An application on soft $\tilde{L}$-fuzzy convergence structure

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Abstract. In this paper, the concept of soft $\tilde{L}$-fuzzy set is introduced. A new notion of soft $\tilde{L}$-fuzzy convergence structure on the basis of soft fuzzy filter is introduced. The main aim of this paper is to present an application on soft $\tilde{L}$-fuzzy convergence structure which has been discussed analogously as in the paper of Thomas Kubiak. Besides, characterization and several properties of soft $\tilde{L}$-fuzzy $C$-$\lim$-convergence extremally disconnected space are established.

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1. Introduction

Zadeh [10] introduced the fundamental concepts of fuzzy sets in his classical paper. The fuzzy concept in various branches of Mathematics was developed by Rodabaugh [11, 12, 13]. Fuzzy sets have applications in many fields such as information [7] and control [9]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Fuzzy topological space was introduced by C.L. Chang[1]. Bruce Hutton[3] constructed an interesting $L$-fuzzy topological space, called $L$-fuzzy unit interval, which plays the same role in fuzzy topology that the unit interval plays in general topology. Thomas Kubiak[14] introduced and studied about the properties of $L$-sets. Thomas Kubiak[15] also investigated the Tietze Extension Theorem using $L$-fuzzy sets in normal spaces. Kent[4] introduced the concept of convergence space whereas Richardson.G [10] established the fuzzy convergence space on the basis of fuzzy filters.

In this paper, the notion of soft L-fuzzy set is introduced. The concept of soft L-fuzzy convergence structure on the basis of soft fuzzy filter is introduced. Using Sostak’s idea [8] on the structure, soft L-fuzzy C-lim-convergence topology is established. The concept of soft L-fuzzy C-lim-convergence extremally disconnected space and its characterizations are discussed. In this connection, Insertion and Extension Theorem are established, which is one of the most applicable one in digital space.

2. Preliminaries

Definition 2.1 ([4]). A convergence space is a pair $(X, q)$, where $X$ is a set and $q : F(X) \to 2^X$ satisfies the following:

(CS1) $x \in q(\hat{x})$, for each $x \in X$, where $\hat{x}$ is the ultrafilter containing $\{x\}$.

(CS2) $q(\mathcal{F}) \subseteq q(\mathcal{G})$, when $\mathcal{F} \subseteq \mathcal{G}$.

(CS3) $q(\mathcal{F}) = \bigcap\{q(\mathcal{G}) : \mathcal{G} \text{ is an ultrafilter containing } \mathcal{F}\}$

Definition 2.2 ([3]). Let $X$ be a set and $L$ be a complete lattice. An $L$-fuzzy set on $X$ is a map from $X$ into $L$. That is, if $\lambda$ is a $L$-fuzzy subset of $X$ then $\lambda \in L^X$, where $L^X$ denotes the collection of all maps from $X$ into $L$.

Definition 2.3 ([10]). The pair $(X, \text{lim})$ is called as a fuzzy convergence space, where $\text{lim} : F(X) \to I^X$ provided:

(FC1) $\forall \mathfrak{F} \in F(X), \text{lim} \mathfrak{F} = \bigcap_{\mathcal{F} \in \mathfrak{F}_{\mu}(\mathfrak{F})} \text{lim} \mathcal{F}$.

(FC2) $\forall \mathfrak{F} \in F_\mu(X), \text{lim} \mathfrak{F} \subseteq c(\mathfrak{F})$.

(FC3) $\forall \mathfrak{F}, \mathfrak{G} \in F_\mu(X)$, when $\mathfrak{F} \subseteq \mathfrak{G} \Rightarrow \text{lim} \mathfrak{G} \subseteq \text{lim} \mathfrak{F}$.

(FC4) $\forall \; x \in X, 0 < \alpha \leq 1, \text{lim} (\alpha \mathfrak{F}) \subseteq (\alpha \text{lim} \mathfrak{F})$

Definition 2.4 ([16] [18]). Let $X$ be a nonempty set and $I=[0,1]$ be the unit interval. Let $\mu$ be a fuzzy subset of $X$ such that $\mu : X \to [0,1]$ and $M$ be any crisp subset of $X$. Then, the pair $(\mu, M)$ is called as a soft fuzzy set in $X$. The family of all soft fuzzy subsets of $X$, will be denoted by $\text{SF}(X)$.

Definition 2.5 ([18]). Let $X$ be a non-empty set. Then the complement of a soft fuzzy set $(\mu, M)$ is defined as $(\mu, M)' = (1 - \mu, X\setminus M)$

Definition 2.6 ([18]). Let $(X, \tau)$ be a SFTS. A soft fuzzy set $(\lambda, N)$ is said to be soft fuzzy $C$-open, if

$$(\lambda, N) = (\mu, M) \cap (\gamma, K)$$

where, $(\mu, M)$ is a soft fuzzy open set and $(\gamma, K)$ is a soft fuzzy $\alpha^*$-open set.

The complement of soft fuzzy $C$-open set (in short, $\text{SFcOS}$) is called as a soft fuzzy $C$-closed set. (in short $\text{SFcCS}$)
3. On soft $\tilde{L}$-fuzzy set

Throughout this paper $<\tilde{L}, \sqcup, \sqcap, '>$ is an infinitely distributive lattice with an order-reversing involution. Such a lattice being complete has a least element $(0,0)$ and greatest element $(1,1)$.

**Definition 3.1.** Let $X$ be a non-empty crisp set. A soft fuzzy filter $\mathfrak{F}$ is a non-empty collection of soft fuzzy sets in $X$ provided:

1. $(\lambda, N) \notin \mathfrak{F}$ such that either $\lambda = 1 \phi$ or $N = \phi$.
2. $(\mu, M), (\lambda, N) \in \mathfrak{F} \Rightarrow (\mu, M) \sqcap (\lambda, N) \in \mathfrak{F}.$
3. $(\lambda, N) \in \mathfrak{F}$ and $(\lambda, N) \sqsubseteq (\mu, M) \Rightarrow (\mu, M) \in \mathfrak{F}.$

**Note:** If $A \subseteq X$, then $1_A$ denotes the characteristic function of $A$, and $1_{\{x\}}$ is written simply as $1_x$.

**Definition 3.2.** A soft fuzzy filter $\mathfrak{F}$ is said to be a soft fuzzy prime filter, whenever $(\lambda, N) \sqcup (\mu, M) \in \mathfrak{F}$ with either $\hat{(\lambda, N)} \neq \phi$ or $\hat{(\mu, M)} \neq \phi \Rightarrow (\mu, M) \in \mathfrak{F}_c$ or $(\lambda, M) \in \mathfrak{F}$.

**Notation:**

- $\hat{(\lambda, N)} = \{x \in X: \lambda(x) > 0 \text{ and } x \in N\}$
- $(\lambda, N)_0 = \{x \in X: \lambda(x) > 0 \text{ or } x \in N\}$

**Note:**

- The set of all soft fuzzy prime filters on $X$ is denoted by $\mathbb{F}_p(X)$.
- The set of all soft fuzzy filters on $X$ is denoted by $\mathbb{F}(X)$.
- The set of all soft fuzzy prime filters containing $\mathfrak{F}$ on $X$ is denoted by $\mathbb{F}_p(\mathfrak{F})$.
- The set of all filters on $X$ is denoted as $\mathbb{F}(X)$.

**Definition 3.3.** The soft fuzzy principal filter $(\mu, M)$ is the soft fuzzy filter generated by singleton set $\{\mu, M\}$, that is,

$$(\mu, M) := \{(\lambda, N) \in SF(X): (\mu, M) \sqsubseteq (\lambda, N)\}.$$

**Definition 3.4.** Let $X$ be a non-empty set and $L$ be any lattice. Let $N \subseteq X$. Associated to each soft fuzzy set $(\lambda, N)$, a soft $\tilde{L}$-fuzzy set is defined. A soft $\tilde{L}$-fuzzy set $(\lambda, \psi_N)$ in $X$ is an element of the set of all functions from $X$ to $L \times L$, where $\lambda$ is $L$-fuzzy set on $X$ and $\psi_N: X \rightarrow L$ such that

$$\psi_N(x) = \begin{cases} 
1, & \text{if } N = X \\
I \in L (0 < I < 1), & \text{if } x \in N \\
0, & \text{if } N = \phi
\end{cases}$$

The family of all soft $\tilde{L}$-fuzzy sets, is denoted by $\tilde{L}^X$.

**Definition 3.5.** Let $X$ be a non-empty set and the soft $\tilde{L}$-fuzzy sets $A$ and $B$ be in the form,
Let \((\mu, \psi_M) : \mu(x) \in L\) and \(\psi_M(x) \in L, \forall x \in X\)\\
\(B = \{(\lambda, \psi_N) : \lambda(x) \in L\) and \(\psi_N(x) \in L, \forall x \in X\)

Then,

1. \(A \subseteq B \Leftrightarrow \mu(x) \leq \lambda(x)\) and \(\psi_M(x) \leq \psi_N(x), \forall x \in X.
2. \(A = B \Leftrightarrow \mu(x) = \lambda(x)\) and \(\psi_M(x) = \psi_N(x), \forall x \in X.
3. \(A \cap B \Leftrightarrow \mu(x) \wedge \lambda(x)\) and \(\psi_M(x) \wedge \psi_N(x), \forall x \in X.
4. \(A \cup B \Leftrightarrow \mu(x) \vee \lambda(x)\) and \(\psi_M(x) \vee \psi_N(x), \forall x \in X.

**Definition 3.6.** The pair \((X, \text{lim})\) is called a soft \(L\)-fuzzy convergence structure, where \(\text{lim} : F(X) \to \hat{L}^X\) provided:

\((SFC1) \forall \bar{a} \in F(X), \text{lim} \bar{a} = \bigcap_{\bar{g} \in \mathcal{P}_M(\bar{a})} \text{lim} \bar{g}.
(SFC2) \forall \bar{g}, \bar{h} \in F_p(X), \text{when} \bar{g} \subseteq \bar{h} \Rightarrow \text{lim} \bar{g} \subseteq \text{lim} \bar{h}.
(SFC3) \forall x \in X, 0 < \alpha < 1, \text{lim} \{(\alpha 1_x, \{x\}) \subseteq (0X, 0X)\}, \text{where} \alpha 1_x = \alpha \land 1_x.
(SFC4) \text{lim}(1X, \bar{X}) = (1X, 1X)

**Definition 3.7.** Let \((X, \text{lim}), (Y, \text{lim})\) be the soft \(L\)-fuzzy convergence structures. Let \(\bar{g} \in F(X). \) If \(f\) is a function from \(X\) to \(Y\) and \(\text{lim} \bar{g} \in \hat{L}^Y\), then the image of \(\text{lim} \bar{g}\), \(f(\text{lim} \bar{g})\) is the soft \(L\)-fuzzy set in \(Y\) defined by

\[ f(\text{lim} \bar{g})(y) = \sup_{x \in f^{-1}(y)} \{\text{lim} \bar{g}(x)\} \]

**Definition 3.8.** Let \((X, \text{lim}), (Y, \text{lim})\) be the soft \(L\)-fuzzy convergence structures. Let \(\bar{g} \in F(Y)\). Let \(\bar{h}\) is a function from \(X\) to \(Y\) and \(\text{lim} \bar{g} \in \hat{L}^Y\), then the inverse image of \(\text{lim} \bar{g}\), \(f^{-1}(\text{lim} \bar{g})\) is the soft \(L\)-fuzzy set in \(Y\) defined by

\[ f^{-1}(\text{lim} \bar{g}) = \text{lim} \bar{g} \circ f \]

**Note:** Analogically, \(\sup_{i \in I}\{\text{lim} \bar{g}\}, \inf_{i \in I}\{\text{lim} \bar{g}\}, \cup_{i \in I}\text{lim} \bar{g}, \cap_{i \in I}\text{lim} \bar{g}\), et c., are defined in usual way as in \(L\)-fuzzy sets.

**Definition 3.9.** Let \((X, \text{lim})\) be a soft \(L\)-fuzzy convergence structure. Then, the interior operator, \(\text{int} : \hat{L}^X \to \hat{L}^X\) is defined for \(\bar{g} \in F(X)\) as,

\[ \text{int}(\text{lim} \bar{g}) = \cup\{\text{lim} \bar{g} \in \hat{L}^X : \text{lim} \bar{g} \subseteq \text{lim} \bar{h}, \bar{h} \in F_p(X)\} \]

**Definition 3.10.** Let \((X, \text{lim})\) be a soft \(L\)-fuzzy convergence structure. Let \(\bar{g} \in F(X)\). Let \(\text{lim} \bar{g} \in \hat{L}^X\). Then, the soft \(L\)-fuzzy lim-convergence topology is defined by

\[ \tau_{\text{lim}} = \{\text{lim} \bar{g} : \text{int}(\text{lim} \bar{g}) = \text{lim} \bar{g}, \bar{g} \in F_p(X)\} \]

Now, the pair \((X, \tau_{\text{lim}})\) is said to be a soft \(L\)-fuzzy lim-convergence topological space.

The member of \(\tau_{\text{lim}}\) is called as a soft \(L\)-fuzzy lim-open set and its complement is defined as

\[ (\text{lim} \bar{g})' = \text{lim}(1X, \bar{X}) - \text{lim} \bar{g} \]

which is called as a soft \(L\)-fuzzy lim-closed set.

**Definition 3.11.** Let \((X, \tau_{\text{lim}})\) be a soft \(L\)-fuzzy lim-convergence topological space. Let \(\bar{g} \in F(X)\). Let \(\text{lim} \bar{g} \in \hat{L}^X\). Then, the soft \(L\)-fuzzy lim-closure and soft \(L\)-fuzzy lim-interior of a soft \(L\)-fuzzy set, \(\text{lim} \bar{g}\) in \(X\) will be defined as

\[ 730 \]
\[ S\hat{L}F\text{-int}(\lim \mathfrak{G}) = \bigcup \{ \lim \mathfrak{G} \in \hat{L}^X : \lim \mathfrak{G} \text{ is a soft } \hat{L} \text{-fuzzy lim-open set}, \lim \mathfrak{G} \subseteq \lim \mathfrak{G}, \mathfrak{G} \in F_p(X) \} \]
\[ S\hat{L}F\text{-cl}(\lim \mathfrak{G}) = \cap \{ \lim \mathfrak{G} \in \hat{L}^X : \lim \mathfrak{G} \text{ is a soft } \hat{L} \text{-fuzzy lim-closed set}, \lim \mathfrak{G} \subseteq \lim \mathfrak{G}, \mathfrak{G} \in F_p(X) \} \]

**Proposition 3.12.** For any \( \mathfrak{G} \in F(X) \), we have
(a) \( S\hat{L}F\text{-cl}(\lim \mathfrak{G}) = (S\hat{L}F\text{-int}(\lim \mathfrak{G}))' \)
(b) \( S\hat{L}F\text{-int}(\lim \mathfrak{G})' = (S\hat{L}F\text{-cl}(\lim \mathfrak{G}))' \)

**Proof.** (a) \[ S\hat{L}F - cl((\lim \mathfrak{G})') = S\hat{L}F - cl((\lim (1_X, X) - lim \mathfrak{G})) \]
\[ = \cap \{ lim(1_X, X) - lim \mathfrak{G} \in \hat{L}^X : lim(1_X, X) - lim \mathfrak{G} \text{ is a soft } \hat{L} \text{-fuzzy lim-closed set} \}
\[ \lim(1_X, X) \subseteq lim(1_X, X) - lim \mathfrak{G}, \mathfrak{G} \in F_p(X) \} \]
\[ = lim(1_X, X) - S\hat{L}F - int(\lim \mathfrak{G}). \]
(b) Similar proof of (a). \( \Box \)

**Note:** If \( \mathfrak{G}, \mathfrak{G} \in F(X) \), and \( \lim \mathfrak{G}, \lim \mathfrak{G} \in \hat{L}^X \), \( \lim \mathfrak{G} \subseteq \lim \mathfrak{G} \) is defined analogously as in L-fuzzy set in usual way.

**Definition 3.13.** Let \( (X, \tau_{lim}) \) be a soft \( \hat{L} \)-fuzzy lim-convergence topological space and \( A \) be a subset of \( X \). If \( \mathfrak{G} \in F_p(X) \) and \( (\lim \mathfrak{G})/A \in \hat{L}^A \), then
\[ \tau_{lim/A} = \{ ((\lim \mathfrak{G})/A : \lim \mathfrak{G} \in \tau_{lim} \} \]
is called as a soft \( \hat{L} \)-fuzzy lim-convergence subspace topology. Now, the pair \( (A, \tau_{lim/A}) \)
is called as a soft \( \hat{L} \)-fuzzy lim-convergence subspace of \( (X, \tau_{lim}) \).

**Definition 3.14.** Let \( (X, lim) \) be a soft \( \hat{L} \)-fuzzy convergence structure. Let \( \mathfrak{G} \in F(X) \). Let \( \lim \mathfrak{G} \in \hat{L}^X \). Then, the soft \( \hat{L} \)-fuzzy real line \( \hat{L}^R \) i.e. \( \hat{R}(L \times L) \) is the set of all monotone decreasing element \( \lim \mathfrak{G} \in \hat{L}^R \) satisfying
\[ \bigcup \{ lim \mathfrak{G}(t) : t \in R \} = (1, 1) \]
\[ \cap \{ lim \mathfrak{G}(t) : t \in R \} = (0, 0) \]
after the identification of \( lim \mathfrak{G}, lim \mathfrak{G} \in \hat{L}^R \) and \( \mathfrak{G}, \mathfrak{G} \in F(X) \) iff
\[ lim \mathfrak{G}(t-) = lim \mathfrak{G}(t-) \]
\[ lim \mathfrak{G}(t+) = lim \mathfrak{G}(t+) \]
for all \( t \in R \), where,
\[ lim \mathfrak{G}(t-) = \bigcap_{s < t} lim \mathfrak{G}(s) = ts_{-} - lim \mathfrak{G}(s). \]
\[ lim \mathfrak{G}(t+) = \bigcup_{s > t} lim \mathfrak{G}(s) = ts_{+} - lim \mathfrak{G}(s). \]

**Definition 3.15.** Let \( (X, lim) \) be a soft \( \hat{L} \)-fuzzy convergence structure. Let \( \mathfrak{G} \in F(X) \). Let \( \lim \mathfrak{G} \in \hat{L}^X \). The natural soft \( \hat{L} \)-fuzzy lim-convergence topology on \( \hat{R}(L \times L) \) is generated from the sub-basis \( \{ L_t, R_t : t \in R \} \), where, \( L_t, R_t : \hat{R} \rightarrow L \times L \) and \( L_t(\lim \mathfrak{G}) = (\lim \mathfrak{G}(t-))'(1, 1) - \lim \mathfrak{G}(t-) \) and \( R_t(\lim \mathfrak{G}) = \lim \mathfrak{G}(t+) \), for all \( lim \mathfrak{G} \in \hat{L}^R \). This topology is called as the usual topology for \( \hat{R}(L \times L) \).
\[ \mathcal{L} = \{ L_t : t \in R \} \cup \{(0_X, 0_X), (1_X, 1_X) \} \text{ and } \mathcal{R} = \{ R_t : t \in R \} \cap \{(0_X, 0_X), (1_X, 1_X) \} \]
are called the left and right hand \( \hat{L} \)-topologies respectively.
Definition 3.16. A partial order on $\mathbb{R}(L \times L)$ is defined by $[\lim \mathfrak{A}] \subseteq [\lim \mathfrak{B}]$ iff $\lim \mathfrak{A}(t-) \subseteq \lim \mathfrak{B}(t-)$ and $\lim \mathfrak{A}(t+) \subseteq \lim \mathfrak{B}(t+)$, for all $t \in \mathbb{R}$ and $\mathfrak{A}, \mathfrak{B} \in \mathcal{F}(X)$.

Definition 3.17. Let $(X, \lim)$ be a soft $\bar{L}$-fuzzy convergence structure. The soft $\bar{L}$-fuzzy unit interval $I(L \times L)$ is a subset of $\mathbb{R}(L \times L)$ such that $[\lim \mathfrak{A}] \in I(L \times L)$ i.e. $[\lim \mathfrak{A}] \in \bar{L}$, if

$$\lim \mathfrak{A}(t) = (1, 1) \text{ for } t < 0, \; t \in \mathbb{R}$$

$$\lim \mathfrak{A}(t) = (0, 0) \text{ for } t > 1, \; t \in \mathbb{R}$$

It is equipped with the soft $\bar{L}$-fuzzy $\lim$-convergence subspace topology.

Definition 3.18. Let $(X, \tau_{lim})$ be a soft $\bar{L}$-fuzzy $\lim$-convergence topological space. A soft $\bar{L}$-fuzzy set, $\lim \mathfrak{A}$ is said to be a soft $\bar{L}$-fuzzy $\alpha^*$-lim-open set (in short., $\bar{SLF} \alpha^*$-limOS), if

$$\bar{SLF} \text{-int}(\lim \mathfrak{A}) = \bar{SLF} \text{-int}(\bar{SLF} \text{-cl}(\bar{SLF} \text{-int}(\lim \mathfrak{A})))$$

for each $\mathfrak{A} \in \mathcal{F}_p(X)$.

The complement of a soft $\bar{L}$-fuzzy $\alpha^*$-lim-open set is called as a soft $\bar{L}$-fuzzy $\alpha^*$-lim-closed set. It is denoted by $\bar{SLF} \alpha^*$-limCS.

Definition 3.19. Let $(X, \tau_{lim})$ be a soft $\bar{L}$-fuzzy $\lim$-convergence topological space. A soft $\bar{L}$-fuzzy set, $\lim \mathfrak{A}$ is said to be a soft $\bar{L}$-fuzzy $C$-lim-open set (in short., $\bar{SLFC} \text{-limOS}$), if

$$\lim \mathfrak{A} = \lim \mathfrak{G}_1 \cap \lim \mathfrak{G}_2, \; \mathfrak{G} \in \mathcal{F}(X), \; \mathfrak{G}_1, \mathfrak{G}_2 \in \mathcal{F}_p(X).$$

where, $\lim \mathfrak{G}_1$ is a soft $\bar{L}$-fuzzy $\lim$-open set and $\lim \mathfrak{G}_2$ is a soft $\bar{L}$-fuzzy $\alpha^*$-lim-open set.

Now, the complement of a soft $\bar{L}$-fuzzy $\lim$-open set is called as a soft $\bar{L}$-fuzzy $C$-lim-closed set. It is denoted by $\bar{SLFC} \text{-limCS}.$

Definition 3.20. Let $(X, \tau_{lim})$ be a soft $\bar{L}$-fuzzy $\lim$-convergence topological space. Let $\mathfrak{A} \in \mathcal{F}(X)$. Let $\lim \mathfrak{A} \in \bar{L}^X$. Then, the soft $\bar{L}$-fuzzy $C$-lim-closure and soft $\bar{L}$-fuzzy $C$-lim-interior of a soft $\bar{L}$-fuzzy set $\lim \mathfrak{A}$ in $X$ will be defined as

$$\bar{SLFC} \text{-cl}(\lim \mathfrak{A}) = \bigcap\{\lim \mathfrak{G} \subseteq \bar{L}^X : \lim \mathfrak{G} \text{ is a soft } \bar{L}\text{-fuzzy } C\text{-lim-open set and }$$

$$\lim \mathfrak{G} \supseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathcal{F}_p(X)\}$$

$$\bar{SLFC} \text{-int}(\lim \mathfrak{A}) = \bigcap\{\lim \mathfrak{G} \subseteq \bar{L}^X : \lim \mathfrak{G} \text{ is a soft } \bar{L}\text{-fuzzy } C\text{-lim-closed set and }$$

$$\lim \mathfrak{G} \subseteq \lim \mathfrak{G}, \mathfrak{G} \in \mathcal{F}_p(X)\}$$

Remark 3.21.
(1) $\bar{SLF} \text{-int}(\lim \mathfrak{A}) \subseteq \lim \mathfrak{A} \subseteq \bar{SLF} \text{-cl}(\lim \mathfrak{A})$
(2) $\bar{SLFC} \text{-int}(\lim \mathfrak{A}) \subseteq \lim \mathfrak{A} \subseteq \bar{SLFC} \text{-cl}(\lim \mathfrak{A})$

Proposition 3.22. For any $\mathfrak{A} \in \mathcal{F}(X)$, we have
(a) $\bar{SLFC} \text{-cl}((\lim \mathfrak{A})') = (\bar{SLFC} \text{-int}(\lim \mathfrak{A}))'$
(b) $\bar{SLFC} \text{-int}((\lim \mathfrak{A})') = (\bar{SLFC} \text{-cl}(\lim \mathfrak{A}))'$

732
Proof. (a)

\[ S\text{-}LF\text{-}C-cl((\lim \mathcal{G})') = S\text{-}LF\text{-}C-cl(\lim(\tilde{1}_X, X) \setminus \lim \mathcal{G}) \]
\[ = \cap \{ \lim(\tilde{1}_X, X) \setminus \lim \mathcal{G} \in \tilde{L}^X : \lim(\tilde{1}_X, X) - \lim \mathcal{G} \in \text{soft } \tilde{L}\text{-fuzzy C-lim-closed set}, \lim(\tilde{1}_X, X) - \lim \mathcal{G} \in \mathbb{F}_p(X) \} \]
\[ = \lim(\tilde{1}_X, X) - \cup \{ \lim \mathcal{G} \in \tilde{L}^X : \lim \mathcal{G} \text{ is a soft } \tilde{L}\text{-fuzzy C-lim-open set}, \lim \mathcal{G} \supseteq \lim \mathcal{G}, \mathcal{G} \in \mathbb{F}_p(X) \} \]
\[ = \lim(\tilde{1}_X, X) - S\text{-}LF\text{-}C-int(\lim \mathcal{G}). \]

(b) Similar proof of (a).

\[\square\]

4. Soft \( \tilde{L} \)-fuzzy C-lim-convergence extremally disconnected space

**Definition 4.1.** Let \((X, \tau_{lim})\) be a soft \( \tilde{L} \)-fuzzy lim-convergence topological space. If the soft \( \tilde{L} \)-fuzzy C-lim-closure of a soft \( \tilde{L} \)-fuzzy C-lim-open set is soft \( \tilde{L} \)-fuzzy C-lim-open, then \((X, \tau_{lim})\) is said to be soft \( \tilde{L} \)-fuzzy C-lim-convergence extremally disconnected space.

**Proposition 4.2.** For any soft \( \tilde{L} \)-fuzzy lim-convergence topological space \((X, \tau_{lim})\), the following are equivalent:

(a) \((X, \tau_{lim})\) is a soft \( \tilde{L} \)-fuzzy C-lim-convergence extremally disconnected space.

(b) For each soft \( \tilde{L} \)-fuzzy C-lim-closed set \( \lim \mathcal{G} \), \( S\text{-}LF\text{-}C-int(\lim \mathcal{G}) \) is a soft \( \tilde{L} \)-fuzzy C-lim-closed set.

(c) For each soft \( \tilde{L} \)-fuzzy C-lim-open set \( \lim \mathcal{G} \), we have \( S\text{-}LF\text{-}C-int(S\text{-}LF\text{-}C-cl(\lim \mathcal{G})) = S\text{-}LF\text{-}C-cl(\lim \mathcal{G}). \)

(d) For each pair of soft \( \tilde{L} \)-fuzzy C-lim-open sets \( \lim \mathcal{G} \) and \( \lim \mathcal{G} \) in \((X, \tau_{lim})\) with \( S\text{-}LF\text{-}C-int(\lim(1_X, X) - \lim \mathcal{G}) = \lim \mathcal{G} \), we have \( \lim(1_X, X) - S\text{-}LF\text{-}C-cl(\lim \mathcal{G}) = S\text{-}LF\text{-}C-cl(\lim \mathcal{G}). \)

**Proof.** (a) \(\Rightarrow\) (b): Let \( \lim \mathcal{G} \) be a soft \( \tilde{L} \)-fuzzy C-lim-closed set. Now, \((\lim \mathcal{G})'\) is a soft \( \tilde{L} \)-fuzzy C-lim-open set. By (a), \( S\text{-}LF\text{-}C-cl((\lim \mathcal{G})') \) is a soft \( \tilde{L} \)-fuzzy C-lim-open set. Now,

\[ S\text{-}LF\text{-}C-cl((\lim \mathcal{G})') = S\text{-}LF\text{-}C-cl(\lim(1_X, X) \setminus \lim \mathcal{G}) \]
\[ = \lim(1_X, X) - S\text{-}LF\text{-}C-int(\lim \mathcal{G}). \]

This implies that, \( S\text{-}LF\text{-}C-int(\lim \mathcal{G}) \) is a soft \( \tilde{L} \)-fuzzy C-lim-closed set.

(b) \(\Rightarrow\) (c): Let \( \lim \mathcal{G} \) be a soft \( \tilde{L} \)-fuzzy C-lim-open set. Then, \((\lim \mathcal{G})'\) is a soft \( \tilde{L} \)-fuzzy C-lim-closed set. By (b), \( S\text{-}LF\text{-}C-int((\lim \mathcal{G})') \) is a soft \( \tilde{L} \)-fuzzy C-lim-closed set.
set. Now,

\[ \lim(\overline{1_X, X}) - \text{S\textsuperscript{LF}C-int(S\textsuperscript{LF}C-cl(\lim\overline{\emptyset}))} \]

\[ = \text{S\textsuperscript{LF}C-cl(S\textsuperscript{LF}C-int(\lim(\overline{1_X, X}) - \lim\overline{\emptyset}))} \]

\[ = \text{S\textsuperscript{LF}C-int(\lim(\overline{1_X, X}) - \lim\overline{\emptyset})} \]

\[ = \lim(\overline{1_X, X}) - \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \]

Hence, \( \text{S\textsuperscript{LF}C-int(S\textsuperscript{LF}C-cl(\lim\overline{\emptyset}))} = \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \).

(c) \to (d): Let \( \lim\overline{\emptyset} \), \( \lim\overline{\emptyset} \) be two soft \( \tilde{L} \)-fuzzy C-limit-open sets such that

\[ \text{S\textsuperscript{LF}C-int(\lim(\overline{1_X, X}) - \lim\overline{\emptyset})} = \lim\overline{\emptyset}. \]

By (c), we have \( \text{S\textsuperscript{LF}C-int(\text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})})} = \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \). Now,

\[ \lim(\overline{1_X, X}) - \text{S\textsuperscript{LF}C-int(\text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})})} = \lim(\overline{1_X, X}) - \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \]

\[ = \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \]

This implies that \( \text{S\textsuperscript{LF}C-cl(\text{S\textsuperscript{LF}C-int(\lim(\overline{1_X, X}) - \lim\overline{\emptyset})})} = \lim\overline{\emptyset} \). It follows that \( \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} = \lim\overline{\emptyset} \). This implies that

\[ \lim(\overline{1_X, X}) - \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} = \text{S\textsuperscript{LF}C-int(\lim(\overline{1_X, X}) - \lim\overline{\emptyset})} \]

\[ = \lim\overline{\emptyset} = \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})}. \]

(d) \to (a): Let \( \lim\overline{\emptyset} \) be any soft \( \tilde{L} \)-fuzzy C-limit-open set. Consider another soft \( \tilde{L} \)-fuzzy C-limit-open set \( \lim\overline{\emptyset} \), with \( \text{S\textsuperscript{LF}C-int(\lim(\overline{1_X, X}) - \lim\overline{\emptyset})} = \lim\overline{\emptyset} \). By (d), \( \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} = \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \). This implies that \( \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} = \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \). It follows that \( \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \) is a soft \( \tilde{L} \)-fuzzy C-limit-open set. Hence, \( (X, \tau_{\text{lim}}) \) is soft \( \tilde{L} \)-fuzzy C-limit-convergence extremally disconnected space.

\( \square \)

**Proposition 4.3.** Let \( (X, \tau_{\text{lim}}) \) be a soft \( \tilde{L} \)-fuzzy \( \lim \)-convergence topological space. Then, \( (X, \tau_{\text{lim}}) \) is soft \( \tilde{L} \)-fuzzy \( \lim \)-convergence extremally disconnected space if and only if for each soft \( \tilde{L} \)-fuzzy C-limit-open set, \( \lim\overline{\emptyset} \) and soft \( \tilde{L} \)-fuzzy C-limit-closed set, \( \lim\overline{\emptyset} \) such that \( \lim\overline{\emptyset} \subseteq \lim\overline{\emptyset} \), we have \( \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \subseteq \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \).

**Proof.** Suppose \( (X, \tau_{\text{lim}}) \) is a soft \( \tilde{L} \)-fuzzy C-limit-convergence extremally disconnected space. Let \( \lim\overline{\emptyset} \) be any soft \( \tilde{L} \)-fuzzy C-limit-open set and \( \lim\overline{\emptyset} \) be any soft \( \tilde{L} \)-fuzzy C-limit-closed set. Then, by (b) of Proposition 4.2, \( \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \) is soft \( \tilde{L} \)-fuzzy C-limit-closed. Since \( \lim\overline{\emptyset} \) is a soft \( \tilde{L} \)-fuzzy C-limit-open set and \( \lim\overline{\emptyset} \subseteq \lim\overline{\emptyset} \), it follows that \( \lim\overline{\emptyset} \subseteq \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \). Thus, it follows that

\[ \text{S\textsuperscript{LF}C-cl(\lim\overline{\emptyset})} \subseteq \text{S\textsuperscript{LF}C-cl(\text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})})} = \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})}. \]

Conversely, let \( \lim\overline{\emptyset} \) be a soft \( \tilde{L} \)-fuzzy C-limit-closed set. Then, \( \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \) is a \( \text{S\textsuperscript{LF}C-limit} \) open set. Also, \( \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \) \( \subseteq \lim\overline{\emptyset} \). Now, by assumption, it follows that \( \text{S\textsuperscript{LF}C-cl(\text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})})} \subseteq \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \). This implies that \( \text{S\textsuperscript{LF}C-cl(\text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})})} = \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \). Thus, \( \text{S\textsuperscript{LF}C-int(\lim\overline{\emptyset})} \) is a soft
By the Proposition 4.3, we have \((X, \tau_{lim})\) is a soft \(L\)-fuzzy \(C\)-lim-convergence extremally disconnected space.

\[ \]

**Definition 4.4.** A soft \(L\)-fuzzy set which is both soft \(L\)-fuzzy \(C\)-lim-open and soft \(L\)-fuzzy \(C\)-lim-closed set is called as a soft \(L\)-fuzzy \(C\)-lim-clopen set. It is denoted by \(SLFC_{-}limCOS\).

**Definition 4.5.** A soft \(L\)-fuzzy \(lim\)-convergence topological space, \((X, \tau_{lim})\) is said to have property \(\nabla\), if the union of any family of \(C\)-lim-open sets is \(C\)-lim-open.

**Remark 4.6.** Let \((X, \tau_{lim})\) be a soft \(L\)-fuzzy \(C\)-lim-convergence extremally disconnected space. Let \((X, \tau_{lim})\) possess the property \(\nabla\). Let \(\{lim_{i}\}_i, lim_{j}\) \(i, j \in \mathbb{N}\) be a collection such that each \(lim_{i}\)'s are \(SLFC_{-}limOS\) and each \(lim_{j}\)'s are \(SLFC_{-}limCS\). Let \(lim_{i}\) \(lim_{j}\) be the \(SLFC\)-lim open set and \(SLFC\)-lim closed set respectively. If \(lim_{i} \subseteq lim_{j} \subseteq lim_{j}\) and \(lim_{i} \subseteq lim_{j} \subseteq lim_{j}\), for all \(i, j \in \mathbb{N}\), then there exists a soft \(L\)-fuzzy \(C\)-lim-clopen set, \(lim_{i}\) such that \(SLFC_{-}cl(lim_{i})\) \(\subseteq lim_{i}\) \(\subseteq SLFC\)-int\((lim_{j})\), for all \(i, j \in \mathbb{N}\).

**Proof.** By the Proposition 4.3,

\[
SLFC_{-}cl(lim_{i}) \subseteq SLFC_{-}cl(lim_{i}) \cap SLFC\text{-}int(lim_{j}) \\
\cap SLFC\text{-}int(lim_{j})
\]

for all \(i, j \in \mathbb{N}\). Therefore, using the hypothesis,

\[
lim_{i} = SLFC_{-}cl(lim_{i}) \cap SLFC\text{-}int(lim_{j})
\]

is a soft \(L\)-fuzzy \(C\)-lim-clopen set satisfying the required condition.

**Proposition 4.7.** Let \((X, \tau_{lim})\) be a soft \(L\)-fuzzy \(C\)-lim-convergence extremally disconnected space. Let \((X, \tau_{lim})\) possess the property \(\nabla\). Let \(\{lim_{i}q_{i}\}_q\in \mathbb{Q}\) and \(\{lim_{j}q_{j}\}_q\in \mathbb{Q}\) be the monotone increasing collections of soft \(L\)-fuzzy \(C\)-lim-open sets and soft \(L\)-fuzzy \(C\)-lim-closed sets of \((X, \tau_{lim})\). \((\mathbb{Q} \text{ is the set of all rational numbers})\).

If \(lim_{i}q_{i} \subseteq lim_{j}q_{i}\), whenever \(q_{1} \leq q_{2} \in \mathbb{Q}\), then, there exists a monotone increasing collection \(\{lim_{i}q_{i}\}_q\in \mathbb{Q}\) of soft \(L\)-fuzzy \(C\)-lim-clopen sets of \((X, \tau_{lim})\) such that \(SLFC_{-}cl(lim_{i}q_{i}) \subseteq lim_{i}q_{i}\) and \(lim_{j}q_{i} \subseteq SLFC\text{-}int(lim_{j}q_{i})\) whenever \(q_{1} < q_{2}\).

**Proof.** Let us arrange into a sequence \(\{q_{n}\}\) of rational numbers without repetitions. For every \(n \geq 2\), define inductively a collection \(\{lim_{j}q_{i} : 1 \leq i < n\} \subseteq \mathbb{Q}\) such that By Proposition 4.3, the countable collections \(\{SLFC_{-}cl(lim_{i}q_{i})\}_q\in \mathbb{Q}\) and \(\{SLFC\text{-}int(lim_{j}q_{i})\}_q\in \mathbb{Q}\) satisfying \(SLFC\text{-}cl(lim_{i}q_{i}) \subseteq SLFC\text{-}int(lim_{j}q_{i})\), if \(q_{1} < q_{2} (q_{1}, q_{2} \in \mathbb{Q})\). By the Remark 4.6, there exists \(SLFC\)-lim clopen set, \(lim_{i}\) such that \(SLFC\text{-}cl(lim_{i}q_{i}) \subseteq lim_{i}\) \(\subseteq SLFC\text{-}int(lim_{j}q_{i})\). By setting \(lim_{i}q_{i} = lim_{i}\), we get \((S_{2})\).

Assume that soft \(L\)-fuzzy sets \(lim_{i}q_{i}\) (already defined), for \(i < n\) and satisfy \((S_{n})\).

Define

\[
\Phi = \cup \{lim_{j}q_{i} : i < n, q_{i} < q_{n}\} \cup lim_{i}q_{n}
\]

\[
\Omega = \cap \{lim_{j}q_{i} : j < n, q_{j} > q_{n}\} \cap lim_{i}q_{n}
\]

735
Let \( \lim \) \( S \) together with \( \Phi \) and \( \Omega \), fulfill all the conditions of the Remark 4.6. Hence, there exists \( n, q \) \( q < q \). Then, we have, \( \text{SLFC-cl}(\lim S) \subseteq \text{SLFC-cl}(\Phi) \subseteq \text{SLFC-cl}(\lim S_q) \) and \( \text{SLFC-cl}(\lim S_q) \subseteq \text{SLFC-cl}(\lim S_q) \), whenever \( q_i < q_n < q_j \) \( (i, j < n) \), as well as \( \lim S_q \subseteq \text{SLFC-cl}(\Phi) \subseteq \lim S_q \) and \( \lim S_q \subseteq \text{SLFC-cl}(\lim S_q) \), whenever \( q < q_n < q_j \). This shows that the countable \( \{ \lim S_i : i < q_n \} \) \( \bigcup \{ \lim S_i : q < q_n \} \) and \( \{ \lim S_i : j < q_n \} \bigcup \{ \lim S_q : q > q_n \} \) together with \( \Phi \) and \( \Omega \), fulfill all the conditions of the Remark 4.6. Hence, there exists a \( \text{SLFC-lim} \) clopen set, \( \lim \mathcal{M}_n \) such that \( \text{SLFC-cl}(\lim \mathcal{M}_n) \subseteq \lim S_q \) if \( q_n < q \), and \( \lim S_q \subseteq \text{SLFC-cl}(\lim \mathcal{M}_n) \) if \( q < q_n \). Also, \( \text{SLFC-cl}(\lim S_n) \subseteq \text{SLFC-cl}(\lim \mathcal{M}_n) \) if \( q_i < q_n \) and \( \text{SLFC-cl}(\lim \mathcal{M}_n) \subseteq \text{SLFC-cl}(\lim S_q) \) if \( q_n < q_j \), where \( 1 \leq i, j \leq n - 1 \). Now setting \( \lim S_n = \lim \mathcal{M}_n \), we obtain the soft \( \text{LF} \)-fuzzy sets \( \lim S_n, \lim S_{q_1}, \ldots, \lim S_{q_n} \) that satisfy \( (S_{n+1}) \). Therefore, the collection \( \{ \lim S_i : i = 1, 2, 3, \ldots \} \) has the required property. This completes the proof. \( \square \)

5. Characterizations of Soft \( \text{L-fuzzy C-lim-convergence} \) Extremely Disconnected Space

Definition 5.1. Let \((X, \tau_{lim})\) and \((Y, \sigma_{lim})\) be two soft \( \text{L-fuzzy lim-convergence} \) topological spaces. Let \( f : (X, \tau_{lim}) \rightarrow (Y, \sigma_{lim}) \) be a function. Let \( \mathfrak{S} \in \mathcal{P}_p(Y) \). Then, \( f \) is said to be soft \( \text{L-fuzzy C-lim-convergence continuous} \) function if, for every soft \( \text{L-fuzzy lim-open} \) set, \( \lim \mathfrak{S} \) in \((Y, \sigma_{lim})\), there exists a soft \( \text{L-fuzzy C-lim-open} \) set \( f^{-1}(\lim \mathfrak{S}) \) of \((X, \tau_{lim})\).

Equivalently, if, for every soft \( \text{L-fuzzy lim-closed} \) set, \( \lim \mathfrak{S} \) in \((Y, \sigma_{lim})\), there exists a soft \( \text{L-fuzzy C-lim-closed} \) set \( f^{-1}(\lim \mathfrak{S}) \) of \((X, \tau_{lim})\).

Proposition 5.2. Let \((X, \tau_{lim})\) and \((Y, \sigma_{lim})\) be two soft \( \text{L-fuzzy lim-convergence} \) topological spaces. A function \( f : (X, \tau_{lim}) \rightarrow (Y, \sigma_{lim}) \) is a soft \( \text{L-fuzzy C-lim-convergence} \) continuous function if

\[
\lim \mathfrak{S} \subseteq f^{-1}(f(\lim \mathfrak{S})) \nsubseteq S \text{LF-cl}(f(\lim \mathfrak{S}))
\]

for all \( \lim \mathfrak{S} \in \mathcal{L}^X, \mathfrak{S} \in \mathcal{P}(X) \).

Proof. Suppose that \( f \) is a soft \( \text{L-fuzzy C-lim-convergence} \) continuous function. Let \( \mathfrak{S} \in \mathcal{P}(X) \). Let \( \lim \mathfrak{S} \in \mathcal{L}^X \). Then \( f(\lim \mathfrak{S}) \in \mathcal{L}^Y \). By the hypothesis,

\[
f^{-1}(S \text{LF-cl}(f(\lim \mathfrak{S})))
\]

is a soft \( \text{L-fuzzy C-lim-closed} \) set in \( X \). Also,

\[
\lim \mathfrak{S} \subseteq f^{-1}(f(\lim \mathfrak{S})) \nsubseteq f^{-1}(S \text{LF-cl}(f(\lim \mathfrak{S}))).
\]

Now, by the definition of soft \( \text{L-fuzzy C-lim-closure} \),

\[
S \text{LF-cl}(\lim \mathfrak{S}) \subseteq f^{-1}(S \text{LF-cl}(f(\lim \mathfrak{S}))).
\]

This implies that, \( f(S \text{LF-cl}(\lim \mathfrak{S})) \subseteq S \text{LF-cl}(f(\lim \mathfrak{S}))) \).

Conversely, let \( \mathfrak{S} \in \mathcal{P}_p(Y) \). Suppose that \( \lim \mathfrak{S} \in \mathcal{L}^Y \) is soft \( \text{L-fuzzy lim-closed} \). Then, \( f^{-1}(\lim \mathfrak{S}) \in \mathcal{L}^X \). By hypothesis,

\[
f(S \text{LF-cl}(f^{-1}(\lim \mathfrak{S}))) \subseteq S \text{LF-cl}(f(f^{-1}(\lim \mathfrak{S}))) \subseteq S \text{LF-cl}(\lim \mathfrak{S}) = \lim \mathfrak{S}.
\]

736
This implies that, $\text{SLFC-cl}(f^{-1}(\text{lim}\mathfrak{F})) \subseteq f^{-1}(\text{lim}\mathfrak{F})$. It follows that, $f^{-1}(\text{lim}\mathfrak{F})$ is soft $\tilde{L}$-fuzzy $\text{C-lim}$-closed. Hence, $f$ is a soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence continuous function. \hfill $\square$

**Proposition 5.5.** Let $(X, \tau_{\text{lim}})$ be a soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence extremally disconnected space and $(Y, \sigma_{\text{lim}})$ be a soft $\tilde{L}$-fuzzy $\text{lim}$-convergence topological space. A function $f : (X, \tau_{\text{lim}}) \rightarrow (Y, \sigma_{\text{lim}})$ is a soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence continuous function. Then, for each $\text{lim}\mathfrak{F} \in \tilde{L}^X$, $\text{SLFC-cl}(\text{SLFin}(\text{lim}\mathfrak{F})) \subseteq \text{SLFC-int}(f^{-1}(\text{SLFcl}(f(\text{SLFin}(\text{lim}\mathfrak{F}))))).

**Proof.** Let $\mathfrak{F} \in \mathcal{F}(X)$. Let $\text{lim}\mathfrak{F} \in \tilde{L}^X$. Then, $\text{SLFin}(\text{lim}\mathfrak{F})$ is a soft $\tilde{L}$-fuzzy $\text{lim}$-open set in $X$. By the hypothesis,

$$f(\text{SLFC-cl}(\text{SLFin}(\text{lim}\mathfrak{F}))) \subseteq \text{SLFcl}(f(\text{SLFin}(\text{lim}\mathfrak{F}))).$$

This implies that, $\text{SLFC-cl}(\text{SLFin}(\text{lim}\mathfrak{F})) \subseteq f^{-1}(\text{SLFcl}(f(\text{SLFin}(\text{lim}\mathfrak{F}))))$. Since $(X, \tau_{\text{lim}})$ is a soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence extremally disconnected space, it follows that, $\text{SLFC-cl}(\text{SLFin}(\text{lim}\mathfrak{F})) \subseteq \text{SLFC-int}(f^{-1}(\text{SLFcl}(f(\text{SLFin}(\text{lim}\mathfrak{F}))))). \hfill \square$

**Definition 5.4.** Let $(X, \tau_{\text{lim}})$ be a soft $\tilde{L}$-fuzzy $\text{lim}$-convergence topological space. A mapping $f : X \rightarrow \mathbb{R}(L \times L)$ is called as the lower (upper) soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence continuous function, if $f^{-1}R_t$ (resp. $f^{-1}L_t$) is a soft $\tilde{L}$-fuzzy $\text{C-lim}$-open set (soft $\tilde{L}$-fuzzy $\text{C-lim}$-closed set), for each $t \in \mathbb{R}$.

**Proposition 5.5.** Let $(X, \tau_{\text{lim}})$ be a soft $\tilde{L}$-fuzzy $\text{lim}$-convergence topological space. Let $\text{lim}\mathfrak{F} \in \tilde{L}^X$. Let $f : X \rightarrow \mathbb{R}(L \times L)$ be such that

$$f(x)(t) = \begin{cases} (1,1), & \text{if } t < 0 \\ \text{lim}\mathfrak{F}(x), & \text{if } t \in [0,1] \\ (0,0), & \text{if } t > 1 \end{cases}$$

for all $x \in X$. Then, $f$ is lower (resp. upper) soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence continuous function iff $\text{lim}\mathfrak{F}$ is soft $\tilde{L}$-fuzzy $\text{C-lim}$-open (resp. $\text{C-lim}$-closed).

**Proof.** Let

$$f^{-1}R_t = \begin{cases} (1,1) & \text{if } t < 0 \\ \text{lim}\mathfrak{F}, & \text{if } t \in [0,1] \\ (0,0), & \text{if } t > 1 \end{cases}$$

This implies that, $f$ is lower soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence continuous function iff $\text{lim}\mathfrak{F}$ is soft $\tilde{L}$-fuzzy $\text{C-lim}$-open. Let

$$f^{-1}L_t = \begin{cases} (1,1), & \text{if } t < 0 \\ \text{lim}\mathfrak{F}, & \text{if } t \in [0,1] \\ (0,0), & \text{if } t > 1 \end{cases}$$

This implies that, $f$ is upper soft $\tilde{L}$-fuzzy $\text{C-lim}$-convergence continuous function iff $\text{lim}\mathfrak{F}$ is soft $\tilde{L}$-fuzzy $\text{C-lim}$-closed. \hfill $\square$
Definition 5.6. The lim-characteristic function of \( \lim \mathfrak{X} \in \mathcal{L}^X \) is the map \( \chi_{\lim \mathfrak{X}} : X \to \mathbb{R}(L \times L) \) defined by

\[
\chi_{\lim \mathfrak{X}}(x) = \begin{cases} 
(1, 1), & \text{if } t < 0 \\
\lim \mathfrak{X}(x), & \text{if } t \in [0, 1] \\
(0, 0), & \text{if } t > 1
\end{cases}
\]

for all \( x \in X \).

Proposition 5.7. Let \( (X, \tau_{\lim}) \) be a soft \( \mathcal{L} \)-fuzzy \( \lim \)-convergence topological space. Let \( \mathfrak{G} \in \mathcal{P}(X) \). Then, \( \chi_{\lim \mathfrak{G}} \) is lower (upper) soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-convergence continuous function iff \( \lim \mathfrak{G} \) is soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-open (\( C \)-\( \lim \)-closed).

Proof. It follows from the Proposition 5.5. \( \square \)

Proposition 5.8. Let \( (X, \tau_{\lim}) \) be a soft \( \mathcal{L} \)-fuzzy \( \lim \)-convergence topological space. Let \( (X, \tau_{\lim}) \) possess the property \( \partial \). Then, the following are equivalent.

(a) \( (X, \tau_{\lim}) \) be a soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-convergence extremally disconnected space.

(b) (Soft \( \mathcal{L} \)-fuzzy Insertion Theorem:) Let \( g, h : X \to \mathbb{R}(L \times L) \). If \( g \) is lower soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-convergence continuous function and \( h \) is upper soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-convergence continuous function with \( g \sqsubseteq h \), then there exists a soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-convergence continuous function \( f : X \to \mathbb{R}(L \times L) \) such that \( g \sqsubseteq f \sqsubseteq h \).

(c) (Soft \( \mathcal{L} \)-fuzzy Urysohn Lemma:) Let \( \mathfrak{G}, \mathfrak{H} \in \mathcal{P}(X) \). If \( (\lim \mathfrak{G})', \lim \mathfrak{H} \) are soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-open sets such that \( \lim \mathfrak{H} \sqsubseteq \lim \mathfrak{G} \), then there exists a soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-convergent continuous function \( f : X \to [0, 1](L \times L) \) such that \( \lim \mathfrak{H} \sqsubseteq (L_1)^r f \sqsubseteq R_0 f \sqsubseteq \lim \mathfrak{G} \).

Proof. (a) \( \Rightarrow \) (b): Define \( \lim \mathfrak{H}_r = L_r h \) and \( \lim \mathfrak{G}_r = R'_r g, r \in \mathbb{Q} \). Then, we have two monotone increasing families of soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-open and soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-closed sets of \( (X, \tau_{\lim}) \). Moreover, \( \lim \mathfrak{H}_r \sqsubseteq \lim \mathfrak{G}_r \), if \( r < s \). By Proposition: 4.7, there exists a monotone increasing family \( \{ \lim \mathfrak{H}_r \}_{r \in \mathbb{Q}} \) of soft \( \mathcal{L} \)-fuzzy \( C \)-\( \lim \)-cufen sets of \( (X, \tau_{\lim}) \) such that \( \mathcal{S}_c \mathcal{L}(\lim \mathfrak{H}_r) \sqsubseteq \lim \mathfrak{G}_r \) and \( \lim \mathfrak{H}_r \sqsubseteq \mathcal{S}_c \mathcal{L}(\mathfrak{H}_r) \), whenever \( r < s \). Let \( \lim \mathcal{U}_t = \cap_{r < t} (\lim \mathfrak{H}_r)' \), for all \( t \in \mathbb{R} \), we define monotone decreasing family \( \{ \lim \mathcal{U}_t : t \in \mathbb{R} \} \sqsubseteq \mathcal{L}^X \). Moreover, we have \( \mathcal{S}_c \mathcal{L}(\lim \mathfrak{U}_t) \sqsubseteq \mathcal{S}_c \mathcal{L}(\mathfrak{H}_r) \), whenever \( s < t \). Now,

\[
\cup_{t \in \mathbb{R}} \lim \mathcal{U}_t = \cup_{t \in \mathbb{R}} \cap_{r < t} (\lim \mathfrak{H}_r)' = \cup_{t \in \mathbb{R}} \cap_{r < t} (\lim \mathfrak{G}_r)' = \cup_{t \in \mathbb{R}} \cap_{r < t} g^{-1} R_r = g^{-1} \cup_{t \in \mathbb{R}} R_t = (1_X, 1_X)
\]

Similarly,

\[
\cap_{t \in \mathbb{R}} \lim \mathcal{U}_t = (0_X, 0_X).
\]

We now define a function \( f : X \to \mathbb{R}(L \times L) \) possessing the required properties. Let \( f(x)(t) = \lim \mathfrak{U}_t(x) \), for all \( x \in X, t \in \mathbb{R} \). By the above discussion, it follows that \( f \)
is well defined. To prove \( f \) is soft \( \tilde{\mathcal{L}} \)-fuzzy \( C \)-lim-convergence continuous function, we observe that,

\[
\bigcup_{s \succ t} \lim_{\mathcal{U}_s} = \bigcup_{s \succ t} \tilde{\mathcal{LFC-int}}(\lim_{\mathcal{U}_s})
\]

and

\[
\bigcap_{s \prec t} \lim_{\mathcal{U}_s} = \bigcap_{s \prec t} \tilde{\mathcal{LFC-cl}}(\lim_{\mathcal{U}_s})
\]

Then, \( f^{-1} R_t = \bigcup_{s \succ t} \lim_{\mathcal{U}_s} = \bigcup_{s \succ t} \tilde{\mathcal{LFC-int}}(\lim_{\mathcal{U}_s}) \) is soft \( \tilde{\mathcal{L}} \)-fuzzy \( C \)-lim-open set and also, \( f^{-1}(L'_t) = \bigcap_{s \prec t} \lim_{\mathcal{U}_s} = \bigcap_{s \prec t} \tilde{\mathcal{LFC-cl}}(\lim_{\mathcal{U}_s}) \) is soft \( \tilde{\mathcal{L}} \)-fuzzy \( C \)-lim-closed set. Therefore, \( f \) is a soft \( \tilde{\mathcal{L}} \)-fuzzy \( C \)-lim-convergence continuous function. To conclude the proof, it remains to show that \( g \subseteq f \subseteq h \). It is enough to show that,

\[
g^{-1}(\lim(\overline{1_X}, \overline{X}) - L_t) \subseteq f^{-1}(L'_t) \subseteq h^{-1}(L'_t)
\]

and

\[
g^{-1}(\lim(\overline{1_X}, \overline{X}) - L_t) \subseteq f^{-1}(L'_t) \subseteq h^{-1}(L'_t)
\]

Now,

\[
f^{-1}(L'_t) = \bigcap_{s \prec t} \lim_{\mathcal{U}_s}
\]

\[
= \bigcap_{s \prec t} \bigcap_{r \prec s} (\lim(\overline{1_X}, \overline{X}) - \lim \overline{\mathcal{S}_r})
\]

\[
\subseteq \bigcap_{s \prec t} \bigcap_{r \prec s} (\lim(\overline{1_X}, \overline{X}) - \lim \overline{\mathcal{S}_r})
\]

\[
= \bigcap_{s \prec t} \bigcap_{r \prec s} (\lim(\overline{1_X}, \overline{X}) - \lim \overline{\mathcal{S}_r})
\]

\[
= \bigcap_{s \prec t} h^{-1}(L'_s)
\]

\[
= h^{-1}(L'_t)
\]

Similarly, we obtain,

\[
g^{-1} R_t = \bigcup_{s \succ t} g^{-1} R_s
\]

\[
= \bigcup_{s \succ t} \bigcup_{r \prec s} g^{-1} R_r
\]

\[
= \bigcup_{s \succ t} \bigcup_{r \prec s} (\lim(\overline{1_X}, \overline{X}) - \lim \overline{\mathcal{S}_r})
\]

\[
\subseteq \bigcup_{s \succ t} \bigcup_{r \prec s} (\lim(\overline{1_X}, \overline{X}) - \lim \overline{\mathcal{S}_r})
\]

\[
= \bigcup_{s \succ t} \lim_{\mathcal{U}_s}
\]

\[
= f^{-1} R_t
\]

739
Now,
\[
\begin{align*}
f^{-1}R_t & = \bigcup_{s \geq t} \lim \Omega_s \\
& = \bigcup_{s > t} \bigcap_{r < s} (\lim (1_X, X) - \lim \delta_r) \\
& \subseteq \bigcup_{s > t} \bigcup_{r > s} (\lim (1_X, X) - \lim \delta_r) \\
& = \bigcup_{s > t} \bigcup_{r > s} h^{-1}(L_r) \\
& = \bigcup_{s > t} h^{-1}R_s \\
& = h^{-1}R_t
\end{align*}
\]

Thus, (b) is proved.

(b) \(\Rightarrow\) (c): Suppose that \(\lim \delta\) is soft \(\hat{L}\)-fuzzy \(C\)-lim-closed set and \(\lim \delta\) is soft \(\hat{L}\)-fuzzy \(C\)-lim-open set such that \(\lim \delta \subseteq \lim \delta\). Then, by the Proposition: 5.7, \(\chi_{\text{lim soft}} \subseteq \chi_{\text{lim soft}}\), where \(\chi_{\text{lim soft}}\) and \(\chi_{\text{lim soft}}\) are the lower and upper soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence continuous functions respectively. Hence, by (b), there exists a soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence continuous function, \(f : X \to \mathbb{R}(L \times L)\) such that \(\chi_{\text{lim soft}} \subseteq f \subseteq \chi_{\text{lim soft}}\). Clearly, \(f(x) \in \hat{L}, \) for all \(x \in \mathbb{R}\) and
\[
\lim \delta = L'_1 \chi_{\text{lim soft}} \subseteq L'_1 f \subseteq R_0 f \subseteq R_0 \chi_{\text{lim soft}} = \lim \delta.
\]

Therefore, \(\lim \delta \subseteq L'_1 f \subseteq R_0 f \subseteq \lim \delta\).

(c) \(\Rightarrow\) (a): Let \(\lim \delta\) be a soft \(\hat{L}\)-fuzzy \(C\)-lim-closed set and \(\lim \delta\) be a soft \(\hat{L}\)-fuzzy \(C\)-lim-open set such that \(\lim \delta \subseteq \lim \delta\). Then, by the hypothesis there exists a soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence continuous function, \(f : X \to [0, 1](L \times L)\) such that \(L'_1 f \subseteq R_0 f\). In fact that, \(L'_1\) is a soft \(\hat{L}\)-fuzzy \(\lim\)-closed set and \(R_0\) is a soft \(\hat{L}\)-fuzzy \(\lim\)-open set. Since \(\lim \delta \subseteq L'_1 f \subseteq R_0 f \subseteq \lim \delta\), it follows that, \(\text{SLFC-cl}(\lim \delta) \subseteq \text{SLFC-cl}((L'_1)f) = (L'_1)f\). Similarly, \(R_0 f = \text{SLFC-int}(R_0 f) \subseteq \text{SLFC-int}(\lim \delta)\). This implies that, \(\text{SLFC-cl}(\lim \delta) \subseteq \text{SLFC-int}(\lim \delta)\). By the Proposition: 4.3, \((X, \tau_{\text{lim}})\) is a soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence extremely disconnected space. \(\square\)

6. Tietze extension theorem on soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence extremely disconnected space

**Definition 6.1.** Let \(X\) be any non-empty crisp set. Let \(A\) be any subset of \(X\) and \(\chi_A^*: X \to \{(1_X, 1_X), (0_X, 0_X)\}\). Then, the characteristic* function of \(A\), \(\chi_A^*\) is defined as
\[
\chi_A^*(x) = \begin{cases} 
(1_X, 1_X), & \text{if } x \in A \\
(0_X, 0_X), & \text{if } x \notin A
\end{cases}
\]
for all \(x \in X\).

**Proposition 6.2.** Let \((X, \tau_{\text{lim}})\) be a soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence extremely disconnected space. Let \((X, \tau_{\text{lim}})\) possess the property \(\nabla\). Let \(A \subseteq X\) such that \(\chi_A^*\) is a soft \(\hat{L}\)-fuzzy \(C\)-lim-open set in \((X, \tau_{\text{lim}})\). Let \(f : (A, \tau_{\text{lim}}/A) \to [0, 1](L \times L)\) be a soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence continuous function. Then, \(f\) has a soft \(\hat{L}\)-fuzzy \(C\)-lim-convergence continuous extension over \((X, \tau_{\text{lim}})\).
Proof. Let \( g, h : X \to [0, 1](L \times L) \) be such that \( g = f = h \) on \( A \) and \( g(x) = (0_X, 0_X), \) if \( x \notin A \) and \( h(x) = (1_X, 1_X) \), if \( x \notin A \). We now have,

\[
R_t g = \begin{cases} 
\lim G_t \cap \chi_A^*, & \text{if } t \geq 0 \\
(1_X, 1_X), & \text{if } t < 0 
\end{cases}
\]

for all \( t \in \mathbb{R} \), where \( \lim G_t \) is a soft \( \tilde{L} \)-fuzzy \( C \)-lim-open set such that \( \lim G_t / A = R_t g \) and

\[
L_t h = \begin{cases} 
\lim H_t \cap \chi_A^*, & \text{if } t \leq 1 \\
(0_X, 0_X), & \text{if } t > 1 
\end{cases}
\]

for all \( t \in \mathbb{R} \), where \( \lim H_t \) is a soft \( \tilde{L} \)-fuzzy \( C \)-lim-closed set such that \( \lim H_t / A = L_t h \). Thus, \( g \) is a lower soft \( \tilde{L} \)-fuzzy \( C \)-lim-convergence continuous function and \( h \) is an upper soft \( \tilde{L} \)-fuzzy \( C \)-lim-convergence continuous function with \( g \subseteq h \). Now, by the Proposition: 5.8, there exists a soft \( \tilde{L} \)-fuzzy \( C \)-lim-convergence continuous function, \( F : X \to [0, 1](L \times L) \) such that \( g \leq F \leq h \). Hence, \( F \equiv f \) on \( A \). \( \square \)

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