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A neutrosophic soft set approach to a decision making problem

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Abstract. The decision making problems in an imprecise environment has found paramount importance in recent years. Here we consider an object recognition problem in an imprecise environment. The recognition strategy is based on multiobserver input parameter data set.

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1. Introduction

Soft set theory is growing very rapidly since its introduction [1]. The nobel concept of soft set theory plays an important role as a mathematical tool for dealing with uncertainties. The basic properties of the theory may be found in [2]. Ali et al. [3] has presented some new algebraic operations on soft sets. Chen et al. [4] has presented a new definition of soft set parameterization reduction and compare this definition with the related concept of knowledge reduction in the rough set. Feng et al. [5] has introduced the concept of semirings whereas we can find the concept of soft groups in [6]. Xu et al. [7] has introduced vague soft sets which is a combination of soft sets and vague sets. Some applications of soft sets may be found in [4, 8, 9, 10].

In the present paper we present some results as an application of neutrosophic soft set in a decision making problem. The problem of object recognition has received paramount importance in recent years. The recognition problem may be viewed as a multiobserver decision making problem, where the final identification of the object is based on the set of inputs from different observers who provide the overall object characterization in terms of the diverse sets of choice parameters. In this paper we present a neutrosophic soft set theoretic approach towards the solution of the above decision making problem.

In section 2 of this paper we briefly present some relevant preliminaries centered around our problem. Some basic definitions on neutrosophic soft set is available in section 3. A decision making problem has been discussed and solved in section 4. Conclusions are there in the concluding section 5.

2. Preliminaries

Most of the real life problems are imprecise in nature. The classical mathematical tools are not capable of dealing with such problems. Molodtsov [1] initiated the nobel concept 'soft set theory' as a new mathematical tool to deal with such problems.

Definition 2.1 ([1])**.** Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U. Consider a nonempty set $A \subset E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. A soft set over U is a parameterized family of subsets of the universe U. For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A).

Definition 2.2 ([2])**.** For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if
(i) $A \subset B$, and
(ii) $\forall\ \epsilon \in A$, $F(\epsilon)$ and $G(\epsilon)$ are identical approximations.
We write $(F, A) \tilde{\subset} (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \tilde{\supset} (G, B)$.

Definition 2.3 ([2])**.** If (F, A) and (G, B) are two soft sets then "(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined by

$$(F, A) \wedge (G, B) = (H, A \times B),$$

where $H(\alpha, \beta) = F(\alpha) \bigcap G(\beta)$, $\forall(\alpha, \beta) \in A \times B$.

Definition 2.4 ([2])**.** If (F, A) and (G, B) be two soft sets then "(F, A) OR (G, B)" denoted by (F,A) $\vee$ (G, B) is defined by

$$(F, A) \vee (G, B) = (O, A \times B),$$

where $O(\alpha, \beta) = F(\alpha) \bigcup G(\beta)$, $\forall(\alpha, \beta) \in A \times B$.

3. Neutrosophic soft set in a decision making problem

In this section we present neutrosophic soft set and some results of it. Let

$$U = \{o_1, o_2, \cdots, o_n\}$$

be a set of n objects which may be characterized by a family of parameter sets $\{A_1, A_2, \cdots, A_i\}$. The parameter space E may be written as $E \supseteq A_1 \cup A_2 \cup \cdots \cup A_i$. Let each parameter set A_i represent the i-th class of parameters and the elements of A_i represents a specific property set. Here we assume that these property sets may be viewed as neutrosophic sets. In view of the above we may now define a neutrosophic soft set (F_i, A_i) which characterises a set of objects having the parameter set A_i of neutrosophic in nature. The values of neutrosophic sets are taken from the non-standard unit interval $]^-0, 1^+[$. The non-standard finite numbers $1^+ = 1 + \delta$, where '1' is the standard part and 'δ' is its non-standard part and $^-0 = 0 - \delta$, where '0' is its standard part and 'δ' is non-standard part. Now we recall the definition of neutrosophic sets.

Definition 3.1 ([11]). A neutrosophic set A on the universe of discourse X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$, where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic soft set which takes the value from the subset of $[0, 1]$.

Definition 3.2 ([12]). Let U be an initial universe set and E be a set of parameters. Consider $A_i \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F_i, A_i) is termed to be the neutrosophic soft set over U, where F_i is a mapping given by $F_i : A_i \to P(U)$.

Definition 3.3 ([12]). Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if $A \subset B$, and $T_{F(e)}(x) \leq T_{G(e)}(x), I_{F(e)}(x) \leq I_{G(e)}(x), F_{F(e)}(x) \geq F_{G(e)}(x), \forall\ e \in A$. We denote it by $(F, A) \subseteq (G, B)$. (F, A) is said to be neutrosophic soft super set of (G, B) if (G, B) is a neutrosophic soft subset of (F, A). We denote it by $(F, A) \supseteq (G, B)$.

Definition 3.4 ([12]). Let (H, A) and (G, B) be two NSSs over the same universe U. Then the 'AND' operation on them is denoted by '$(H, A) \bigwedge (G, B)$' and is defined by $(H, A) \bigwedge (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$T_{K(\alpha,\beta)}(m) = min(T_{H(\alpha)}(m), T_{G(\beta)}(m)),$$
$$I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2},$$
$$F_{K(\alpha,\beta)}(m) = max(F_{H(\alpha)}(m), F_{G(\beta)}(m))$$

for all $\alpha \in A$ and $\beta \in B$.

Some membership and non-membership values may be there in indeterminacy part. Considering this point of view the arithmetic mean has been taken to calculate the indeterminacy-membership value of AND operation.

Definition 3.5 ([12]). (Comparison Matrix) It is a matrix whose rows are labelled by the object names $h_1, h_2, \cdots h_n$ and the columns are labelled by the parameters $e_1, e_2, \cdots e_m$. The entries c_{ij} are calculated by $c_{ij} = a + b - c$, where 'a' is the integer calculated as 'how many times $T_{h_i}(e_j)$ exceeds or equal to $T_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall\ h_k \in U$, 'b' is the integer calculated as 'how many times $I_{h_i}(e_j)$ exceeds or equal to $I_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in U$ and 'c' is the integer 'how many times $F_{h_i}(e_j)$ exceeds or equal to $F_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall\ h_k \in U$.

Definition 3.6 ([12]). The score of an object h_i is S_i and is calculated as $S_i = \sum_j c_{ij}$. The problem we consider here for choosing an object from the set of given objects with respect to a set of choice parameter P. We follow an algorithm to identify an object based on multiobserver (considered here three observers with their own choices) input data characterized by colours (F, A), size (G, B) and surface textures (H, C) features. The algorithm for choosing an appropriate object depending upon the choice parameters is given below.

3.1. **Algorithm.**

(1) input the neutrosophic soft sets (H, A), (G, B) and (H, C) for three observers
(2) input the parameter set P as preferred by the decision maker
(3) compute the corresponding NSS (S, P) from the NSSs (H, A), (G, B) and (H, C) and place in tabular form
(4) compute the comparison matrix of the NSS (S, P)
(5) compute the score S_i of $o_i, \forall i$
(6) the decision is o_k if $S_k = \max_i S_i$
(7) if k has more than one value then any one of o_i may be chosen.

4. Application in a decision making problem

Let $U = \{o_1, o_2, o_3, o_4, o_5\}$ be the set of objects characterized by different sizes, texture and colours. Consider the parameter set

$$E = \{\text{blackish, dark brown, yellowish, reddish, large, small, very small, average, very large, course, moderately course, fine, extra fine}\}.$$

Also consider $A(\subset E)$ to represent the size of the objects and $B(\subset E)$ represents the texture granularity while $C(\subset E)$ represents different colours of the objects. Let
$A = \{\text{large, very large, small, average, very small}\}$,
$B = \{\text{course, moderately course, fine, extra fine}\}$ and
$C = \{\text{blackish, yellowish, reddish}\}$
be three subsets of the set of parameters E. Now, suppose the NSS (F, A) describes the 'objects having size', the NSS (G, B) describes the 'surface texture of the objects' and the NSS (H, C) describes the 'objects having colour space'. The problem is to identify an unknown object from the multiobservers neutrosophic data, specified by different observers (we consider here three observers), in terms of NSSs (F, A), (G, B) and (H, C) as described above. These NSSs as computed by the three observers are given below in their respective tabular forms.

U	large=a_1	very large =a_2	small =a_3	average=a_4	very small=a_5
o_1	(0.6, 0.4, 0.8)	(0.5, 0.8, 0.7)	(0.3, 0.5, 0.6)	(0.6, 0.7, 0.3)	(0.3, 0.6, 0.7)
o_2	(0.7, 0.5, 0.6)	(0.6, 0.7, 0.8)	(0.7, 0.6, 0.3)	(0.8, 0.2, 0.7)	(0.7, 0.2, 0.3)
o_3	(0.8, 0.3, 0.4)	(0.8, 0.4, 0.5)	(0.7, 0.8, 0.4)	(0.6, 0.1, 0.7)	(0.8, 0.1, 0.4)
o_4	(0.7, 0.8, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.4, 0.5)	(0.8, 0.2, 0.3)	(0.7, 0.5, 0.6)
o_5	(0.8, 0.6, 0.7)	(0.7, 0.7, 0.8)	(0.4, 0.7, 0.3)	(0.7, 0.8, 0.7)	(0.8, 0.3, 0.4)

Table 1: Tabular form of the NSS (F, A).

U	course=b_1	moderately course=b_2	fine=b_3	extra fine=b_4
o_1	(0.6, 0.8, 0.7)	(0.7, 0.3, 0.8)	(0.6, 0.3, 0.4)	(0.3, 0.7, 0.8)
o_2	(0.8, 0.4, 0.6)	(0.8, 0.2, 0.8)	(0.8, 0.5, 0.6)	(0.5, 0.8, 0.6)
o_3	(0.8, 0.5, 0.7)	(0.7, 0.6, 0.5)	(0.7, 0.2, 0.8)	(0.7, 0.2, 0.6)
o_4	(0.7, 0.3, 0.8)	(0.8, 0.1, 0.3)	(0.6, 0.3, 0.7)	(0.8, 0.3, 0.7)
o_5	(0.6, 0.5, 0.7)	(0.8, 0.3, 0.5)	(0.8, 0.1, 0.3)	(0.8, 0.2, 0.8)

Table 2: Tabular form of the NSS (G, B).

U	blackish = c_1	yellowish = c_2	reddish = c_3
o_1	(0.6, 0.4, 0.8)	(0.8, 0.3, 0.8)	(0.7, 0.2, 0.8)
o_2	(0.8, 0.5, 0.6)	(0.7, 0.8, 0.4)	(0.3, 0.5, 0.7)
o_3	(0.7, 0.6, 0.7)	(0.8, 0.2, 0.6)	(0.4, 0.7, 0.8)
o_4	(0.8, 0.3, 0.8)	(0.7, 0.8, 0.3)	(0.5, 0.6, 0.7)
o_5	(0.7, 0.4, 0.7)	(0.3, 0.5, 0.7)	(0.8, 0.3, 0.6)

Table 3: Tabular form of the NSS (H, C).

Consider the above two NSSs (F, A) and (G, B) if we perform (F, A) AND (G, B) then we will have $5 \times 4 = 20$ parameters of the form e_{ij}, where $e_{ij} = a_i \wedge b_j$, $\forall i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4$. If we require the NSS for the parameters $R = \{e_{13}, e_{21}, e_{24}, e_{33}, e_{51}, e_{53}\}$, then the resultant NSS for the NSSs (F, A) and (G, B) is (K, R) say. Computing '(F, A) AND (G, B)' for the choice parameters R, we have the tabular representation of the resultant NSS (K, R) as below.

U	e_{13}	e_{21}	e_{24}	e_{33}	e_{51}	e_{53}
o_1	(0.6, 0.35, 0.8)	(0.5, 0.8, 0.7)	(0.3, 0.75, 0.8)	(0.3, 0.4, 0.6)	(0.3, 0.7, 0.7)	(0.3, 0.7, 0.7)
o_2	(0.7, 0.5, 0.6)	(0.6, 0.55, 0.8)	(0.5, 0.75, 0.8)	(0.7, 0.55, 0.6)	(0.7, 0.3, 0.6)	(0.7, 0.35, 0.6)
o_3	(0.7, 0.25, 0.8)	(0.8, 0.45, 0.7)	(0.7, 0.3, 0.6)	(0.7, 0.5, 0.8)	(0.8, 0.3, 0.7)	(0.7, 0.15, 0.8)
o_4	(0.6, 0.55, 0.8)	(0.7, 0.3, 0.8)	(0.8, 0.3, 0.7)	(0.6, 0.35, 0.6)	(0.7, 0.4, 0.8)	(0.6, 0.4, 0.7)
o_5	(0.8, 0.35, 0.7)	(0.6, 0.6, 0.8)	(0.7, 0.45, 0.8)	(0.4, 0.4, 0.3)	(0.6, 0.4, 0.7)	(0.8, 0.2, 0.4)

Table 4: Tabular form of the NSS (K, R).

Considering the NSSs (F, A), (G, B) and (H, C) defined above, suppose that

$$P = \{e_{13} \wedge c_1, e_{24} \wedge c_2, e_{33} \wedge c_3, e_{51} \wedge c_1, e_{53} \wedge c_3\}.$$

Then computing 'AND' operation for the specified parameters we have the NSS (S, P).

U	$e_{13} \wedge c_1$	$e_{24} \wedge c_2$	$e_{33} \wedge c_3$	$e_{51} \wedge c_1$	$e_{53} \wedge c_3$
o_1	(0.6, 0.375, 0.8)	(0.5, 0.55, 0.8)	(0.3, 0.3, 0.8)	(0.3, 0.55, 0.8)	(0.3, 0.325, 0.8)
o_2	(0.7, 0.5, 0.6)	(0.6, 0.675, 0.8)	(0.3, 0.525, 0.7)	(0.7, 0.4, 0.6)	(0.3, 0.425, 0.7)
o_3	(0.7, 0.425, 0.7)	(0.8, 0.325, 0.7)	(0.7, 0.6, 0.8)	(0.7, 0.45, 0.7)	(0.4, 0.425, 0.8)
o_4	(0.6, 0.425, 0.8)	(0.7, 0.55, 0.8)	(0.6, 0.475, 0.7)	(0.7, 0.35, 0.8)	(0.5, 0.5, 0.7)
o_5	(0.7, 0.375, 0.7)	(0.3, 0.55, 0.8)	(0.4, 0.35, 0.6)	(0.6, 0.4, 0.7)	(0.8, 0.25, 0.6)

Table 5: Tabular form of the NSS (S, P).

To compute the score for each o_i we shall calculate the entries of the comparison matrix for the NSS (S, P).

U	$e_{13} \wedge c_1$	$e_{24} \wedge c_2$	$e_{33} \wedge c_3$	$e_{51} \wedge c_1$	$e_{53} \wedge c_3$
o_1	-2	0	-3	0	-2
o_2	8	2	2	5	2
o_3	5	4	4	5	1
o_4	0	2	3	0	5
o_5	3	-1	3	1	4

Table 6: comparison matrix of the NSS (S, P).

Computing the score for each of the objects we have the scores as below.

U	score
o_1	-7
o_2	19
o_3	19
o_4	10
o_5	10

Clearly, the maximum score is 19 and scored by two objects o_2 and o_3. The selection will be in favour of either o_2 or o_3. In case if the decision maker does not choose them then his next choice will go for the object having next score ie. 10. So his next choice will be either o_4 or o_5.

5. Conclusions

Since its introduction the soft set theory plays an important role as a mathematical tool for dealing with problems involving uncertain, vague data. In this paper we present an application of neutrosophic soft set in object recognition problem. The recognition strategy is based on multiobserver input data set. We introduce an algorithm to choose an appropriate object from a set of objects depending on some specified parameters.

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