A new concept of $(\lambda, \mu)$-fuzzy ideals and $(\lambda, \mu)$-fuzzy interior ideals of an ordered semigroup was introduced. It was proved that in regular and intra-regular ordered semigroups, the $(\lambda, \mu)$-fuzzy ideals and the $(\lambda, \mu)$-fuzzy interior ideals coincide. The concept of a $(\lambda, \mu)$-fuzzy simple ordered semigroup was also introduced and it was proved that an ordered semigroup is simple if and only if it is $(\lambda, \mu)$-fuzzy simple.

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1. Introduction and preliminaries

The concept of fuzzy sets was first introduced by Zadeh [10] in 1965 and then the fuzzy sets have been used in the reconsideration of classical mathematics. Recently, Yuan et al. [9] introduced the concept of fuzzy subfield with thresholds. A fuzzy subfield with thresholds $\lambda$ and $\mu$ is also called a $(\lambda, \mu)$-fuzzy subfield. Yao continued to research $(\lambda, \mu)$-fuzzy normal subfields, $(\lambda, \mu)$-fuzzy quotient subfields, $(\lambda, \mu)$-fuzzy subrings and $(\lambda, \mu)$-fuzzy ideals in [5, 6, 7, 8]. Feng et al. researched $(\lambda, \mu)$-fuzzy sublattices and $(\lambda, \mu)$-fuzzy subhyperlattices in [1].

In this paper, we studied $(\lambda, \mu)$-fuzzy ideals of ordered semigroups. This can be seen as an application of [8] and as a generalization of [3]. We first introduced definitions of $(\lambda, \mu)$-fuzzy ideals and $(\lambda, \mu)$-fuzzy interior ideals of an ordered semigroup. Then we proved that in regular and in intra-regular ordered semigroups the $(\lambda, \mu)$-fuzzy ideals and the $(\lambda, \mu)$-fuzzy interior ideals coincide. Lastly, we introduced the concept of a $(\lambda, \mu)$-fuzzy simple ordered semigroup, proved that an ordered semigroup is simple if and only if it is $(\lambda, \mu)$-fuzzy simple and characterized the simple ordered semigroups in terms of $(\lambda, \mu)$-fuzzy interior ideal.

An ordered semigroup $(S, \circ, \leq)$ is a poset $(S, \leq)$ equipped with a binary operation $\circ$ such that
(1) \((S, \circ)\) is a semigroup, and

(2) If \(x, a, b \in S\), then \(a \leq b \Rightarrow \{a \circ x \leq b \circ x \}
\)
\(\circ a \leq x \circ b\).

If \((S, \circ, \leq)\) is an ordered semigroup, and \(A\) is a subset of \(S\), we denote by \((A]\) the subset of \(S\) defined as follows:\n\[(A] = \{t \in S | t \leq a \text{ for some } a \in A\}.\]

Given an ordered semigroup \(S\), a fuzzy subset of \(S\) (or a fuzzy set in \(S\)) is an arbitrary mapping \(f : S \to [0, 1]\), where \([0, 1]\) is the usual closed interval of real numbers. For any \(\alpha \in [0, 1]\), \(f_\alpha\) is defined by
\[f_\alpha = \{x \in S | f(x) \geq \alpha\}.
\]

For each subset \(A\) of \(S\), the characteristic function \(f_A\) is a fuzzy subset of \(S\) defined by
\[f_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \not\in A. \end{cases}\]

In the following, we will use \(S\) or \((S, \circ, \leq)\) to denote an ordered semigroup and the multiplication of \(x, y\) will be \(xy\) instead of \(x \circ y\).

In the rest of this paper, we will always assume that \(0 \leq \lambda < \mu \leq 1\).

2. \((\lambda, \mu)\)-Fuzzy Ideals and \((\lambda, \mu)\)-Fuzzy Interior Ideals

In this section, we first introduce the concepts of \((\lambda, \mu)\)-fuzzy ideals and \((\lambda, \mu)\)-fuzzy interior ideals of an ordered semigroup. And then show that every \((\lambda, \mu)\)-fuzzy ideal is a \((\lambda, \mu)\)-fuzzy interior ideal.

Definition 2.1. A fuzzy subset \(f\) of an ordered semigroup \(S\) is called a \((\lambda, \mu)\)-fuzzy right ideal of \(S\) if

(1) \(f(xy) \vee \lambda \geq f(x) \wedge \mu\) for all \(x, y \in S\) and

(2) If \(x \leq y\), then \(f(x) \vee \lambda \geq f(y) \wedge \mu\) for all \(x, y \in S\).

A fuzzy subset \(f\) of \(S\) is called a \((\lambda, \mu)\)-fuzzy left ideal of \(S\) if

(1) \(f(xy) \vee \lambda \geq f(y) \wedge \mu\) for all \(x, y \in S\) and

(2) If \(x \leq y\), then \(f(x) \vee \lambda \geq f(y) \wedge \mu\) for all \(x, y \in S\).

A fuzzy subset \(f\) of \(S\) is called a \((\lambda, \mu)\)-fuzzy ideal of \(S\) if it is both a \((\lambda, \mu)\)-fuzzy right and a \((\lambda, \mu)\)-fuzzy left ideal of \(S\).

Example 2.2. Let \((S, *, \leq)\) be an ordered semigroup defined by \(e \leq a\) and the following table:

<table>
<thead>
<tr>
<th></th>
<th>(e)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e)</td>
<td>(e)</td>
<td>(a)</td>
</tr>
<tr>
<td>(a)</td>
<td>(e)</td>
<td>(a)</td>
</tr>
</tbody>
</table>

If we define a fuzzy set \(f\) as following

<table>
<thead>
<tr>
<th>(S)</th>
<th>(e)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Then \(f\) is a \((0.5, 0.7)\)-fuzzy ideal of \(S\).

Definition 2.3 ([2]). If \((S, \circ, \leq)\) is an ordered semigroup, a nonempty subset \(A\) of \(S\) is called an interior ideal of \(S\) if

(1) \(SAS \subseteq A\) and

(2) If \(x, a, b \in S\), then \(a \leq b \Rightarrow \{a \circ x \leq b \circ x \}
\)
\(\circ a \leq x \circ b\).
(2) If \( a \in A, b \in S \) and \( b \leq a \), then \( b \in A \).

**Definition 2.4.** If \((S, \circ, \leq)\) is an ordered semigroup, a fuzzy subset \( f \) of \( S \) is called a \((\lambda, \mu)\)-fuzzy interior ideal of \( S \) if the following assertions are satisfied:

1. \( f(xay) \vee \lambda \geq f(a) \wedge \mu \) for all \( x, a, y \in S \) and
2. If \( x \leq y \), then \( f(x) \vee \lambda \geq f(y) \wedge \mu \).

In Example 2.2, it is easy to know that \( f \) is also a \((0.5, 0.7)\)-fuzzy interior ideal of \( S \).

**Theorem 2.5.** Let \((S, \circ, \leq)\) be an ordered semigroup, Then \( f \) is a \((\lambda, \mu)\)-fuzzy interior ideal of \( S \) if and only if \( f_\alpha \) is an interior ideal of \( S \) for all \( \alpha \in (\lambda, \mu] \).

**Proof.** Let \( f \) be a \((\lambda, \mu)\)-fuzzy interior ideal of \( S \) and \( \alpha \in (\lambda, \mu] \). First of all, we need to show that \( xay \in f_\alpha \), for all \( a \in f_\alpha \), \( x, y \in S \). From \( f(xay) \vee \lambda \geq f(a) \wedge \mu \geq \alpha \wedge \mu = \alpha \) and \( \lambda < \alpha \), we conclude that \( f(xay) \geq \alpha \), that is \( xay \in f_\alpha \). Then, we need to show that \( b \in f_\alpha \) for all \( a \in f_\alpha \), \( b \in S \) such that \( b \leq a \). From \( b \leq a \) we know that \( f(b) \vee \lambda \geq f(a) \wedge \mu \) and from \( a \in f_\alpha \), we have \( f(a) \geq \alpha \). Thus \( f(b) \vee \lambda \geq \alpha \wedge \mu = \alpha \).

Notice that \( \lambda < \alpha \), we conclude that \( f(b) \geq \alpha \), that is, \( b \in f_\alpha \).

Conversely, let \( f_\alpha \) be an interior ideal of \( S \) for all \( \alpha \in (\lambda, \mu] \). If there are \( x_0, a_0, y_0 \in S \), such that \( f(x_0a_0y_0) \vee \lambda < \alpha = f(a_0) \wedge \mu \), then \( \alpha \in (\lambda, \mu], f(a_0) \geq \alpha \) and \( f(x_0a_0y_0) < \alpha \). That is \( a_0 \in f_\alpha \) and \( x_0a_0y_0 \notin f_\alpha \). This is a contradiction with that \( f_\alpha \) is an interior ideal of \( S \). Hence \( f(xay) \vee \lambda \geq f(a) \wedge \mu \) holds for all \( x, a, y \in S \). If there are \( x_0, y_0 \in S \) such that \( x_0 \leq y_0 \) and \( f(x_0) \vee \lambda < \alpha = f(y_0) \wedge \mu \), then \( \alpha \in (\lambda, \mu], f(y_0) \geq \alpha \) and \( f(x_0) < \alpha \), that is \( y_0 \in f_\alpha \) and \( x_0 \notin f_\alpha \). This is a contradiction with that \( f_\alpha \) is an interior ideal of \( S \). Hence if \( x \leq y \), then \( f(x) \vee \lambda \geq f(y) \wedge \mu \). \( \square \)

**Theorem 2.6.** Let \((S, \circ, \leq)\) be an ordered semigroup and \( f \) a \((\lambda, \mu)\)-fuzzy ideal of \( S \). Then \( f \) is a \((\lambda, \mu)\)-fuzzy interior ideal of \( S \).

**Proof.** Let \( x, a, y \in S \). Since \( f \) is a \((\lambda, \mu)\)-fuzzy left ideal of \( S \) and \( x, ay \in S \), we have that

\[
f(x(ay)) \vee \lambda \geq f(ay) \wedge \mu. \tag{1}
\]

Since \( f \) is a \((\lambda, \mu)\)-fuzzy right ideal of \( S \), we have that

\[
f(ay) \vee \lambda \geq f(a) \wedge \mu. \tag{2}
\]

From (1) and (2) we know that \( f(xay) \vee \lambda = (f(x(ay)) \vee \lambda) \vee \lambda \geq (f(ay) \wedge \mu) \vee \lambda = (f(ay) \vee \lambda) \wedge (\mu \vee \lambda) \geq f(a) \wedge \mu. \)

\( \square \)

3. \((\lambda, \mu)\)-Fuzzy Interior Ideals of Regular/Intra-Regular Ordered Semigroups

We prove here that in regular and in intra-regular ordered semigroups the \((\lambda, \mu)\)-fuzzy ideals and the \((\lambda, \mu)\)-fuzzy interior ideals coincide.

**Definition 3.1 ([3]).** An ordered semigroup \((S, \circ, \leq)\) is called regular if for all \( a \in S \) there exists \( x \in S \) such that \( a \leq axa \).

**Definition 3.2 ([3]).** An ordered semigroup \((S, \circ, \leq)\) is called intra-regular if for all \( a \in S \) there exists \( x, y \in S \) such that \( a \leq xay \).
Theorem 3.3. Let \((S, \circ, \leq)\) be a regular ordered semigroup and \(f\) a \((\lambda, \mu)\)-fuzzy interior ideal of \(S\). Then \(f\) is a \((\lambda, \mu)\)-fuzzy ideal of \(S\).

Proof. Let \(x, y \in S\). Then \(f(xy) \vee \lambda \geq f(x) \wedge \mu\). Indeed, since \(S\) is regular and \(x \in S\), there exist \(z \in S\) such that \(x \leq xzx\). Thus we have that \(xy \leq (xzx)y = (xz)xy\). So

\[
    f(xy) \vee \lambda \geq f((xz)xy) \wedge \mu
\]

for \(f\) is a \((\lambda, \mu)\)-fuzzy interior ideal. Again since \(f\) is a \((\lambda, \mu)\)-fuzzy interior ideal of \(S\), we have

\[
    f((xz)xy) \vee \lambda \geq f(x) \wedge \mu.
\]

From (3) and (4) we have that \(f(xy) \vee \lambda = (f(xy) \vee \lambda) \vee (f((xz)xy) \wedge \mu) \geq f(x) \wedge \mu\), and \(f\) is a \((\lambda, \mu)\)-fuzzy right ideal of \(S\). In a similar way, we can prove that \(f\) is a \((\lambda, \mu)\)-fuzzy left ideal of \(S\). Thus \(f\) is a \((\lambda, \mu)\)-fuzzy ideal of \(S\). \(\square\)

Theorem 3.4. Let \((S, \circ, \leq)\) be a intra-regular ordered semigroup and \(f\) a \((\lambda, \mu)\)-fuzzy interior ideal of \(S\). Then \(f\) is a \((\lambda, \mu)\)-fuzzy ideal of \(S\).

Proof. Let \(a, b \in S\). Then \(f(ab) \vee \lambda \geq f(a) \wedge \mu\). Indeed, since \(S\) is intra-regular and \(a \in S\), there exist \(x, y \in S\) such that \(a \leq xa^2y\). Then \(ab \leq (xa^2y)b\). Since \(f\) is a \((\lambda, \mu)\)-fuzzy interior ideal of \(S\), we have that \(f(ab) \vee \lambda = (f(ab) \vee \lambda) \vee (f(xa^2y) \wedge \mu) \geq (f(xa^2y) \wedge \mu) \vee \lambda = (f(xa^2y) \vee \lambda) \wedge (\mu \vee \lambda)\). Again since \(f\) is a \((\lambda, \mu)\)-fuzzy interior ideal of \(S\), we have \(f(xa^2y) \vee \lambda = f((xa)a(yb)) \vee \lambda \geq f(a) \wedge \mu\). Thus we have that \(f(ab) \vee \lambda \geq f(a) \wedge \mu\), and \(f\) is a \((\lambda, \mu)\)-fuzzy right ideal of \(S\). In a similar way we can prove that \(f\) is a \((\lambda, \mu)\)-fuzzy left ideal of \(S\). Therefore, \(f\) is a \((\lambda, \mu)\)-fuzzy ideal of \(S\). \(\square\)

Remark 3.5. From previous theorems we know that in regular or intra-regular ordered semigroups the concepts of \((\lambda, \mu)\)-fuzzy ideals and \((\lambda, \mu)\)-fuzzy interior ideals coincide.

4. \((\lambda, \mu)\)-Fuzzy Simple Ordered Semigroups

In this section, we introduce the concept of \((\lambda, \mu)\)-fuzzy simple ordered semigroups and characterize this type of ordered semigroups in terms of \((\lambda, \mu)\)-fuzzy interior ideals.

Definition 4.1 ([3]). An ordered semigroup \(S\) is called simple if it does not contain proper ideals, that is, for any ideal \(A \neq \emptyset\) of \(S\), we have \(A = S\).

Definition 4.2. An ordered semigroup \(S\) is called \((\lambda, \mu)\)-fuzzy simple if for any \((\lambda, \mu)\)-fuzzy ideal \(f\) of \(S\), we have \(f(a) \vee \lambda \geq f(b) \wedge \mu\), for all \(a, b \in S\).

Remark 4.3. In [3], Kehayopulu and Tsingelis studied \((0, 1)\)-fuzzy simple ordered semigroup, which was called fuzzy simple ordered semigroup. (see Definition 3.1 of [3]).

Theorem 4.4. Let \(S\) be an ordered semigroup. Then \(S\) is \((\lambda, \mu)\)-fuzzy simple if and only if for any \((\lambda, \mu)\)-fuzzy ideal \(f\) of \(S\), if \(f_\alpha \neq \emptyset\), then \(f_\alpha = S\), for all \(\alpha \in (\lambda, \mu)\).
Proof. For any $(\lambda, \mu)$-fuzzy ideal $f$ of $S$, suppose that $f_\alpha \neq \emptyset$. We need to prove that $x \in f_\alpha$ for all $x \in S$, where $\alpha \in (\lambda, \mu)$. Since $f_\alpha \neq \emptyset$, we can suppose that there exists $y \in f_\alpha$, that is $f(y) \geq \alpha$. So $f(x) \lor \lambda \geq f(y) \land \mu \geq \alpha \land \mu = \alpha$. Notice that $\lambda < \alpha$, we have that $f(x) \geq \alpha$, that is $x \in f_\alpha$.

Conversely, for any $(\lambda, \mu)$-fuzzy ideal $f$ of $S$, suppose that $f_\alpha = S$, for all $\alpha \in (\lambda, \mu)$. We need to prove that $f(a) \lor \lambda \geq f(b) \land \mu$, for all $a, b \in S$. If there exist $a_0, b_0 \in S$, such that $f(a_0) \lor \lambda < \alpha = f(b_0) \land \mu$, then $\alpha \in (\lambda, \mu]$, $f(a_0) < \alpha$ and $f(b_0) \geq \alpha$. Thus $a_0 \notin f_\alpha = S$. This is a contradiction. So $f(a) \lor \lambda \geq f(b) \land \mu$ holds, for all $a, b \in S$. □

Proposition 4.5. Let $S$ be an ordered semigroup and $f$ a $(\lambda, \mu)$-fuzzy right ideal of $S$. Then $I_a = \{b \in S | f(b) \lor \lambda \geq f(a) \land \mu\}$ is a right ideal of $S$ for every $a \in S$.

Proof. Let $a \in S$. Then $I_a \neq \emptyset$ since $a \in I_a$.

(1) Let $b \in I_a$ and $s \in S$, then $bs \in I_a$. Indeed, since $f$ is a $(\lambda, \mu)$-fuzzy right ideal of $S$ and $b, s \in S$, we have

$$f(bs) \lor \lambda \geq f(b) \land \mu.$$  

(5)

Since $b \in I_a$, we have that

$$f(b) \lor \lambda \geq f(a) \land \mu.$$  

(6)

From (5) and (6) we conclude that $f(bs) \lor \lambda = (f(bs) \lor \lambda) \lor \lambda \geq (f(b) \land \mu) \lor \lambda = (f(b) \lor \lambda) \land (\mu \lor \lambda) \geq f(a) \land \mu$. So $bs \in I_a$.

(2) Let $b \in I_a$ and $S \ni s \leq b$, then $s \in I_a$. Indeed, since $f$ is a $(\lambda, \mu)$-fuzzy right ideal of $S$, $s, b \in S$ and $s \leq b$, we have

$$f(s) \lor \lambda \geq f(b) \land \mu.$$  

(7)

Since $b \in I_a$, we have

$$f(b) \lor \lambda \geq f(a) \land \mu.$$  

(8)

From (7) and (8) we obtain that $f(s) \lor \lambda = (f(s) \lor \lambda) \lor \lambda \geq (f(b) \land \mu) \lor \lambda = (f(b) \lor \lambda) \land (\mu \lor \lambda) \geq f(a) \land \mu$. So $s \in I_a$. □

Similarly, we have

Proposition 4.6. Let $S$ be an ordered semigroup and $f$ a $(\lambda, \mu)$-fuzzy left ideal of $S$. Then $I_a = \{b \in S | f(b) \lor \lambda \geq f(a) \land \mu\}$ is a left ideal of $S$ for every $a \in S$.

By the previous propositions, we have

Proposition 4.7. Let $S$ be an ordered semigroup and $f$ a $(\lambda, \mu)$-fuzzy ideal of $S$. Then $I_a = \{b \in S | f(b) \lor \lambda \geq f(a) \land \mu\}$ is an ideal of $S$ for every $a \in S$.

Lemma 4.8. Let $S$ be an ordered semigroup and $\emptyset \neq I \subseteq S$, then $I$ is an ideal of $S$ if and only if the characteristic function $f_I$ is a $(\lambda, \mu)$-fuzzy ideal of $S$.

Proof. Similar to the proof of Theorem 1 of Section 2. One can also see the proof of Proposition 3.2 of [8]. □

Theorem 4.9. An ordered semigroup $S$ is simple if and only if it is $(\lambda, \mu)$-fuzzy simple.
Proof. Suppose $S$ is simple, let $f$ be a $(\lambda, \mu)$-fuzzy ideal of $S$ and $a, b \in S$. By previous proposition, the set $I_a$ is an ideal of $S$. Since $S$ is simple, we have $I_a = S$. Then $b \in I_a$, from which we have that $f(b) \lor \lambda \geq f(a) \land \mu$. Thus $S$ is $(\lambda, \mu)$-fuzzy simple.

Conversely, suppose $S$ contains proper ideals and let $I$ be such ideal of $S$. By the previous lemma, we know that $f_I$ is a $(\lambda, \mu)$-fuzzy ideal of $S$. We have that $S \subseteq I$. Indeed, let $x \in S$. Since $S$ is $(\lambda, \mu)$-fuzzy simple, $f_I(x) \lor \lambda \geq f_I(b) \land \mu$ for all $b \in S$. Now let $a \in I$. Then we have $f_I(x) \lor \lambda \geq f_I(a) \land \mu = 1 \land \mu = \mu$. Notice that $\lambda < \mu$, we conclude that $f_I(x) \geq \mu$, which implies that $f_I(x) = 1$, that is $x \in I$. Thus we have that $S \subseteq I$, and so $S = I$. We get a contradiction. □

Lemma 4.10 ([3, 4]). An ordered semigroup $S$ is simple if and only if for every $a \in S$, we have $S = (S a S)$.

Theorem 4.11. Let $S$ be an ordered semigroup. Then $S$ is simple if and only if for every $(\lambda, \mu)$-fuzzy interior ideal $f$ of $S$, we have $f(a) \lor \lambda \geq f(b) \land \mu$, for all $a, b \in S$.

Proof. Suppose $S$ is simple. Let $f$ be a $(\lambda, \mu)$-fuzzy interior ideal of $S$ and $a, b \in S$. Since $S$ is simple and $b \in S$, by the previous lemma, we have that $S = (S b S)$. Since $a \in S$, we have that $a \in (S b S)$. Then there exist $x, y \in S$ such that $a \leq x y$. Since $a, x y \in S, a \leq x y$ and $f$ is a $(\lambda, \mu)$-fuzzy interior ideal of $S$, we have that

$$f(a) \lor \lambda \geq f(x y) \land \mu. \quad (9)$$

Since $x, b, y \in S$ and $f$ is a $(\lambda, \mu)$-fuzzy interior ideal of $S$, we have that

$$f(x y) \lor \lambda \geq f(b) \land \mu. \quad (10)$$

From (9) and (10) we conclude that $f(a) \lor \lambda = (f(a) \lor \lambda) \lor \lambda \geq (f(x y) \land \mu) \lor \lambda = (f(x y) \lor \lambda) \land (\mu \lor \lambda) \geq f(b) \land \mu$.

Conversely, Suppose that for every $(\lambda, \mu)$-fuzzy interior ideal $f$ of $S$, we have $f(a) \lor \lambda \geq f(b) \land \mu$, for all $a, b \in S$. Now let $f$ be any $(\lambda, \mu)$-fuzzy ideal $f$ of $S$, then it is a $(\lambda, \mu)$-fuzzy interior ideal of $S$. So we have $f(a) \lor \lambda \geq f(b) \land \mu$, for all $a, b \in S$. Thus $S$ is $(\lambda, \mu)$-fuzzy simple by its definition. And from the previous theorem, we conclude that $S$ is simple. □

As a consequence we have

Theorem 4.12. For an ordered semigroup $S$, the following are equivalent:

(1) $S$ is simple.
(2) $S = (S a S)$ for every $a \in S$.
(3) $S$ is $(\lambda, \mu)$-fuzzy simple.
(4) For every $(\lambda, \mu)$-fuzzy interior ideal $f$ of $S$, we have $f(a) \lor \lambda \geq f(b) \land \mu$, for all $a, b \in S$.

5. CONCLUSION AND FURTHER RESEARCH

In this paper, we generalized Kehayopulu and Tsingelis’ results. We introduced $(\lambda, \mu)$-fuzzy ideals and $(\lambda, \mu)$-fuzzy interior ideals of an ordered semigroup and studied them. When $\lambda = 0$ and $\mu = 1$, we meet ordinary fuzzy ideals and fuzzy interior ideals. From this view, we say that $(\lambda, \mu)$-fuzzy ideals and $(\lambda, \mu)$-fuzzy interior ideals are more general concepts than fuzzy ones.
In [8], Yao gave the definition of $(\lambda, \mu)$-fuzzy bi-ideals in semigroups. One can study $(\lambda, \mu)$-fuzzy bi-ideals in ordered semigroups. For example, one can research the relationship among $(\lambda, \mu)$-fuzzy ideals, $(\lambda, \mu)$-fuzzy interior ideals and $(\lambda, \mu)$-fuzzy bi-ideals. We would like to explore this in next papers.

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