

Fuzzy EOQ model under bi-level trade credit policy

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ABSTRACT. This paper deals with development of an Economic Order Quantity (EOQ) model for an item under a bi-level trade credit policy. According to the policy, supplier offers a delay period (M) on payment to the retailer. To boost the demand, retailer also offers a trade credit period (N) to customers. In the proposed model, we assume that customer's trade credit period has a positive effect on demand and demand decreases inversely on some exponent of selling price. Estimated different parameters of the demand function are turns out to be fuzzy numbers. Due to inherent impreciseness of demand, average profit function is also imprecise. This imprecise profit function is maximized for optimal decision making. A numerical example is supported to illustrate the proposed model. Sensitivity analysis on the proposed model has also been reported.

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1. INTRODUCTION

Inventory models are usually developed under the assumption that the retailer must be paid fully for the items at the time of receiving. However, due to introduction of multinationals there are keen competition among the franchisees of different companies of a type of product. As a result to capture the market, in reality it is observed for many items that supplier offers the retailer a delay period, that is trade credit period, in setting the accounts. Due to this phenomenon, the effect of supplier's credit policies on optimal order quantity has received an extensive attention by many researchers ([1], [15], [6], [19], [12],[7], [8], [13], [14], etc.). Due to same reason, retailer also offers a credit period to his/her customers to stimulate demand which is termed as customer's trade credit period. Though customer's trade credit

period is a realistic phenomenon, not much attention has been paid on it by the researcher. The inventory policy which incorporates both the retailer's trade credit period as well as customer's trade credit period is named as bi-level trade credit policy.

Initial attention in this direction is made recently by Huang [13]. He proposed a single item production inventory model where producer purchases parts of the item from a supplier and after production of the item he/she sells it to the customers. The model was developed under two levels of trade credit policy where the supplier offers the retailer a delay period for payment and the retailer also adopts the trade credit policy to stimulate his/her product's demand. Moreover, he assumed that the credit period offered by supplier M is not shorter than the customer's trade credit period offered by the retailer N ($M \geq N$). A drawback of the model is that here credit-period of customer is not same for all the customers. The persons who purchase the item earlier will get more credit-period than those purchase later. Moreover, in the model, retailer's credit period has no effect on the demand of the item. Incorporating these shortcomings, Jaggi *et al.* [14] developed an inventory model under two-level trade credit policy where demand of the item depends on retailer's credit-period. Besides, this, this paper also extended the Huang's [13] model by relaxing the assumption $M \geq N$. In these papers, a retailer allows each customer with the same trade period N . Recently, Annadurai [2] extended the Jaggi's [14] model for deteriorating items including shortages under trade credit.

It is a formidable task for the retailers to estimate different inventory parameters as crisp or stochastic as due to rapid changes of product specification and the introduction of new products; in the market sufficient past data is not available for such an estimation. In the above inventory models, it was assumed that the demand rate and the inventory costs are constant in nature. Due to various uncertainties, the annual demand rate may have a little fluctuation, especially in a perfect competitive market. In the present day scenario, it is tough to decide the exact annual demand rate, namely, how many items customers will purchase during the whole year. As a result it is very difficult to estimate the parameters of an item precisely. On this view, several researchers have developed inventory models in imprecise environment ([20], [16], [11], [9], [3], [4],[17],[18] etc.). But in the above models ([14], [2], etc.) it is assumed that demand is crisp in nature.

Here an inventory model of an item is developed where the item is supplied in a lot to the retailer by the supplier. The supplier offers the retailer a delay period (M) for payment and the retailer also offers his/her customers a delay period (N) for payment to stimulate his/her customer's demand. i.e., item purchased by the customers during this period has to pay final payment after a time N from purchasing the item. Here the demand not only depends on customer's credit period offered by retailer, but also depends on selling price of the items to be sold, to reflect the real life situation. Different parameters of the demand function is estimated as fuzzy number to get more realistic inventory model. During credit period, retailer earns interest (at a rate of I_e) from the selling price. After credit period, retailer has to pay some interest (at a rate of $I_k(> I_e)$) to the supplier on stocked item. Model is formulated as profit maximization principle. Here the average profit is fuzzy in nature. The possibilistic mean value of the fuzzy profit [5] is found and

then optimized. Numerical examples are given to solve the model. Some sensitivity analysis on some inventory parameters are presented.

The outline of this paper is as follows. Section 2 contains relevant notations and assumptions connected to the model. Section 3 presents the mathematical formulation of the proposed fuzzy EOQ model under bi-level trade credit policy. Section 4 contains the algorithm of the proposed model. Section 5 and Section 6 are the illustration with numerical example and some sensitivity analysis respectively. Final Section 7 contains the concluding remarks.

2. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used in developing the model.

2.1. Notations.

- (i) A = ordering cost per order.
- (ii) c_p = unit purchasing price.
- (iii) s_p = unit selling price, ($s > c$).
- (iv) h = stock holding cost per item per year excluding interest charges.
- (v) $q(t)$ = inventory level at time t .
- (vi) Q = order quantity where $q(0) = Q$.
- (vii) I_e = interest earned per dollar per year.
- (viii) I_k = interest charged per dollar in stocks per year by the supplier.
- (ix) M = Retailer's trade credit period offered by supplier in years where $M \leq T$.
- (x) N = Customers trade credit period offered by retailer in years where $N \leq T$.
- (xi) m is the mark-up of selling price, s , i.e., $s = mc$, $m > 1$.
- (xii) $\tilde{D}(N)$ demand for any N per unit time and is of the form:

$$\begin{aligned}\tilde{D} &= \frac{\tilde{S} - (\tilde{S} - \tilde{s})r^N}{(mc)^\nu}, \quad 0 < r < 1 \\ &= \frac{(1 - r^N)\tilde{S} + \tilde{s}r^N}{(mc)^\nu}, \quad 0 < r < 1\end{aligned}$$

where \tilde{S} is maximum demand, r is inverse rate of saturation of demand (which can be estimated using the past data) under the assumption that the marginal effect of credit period on sales is proportional to the unrealized potential of the market demand without any delay. \tilde{s} is initial demand, keeping other attributes like price, quantity, etc. at constant level.

- (xiii) \tilde{T} = Cycle length in years which is fuzzy in nature.
- (xiv) \tilde{P}_i ($i = 1, 2$) = annual total average profit, which are functions of Q , N , m , for two scenarios.

2.2. Assumptions.

- (i) Inventory system involves only one item.
- (ii) Demand rate is fuzzy in nature.
- (iii) Shortages are not allowed.
- (iv) Time horizon is infinite.
- (v) Supplier offers a credit period M in each cycle to the retailer and retailer offers a credit period N to its customers to boost the demand.

- (vi) $I_k \geq I_e$.
- (vii) Q, N, m are decision variables.
- (viii) Lead time is negligible.
- (ix) \tilde{S} is assumed as a TFN (S_1, S_2, S_3) and \tilde{s} is assumed as a TFN (s_1, s_2, s_3) .

3. MATHEMATICAL FORMULATION

In the development of the model, it is assumed that at the beginning of each cycle retailer purchases an amount Q of the item which is sold during T according to customer's demand \tilde{D} , i.e., \tilde{T} is cycle length. Here instantaneous state $q(t)$ of stock is given by

$$\frac{dq(t)}{dt} = -\tilde{D}$$

where $q(0) = Q, q(\tilde{T}) = 0$.

Solving [following [10]] we get

$$\begin{aligned} \tilde{q}(t) &= Q - \tilde{D}t \\ \tilde{T} &= \frac{Q}{\tilde{D}} \end{aligned}$$

Total purchase price (PC) = $c_p Q$ and selling price (SP) = $mc_p Q$.

Holding cost (\widetilde{HOC}) is given by

$$\widetilde{HOC} = h \int_0^{\tilde{T}} [Q - \tilde{D}t] dt = \frac{hQ^2}{2\tilde{D}}$$

According to the assumptions two scenarios may arise in calculation of interest earned by the retailer from the customers and interest paid to the supplier by the retailer:

Scenario-3.1: $M > N$

In this case interest to be paid, $\widetilde{IP} = (\widetilde{IP}_1 \oplus \widetilde{IP}_2 \oplus \widetilde{IP}_3)$, where,

$$\begin{aligned} \widetilde{IP}_1 &= \text{Interest to be paid due to item stocked during } [M, \tilde{T}] \\ &= I_k c_p \int_M^{\tilde{T}} q(t) dt \\ &= I_k c_p \int_M^{\tilde{T}} [Q - \tilde{D}t] dt \\ &= I_k c_p \left[\frac{\tilde{D}(\tilde{T} - M)^2}{2} \right] \end{aligned}$$

$$\begin{aligned} \widetilde{IP}_2 &= \text{Interest to be paid due to item sold during } [M, \tilde{T}] \\ &= I_k c_p \int_M^{\tilde{T}} \tilde{D}N dt \\ &= I_k c_p [\tilde{D}N(\tilde{T} - M)] \end{aligned}$$

$$\begin{aligned} \widetilde{IP}_3 &= \text{Interest to be paid due to item sold during } [M - N, M] \\ &= I_k c_p \int_{M-N}^M \tilde{D}(t + N - M) dt \\ &= I_k c_p \left[\frac{\tilde{D}N^2}{2} \right] \end{aligned}$$

So
$$\begin{aligned} \widetilde{IP} &= (\widetilde{IP}_1 \oplus \widetilde{IP}_2 \oplus \widetilde{IP}_3) \\ &= I_k c_p \left[\left[\frac{\tilde{D}(T - M)^2}{2} \right] + [\tilde{D}N(T - M)] + \left[\frac{\tilde{D}N^2}{2} \right] \right] \end{aligned}$$

Total interest to be earned during $[0, \tilde{T}]$ is, \widetilde{IE} , where

$$\begin{aligned} \widetilde{IE} &= I_e c_p \int_0^{M-N} \tilde{D}(M - t - N) dt \\ &= I_e c_p \left[\frac{\tilde{D}(M - N)^2}{2} \right] \end{aligned}$$

So in this scenario total profit, $\tilde{P}_1(Q, N, m)$, is given by

$$\begin{aligned} \tilde{P}_1(Q, N, m) &= \text{Selling price} - \text{Purchase cost} - \text{Ordering cost} - \text{Holding cost} \\ &\quad - \text{Interest paid} + \text{Interest earned} \\ &= mc_p Q - c_p Q - A - \frac{hQ^2}{2\tilde{D}} \\ &\quad - \frac{I_k c_p \tilde{D}}{2} [(T - M)^2 + 2N(T - M) + N^2] + I_e c_p \left[\frac{\tilde{D}(M - N)^2}{2} \right] \\ &= [c_p(m - 1) + I_k c_p(M - N)]\tilde{D} - \frac{A\tilde{D}}{Q} - \frac{c_k(M - N)^2(I_k - I_e)}{2Q} \tilde{D}^2 \\ &\quad - \frac{hQ}{2} - \frac{I_k c_p Q}{2} \end{aligned} \tag{3.1}$$

Taking α -cut on both sides of demand function we have

$$\tilde{D}(\alpha) = \alpha\text{-cut of } \left[\frac{(1-r^N)\tilde{S} + \tilde{s}r^N}{(mc)^\nu} \right] = [D_L, D_R] \text{ (say)}$$

$$\text{So, } D_L = e_1 + e_2\alpha \text{ and } D_R = e_3 - e_4\alpha$$

where $e_1 = d_1S_1 + d_2s_1$, $e_2 = d_1(S_2 - S_1) + d_2(s_2 - s_1)$, $e_3 = d_1S_3 + d_2s_3$, $e_4 = d_1(S_3 - S_2) + d_2(s_3 - s_2)$, $d_1 = \frac{(1-r^N)}{(mc)^\nu}$, $d_2 = \frac{r^N}{(mc)^\nu}$.

Taking α -cut on both sides of (4), we get, $\tilde{P}_1(\alpha) = [P_{1L}(\alpha), P_{1R}(\alpha)]$, where

$$\begin{aligned} P_{1L}(\alpha) &= [c_p(m-1) + I_k c_p(M-N)]D_L - \frac{AD_R}{Q} - \frac{c_k(M-N)^2(I_k - I_e)}{2Q} D_R^2 \\ &\quad - \frac{hQ}{2} - \frac{I_k c_p Q}{2} \\ &= l_1 D_L - l_2 D_R - l_3 D_R^2 - l_4 \end{aligned}$$

$$\begin{aligned} P_{1R}(\alpha) &= [c_p(m-1) + I_k c(M-N)]D_R - \frac{AD_L}{Q} - \frac{c_k(M-N)^2(I_k - I_e)}{2Q} D_L^2 \\ &\quad - \frac{hQ}{2} - \frac{I_k c Q}{2} \\ &= l_1 D_R - l_2 D_L - l_3 D_L^2 - l_4 \end{aligned}$$

where,

$$\begin{aligned} l_1 &= [c_p(m-1) + I_k c_p(M-N)], \\ l_2 &= \frac{A}{Q}, \\ l_3 &= \frac{c_k(M-N)^2(I_k - I_e)}{2Q}, \\ l_4 &= \frac{hQ}{2} + \frac{I_k c_p Q}{2}. \end{aligned}$$

So the possibilistic mean value of the fuzzy profit function \tilde{P}_1 is given by

$$\overline{M}(\tilde{P}_1) = \int_0^1 \alpha [P_{1L}(\alpha) + P_{1R}(\alpha)] d\alpha.$$

Substituting the above values of $P_{1L}(\alpha)$ and $P_{1R}(\alpha)$ and then after simplification, we get,

$$\begin{aligned} \overline{M}(\tilde{P}_1) &= \frac{1}{2} [e_1 l_1 + e_3 l_1 - (e_3 l_2 + e_3^2 l_3 + e_1 l^2 + e_1^2 l_3 + 2l_4)] \\ (3.2) \quad &+ \frac{1}{3} l_1 (e_2 - e_4) + l_2 (e_4 - e_2) + 2l_3 (e_3 e_4 - e_1 e_2) - \frac{1}{4} (e_4^2 + e_2^2) l_3 \end{aligned}$$

Scenario-3.2: $M \leq N$

In this case also interest to be paid, $\widetilde{IP}=(\widetilde{IP}_1\oplus\widetilde{IP}_2\oplus\widetilde{IP}_3)$, where, $\widetilde{IP}_1, \widetilde{IP}_2$ are same as in Scenario 4.1 and

$$\begin{aligned} \widetilde{IP}_3 &= \text{Interest to be paid due to item sold during } [0, M] \\ &= I_k c_p \int_0^M \widetilde{D}(t + N - M) dt \\ &= I_k c_p \left[\frac{\widetilde{D}(2NM - M^2)}{2} \right] \end{aligned}$$

So in this case total interest to be paid, \widetilde{IP} , is given by

$$\begin{aligned} \widetilde{IP} &= (\widetilde{IP}_1 \oplus \widetilde{IP}_2 \oplus \widetilde{IP}_3) \\ &= I_k c_p \left[\left[\frac{\widetilde{D}(T - M)^2}{2} \right] + [\widetilde{D}N(T - M)] + \left[\frac{\widetilde{D}(2NM - M^2)}{2} \right] \right] \end{aligned}$$

In this case no interest to be earned, i.e., $\widetilde{IE} = 0$

So in this scenario total profit, $\widetilde{P}_2(Q, N, m)$, is given by

$$\begin{aligned} \widetilde{P}_2(Q, N, m) &= \text{Selling price} - \text{Purchase cost} - \text{Ordering cost} - \text{Holding cost} \\ &\quad - \text{Interest paid} + \text{Interest earned} \\ &= \left[mc_p Q - c_p Q - A - \frac{hQ^2}{2\widetilde{D}} - \frac{I_k c_p \widetilde{D}}{2} [(T - M)^2 + 2N(T - M) \right. \\ &\quad \left. + 2NM - M_2] \right] \\ &= [c_p(m - 1)]\widetilde{D} - \left[\frac{A}{Q} + I_k c_p(N - M) \right] \widetilde{D} - \frac{hQ}{2} - \frac{I_k c_p Q}{2} \\ (3.3) \quad &= u_1 \widetilde{D} - u_2 \widetilde{D} - u_3 \end{aligned}$$

where

$$\begin{aligned} u_1 &= c_p(m - 1), \\ u_2 &= \frac{A}{Q} + I_k c_p(N - M), \\ u_3 &= \frac{hQ}{2} + \frac{I_k c_p Q}{2}. \end{aligned}$$

Taking α -cut on both sides, we get,

$$\widetilde{P}_2(\alpha) = [P_{2L}(\alpha), P_{2R}(\alpha)],$$

where

$$\begin{aligned} P_{2L}(\alpha) &= u_1 D_L - u_2 D_R - u_3 \\ P_{2R}(\alpha) &= u_1 D_R - u_2 D_L - u_3 \end{aligned}$$

So the possibilistic mean value of the fuzzy profit function \widetilde{P}_2 is given by

$$\overline{M}(\widetilde{P}_2) = \int_0^1 \alpha(P_{2L}(\alpha) + P_{2R}(\alpha))d\alpha$$

Substituting the above values of $P_{2L}(\alpha)$ and $P_{2R}(\alpha)$ and then after simplification, we get,

$$(3.4) \quad \overline{M}(\widetilde{P}_2) = \frac{1}{2}[e_1u_1 - e_3u_2 + e_3u_1 - e_1u_2 - 2u_3] + \frac{1}{3}[e_2u_1 + e_4u_2 - e_4u_1 - e_2u_2].$$

The problems are solved using LINGO software.

4. ALGORITHM

Depending upon the values of M and N two scenarios may occur (i) $M > N$ and (ii) $M \leq N$

$$(4.1) \quad \text{Find optimal } Q, N, m \text{ (say } Q^*, N^* \text{ and } m^* \text{ resp.) to maximize } \overline{M}(\widetilde{P}_i(Q, N, m)), i = 1, 2 \text{ where } Q, N, m > 0$$

If $M > N$

Perform (4.1)(Optimal Profit is given by (3.2))

Else

If $M \leq N$

Perform (4.1)(Optimal Profit is given by (3.4))

5. NUMERICAL ILLUSTRATION

Scenario 5.1. The following numerical example (Example-5.1), is used to illustrate this scenario.

Example 5.1. Here it is assumed that $h = .1$, $c_p = 1.5$, $S_1 = 1550$, $S_2 = 1600$, $S_3 = 1650$, $s_1 = 870$, $s_2 = 900$, $s_3 = 930$, $r = 0.80$, $\gamma = 2.5$, $I_e = .1$, $I_k = .12$, $A = 10$, $M = 0.5$ in respective units.

Using LINGO software for these parametric values, we obtain the optimal solution $Q^* = 77.10$, $N^* = 0.202$, $m^* = 1.76$ and the corresponding maximum profit $(\overline{M}(\widetilde{P}_1))^* = 76.55$.

Scenario 5.2. The following numerical example (Example 5.2), is used to illustrate this scenario.

Example 5.2. Here it is assumed that $h = .1$, $c_p = 1.5$, $S_1 = 1550$, $S_2 = 1600$, $S_3 = 1650$, $s_1 = 870$, $s_2 = 900$, $s_3 = 930$, $r = 0.70$, $\gamma = 2.75$, $I_e = .1$, $I_k = .12$, $A = 10$, $M = 0.5$ in respective units.

Using LINGO software for these parametric values, we obtain the optimal solution $Q^*=72.84$, $N^*=0.6522$, $m^*=1.74$ and the corresponding maximum profit $(\overline{M}(\tilde{P}_2))^*=60.44$.

6. SENSITIVITY ANALYSIS

For the both the examples results are obtained due to different values of ‘inverse rate of saturation of demand’, r , and are presented in Table-6.1 and Table-6.2 respectively. In both the scenarios it is observed that profit decreases with increase of r . Increase of r decreases the demand rate of the item which in turn decreases the profit. For this reason r is called inverse rate of increase of demand. Moreover it is observed that N decreases as r increases. It happens because increase of r decreases the demand, so to keep the profit high N decreases. It should be noted that decrease of N also decreases demand, but here increase of profit due to decrease of N , dominates the loss due to decrease of N .

Table-6.1
Results for Example-5.1 due to different r

r	Q	N	m	$\overline{M}(\tilde{P}_1)$
0.78	76.44	0.3817	1.79	76.98
0.79	76.89	0.2816	1.77	76.73
0.80	77.10	0.2020	1.76	76.54
0.81	77.57	0.0855	1.74	76.44
0.82	78.11	0.0063	1.72	76.43

Table-6.2
Results for Example-5.2 due to different r

r	Q	N	m	$\overline{M}(\tilde{P}_2)$
0.65	72.98	0.8730	1.79	62.39
0.66	72.88	0.8377	1.78	61.97
0.67	73.19	0.7935	1.77	61.57
0.68	72.83	0.7579	1.76	61.18
0.69	72.80	0.7080	1.75	60.80

7. CONCLUSIONS

Here for the first time in an inventory model, customer’s credit period is attached with the imprecise demand of an item in terms of selling price and credit linked. Here the item is supplied to the retailer by the supplier at a lot. The supplier offers the retailer a delay period for payment and the retailer also offers his/her customers a delay period for payment to stimulate his/her customer’s demand. To make more realistic inventory model here customer’s credit period and selling price are taken as decisive factor of the decision maker. Due to highly non linear nature of the objective function optimal decision is made using LINGO software. Some interesting results are also obtained. Two different models and corresponding examples are used to explain the nature of the demand function. This retailer’s model has wide range of

applications in wholesale-retail-customer business where the competition is stiff. The proposed model can be extended with non-neglected lead time, shortages with fully and partially backlogged, mark up, deterioration etc, which is kept in our mind for future research work.

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