

## On algebraic structure of soft sets

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**ABSTRACT.** In this paper, we shall study the algebraic structure of the set of all soft sets defined on a fixed universe. We shall show that the set of all soft sets on a fixed set of parameters is actually a Boolean algebra. Properties of the set of all soft sets on a fixed set of parameters are studied.

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### 1. INTRODUCTION

The concept of soft sets was introduced by Molodtsov [13] in 1999, which is a new mathematical tool for dealing with uncertainties. Since the inception of this concept a large amount of papers devoted to development of this subject ([3, 6, 7, 8, 9, 10, 12, 15]) have appeared. Subsequently, various structures based on soft sets are developed. Some very recent works on soft sets can be found in [5, 18].

Maji *et al.* [12] defined various operations on soft set. Ali *et al.* [3] shows by counterexamples that various concepts defined in [12] are not true and they defined some new operations in soft set theory. In a subsequent paper Qin *et al.* [14] defined soft equality by two ways. In [14], it is proved that set of some soft sets with some suitable operations is a distributive bounded lattice. In this paper, we check which of these structures form a Boolean algebra. Sezgin *et al.* [16] studied several soft set operations. In a very recent paper, Rehman *et al.* [15] also discussed on some operations of soft sets. Also in a recent paper Zhu *et al.* [19] revisited operations on soft sets.

Aktaş and N. Çağman [2] introduced the notion of soft group and discussed various properties. Jun [8] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Jun and Park [9] presented the notion of soft ideals, idealistic soft and idealistic soft BCK/BCI-algebras. Jun *et al.* [10] applied soft set theory to commutative ideals in BCK-algebras. Kazanci *et al.* discussed soft BCH-algebras in [11]. Feng *et al.* [6] worked on soft semirings, soft ideals and idealistic

soft semirings. Ali *et al.* and Shabir and Irfan Ali ([3, 17]) studied soft semigroups and soft ideals over a semi group which characterize generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup. Acar *et al.* [1] worked on soft rings. Recently, Feng *et al.* dealt with soft subsets and soft product operations in [7]. In the present paper we shall discuss the algebraic structure of the set of all soft sets defined on a fixed universe and investigate the properties of it. In a recent paper, Ali *et al.* [4] also discussed soft sets on a fixed set of parameters. Here we obtained some new results.

The organization of the paper is as follows:

Section 2 is the preliminary part where soft set and some operations of soft sets are defined. In section 3, we have studied whether a soft algebraic structure is a Boolean algebra or not. Also in this section we discuss the properties of all soft sets with a fixed set of parameters. In section 4, we define a new equivalence relation on the set of all soft sets on a fixed universe. The quotient algebra formed by this relation will become a Boolean algebra.

## 2. PRELIMINARIES

In this section, we recall some basic notions in soft set theory. Let  $U$  be an initial universe set and  $E$  the set of all possible parameters under consideration with respect to  $U$ . The power set of  $U$  is denoted by  $P(U)$ . Molodtsov [13] defined the notion of a soft set in the following way:

**Definition 2.1** ([13]). A pair  $(F, A)$  is called a soft set over  $U$ , where  $A \subseteq E$  and  $F$  is a mapping given by  $F : A \rightarrow P(U)$ .

If  $A = \emptyset$  then also we consider  $(F, A)$  as a soft set. In fact, in this case, all functions from an empty set to  $P(U)$  are same and taken as empty function, so, all soft sets  $(F, \emptyset)$  are same. If in stead of  $F$ , we write  $G$ , it does not matter. Qin *et al.* already considered such soft sets in [14]. If we do not consider such soft sets, restricted union and intersection cannot be defined for any two soft sets.

For  $A \subseteq E, \uparrow A$  is the set of all *not e*'s, where  $e \in A$ . But in this case, we have to consider '*not e*' also as a member of  $E$ , which may not hold in general. Since in the definition of the complement of a soft set  $(F, A)$  is taken as  $(F^c, \uparrow A)$ , we do not consider this complement. In stead we consider only relative complement.

**Definition 2.2** ([3]). The relative complement of a soft set  $(F, A)$  is denoted by  $(F, A)^c$  and is defined by  $(F, A)^c = (F^c, A)$  where  $F^c : A \rightarrow P(U)$  is defined as  $F^c(e) = U - F(e)$  for all  $e \in A$ .

**Definition 2.3** ([12]). The union of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,  $H(e) = F(e)$ , if  $e \in A - B$ ,  $H(e) = G(e)$ , if  $e \in B - A$  and  $H(e) = F(e) \cup G(e)$ , otherwise.

We shall denote this soft set as  $(F, A) \tilde{\cup} (G, B)$ .

**Definition 2.4** ([14]). The restricted intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cap B$  and for all  $e \in C$ ,  $H(e) = F(e) \cap G(e)$ .

Here  $C$  may be  $\emptyset$ , even when  $A$  and  $B$  are nonempty sets. In this paper, we call this operation as intersection and denote this soft set as  $(F, A)\tilde{\cap}(G, B)$ .

**Definition 2.5** ([14]). The restricted union of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cap B$  and for all  $e \in C$ ,  $H(e) = F(e) \cup G(e)$ .

Here also  $C$  may be  $\emptyset$ , even when  $A$  and  $B$  are nonempty sets. We shall denote it by  $(F, A) \cup_r (G, B)$ .

**Definition 2.6** ([14]). The extended intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,  $H(e) = F(e)$ , if  $e \in A - B$ ,  $H(e) = G(e)$ , if  $e \in B - A$  and  $H(e) = F(e) \cap G(e)$ , otherwise.

We shall denote it by  $(F, A) \cap_e (G, B)$ .

**Definition 2.7** ([14]).  $(F, A)$  is called a relative null soft set (with respect to the parameter set  $A$ ), denoted by  $\emptyset_A$ , if  $F(e) = \emptyset$  for all  $e \in A$ .

$(F, A)$  is called a relative whole soft set (with respect to the parameter set  $A$ ), denoted by  $U_A$ , if  $F(e) = U$  for all  $e \in A$ .

If  $A = \emptyset$  then any soft set  $(F, A)$  is a relative null soft set and this set will be denoted by  $\emptyset_\emptyset$ . Here it may be noted that the relative whole soft set with respect to the parameter set  $\emptyset$  is undefined.

**Definition 2.8.** The relative null soft set  $(F, E)$  is called null soft set and the relative whole soft set  $(F, E)$  is called whole soft set.

### 3. SOFT SETS ON FIXED PARAMETERS

In [14] the lattice structures of soft sets are discussed. It is proved soft sets form bounded distributive lattice under suitable operations. Here we shall further investigate on these structures.

**Theorem 3.1** ([14]).  $(S(U, E), \tilde{\cup}, \tilde{\cap})$  is a bounded distributive lattice, where  $S(U, E) = \{(F, A) : A \subseteq E, F : A \rightarrow P(U)\}$  and  $U_E, \emptyset_\emptyset$  are the greatest and least elements of the lattice respectively.

**Theorem 3.2** ([14]).  $S_A$  is a sublattice of  $(S(U, E), \tilde{\cup}, \tilde{\cap})$ , where  $S_A = \{(F, A); F : A \rightarrow P(U)\}$ , i.e.,  $S_A$  is the set of all soft sets where the parameter set  $A$  is fixed. In this lattice,  $U_A, \emptyset_A$  are the greatest and least elements of the lattice respectively.

**Theorem 3.3.**  $(S_A, \tilde{\cup}, \tilde{\cap}, {}^c)$  is a complemented distributive lattice. In other words,  $(S_A, \tilde{\cup}, \tilde{\cap}, {}^c)$  is a Boolean algebra.

*Proof.* As  $S(U, E)$  is a distributive lattice, by hereditary property, the lattice of  $S_A$  is also distributive.

Now from the definition 2.2 of  $(F, A)^c$ , it follows that  $(F, A)^c \in S_A$ , as  $(F, A) \in S_A$ . On the set  $S_A$ ,  $(F, A)\tilde{\cup}(F, A)^c = (F, A)\tilde{\cup}(F^c, A) = (G, B)$ , say. Then  $B = A \cup A = A$ , and for all  $e \in B$ ,  $G(e) = F(e) \cup F^c(e) = F(e) \cup (U - F(e)) = U$ . Again,  $(F, A)\tilde{\cap}(F, A)^c = (F, A)\tilde{\cap}(F^c, A) = (H, C)$ , say. Then  $C = A \cap A = A$  and for all  $e \in C$ ,  $H(e) = F(e) \cap F^c(e) = F(e) \cap (U - F(e)) = \emptyset$ . Hence  $S_A$  is a complemented lattice. So,  $(S_A, \tilde{\cup}, \tilde{\cap}, {}^c)$  is a Boolean algebra.  $\square$

With respect to other union and intersection,  $S(U, E)$  is also a distributive lattice. In fact,

**Theorem 3.4** ([14]).  $(S(U, E), \cup_r, \cap_e)$  is a distributive lattice.

**Theorem 3.5.** The lattice  $(S(U, E), \cup_r, \cap_e)$  has a least element  $\emptyset_E$  but does not have any greatest element.

*Proof.* Let us consider the soft set  $(G, E)$ , where  $G(e) = \emptyset$  for all  $e \in E$ . So,  $(G, E) = \emptyset_E$ . Now, for any soft set  $(F, A) \in S(U, E)$ ,  $(F, A) \cup_r (G, E) = (H, C)$ , say. Then  $C = A \cap E = A$  and for all  $e \in A$ ,  $H(e) = F(e) \cup G(e) = F(e) \cup \emptyset = F(e)$ . So, in this lattice,  $\emptyset_E$  is the least element.

Let  $(G, B)$  be the greatest element of this lattice. Let  $(F, A) \in S(U, E)$ . Then  $(F, A) \cup_r (G, B) = (G, B)$ . So,  $A \cap B = B$ , i.e.,  $B \subseteq A$ . This has to be true for any  $A$ , consequently  $B = \emptyset$ . Also, for all  $e \in B$ ,  $G(e) = F(e) \cup G(e)$ . So,  $F(e) \subseteq G(e)$ . But  $G(e) = \emptyset$  as  $B = \emptyset$ . So,  $F(e) = \emptyset$ , for all  $e \in A$  and for all  $F$  — which is absurd for nonempty  $E$ .  $\square$

**Theorem 3.6** ([14]).  $S_A$  is a sublattice of  $(S(U, E), \cup_r, \cap_e)$ .

Although the lattice  $(S(U, E), \cup_r, \cap_e)$  is not bounded, the sublattice  $S_A$  is bounded. In fact,  $U_A, \emptyset_A$  are the greatest and least elements of this sublattice respectively.

**Theorem 3.7.**  $(S_A, \cup_r, \cap_e, {}^c)$  is a Boolean algebra.

*Proof.* On the set  $S_A$ ,  $(F, A) \cup_r (F, A)^c = (F, A) \cup_r (F^c, A) = (G, B)$ , say. Then  $B = A \cap A = A$ , and for all  $e \in B$ ,  $G(e) = F(e) \cup F^c(e) = F(e) \cup (U - F(e)) = U$ . Again,  $(F, A) \cap_e (F, A)^c = (F, A) \cap_e (F^c, A) = (H, C)$ , say. Then  $C = A \cup A = A$  and for all  $e \in C$ ,  $H(e) = F(e) \cap F^c(e) = F(e) \cap (U - F(e)) = \emptyset$ .  $\square$

This specialty of  $S_A$  motivates us to study such collection.

Henceforth, in this section, we consider all soft sets taken from  $S_A$ , unless otherwise stated.

**Theorem 3.8.** In  $S_A$ ,  $(F, A) \tilde{\cup} (G, A) = (F, A) \cup_r (G, A)$  and  $(F, A) \tilde{\cap} (G, A) = (F, A) \cap_e (G, A)$ .

*Proof.* We already know that,  $S_A$  is closed under both unions and both intersections. Now, let  $(F, A) \tilde{\cup} (G, A) = (H, A)$  and  $(F, A) \cup_r (G, A) = (I, A)$ . Then for all  $e \in A$ ,  $H(e) = F(e) \cup G(e) = I(e)$ .

Also if  $(F, A) \tilde{\cap} (G, A) = (H, A)$  and  $(F, A) \cap_e (G, A) = (I, A)$ . Then for all  $e \in A$ ,  $H(e) = F(e) \cap G(e) = I(e)$ .  $\square$

We already have the following De Morgan's laws [3, 14].

**Theorem 3.9.** [3] Let  $(F, A)$  and  $(G, B)$  be two soft sets over the same universe  $U$ . Then

- $((F, A) \cup_r (G, B))^c = (F, A)^c \tilde{\cap} (G, B)^c$ .
- $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \cup_r (G, B)^c$ .

**Theorem 3.10.** [14] Let  $(F, A)$  and  $(G, B)$  be two soft sets over the same universe  $U$ . Then

- $((F, A) \tilde{\cup} (G, B))^c = (F, A)^c \cap_e (G, B)^c$ .

- $((F, A) \cap_e (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$ .

**Theorem 3.11.** *Let  $(F, A)$  and  $(G, A)$  be two soft sets over the same universe  $U$ . Then*

- $((F, A) \cup_r (G, A))^c = ((F, A) \tilde{\cup} (G, A))^c = (F, A)^c \tilde{\cap} (G, A)^c = (F, A)^c \cap_e (G, A)^c$ .
- $((F, A) \tilde{\cap} (G, A))^c = ((F, A) \cap_e (G, A))^c = (F, A)^c \cup_r (G, A)^c = (F, A)^c \tilde{\cup} (G, A)^c$ .

*Proof.* It follows from Theorem 3.8. □

In literature, we get three types of soft subsets.

Let  $(F, A)$  and  $(G, B)$  be two soft sets over the same universe  $U$ . Here  $A$  and  $B$  are not necessarily same. Then

- $(F, A)$  is called a soft  $M$ -subset of  $(G, B)$  if and only if  $A \subseteq B$  and for all  $e \in A$ ,  $F(e) = G(e)$ . This is denoted by  $(F, A) \tilde{\subseteq}_M (G, B)$  [12].
- $(F, A)$  is called a soft  $F$ -subset of  $(G, B)$  if and only if  $A \subseteq B$  and for all  $e \in A$ ,  $F(e) \subseteq G(e)$ . This is denoted by  $(F, A) \tilde{\subseteq}_F (G, B)$  [7]. In fact, these relation is same with the lattice order of  $(S(U, E), \cup, \tilde{\cap})$ .

Another type of soft subset relation also present in the lattice  $(S(U, E), \cup_r, \cap_e)$  [14].

- $(F, A)$  is called a soft  $Q$ -subset of  $(G, B)$  if and only if  $B \subseteq A$  and for all  $e \in B$ ,  $F(e) \subseteq G(e)$ . This is denoted by  $(F, A) \tilde{\subseteq}_Q (G, B)$ . In fact, these relation is same with the lattice order of  $(S(U, E), \cup_r, \cap_e)$ .

It is easy to see that the concepts of soft  $F$ -subset and soft  $Q$ -subset coincide in  $S_A$ . Also in  $S_A$ , if  $(F, A)$  is a soft  $M$ -subset of  $(G, A)$  then  $(F, A)$  is also a soft  $F$ -subset as well as soft  $Q$ -subset of  $(G, A)$ . But the converse may not be true as illustrated by the following example.

**Example 3.12.** Suppose there are five houses under consideration, which constitutes the universe  $U = \{h_1, h_2, h_3, h_4, h_5\}$ . Also we have a universal parameter set  $E = \{e_1, e_2, e_3, e_4\}$ , where  $e_i (i = 1, 2, 3, 4)$  stand for “beautiful”, “attractive”, “expensive” and “in good repair” respectively. For subset  $A = \{e_1, e_3\}$  of  $E$ , let  $(F, A)$  and  $(G, A)$  be two soft sets over  $U$ , where  $F(e_1) = G(e_1) = \{h_1, h_3, h_4\}$ ,  $F(e_3) = \{h_3, h_4, h_5\}$  and  $G(e_3) = \{h_1, h_3, h_4, h_5\}$ . Then  $(F, A)$  is a soft  $F$ -subset as well as soft  $Q$ -subset of  $(G, A)$  but  $(F, A)$  is not a soft  $M$ -subset of  $(G, A)$ .

Also we get three types of soft equality relation in literature.

Let  $(F, A)$  and  $(G, B)$  be two soft sets over the same universe  $U$ . Here  $A$  and  $B$  are not necessarily same. Then

- $(F, A)$  and  $(G, B)$  are soft  $M$ -equal if and only if  $(F, A) \tilde{\subseteq}_M (G, B)$  and  $(G, B) \tilde{\subseteq}_M (F, A)$  [12].
- $(F, A)$  and  $(G, B)$  are soft  $F$ -equal if and only if  $(F, A) \tilde{\subseteq}_F (G, B)$  and  $(G, B) \tilde{\subseteq}_F (F, A)$  [7].
- $(F, A)$  and  $(G, B)$  are soft  $Q$ -equal if and only if  $(F, A) \tilde{\subseteq}_Q (G, B)$  and  $(G, B) \tilde{\subseteq}_Q (F, A)$  [14].

**Theorem 3.13.** *Let  $(F, A)$  and  $(G, B)$  be two soft sets over  $U$ . Then the following conditions are equivalent.*

- $(F, A)$  and  $(G, B)$  are soft  $Q$ -equal.
- $(F, A)$  and  $(G, B)$  are soft  $M$ -equal.
- $(F, A)$  and  $(G, B)$  are soft  $F$ -equal.
- $A = B$  and  $F = G$ .

*Proof.* It is easy to observe that all three concepts of soft equality relations coincide not only in  $S_A$  but also in  $S(U, E)$ . In fact, the equivalence of last three are already proved in [7].  $\square$

Another two types of soft equality relations are present in [14].

Let  $(F, A)$  and  $(G, B)$  be two soft sets over the same universe  $U$ . Here  $A$  and  $B$  are not necessarily same. Then

- $(F, A) \approx_S (G, B)$  if and only if for all  $e \in A \cap B$ ,  $F(e) = G(e)$ , for  $e \in A - B$ ,  $F(e) = \emptyset$  and for  $e \in B - A$ ,  $G(e) = \emptyset$ .
- $(F, A) \approx^S (G, B)$  if and only if for all  $e \in A \cap B$ ,  $F(e) = G(e)$ , for  $e \in A - B$ ,  $F(e) = U$  and for  $e \in B - A$ ,  $G(e) = U$ .

**Theorem 3.14.** In  $S_A$ ,  $(F, A) \approx_S (G, A)$  if and only if  $(F, A) \approx^S (G, A)$ .

*Proof.* It follows from the definitions of these soft equality relations.  $\square$

In fact in  $S_A$ , all five types of soft equality coincide. So we simply call it soft equality and denote it by  $=$ , i.e., if  $(F, A)$  is soft equal to  $(G, A)$ , then we write  $(F, A) = (G, A)$ .

In [4] soft sets on a fixed set of parameters are also discussed. Here we compare various types of soft subsets and soft equalities which are not covered in [4].

It is trivial that ‘ $=$ ’ is an equivalence relation on  $S_A$ . In fact, all the equivalent class of the quotient set  $S_A/=$  is singleton.

#### 4. AN EQUIVALENCE RELATION ON $S(U, E)$

In this section, we shall discuss another equivalence relation on  $S(U, E)$  and study the properties of the corresponding quotient algebra. Here we can observe the structure  $S(U, E)$  very rigorously. In fact, the equivalence class we shall discuss in this section are precisely the  $S_A$  corresponding to every  $A \subseteq E$ . This type of study is also not under consideration in [4].

**Definition 4.1.** A relation  $\rho$  on  $S(U, E)$  is a subset of  $S(U, E) \times S(U, E)$ .  $\rho$  is said to be an equivalence relation on  $S(U, E)$  if it is reflexive, symmetric and transitive, in other words,

- $(F, A)\rho(F, A)$  for all  $(F, A) \in S(U, E)$
- If  $(F, A)\rho(G, B)$  then  $(G, B)\rho(F, A)$  for all  $(F, A), (G, B) \in S(U, E)$  and
- If  $(F, A)\rho(G, B)$  and  $(G, B)\rho(H, C)$  then  $(F, A)\rho(H, C)$  for all  $(F, A), (G, B), (H, C) \in S(U, E)$ .

Let  $(F, A)$  and  $(G, B)$  be two soft sets over the same universe  $U$  and let the parameter set is  $E$ . We define  $(F, A)$  and  $(G, B)$  are  $\rho$ -related and write as  $(F, A)\rho(G, B)$  if and only if  $A = B$ . So the relation  $\rho$  depends on the parameter set. It is easy to verify that  $\rho$  is an equivalence relation. The quotient set is denoted by

$S(U, E)/\rho$ . We shall denote the equivalence class corresponding to  $(F, A)$  by  $[(F, A)]$ . So  $[(F, A)] = \{(G, B) : A = B\} = S_A$ . It may be noted that  $S_\emptyset$  is singleton.

We define  $[(F, A)] \cup [(G, B)] := [(F, A) \tilde{\cup} (G, B)]$ . It is routine and also easy to check that  $\cup$  is well-defined. Actually, the operation depends on the parameter set of the class. So,  $[(F, A)] \cup [(G, B)] = [(H, A \cup B)]$ . If one define  $[(F, A)] \cup [(G, B)]$  as  $[(F, A) \cap_e (G, B)]$  then also we get the same  $\cup$ . Hence, here  $S_A \cup S_B = S_{A \cup B}$ .

In a similar way, we define  $[(F, A)] \cap [(G, B)] := [(F, A) \tilde{\cap} (G, B)]$ . It is well-defined too. For a similar reason,  $[(F, A)] \cap [(G, B)] = [(F, A) \cup_r (G, B)]$ . Hence, here  $S_A \cap S_B = S_{A \cap B}$ .

**Theorem 4.2.**  $(S(U, E)/\rho, \cup, \cap, S_\emptyset, S_E)$  is a bounded distributive lattice.

*Proof.* Here we shall sketch a proof. The operations  $\cup$  and  $\cap$  defined on  $S(U, E)$  are actually depend on the ordinary set theoretic union and intersection of the corresponding parameter sets respectively. Since the power set  $P(E)$  of the mother parameter set  $E$  is a distributive lattice with respect to set theoretic union and intersection,  $(S(U, E)/\rho, \cup, \cap)$  is a distributive lattice. As  $\emptyset \subseteq A \subseteq E$ ,  $S_\emptyset$  and  $S_E$  are the least element and greatest elements of this lattice respectively.  $\square$

Now we shall define the complement  $^c$  on  $S(U, E)/\rho$  as follows.  $[(F, A)]^c := [(G, E - A)]$ . It is also easy to check that the unary operation is well-defined.

**Theorem 4.3.**  $(S(U, E)/\rho, \cup, \cap, ^c, S_\emptyset, S_E)$  is a Boolean algebra.

*Proof.* It is clear that  $(S_A)^c = S_{A^c}$ . From this definition it follows that  $(S_\emptyset)^c = S_E$ . Also,  $((S_A)^c)^c = S_A$ ,  $S_A \cup (S_A)^c = S_E$  and  $S_A \cap (S_A)^c = S_\emptyset$ . Thus  $(S(U, E)/\rho, \cup, \cap, ^c, S_\emptyset, S_E)$  is a Boolean algebra.  $\square$

In this section we have studied  $S(U, E)$  in a different way.  $S(U, E)$  is partitioned into  $S_A$ 's for all different  $A \subseteq E$ . After that it is observed that  $S(U, E)$  with respect to this partition form a Boolean algebra.  $S_\emptyset$  is the least element and  $S_E$  is the greatest. So here the importance of studying soft sets relative to a fixed parameter set gets a new height.

## 5. CONCLUSIONS

Algebraic structures of soft sets are investigated thoroughly in this paper. It is shown that soft sets on a fixed parameter set is a Boolean algebra. Here many other interesting properties of these types of sets are discussed. Soft sets on a fixed universe are partitioned in such a way that each partition is a set of soft sets on fixed parameter. Also under this partition a Boolean algebra is formed. Investigation in these directions may be a good area of research. We shall study in future the connection between soft sets and Boolean algebra that revealed in this paper.

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