarrow$ $\beta g\beta lc$-irresolute,*

(3) *$\alpha rg\beta lc$-irresolute $\Longleftrightarrow$ $srg\beta lc$-irresolute $\Longleftrightarrow$ $prg\beta lc$-irresolute $\Longleftrightarrow$ $\beta rg\beta lc$-irresolute.*

Proof. It is obvious from Theorem 2.15. □

Theorem 3.26. *For a function $f : (X, \tau) \to (Y, \upsilon)$ the following hold;*

(1) *f is semi α-irresolute if and only if strongly $\alpha lc\beta$-irresolute and semi αlc-continuous,*

(2) *f is strongly semi β-irresolute if and only if strongly $\alpha lc\beta$-irresolute and strongly αlc- irresolute,*

(3) *f is irresolute if and only if strongly $\alpha lc\beta$-irresolute and strongly αlc- irresolute.*

Proof. It is obvious from Theorem 2.15. □

References

[1] M. Ganster and I. L. Reilly, Locally closed sets and LC-continuous functions, Int. J. Math. Math. Sci. 12 (1989) 417–424.

[2] O. Njåstad, On some classes of nearly open sets, Pacific J. Math. 15 (1965) 961–970.

[3] N. Bourbaki, Elements of mathematics, General Topology, Part 1, Paris, Hermann, Addison-Wesley Publishing Co., Reading, Mass. 1966.

[4] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70 (1963) 36–41.

[5] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt 53 (1982) 47–53.

[6] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo 19 (2) (1970) 89–96.

[7] N. Palaniappan and K. C. Rao, Regular generalized closed sets, Kyungpook Math. J. 33 (1993) 211–219.

[8] B. Al-Nashef, A decomposition of α-continuity and semi-continuity, Acta. Math. Hungar. 97 (2002) 115–120.

[9] G. Lo Faro, On strongly α-irresolute mappings, Indian J. Pure Appl. Math. 1 (1987) 146–151.

[10] Y. Beceren, On strongly α-continuous functions, Far East J. Math. Sci. (FJMS) Special Vol. Part I (2000) 51–58.

[11] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Int. J. Math. Math. Sci. 19 (2) (1996) 303–310.

[12] A. I. El-Maghrabi and M. A. Al-Juhani, Further properties on M-continuity, J. Egypt. Math. Soc. 22 (1) (2014) 63–69.

[13] Ali M. Mubarki, Massed M. Al-Rshudi and Mohammad A. Al-Juhani, β^*-open sets and β^*-continuity in topological spaces, Journal of Taibah University for Science 8 (2014) 142–148.
[14] M. E. Abd El-Monsef, S. N. El-Deepand and R. A. Mahmuoud, β-open sets and β-continuous mapping, Bull. Fac. Sci. Assiut Univ. 12 (1983) 77-90.
[15] D. Andrijević, Semi-preopen sets, Mat. Vesnik 38 (1986) 24–32.
[16] T. Noiri, Mildly normal spaces and some functions, Kyungpook Math. J. 36 (1996) 183–190.
[17] Y. Beceren, T. Noiri, M. C. Fidancı and K. Arslan, On some generalizations of locally closed sets and lc-continuous functions, Far East J. Math. Sci. (FJMS) 22 (3) (2006) 333–344.
[18] Y. Beceren and T. Noiri, Some functions defined by semi-open and β-open sets, Chaos Solitons Fractals 36 (2008) 1225–1231.
[19] I. Arockia Rani and K. Balachandran, On regular generalized continuous maps in topological spaces, Kyungpook Math. J. 37 (1997) 305–314.
[20] A. H. Kocaman, S. Yüksel and A. Açıkgöz, On some strongly functions defined by α-open, Chaos Solitons Fractals 39 (2009) 1346–1355.
[21] G. Aslım and Y. Ayhan, Decompositions of some weaker forms of continuity, cta Math. Hungar. 105 (2004) 85–91.
[22] D. S. Janković, On locally irreducible spaces, Ann. Soc. Sci. Bruxelles Sér. I 97 (1983) 59–72.
[23] A. A. Nasef and T. Noiri, Strong forms of faint continuity, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 19 (1998) 21–28.
[24] S. G. Crossley and S. K. Hildebrand, Semi-topological properties, Fund. Math. 74 (1972) 233–254.
[25] R. A. Mahmoud and M. E. Abd El-Monsef, R. A. Mahmoud and M. E. Abd El-Monsef, β-irresolute and β-topological invariant. Proc. Pakistan Acad Sci. 27 (1990) 285–296.
[26] Y. Beceren, strongly semi β-irresolute functions and semi α-preirresolute functions, Far East J. Math. Sci. (FJMS) 22 (2001) 973–983.

Ali H. Kocaman (akocaman@ohu.edu.tr)
Nigde Omer Halisdemir University, Faculty of Sciences, Department of Mathematics. Nigde, 51100 Türkiye