Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (x 2023), pp. 1–x
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

@FMI
© Research Institute for Basic Science, Wonkwang University
http://ribs.wonkwang.ac.kr

Fuzzy α-b-almost compact space

Anjana Bhattacharyya

Received 16 December 2023; Revised 2 January 2024; Accepted 31 January 2024

Abstract. This paper deals with some applications of fuzzy α-b-open set. Here we introduce fuzzy α-b-almost compactness and characterize this concept via fuzzy net and prefilterbase. Also we introduce fuzzy regularly α-b-open set which characterizes fuzzy α-b-almost compactness. It is shown that fuzzy α-b-almost compactness implies fuzzy almost compactness and the converse is true only on fuzzy α-b-regular space.

2020 AMS Classification: 54A40, 03E72

Keywords: Fuzzy α-b-open set, Fuzzy α-b-regular space, Fuzzy regularly α-b-closed set, Fuzzy α-b-almost compact set (space), αb-adherent point of a prefilterbase, αb-cluster point of a fuzzy net.

Corresponding Author: Anjana Bhattacharyya (anjanabhattacharyya@hotmail.com)

1. Introduction

Fuzzy α-b-open set was introduced in [1] using fuzzy α-open set as a basic tool. After introducing fuzzy compactness by Chang [2], many mathematicians have engaged themselves to introduce different types of fuzzy compactness. In [3], fuzzy almost compactness was introduced.

In this paper, we introduce the concept of fuzzy α-b-almost compactness which is weaker than fuzzy almost compactness. Here we use fuzzy net [4] and prefilterbase [5] to characterize fuzzy α-b-almost compactness.

In recent time, different types of fuzzy sets, viz., fuzzy soft set and fuzzy octahedron set are introduced and studied. A new branch of fuzzy topology is developed using these types of fuzzy sets. In this context we have to mention [6, 7, 8, 9, 10, 11].

2. Preliminaries

In 1965, Zadeh [12] introduced a *fuzzy set* A which is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The *support* of a fuzzy set A in X, denoted by $suppA$, is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The *complement* of a fuzzy set A in X, denoted by $1_X \setminus A$, is defined by $(1_X \setminus A)(x) = 1 - A(x)$ for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ while AqB means A is quasi-coincident (q-coincident, for short) [4] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. The *intersection* and the *union* of two fuzzy sets A, B in X, denoted by $A \wedge B$ and $A \vee B$, are fuzzy sets in X defined by: for each $x \in X$,

$$(A \wedge B)(x) = min\{A(x), B(x)\} \text{ and } (A \vee B)(x) = max\{A(x), B(x)\}.$$

Throughout this paper, (X, τ) or simply by X we shall mean an fts. For a fuzzy set A in X, clA and $intA$ will stand for the *fuzzy closure* and the *fuzzy interior* of A respectively (See [2]). A fuzzy set A in X is called a *fuzzy neighbourhood* (fuzzy nbd, for short) of a fuzzy point x_t, if there exists a fuzzy open set G in X such that $x_t \in G \leq A$. If, in addition, A is fuzzy open, then A is called a *fuzzy open nbd* of x_t (See [4]). A fuzzy set A is said to be a *fuzzy q-nbd* of a fuzzy point x_t in an fts X, if there is a fuzzy open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy open, then A is called a *fuzzy open q-nbd* of x_t (See [4]).

A fuzzy set A in an fts (X, τ) is called a *fuzzy α-open set* in X, if $A \leq int(cl(intA))$. The complement of a fuzzy α-open set is called a *fuzzy α-closed set* in X. The union (the intersection) of all fuzzy α-open (resp. fuzzy α-closed) sets contained in (resp. containing) a fuzzy set A is called the *fuzzy α-interior* (resp. the *fuzzy α-closure* of A, denoted by $\alpha intA$ (resp. αclA) (See [13]).

Let $(D, \geq)$ be a directed set and X be an ordinary set. Let J denote the collection of all fuzzy points in X. A function $S : D \to J$ is called a *fuzzy net* in X [4]. It is denoted by $\{S_n : n \in (D, \geq)\}$. A non empty family $\mathcal{F}$ of fuzzy sets in X is called a *prefilterbase* on X, if (i) $0_X \notin \mathcal{F}$ and (ii) for any U, $V \in \mathcal{F}$, there exists $W \in \mathcal{F}$ such that $W \leq U \wedge V$ [5].

3. Fuzzy α-b-Open Sets : Some Results

In this section we recall some definitions and results from [1, 2, 3, 14, 15] for ready references. Also some properties of fuzzy α-b-open sets are discussed here.

Definition 3.1 ([1]). A fuzzy set A in an fts (X, τ) called a *fuzzy α-b-open set* in X, if $A \leq cl(\alpha int(clA))$.
The complement of a fuzzy α-b-open set is called a *fuzzy α-b-closed set* in X. The collection of all fuzzy α-b-open (resp. fuzzy α-b-closed) sets in X is denoted by $F\alpha bO(X)$ (resp. $F\alpha bC(X)$).

Definition 3.2 ([1]). Let (X, τ) be an fts and $A \in I^X$. Then the *fuzzy α-b-closure* of A, denoted by $\alpha bclA$, is defined by

$$\alpha bclA = \bigwedge\{U \in I^X : A \leq U, \ U \in F\alpha bC(X)\}$$

and the *fuzzy α-b-interior* of A, denoted by $\alpha bintA$, is defined by

$$\alpha bintA = \bigvee\{G : G \leq A,\ G \in F\alpha bO(X)\}.$$

Definition 3.3 ([1])**.** A fuzzy set A in an fts (X, τ) is called a *fuzzy α-b-nbd* of a fuzzy point x_t in X, if there exists a fuzzy α-b-open set U in X such that $x_t \in U \leq A$. If, in addition, A is fuzzy α-b-open, then A is called a *fuzzy α-b-open nbd* of x_t.

Definition 3.4 ([1])**.** A fuzzy set A in an fts (X, τ) is called a *fuzzy α-b-q-nbd* of a fuzzy point x_t in X, if there exists a fuzzy α-b-open set U in X such that $x_t qU \leq A$. If, in addition, A is fuzzy α-b-open, then A is called a *fuzzy α-b-open q-nbd* of x_t.

Result 3.5 ([1])**.** *The union* (resp. *intersection*) *of any two fuzzy α-b-open* (resp. fuzzy α-b-closed) *sets is also so.*

Result 3.6 ([1])**.** *$x_t \in \alpha bclA$ if and only if every fuzzy α-b-open q-nbd U of x_t, UqA.*

Result 3.7. *$\alpha bcl(\alpha bclA) = \alpha bclA$ for any fuzzy set A in an fts X.*

Proof. Let $A \in I^X$. Then clearly, $A \leq \alpha bclA$. Thus we have

$$\alpha bclA \leq \alpha bcl(\alpha bclA). \tag{3.1}$$

Conversely, let $x_t \in \alpha bcl(\alpha bclA)$. Assume that $x_t \notin \alpha bclA$. Then there exists $U \in F\alpha bO(X)$ such that

$$x_t qU,\ U \not q A. \tag{3.2}$$

Since $x_t \in \alpha bcl(\alpha bclA)$, $Uq(\alpha bclA)$. Thus there exists $y \in X$ such that $U(y) + (\alpha bclA)(y) > 1$. So $U(y) + s > 1$, where $s = (\alpha bclA)(y)$. Then $y_s \in \alpha bclA$ and $y_s qU$ where $U \in F\alpha bO(X)$. By definition, UqA. This contradicts (3.2). Hence we get

$$\alpha bcl(\alpha bclA) \leq \alpha bclA. \tag{3.3}$$

Therefore combining (3.1) and (3.3), we get the result. □

Result 3.8. *$\alpha bcl(A \vee B) = \alpha bclA \bigvee \alpha bclB$ for any two fuzzy sets $A,\ B$ in X.*

Proof. It is clear that

$$\alpha bclA \bigvee \alpha bclB \leq \alpha bcl(A \vee B). \tag{3.4}$$

Conversely, let $x_t \in \alpha bcl(A \bigvee B)$ and let U be any α-b-open q-nbd of x_t. Then clearly, $Uq(A \bigvee B)$. Thus there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1$. So either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1$, i.e., either UqA or UqB. Hence either $x_t \in \alpha bclA$ or $x_t \in \alpha bclB$. Therefore $x_t \in \alpha bclA \vee \alpha bclB$. □

Result 3.9. *For any fuzzy set A in an fts (X, τ),*
(1) *$\alpha bcl(1_X \setminus A) = 1_X \setminus \alpha bintA$,*
(2)*$\alpha bint(1_X \setminus A) = 1_X \setminus \alpha bclA$.*

Proof. (1) Let $x_t \in \alpha bcl(1_X \setminus A)$ and assume that $x_t \notin 1_X \setminus \alpha bintA$. Then $x_t q \alpha bintA$. Thus there exists a fuzzy α-b-open set B in X with $B \leq A$ such that $x_t qB$. So B is a fuzzy α-b-open q-nbd of x_t. By the assumption, $Bq(1_X \setminus A) \Rightarrow Aq(1_X \setminus A)$, which is absurd.

Conversely, let $x_t \in 1_X \setminus \alpha bintA$. Then $x_t \not{q} \alpha bintA$. Thus $x_t \not{q} U$ for any fuzzy α-b-open set U in X with $U \leq A$. So $x_t \in 1_X \setminus U$ which is fuzzy α-b-closed set in X with $1_X \setminus A \leq 1_X \setminus U$. Hence $x_t \in \alpha bcl(1_X \setminus A)$.

(2) Writing $1_X \setminus A$ for A in (1), we get the result. □

Definition 3.10. Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a *fuzzy cover* of A, if $sup\{U(x) : U \in \mathcal{U}\} = 1$ for each $x \in suppA$ [14]. If each member of $\mathcal{U}$ is fuzzy open (resp. fuzzy α-b-open), we call $\mathcal{U}$ is a *fuzzy open* [14] (resp. *fuzzy α-b-open*) *cover* of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of X [2].

Definition 3.11. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to *have a finite* (resp. *finite proximate*) *subcover* $\mathcal{U}_0$, if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigvee \mathcal{U}_0 \geq A$ [14] (resp. $\bigvee\{clU : U \in \mathcal{U}_0\} \geq A$ [15]). In particular, if $A = 1_X$, we get $\bigvee \mathcal{U}_0 = 1_X$ (resp. $\bigvee\{clU : U \in \mathcal{U}_0\} = 1_X$ [3]).

Definition 3.12 ([3])**.** An fts (X, τ) is called a *fuzzy almost compact space*, if every fuzzy open cover has a finite proximate subcover.

4. Fuzzy α-b-almost compact space : Some characterizations

In this section, the concept of fuzzy α-b-almost compactness is introduced and studied by fuzzy α-b-open and fuzzy regularly α-b-open sets and characterize this space via fuzzy net and prefilter base.

Definition 4.1. A fuzzy set A in an fts (X, τ) is said to be a *fuzzy α-b-almost compact set*, if every fuzzy α-b-open cover $\mathcal{U}$ of A has a finite αb-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{\alpha bclU : U \in \mathcal{U}_0\} \geq A$. If, in addition, $A = 1_X$, we say that the fts X is fuzzy α-b-almost compact space.

Definition 4.2. A fuzzy point x_t in an fts X is said to *belong to* the αb-closure of a fuzzy set A in X, denoted by $x_t \in \alpha b$-clA, if for every fuzzy α-b-open q-nbd U of x_t, $\alpha bclUqA$.

Definition 4.3. Let x_t be a fuzzy point in an fts (X, τ). A prefilterbase $\mathcal{F}$ on X is said to be:

(i) *αb-adhere at x_t*, written as $x_t \in \alpha b$-ad $\mathcal{F}$, if for each fuzzy α-b-open q-nbd U of x_t and each $F \in \mathcal{F}$, $Fq(\alpha bclU)$, i.e., $x_t \in \alpha b$-clF, for each $F \in \mathcal{F}$,

(ii) *αb-converge to x_t*, written as $\mathcal{F} \overrightarrow{\alpha b} x_t$, if to each fuzzy α-b-open q-nbd U of x_t, there corresponds some $F \in \mathcal{F}$ such that $F \leq \alpha bclU$.

Definition 4.4. Let x_t be a fuzzy point in an fts (X, τ). A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to be:

(i) *αb-adhere at x_t*, denoted by $x_t \in \alpha b$-ad(S_n), if for each fuzzy α-b-open q-nbd U of x_t and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m q \alpha bclU$,

(ii) *αb-converge to x_t*, denoted by $S_n \overrightarrow{\alpha b} x_t$, if for each fuzzy α-b-open q-nbd U of x_t, there exists $m \in D$ such that $S_n q \alpha bclU$, for all $n \geq m (n \in D)$.

Theorem 4.5. *For a fuzzy set A in an fts X, the following statements are equivalent:*

(1) *A is a fuzzy α-b-almost compact set,*

(2) *for every prefilterbase* $\mathcal{B}$ *in* X, $[\bigwedge\{\alpha bclB : B \in \mathcal{B}\}] \wedge A = 0_X$ *implies that there exists a finite subcollection* $\mathcal{B}_0$ *of* $\mathcal{B}$ *such that* $\bigwedge\{\alpha bintB : B \in \mathcal{B}_0\} \not{q} A$,

(3) *for any family* $\mathcal{F}$ *of fuzzy* α*-b-closed sets in* X *with* $\bigwedge\{F : F \in \mathcal{F}\} \wedge A = 0_X$, *there exists a finite subcollection* $\mathcal{F}_0$ *of* $\mathcal{F}$ *such that* $\bigwedge\{\alpha bintF : F \in \mathcal{F}_0\} \not{q} A$,

(4) *every prefilterbase on* X, *each member of which is q-coincident with* A, αb*-adheres at some fuzzy point in* A.

Proof. (1) $\Rightarrow$ (2): Suppose (1) holds and let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{\alpha bclB : B \in \mathcal{B}\}] \wedge A = 0_X$. Then we have: for any $x \in suppA$,

$$\begin{aligned} &[\bigwedge\{\alpha bclB : B \in \mathcal{B}\}](x) = 0 \\ &\Rightarrow 1 - [\bigwedge\{\alpha bclB(x) : B \in \mathcal{B}\}] = 1 \\ &\Rightarrow \bigvee[(1_X \setminus \alpha bclB)(x) : B \in \mathcal{B}] = 1 \\ &\Rightarrow sup\{\alpha bint(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1 \\ &\Rightarrow \{\alpha bint(1_X \setminus B) : B \in \mathcal{B}\} \text{ is a fuzzy } \alpha\text{-}b\text{-open cover of } A. \end{aligned}$$

By (1), there exists a finite αb-proximate subcover of it for A

$$\{\alpha bint(1_X \setminus B_1), \alpha bint(1_X \setminus B_2), \cdots, \alpha bint(1_X \setminus B_n)\}.$$

Thus we get

$$\begin{aligned} A &\leq \bigvee_{i=1}^{n} \alpha bcl(\alpha bint(1_X \setminus B_i)) \\ &= \bigvee_{i=1}^{n} [1_X \setminus \alpha bint(\alpha bclB_i)] \\ &= 1_X \setminus \bigwedge_{i=1}^{n} \alpha bint(\alpha bclB_i). \end{aligned}$$

Thus $\bigwedge_{i=1}^{n} \alpha bint(\alpha bclB_i) \leq 1_X \setminus A$. So $A \not{q} \bigwedge_{i=1}^{n} \alpha bint(\alpha bclB_i)$. Hence $A \not{q} \bigwedge_{i=1}^{n} \alpha bintB_i$.

(2) $\Rightarrow$ (1): Suppose (2) holds and assume that there exists a fuzzy α-b-open cover $\mathcal{U}$ of A having no finite αb-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $sup\{\alpha bclU(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - sup\{(\alpha bclU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0$, i.e., $inf\{(1_X \setminus \alpha bclU)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0} (1_X \setminus \alpha bclU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ (=$\mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, U_2, \cdots, U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^{n} \alpha bint(1_X \setminus \alpha bclU_i) \not{q} A$, then Result 3.7, we have

$$\begin{aligned} A &\leq 1_X \setminus \bigwedge_{i=1}^{n} \alpha bint(1_X \setminus \alpha bclU_i) \\ &= \bigvee_{i=1}^{n} [1_X \setminus \alpha bint(1_X \setminus \alpha bclU_i)] \\ &= \bigvee_{i=1}^{n} \alpha bcl(\alpha bclU_i) \end{aligned}$$

$$= \bigvee_{i=1}^{n} \alpha bclU_i.$$

Thus $\mathcal{U}$ has a finite αb-proximate subcover for A, contradicts our hypothesis. So for every finite subcollection $\{\bigwedge_{U\in\mathcal{U}_1}(1_X \setminus \alpha bclU), \cdots, \bigwedge_{U\in\mathcal{U}_k}(1_X \setminus \alpha bclU)\}$ of $\mathcal{B}$, where $\mathcal{U}_1, \cdots, \mathcal{U}_k$ are finite subset of $\mathcal{U}$, we have $[\bigwedge_{U\in\mathcal{U}_1\bigvee\cdots\bigvee\mathcal{U}_k} \alpha bint(1_X \setminus \alpha bclU)]qA$. By (2), $[\bigwedge_{U\in\mathcal{U}} \alpha bcl(1_X \setminus \alpha bclU)] \wedge A \neq 0_X$. Hence there exists $x \in suppA$, such that

$$\inf_{U\in\mathcal{U}}[\alpha bcl(1_X \setminus \alpha bclU)](x) > 0$$
$$\Rightarrow 1 - \inf_{U\in\mathcal{U}}[\alpha bcl(1_X \setminus \alpha bclU)](x) < 1$$
$$\Rightarrow \sup_{U\in\mathcal{U}}[1_X \setminus \alpha bcl(1_X \setminus \alpha bclU)](x) < 1$$
$$\Rightarrow \sup_{U\in\mathcal{U}} U(x) \leq \sup_{U\in\mathcal{U}} \alpha bint(\alpha bclU)(x) < 1.$$

This contradicts that $\mathcal{U}$ is a fuzzy α-b-open cover of A.

(1) $\Rightarrow$ (3): Suppose (1) holds and let $\mathcal{F}$ be a family of fuzzy α-b-closed sets in X such that $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$. Then for each $x \in suppA$ and for each positive integer n, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n$, i.e., $1 - F_n(x) > 1 - 1/n$, i.e., $\sup_{F\in\mathcal{F}}[(1_X \setminus F)(x)] = 1$. Thus $\{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy α-b-open cover of A. By (1), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigvee_{F\in\mathcal{F}_0} \alpha bcl(1_X \setminus F)$. So $1_X \setminus A \geq 1_X \setminus \bigvee_{F\in\mathcal{F}_0} \alpha bcl(1_X \setminus F) = \bigwedge_{F\in\mathcal{F}_0}(1_X \setminus \alpha bcl(1_X \setminus F)) = \bigwedge_{F\in\mathcal{F}_0} \alpha bintF$. Hence $A \not q (\bigwedge_{F\in\mathcal{F}_0} \alpha bintF)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(3) $\Rightarrow$ (2): Suppose (3) holds and let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{\alpha bclB : B \in \mathcal{B}\}] \wedge A = 0_X$. Then the family $\mathcal{F} = \{\alpha bclB : B \in \mathcal{B}\}$ is a family of fuzzy α-b-closed sets in X with $(\bigwedge F) \wedge A = 0_X$. Thus by (3), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigwedge\{\alpha bint(\alpha bclB) : B \in \mathcal{B}_0\}] \not q A$. So $(\bigwedge_{B\in\mathcal{B}_0} \alpha bintB) \not q A$.

(1) $\Rightarrow$ (4): Suppose (1) holds and let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. Assume that $\mathcal{F}$ do not αb-adhere at any fuzzy point in A. Then for each $x \in suppA$, there exists $n_x \in \mathcal{N}$ (the set of all natural numbers) such that $x_{1/n_x} \in A$. Thus there are a fuzzy α-b-open set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x} q U^x_{n_x}$ and $\alpha bclU^x_{n_x} \not q F^x_{n_x}$. So $U^x_{n_x}(x) > 1 - 1/n_x$. Hence $sup\{U^x_n(x) : n \in \mathcal{N}, n \geq n_x\} = 1$. Therefore $\{U^x_n : n \in \mathcal{N}, n \geq n_x, x \in suppA\}$ forms a fuzzy α-b-open cover of A. By (1), there exist finitely many points $x_1, x_2, \cdots, x_k \in suppA$ and $n_1, n_2, \cdots, n_k \in \mathcal{N}$ such that $A \leq \bigvee_{i=1}^{k} \alpha bclU^{x_i}_{n_{x_i}}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigwedge_{i=1}^{k} F^{x_i}_{n_i}$. Then $F \not q [\bigvee_{i=1}^{k} \alpha bclU^{x_i}_{n_{x_i}}]$, i.e., $F \not q A$, a contradiction.

(4) ⇒ (1): Suppose (4) holds and assume that there exist a fuzzy α-b-open cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\bigvee\{\alpha bclU : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigvee\limits_{U\in\mathcal{U}_0} \alpha bclU : \mathcal{U}_0 \text{ is a finite subset of } \mathcal{U}\}$ is a prefilterbase on X such that FqA for each $F \in \mathcal{F}$. By (4), $\mathcal{F}$ αb-adheres at some fuzzy point $x_t \in A$. As $\mathcal{U}$ is a fuzzy cover of A, $\sup\limits_{U\in\mathcal{U}} U(x) = 1$. Thus there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1 - t$, i.e., $x_t q U_0$. As $x_t \in \alpha b\text{-ad}\mathcal{F}$ and $1_X \setminus \alpha bclU_0 \in \mathcal{F}$, we have $\alpha bclU_0 q(1_X \setminus \alpha bclU_0)$, a contradiction. □

Theorem 4.6. *For a fuzzy set A in an fts X, Consider the following statements:*

(1) *every fuzzy net in A αb-adheres at some fuzzy point in A,*

(2) *every fuzzy net in A has a αb-convergent fuzzy subnet,*

(3) *every prefilterbase in A αb-adheres at some fuzzy point in A,*

(4) *for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge\limits_{\alpha\in\Lambda} \alpha b\text{-}clB_\alpha] \wedge A = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge\limits_{\alpha\in\Lambda_0} B_\alpha) \wedge A = 0_X$,*

(5) *A is fuzzy α-b-almost compact set.*

Then (1), (2) *and* (3) *are equivalent, and* (3)⇒ (4) *and* (4)⇒ (5).

Proof. (1) ⇒ (2): Suppose (1) holds and let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A, where $(D, \geq)$ is a directed set, αb-adhere at a fuzzy point $x_\alpha \in A$. Let Q_{x_α} denote the set of the fuzzy α-b-closures of all fuzzy α-b-open q-nbds of x_α. For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n q B$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n q B$. Then $(E, \gg)$ is a directed set, where $(m, C) \gg (n, B)$ if and only if $m \geq n$ in D and $C \leq B$. Thus $T : (E, \gg) \to (X, \tau)$ given by $T(n, B) = S_n$ is a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$. Let V be any fuzzy α-b-open q-nbd of x_α. Then there is $n \in D$ such that that $(n, \alpha bclV) \in E$. Thus $S_n q(\alpha bclV)$. Now, for any $(m, U) \gg (n, \alpha bclV)$, $T(m, U) = S_m q U \leq \alpha bclV$. So $T(m, U) q(\alpha bclV)$. Hence $T \overrightarrow{\alpha b} x_\alpha$.

(2) ⇒ (1). Suppose (2) holds and assume that a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not αb-adhere at a fuzzy point x_α. Then there is a fuzzy α-b-open q-nbd U of x_α and an $n \in D$ such that $S_m \not{q}(\alpha bclU)$ for all $m \geq n$. Thus obviously no fuzzy subnet of the fuzzy net can αb-converge to x_α. This is a contradiction.

(1) ⇒ (3): Suppose (1) holds and let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A. For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \in F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ if and only if $F_\alpha \leq F_\beta$. By (1), the fuzzy net S αb-adheres at some fuzzy point x_t $(0 < t \leq 1) \in A$. Then for any fuzzy α-b-open q-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} q(\alpha bclU)$. Thus $F_\beta q(\alpha bclU)$. So $F_\alpha q(\alpha bclU)$. Hence $\mathcal{F}$ αb-adheres at x_t.

(3) ⇒ (1): Suppose (3) holds and let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. Then by (3), there exists a fuzzy point $a_\alpha \in A$ such that $\mathcal{F}$ αb-adheres at a_α. Thus for each fuzzy α-b-open q-nbd U of a_α and each

$F \in \mathcal{F}$, $Fq(\alpha bclU)$, i.e., $(\alpha bclU)qT_n$ for all $n \in D$. So the given fuzzy net αb-adheres at a_α.

(3) $\Rightarrow$ (4): Suppose (3) holds and let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ, $(\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha) \wedge A \neq 0_X$. Then $\mathcal{F} = \{(\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha) \wedge A : \Lambda_0$ is a finite subset of $\Lambda\}$ is a prefilterbase in A. By (3), $\mathcal{F}$ αb-adheres at some fuzzy point $a_t \in A$ $(0 < t \leq 1)$. Thus for each $\alpha \in \Lambda$ and each fuzzy α-b-open q-nbd U of a_t, $B_\alpha q(\alpha bclU)$, i.e., $a_t \in \alpha b$-clB_α for each $\alpha \in \Lambda$. So $(\bigwedge\limits_{\alpha \in \Lambda} \alpha b\text{-}clB_\alpha) \bigwedge A \neq 0_X$.

(4) $\Rightarrow$ (5): Suppose (4) holds and let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy α-b-open cover of a fuzzy set A. Then by (4), $A \wedge [\bigwedge\limits_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = A \wedge [1_X \setminus \bigvee\limits_{\alpha \in \Lambda} U_\alpha] = 0_X$. If for some $\alpha \in \Lambda$, $1_X \setminus \alpha bclU_\alpha = 0_X$, then we are done. If $1_X \setminus \alpha bclU_\alpha$ $(=B_\alpha$, say$) \neq 0_X$, then for each $\alpha \in \Lambda$, $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigwedge\limits_{\alpha \in \Lambda} \alpha b\text{-}clB_\alpha \leq \bigwedge\limits_{\alpha \in \Lambda} (1_X \setminus U_\alpha)$. In fact, let x_t $(0 < t \leq 1)$ be a fuzzy point such that $x_t \in \alpha b\text{-}clB_\alpha = \alpha b\text{-}cl(1_X \setminus \alpha bclU_\alpha)$. If $x_t qU_\alpha$, then $\alpha bclU_\alpha q(1_X \setminus \alpha bclU_\alpha)$, which is absurd. Thus $x_t \not{q} U_\alpha \Rightarrow x_t \in 1_X \setminus U_\alpha$. So $[\bigwedge\limits_{\alpha \in \Lambda} \alpha b\text{-}clB_\alpha] \wedge A \leq A \bigwedge [\bigwedge\limits_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = 0_X$. By (4), there exists a finite subset Λ_0 of Λ such that $[\bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha] \bigwedge A = 0_X$, i.e., $A \leq 1_X \setminus \bigwedge\limits_{\alpha \in \Lambda_0} B_\alpha = \bigvee\limits_{\alpha \in \Lambda_0} (1_X \setminus B_\alpha) = \bigvee\limits_{\alpha \in \Lambda_0} \alpha bclU_\alpha$. Hence (5) holds. □

Definition 4.7. A fuzzy set A in an fts (X, τ) is said to be *fuzzy regularly α-b-open*, if $A = \alpha bint(\alpha bclA)$. The complement of such a set is called *fuzzy regularly α-b-closed*.

Note 4.8. It is clear from definitions that fuzzy regularly α-b-open set is fuzzy α-b-open set. But the converse need not be true, as it follows from the next example.

Example 4.9. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$, where $A(a) = 0.5, A(b) = 0.6$. Then (X, τ) is an fts. Here $F\alpha bO(X) = \{0_X, 1_X, U\}$, where $U \not\leq 1_X \setminus A$ and thus $F\alpha bC(X) = \{0_X, 1_X, 1_X \setminus U\}$, where $1_X \setminus U \not\geq A$. Consider the fuzzy set B defined by $B(a) = B(b) = 0.6$. Then clearly B is fuzzy α-b-open set, but not fuzzy regularly α-b-open set.

Definition 4.10. A fuzzy point x_α in X is said to be a *fuzzy αb-cluster point* of a prefilterbase $\mathcal{B}$, if $x_\alpha \in \alpha bclB$, for all $B \in \mathcal{B}$. If, in addition, $x_\alpha \in A$ for a fuzzy set A, then $\mathcal{B}$ is said to *have a fuzzy αb-cluster point* in A.

Theorem 4.11. *A fuzzy set A in an fts (X, τ) is fuzzy α-b-almost compact if and only if for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2, \cdots, F_n$ from $\mathcal{F}$ and for any fuzzy regularly α-b-closed set C containing A, one has $(F_1 \bigwedge \cdots \bigwedge F_n)qC$, $\mathcal{F}$ has a fuzzy αb-cluster point in A.*

Proof. Suppose A is a fuzzy α-b-almost compact set and let $\mathcal{F}$ be a prefilterbase in X such that

$$[\bigwedge\{\alpha bclF : F \in \mathcal{F}\}] \wedge A = 0_X. \tag{4.1}$$

Let $x \in suppA$. Consider any $n \in \mathcal{N}$ (the set of all natural numbers) such that $1/n < A(x)$, i.e., $x_{1/n} \in A$. Then by (4.1), $x_{1/n} \notin \alpha bclF^n_x$ for some $F^n_x \in \mathcal{F}$. Thus there exists a fuzzy α-b-open q-nbd U^n_x of $x_{1/n}$ such that $U^n_x \not{q} F^n_x$. Now $U^n_x(x) > 1 - 1/n \Rightarrow sup\{U^n_x(x) : 1/n < A(x), n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{U} = \{U^n_x : x \in suppA, n \in \mathcal{N}\}$ forms a fuzzy α-b-open cover of A such that for U^n_x, there exists $F^n_x \in \mathcal{F}$ with $U^n_x \not{q} F^n_x$. Since A is fuzzy α-b-almost compact, there exist finitely many members $U^{n_1}_{x_1}, \cdots, U^{n_k}_{x_k}$ of $\mathcal{U}$ such that $A \leq \bigvee_{i=1}^{k} \alpha bclU^{n_i}_{x_i} = \alpha bcl(\bigvee_{i=1}^{k} U^{n_i}_{x_i})$ (by Result 3.8) (=U, say). Now $F^{n_1}_{x_1}, \cdots, F^{n_k}_{x_k} \in \mathcal{F}$ such that $U^{n_i}_{x_i} \not{q} F^{n_i}_{x_i}$ for $i = 1, 2, \cdots, k$. Now U is a fuzzy regularly α-b-closed set containing A such that $U \not{q} (F^{n_1}_{x_1} \bigwedge \cdots \bigwedge F^{n_k}_{x_k})$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy αb-cluster point in A. Then by the hypothesis, there is a fuzzy regularly α-b-closed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigwedge \mathcal{B}_0) \not{q} C$. Thus $(\bigwedge \mathcal{B}_0) \not{q} A$. By (2) $\Rightarrow$ (1) of Theorem 4.5, A is fuzzy α-b-almost compact set. □

From Theorem 4.5, Theorem 4.6 and Theorem 4.11, we have the characterizations of fuzzy α-b-almost compact space as follows.

Theorem 4.12. *For an fts X, the following statements are equivalent:*
(1) *X is fuzzy α-b-almost compact,*
(2) *every fuzzy net in X αb-adheres at some fuzzy point in X,*
(3) *every fuzzy net in X has a αb-convergent fuzzy subnet,*
(4) *every prefilterbase in X αb-adheres at some fuzzy point in X,*
(5) *for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha \in \Lambda} \alpha b\text{-}clB_\alpha] = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) = 0_X$,*
(6) *for every prefilterbase $\mathcal{B}$ in X with $\bigwedge\{\alpha bclB : B \in \mathcal{B}\} = 0_X$, there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{\alpha bintB : B \in \mathcal{B}_0\} = 0_X$,*
(7) *for any family $\mathcal{F}$ of fuzzy α-b-closed sets in X with $\bigwedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{\alpha bintF : F \in \mathcal{F}_0\} = 0_X$.*

Theorem 4.13. *An fts X is fuzzy α-b-almost compact if and only if for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy α-b-open sets in X having finite intersection property $\bigwedge\{\alpha bclF_\alpha : \alpha \in \Lambda\} \neq 0_X$.*

Proof. Suppose X is a fuzzy α-b-almost compact space and let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy α-b-open sets in X with finite intersection property. Assume that $\bigwedge\{\alpha bclF_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\{1_X \setminus \alpha bclF_\alpha : \alpha \in \Lambda\}$ is a fuzzy α-b-open cover of X. Thus by the hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \bigvee\{\alpha bcl(1_X \setminus \alpha bclF_\alpha) : \alpha \in \Lambda_0\} = \bigvee\{1_X \setminus \alpha bint(\alpha bclF_\alpha) : \alpha \in \Lambda_0\} \leq \bigvee\{1_X \setminus F_\alpha : \alpha \in \Lambda_0\} = 1_X \setminus \bigwedge_{\alpha \in \Lambda_0} F_\alpha \Rightarrow \bigwedge_{\alpha \in \Lambda_0} F_\alpha = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.

Conversely, suppose the necessary condition holds and assume that X is not fuzzy α-b-almost compact space. Then there is a fuzzy α-b-open cover $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of X such that for every finite subset Λ_0 of Λ, $\bigvee\{\alpha bclF_\alpha : \alpha \in \Lambda_0\} \neq 1_X$. Thus $1_X \setminus \bigvee\{\alpha bclF_\alpha : \alpha \in \Lambda_0\} \neq 0_X$. So $\bigwedge\limits_{\alpha\in\Lambda_0} (1_X \setminus \alpha bclF_\alpha) \neq 0_X$. Hence $\{1_X \setminus \alpha bclF_\alpha : \alpha \in \Lambda\}$ is a collection of fuzzy α-b-open sets with finite intersection property. By the hypothesis, $\bigwedge\limits_{\alpha\in\Lambda} \alpha bcl(1_X \setminus \alpha bclF_\alpha) \neq 0_X$, i.e., $1_X \setminus \bigvee\limits_{\alpha\in\Lambda} \alpha bint(\alpha bclF_\alpha) \neq 0_X$, i.e., $\bigvee\limits_{\alpha\in\Lambda} \alpha bint(\alpha bclF_\alpha) \neq 1_X$. Therefore $\bigvee\limits_{\alpha\in\Lambda} F_\alpha \neq 1_X$. This is a contradiction as $\mathcal{F}$ is a fuzzy α-b-open cover of X. □

Definition 4.14. Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy α-b-open sets in X, i.e., for each member n of a directed set $(D, \geq)$, S_n is a fuzzy α-b-open set in X. A fuzzy point x_α in X is said to be a *fuzzy αb-cluster point* of the fuzzy net, if for every $n \in D$ and every fuzzy α-b-open q-nbd V of x_α, there exists $m \in D$ with $m \geq n$ such that $S_m qV$.

Theorem 4.15. *An fts X is fuzzy α-b-almost compact if and only if every fuzzy net of fuzzy α-b-open sets in X has a fuzzy αb-cluster point in X.*

Proof. Suppose X is fuzzy α-b-almost compact and let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy α-b-open sets in a fuzzy α-b-almost compact space X. For each $n \in D$, let $F_n = \alpha bcl[\bigvee\{S_m : m \in D \text{ and } m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$ is a family of fuzzy α-b-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{\alpha bintF : F \in \mathcal{F}_0\} \neq 0_X$. By (1)$\Rightarrow$ (7) of Theorem 4.12, $\bigwedge\limits_{n\in D} F_n \neq 0_X$. Let $x_\alpha \in \bigwedge\limits_{n\in D} F_n$. Then $x_\alpha \in F_n$ for all $n \in D$. Thus for any fuzzy α-b-open q-nbd A of x_α and any $n \in D$, $Aq[\bigvee\{S_m : m \geq n\}]$. So there exists some $m \in D$ with $m \geq n$ and AqS_m. Hence x_α is a fuzzy αb-cluster point of $\mathcal{U}$.

Conversely, suppose the necessary condition holds and let $\mathcal{F}$ be a collection of fuzzy α-b-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{\alpha bintF : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '$\gg$' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ if and only if $F_1 \leq F_2$. Let $F^* = \alpha bintF$ for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy α-b-open sets of X. By the hypothesis, $\mathcal{U}$ has a fuzzy αb-cluster point, say x_α. We claim that $x_\alpha \in \bigwedge \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy α-b-open q-nbd of x_α. Since $F \in \mathcal{F}^*$ and x_α is a fuzzy αb-cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and $G^* qA \Rightarrow GqA \Rightarrow FqA \Rightarrow x_\alpha \in \alpha bclF = F$ for each $F \in \mathcal{F} \Rightarrow x_\alpha \in \bigwedge \mathcal{F} \Rightarrow \bigwedge \mathcal{F} \neq 0_X$. By (7)$\Rightarrow$ (1) of Theorem 4.12, X is a fuzzy α-b-almost compact space. □

Definition 4.16. A fuzzy cover $\mathcal{U}$ by fuzzy α-b-closed sets of an fts (X, τ) is called a *fuzzy αb-cover* of X, if for each fuzzy point x_α $(0 < \alpha < 1)$ in X, there exists $U \in \mathcal{U}$ such that U is a fuzzy α-b-open nbd of x_α.

Theorem 4.17. *An fts (X, τ) is fuzzy α-b-almost compact if and only if every fuzzy αb-cover of X has a finite subcover.*

Proof. Suppose X is fuzzy α-b-almost compact and let $\mathcal{U}$ be any fuzzy αb-cover of X. Then for each $n \in \mathcal{N}$ (the set of all natural numbers) with $n > 1$, there exist $U_x^n \in \mathcal{U}$ and a fuzzy α-b-open set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Thus $V_x^n(x) \geq 1 - 1/n$. So $sup\{V_x^n(x) : n \in \mathcal{N}\} = 1$. Hence $\mathcal{V} = \{V_x^n : x \in X, n \in \mathcal{N}, n > 1\}$ is a fuzzy α-b-open cover of X. As X is fuzzy α-b-almost compact, there exist finitely many points $x_1, x_2, \cdots, x_m \in X$ and $n_1, n_2, \cdots, n_m \in N \setminus \{1\}$ such that $1_X = \bigvee_{k=1}^{m} \alpha bclV_{x_k}^{n_k} \leq \bigvee_{k=1}^{m} \alpha bclU_{x_k}^{n_k} = \bigvee_{k=1}^{m} U_{x_k}^{n_k}$.

Conversely, suppose the necessary condition holds and let $\mathcal{U}$ be fuzzy α-b-open cover of X. For any fuzzy point x_α $(0 < \alpha < 1)$ in X, as $\sup_{U \in \mathcal{U}} U(x) = 1$, there exists $U_{x_\alpha} \in \mathcal{U}$ such that $U_{x_\alpha}(x) \geq \alpha$ $(0 < \alpha < 1)$. Then $\mathcal{V} = \{\alpha bclU : U \in \mathcal{U}\}$ is a fuzzy αb-cover of X and the rest is clear . □

The following theorem gives a necessary condition for an fts to be fuzzy α-b-almost compact.

Theorem 4.18. *If an fts X is fuzzy α-b-almost compact, then every prefilterbase on X with at most one αb-adherent point is αb-convergent*

Proof. Suppose X is fuzzy α-b-almost compact and let $\mathcal{F}$ be a prefilterbase with at most one αb-adherent point in X. Then by Theorem 4.12, $\mathcal{F}$ has at least one αb-adherent point in X. Let x_α be the unique αb-adherent point of $\mathcal{F}$ and assume that $\mathcal{F}$ do not αb-converge to x_α. Then for some fuzzy α-b-open q-nbd U of x_α and for each $F \in \mathcal{F}$, $F \not\leq \alpha bclU$. Thus $F \bigwedge \{1_X \setminus \alpha bclU\} \neq 0_X$. So $\mathcal{G} = \{F \bigwedge (1_X \setminus \alpha bclU) : F \in \mathcal{F}\}$ is a prefilterbase in X. Hence $\mathcal{G}$ has a αb-adherent point y_t (say) in X. Now $\alpha bclU \not{q} G$ for all $G \in \mathcal{G}$ so that $x_\alpha \neq y_t$. Again, for each fuzzy α-b-open q-nbd V of y_t and each $F \in \mathcal{F}$, $\alpha bclV q (F \bigwedge (1_X \setminus \alpha bclU))$. Then $\alpha bclV qF$. Thus y_t is a fuzzy αb-adherent point of $\mathcal{F}$, where $x_\alpha \neq y_t$. This contradicts the fact that x_α is the only fuzzy αb-adherent point of $\mathcal{F}$. □

Some results on fuzzy α-b-almost compactness of an fts are given by the following theorem.

Theorem 4.19. *Let (X, τ) be an fts and $A \in I^X$. Then the following statements are true:*

(1) *if A is fuzzy α-b-almost compact, then so is $\alpha bclA$,*

(2) *the union of two fuzzy α-b-almost compact sets is also so,*

(3) *if X is fuzzy α-b-almost compact, then every fuzzy regularly α-b-closed set A in X is fuzzy α-b-almost compact.*

Proof. (1) Suppose A is fuzzy α-b-almost compact and let $\mathcal{U}$ be a fuzzy α-b-open cover of $\alpha bclA$. Then $\mathcal{U}$ is also a fuzzy α-b-open cover of A. As A is fuzzy α-b-almost compact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that

$$A \leq \bigvee \{\alpha bclU : U \in \mathcal{U}_0\} = \alpha bcl\{\bigvee U : U \in \mathcal{U}_0\}.$$

Thus we have

$$\begin{aligned} \alpha bclA &\leq \alpha bcl\{\alpha bcl[\bigvee\{U : U \in \mathcal{U}_0\}]\} \\ &= \alpha bcl\{\bigvee U : U \in \mathcal{U}_0\} \end{aligned}$$

$= \bigvee\{\alpha bclU : U \in \mathcal{U}_0\}$.
So $\alpha bclA$ is fuzzy α-b-almost compact.

(2) Obvious.

(3) Suppose X is fuzzy α-b-almost compact and let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy α-b-open cover of a fuzzy regularly α-b-closed set A in X. Then for each $x \notin suppA$, $A(x) = 0$, i.e., $(1_X \setminus A)(x) = 1$. Thus $\mathcal{U} \bigvee\{(1_X \setminus A)\}$ is a fuzzy α-b-open cover of X. Since X is fuzzy α-b-almost compact, there are finitely many members $U_1, U_2, \cdots, U_n$ in $\mathcal{U}$ such that

$$1_X = (\alpha bclU_1 \bigvee \cdots \bigvee \alpha bclU_n) \bigvee \alpha bcl(1_X \setminus A).$$

We claim that $\alpha bintA \leq \alpha bclU_1 \bigvee \cdots \bigvee \alpha bclU_n$. If not, there exists a fuzzy point $x_t \in \alpha bintA$, but $x_t \notin (\alpha bclU_1 \bigvee \cdots \bigvee \alpha bclU_n)$, i.e., $t > max\{(\alpha bclU_1)(x), \cdots, (\alpha bclU_n)(x)\}$. As $1_X = (\alpha bclU_1 \bigvee \cdots \bigvee \alpha bclU_n) \bigvee \alpha bcl(1_X \setminus A)$, we have

$$[\alpha bcl(1_X \setminus A)](x) = 1.$$

Then $1 - \alpha bintA(x) = 1$. Thus $\alpha bintA(x) = 0$. So $x_t \notin \alpha bintA$. This is a contradiction. Hence by Results 3.7 and 3.8, we get

$$A = \alpha bcl(\alpha bintA) \leq \alpha bcl(\alpha bclU_1 \bigvee \cdots \bigvee \alpha bclU_n) = \alpha bclU_1 \bigvee \cdots \bigvee \alpha bclU_n.$$

Therefore A is a fuzzy α-b-almost compact set. □

5. Mutual relationship

Here we establish the mutual relationship between fuzzy almost compactness [3] and fuzzy α-b-almost compactness. Then it is shown that fuzzy α-b-almost compactness implies fuzzy almost compactness, but converse is true in fuzzy α-b-regular space [1]. It is also established that fuzzy α-b-almost compactness remains invariant under fuzzy α-b-irresolute function [1].

Since for any fuzzy set A in an fts X, $\alpha bclA \leq clA$ (as every fuzzy closed set is fuzzy α-b-closed [1]), we can state the following theorem easily.

Theorem 5.1. *Every fuzzy α-b-almost compact space is fuzzy almost compact.*

To get the converse we have to recall the following definition and theorem for ready references.

Definition 5.2 ([1])**.** An fts (X, τ) is said to be *fuzzy α-b-regular*, if for each fuzzy α-b-closed set F in X and each fuzzy point x_α in X with $x_\alpha q(1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy α-b-open set V in X such that $x_\alpha qU$, $F \leq V$ and $U \not{q} V$.

Theorem 5.3 ([1])**.** *An fts (X, τ) is fuzzy α-b-regular iff every fuzzy α-b-closed set is fuzzy closed.*

Theorem 5.4. *A fuzzy α-b-regular, fuzzy almost compact space X is fuzzy α-b-almost compact.*

Proof. Let $\mathcal{U}$ be a fuzzy α-b-open cover of a fuzzy α-b-regular, fuzzy almost compact space X. Then by Theorem 5.3, $\mathcal{U}$ is a fuzzy open cover of X. As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{clU : U \in \mathcal{U}_0\} =$

$\bigvee\{\alpha bclU : U \in \mathcal{U}_0\}$ (by Theorem 5.3) $= 1_X$. Thus X is fuzzy α-b-almost compact space. □

Next we recall the following definition and theorem for ready references.

Definition 5.5 ([1])**.** A function $f : X \to Y$ is said to be *fuzzy α-b-irresolute*, if the inverse image of every fuzzy α-b-open set in Y is fuzzy α-b-open in X.

Theorem 5.6 ([1])**.** *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is fuzzy α-b-irresolute,*

(2) *$f(\alpha bclA) \le \alpha bcl(f(A))$ for all $A \in I^X$,*

(3) *for each fuzzy point x_α in X and each fuzzy α-b-open q-nbd V of $f(x_\alpha)$ in Y, there exists a fuzzy α-b-open q-nbd U of x_α in X such that $f(U) \le V$.*

Theorem 5.7. *Fuzzy α-b-irresolute image of a fuzzy α-b-almost compact space is fuzzy α-b-almost compact.*

Proof. Let $f : X \to Y$ be fuzzy α-b-irresolute surjective function from a fuzzy α-b-almost compact space X onto an fts Y and let $\mathcal{V}$ be a fuzzy α-b-open cover of Y. Let $x \in X$ and $f(x) = y$. Since $sup\{V(y) : V \in \mathcal{V}\} = 1$ for each $n \in \mathcal{N}$ (the set of all natural numbers), there exists some $V^n_x \in \mathcal{V}$ with $V^n_x(y) > 1 - 1/n$, i.e., $y_{1/n} q V^n_x$. Since f is fuzzy α-b-irresolute, by (1)⇒ (3) of Theorem 5.6, $f(U^n_x) \le V^n_x$ for some fuzzy α-b-open set U^n_x in X q-coincident with $x_{1/n}$. Since $U^n_x(x) > 1-1/n$, $sup\{U^n_x(x) : n \in \mathcal{N}\} = 1$. Thus $\mathcal{U} = \{U^n_x : n \in \mathcal{N}, x \in X\}$ is a fuzzy α-b-open cover of X. By fuzzy α-b-almost compactness of X, $\bigvee\limits_{i=1}^{k} \alpha bclU^{n_i}_{x_i} = 1_X$ for some finite subcollection $\{U^{n_1}_{x_1}, \cdots, U^{n_k}_{x_k}\}$ of $\mathcal{U}$. So we get

$$1_Y = f(\bigvee_{i=1}^{k} \alpha bclU^{n_i}_{x_i}) = \bigvee_{i=1}^{k} f(\alpha bclU^{n_i}_{x_i}) \le \bigvee_{i=1}^{k} \alpha bcl(f(U^{n_i}_{x_i})).$$

Then by (1)⇒ (2) of Theorem 5.6, $\bigvee\limits_{i=1}^{k} \alpha bcl(f(U^{n_i}_{x_i})) \le \bigvee\limits_{i=1}^{k} \alpha bclV^{n_i}_{x_i}$. Hence Y is a fuzzy α-b-almost compact space. □

6. Conclusions

This paper is a continuation of [1]. The main goal of this paper is to establish the various results of fuzzy α-b-open sets and fuzzy covering properties. We further want to establish the inter-relations of various types of fuzzy covering properties.

Acknowledgements. I express my sincere gratitude to the referees for their valuable remark.

References

[1] Anjana Bhattacharyya, Fuzzy α-b-regularity in a fuzzy topological space, Ann. Fuzzy Math. Inform. Accepted for Publication.

[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.

[3] A. DiConcillio and G. Gerla, Almost compactness in fuzzy topological spaces, Fuzzy Sets and Systems 13 (1984) 187–192.
[4] Pao Ming Pu and Ying Ming Liu, Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith Convergence, J. Math Anal. Appl. 76 (1980) 571–599.
[5] R. Lowen, Convergence in fuzzy topological spaces, General Topology and Its Applications 10 (1979) 147–160.
[6] J. G. Lee, G. Şenel, P. K. Lim, J. Kim and K. Hur, Octahedron sets, Ann. Fuzzy Math. Inform. 19 (3) (2020) 211–238.
[7] G. Şenel and N. Çağman, Soft closed sets on soft bitopological space, Journal of New Results in Science 3 (5) (2014) 57–66.
[8] G. Şenel and N. Çağman, Soft topological subspaces, Ann. Fuzzy Math.Inform. 10 (4) (2015) 525–535.
[9] G. Şenel, A new approach to Hausdorff space theory via the soft sets, Mathematical Problems in Engineering 1–6. Doi:10.1155/2016/2196743.
[10] G. Şenel, Soft topology generated by L-Soft sets, Journal of New Theory 24 (4) (2018) 88–100.
[11] G. Şenel, J. G. Lee and K. Hur, Distance and similarity measures for octahedron sets and their applications to MCGDM problems, Mathematics 8 (2020) 16–90.
[12] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.
[13] A. S. Bin Shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, Fuzzy Sets and Systems 44 (1991) 303–308.
[14] S. Ganguly and S. Saha, A note on compactness in fuzzy setting, Fuzzy Sets and Systems 34 (1990) 117–124.
[15] M. N. Mukherjee and S. P. Sinha, Almost compact fuzzy sets in fuzzy topological spaces, Fuzzy Sets and Systems 38 (1990) 389–396.

Anjana Bhattacharyya (anjanabhattacharyya@hotmail.com)
Department of Mathematics
Victoria Institution (College)
78 B, A.P.C. Road
Kolkata - 700009, India