Some properties of soft topologies and group soft topologies

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Abstract. This paper aims to study some properties of soft neighbourhood system of identity soft element of group soft topology. Also some results on soft topologies and enriched soft topologies are established using soft elements. For this we define finite soft set, pseudo constant soft set, cofinite soft topology etc. and study their properties.

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1. Introduction

After the introduction of soft set theory in 1999 by Molodtsov [17] for dealing with uncertainties inherent in the problems of physical science, biological science, engineering, economics, social science, medical science etc., many researchers have worked in the different fields of mathematics such as algebra, analysis, topology etc. in this setting. In 2003, Maji et al. [13] worked on some mathematical aspects of soft sets, and, in 2007, Aktas et al. [1] introduced a basic version of soft group theory. Feng [5] in 2008, dealt with the concept of soft semirings; Shabir et al. (2009) [22] studied soft semigroups and soft ideals; Kharal et al. [12] as well as Majumdar et al. [15] defined soft mappings, Babitha et al. [3] worked on soft relations and functions. Recently Feng et al. [8] ascertained the relationships among five different types of soft subsets and considered the free soft algebras associated with soft product operations. In [8] it has been shown that soft sets have some non-classical algebraic properties which are distinct from those of crisp sets or fuzzy sets. It is to be noted that soft set theory has applications in different real life problems including decision making problems [14, 24, 6, 7, 16].
In 2011, Shabir et al. [23] came up with an idea of soft topological spaces. Later Aygun et al. [2], Zorlutuna et al. [25], Cagman et al. [4], Hussain et al. [11] and Hazra et al. [9] studied on soft topological spaces. As a continuation of this, it is natural to investigate the behaviour of a combination of algebraic and topological structures in soft-set theoretic form. In view of this, and also considering the importance of topological groups in developing Haar measure and Haar integral [10], our aim is to extend the theory of topological groups to soft setting. In this direction we have studied some continuity and separation properties of soft topologies, enriched soft topologies and the neighbourhood properties of group soft topologies which was introduced by us in [18] using the concept of soft elements. For this, we define finite soft set, pseudo constant soft set, cofinite soft topology etc. and investigate their properties. In this connection, it is also worth noting that, in soft setting, some significant works have been done on topological groups in [19, 20] approaching from different perspectives than that of the present paper. The organization of the paper is as follows:

Section 2 is the preliminary part where definitions and some properties of soft sets and soft topologies are given and features of soft topologies are studied. In section 3, we study some continuity properties of soft topological spaces and enriched soft topological spaces. In section 4, some basic facets of the system of neighbourhood of the identity soft elements of group soft topology are established. Section 5, concludes the paper. For the sake of the economy of space, few straightforward proofs are omitted throughout this paper.

2. Preliminaries

This section consists of some definitions and results which will be used in the rest of the paper.

Definition 2.1 ([13]). Let $U$ be the initial universal set, $P(U)$ be the power set of $U$ and $E$ be the set of parameters. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$ and $A \subseteq E$.

Let $(F, A)$ and $(G, B)$ be two soft sets over $U$. Then their union is a soft set $(H, C)$ over $U$ where $C = A \cup B$ and $\forall \alpha \in C$,

$$H(\alpha) = \begin{cases} F(\alpha) & \text{if } \alpha \in A/B \\ G(\alpha) & \text{if } \alpha \in B/A \\ F(\alpha) \cup G(\alpha) & \text{if } \alpha \in A \cap B \end{cases}$$

This relationship is written as $(F, A) \cup (G, B) = (H, C)$.

Let $(F, A)$ and $(G, B)$ be two soft sets over $U$. Then their intersection is a soft set $(H, C)$ over $U$ where $C = A \cap B$ and $\forall \alpha \in C$, $H(\alpha) = F(\alpha) \cap G(\alpha)$.

This relationship is written as $(F, A) \cap (G, B) = (H, C)$.

In order to simplify the fundamental operations on soft sets in [13], redefined the soft sets as follows:

Let $E$ be the set of parameters and $A \subseteq E$. Then for each soft set $(F, A)$ over $U$ a soft set $(H, E)$ is constructed over $U$, where $\forall \alpha \in E$,

$$H(\alpha) = \begin{cases} F(\alpha) & \text{if } \alpha \in A \\ \phi & \text{if } \alpha \in E/A \end{cases}$$
Thus the soft sets \((F, A)\) and \((H, E)\) are equivalent to each other and the usual set operations of the soft sets \((F_i, A_i), \ i \in \Delta\) is the same as those of the soft sets \((H_i, E), \ i \in \Delta\). For this reason, in this paper, we have considered our soft sets over the same parameter set \(A\).

Following Molodtsov [17] and Maji et al. [13] some definitions and preliminary results of soft sets are presented in this section in our form. Unless otherwise stated, \(U\) will be assumed to be an initial universal set and \(A\) will be taken to be a set of parameters. Let \(P(U)\) denote the power set of \(U\) and \(S(U, A)\) denote the set of all soft sets over \(U\). In particular, if \(U\) is a group then \(P(U)\) will denote the set of all subgroups of \(U\).

A pair \((F, A)\), where \(F\) is a mapping from \(A\) to \(P(U)\), is called a soft set over \(U\). Let \((F_1, A)\) and \((F_2, A)\) be two soft sets over a common universe \(U\). Then \((F_1, A)\) is said to be a soft subset of \((F_2, A)\) if \(F_1(\alpha) \subseteq F_2(\alpha), \ \forall \alpha \in A\). This relation is denoted by \((F_1, A) \subseteq (F_2, A)\). \((F_1, A)\) is said to be soft equal to \((F_2, A)\) if \(F_1(\alpha) = F_2(\alpha), \ \forall \alpha \in A\). It is denoted by \((F_1, A) = (F_2, A)\).

The complement of a soft set \((F, A)\) is defined as \((F, A)^c = (F^c, A)\), where \(F^c(\alpha) = (F(\alpha))^c = U - F(\alpha), \ \forall \alpha \in A\).

A soft set \((F, A)\) over \(U\) is said to be a null soft set (an absolute soft set) if \(F(\alpha) = \emptyset (F(\alpha) = U)\), \(\forall \alpha \in A\). This is denoted by \(\exists (\overline{F})\).

Definition 2.2. Let \(\{(F_i, A); i \in \Delta\}\) be a nonempty family of soft sets over a common universe \(U\). Then their

(i) Intersection, denoted by \(\cap_{i \in \Delta} (F_i, A)\), is defined by \(\cap_{i \in \Delta} (F_i, A) = (\cap_{i \in \Delta} F_i, A)\), where \(\cap_{i \in \Delta} F_i(\alpha) = \cap_{i \in \Delta} F_i(\alpha)\), \(\forall \alpha \in A\).

(ii) Union, denoted by \(\cup_{i \in \Delta} (F_i, A)\), is defined by \(\cup_{i \in \Delta} (F_i, A) = (\cup_{i \in \Delta} F_i, A)\), where \(\cup_{i \in \Delta} F_i(\alpha) = \cup_{i \in \Delta} F_i(\alpha)\), \(\forall \alpha \in A\).

Definition 2.3. Let \(X\) and \(Y\) be two nonempty sets and \(f : X \to Y\) be a mapping. Then

(i) the image of a soft set \((F, A) \in S(X, A)\) under the mapping \(f\) is defined by \(f[(F, A)] = (f(F), A)\), where \([f(F)](\alpha) = f[F(\alpha)], \ \forall \alpha \in A\).

(ii) the inverse image of a soft set \((G, A) \in S(Y, A)\) under the mapping \(f\) is defined by \(f^{-1}[(G, A)] = (f^{-1}(G), A)\), where \([f^{-1}(G)](\alpha) = f^{-1}[G(\alpha)], \ \forall \alpha \in A\).

Definition 2.4 ([21]). A soft set \((E, A)\) over \(X\) is said to be a soft element if \(\exists \alpha \in A\) such that \(E(\alpha)\) is a singleton, say, \(\{x\}\) and \(E(\beta) = \emptyset, \ \forall \beta (\neq \alpha) \in A\). Such a soft element is denoted by \(E^x_\alpha\). Let \(E\) be the set of all soft elements of the universal set \(X\).

Also we denote the soft set \((E, A)\) where \(E(\alpha) = \{x\}, \ \forall \alpha \in A\) by \(E_x\).

Definition 2.5 ([21]). The soft element \(E^x_\alpha\) is said to be in the soft set \((G, A)\), denoted by \(E^x_\alpha \in (G, A)\), if \(x \in G(\alpha)\).

Proposition 2.6 ([21]). \(E^x_\alpha \in (F, A)\) iff \(E^x_\alpha \not\subseteq (F^c, A)\).

Definition 2.7 ([18]). Let \((F, A)\) and \((G, A)\) be two soft sets over \(X\). The parallel product of \((F, A)\) and \((G, A)\) is defined as \((F, A) \times (G, A) = (F \times G, A)\) where
$[F \times G](\alpha) = F(\alpha) \times G(\alpha), \ \forall \ \alpha \in A$. It is clear that $(F \times G, A)$ is a soft set over $X \times X$.

**Definition 2.8.** A soft set $(F, A)$ over $X$ is said to be a finite soft set if $F(\alpha)$ is a finite subset of $X$, $\forall \ \alpha \in A$.

**Definition 2.9.** A soft set $(F, A) \in S(X, A)$ is said to be pseudo constant soft set if $F(\alpha) = X$ or $\phi$, $\forall \ \alpha \in A$. Let $CS(X, A)$ denote the set of all pseudo constant soft sets over $X$ under the parameter set $A$.

**Definition 2.10 ([23]).** Let $\tau$ be the collection of soft sets over $X$. Then $\tau$ is said to be a soft topology on $X$ if

(i) $(\bar{\phi}, A), (\bar{X}, A) \in \tau$ where $\bar{\phi}(\alpha) = \phi$ and $\bar{X}(\alpha) = X$, for all $\alpha \in A$.

(ii) the intersection of any two soft sets in $\tau$ belongs to $\tau$.

(iii) the union of any number of soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, A, \tau)$ is called a soft topological space over $X$.

**Definition 2.11 ([2]).** A sub collection $\tau$ of $S(X, A)$ is said to be an enriched soft topology on $X$ if

(i) $(F, A) \in \tau, \ \forall (F, A) \in CS(X, A)$;

(ii) the intersection of any two soft sets in $\tau$ belongs to $\tau$;

(iii) the union of any number of soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, A, \tau)$ is called an enriched soft topological space over $X$.

**Proposition 2.12 ([23]).** Let $(X, A, \tau)$ be a soft topological space (an enriched soft topological space) over $X$. Then the collection $\tau^\alpha = \{F(\alpha) : (F, A) \in \tau\}$ for each $\alpha \in A$, defines a topology on $X$.

**Proposition 2.13 ([21]).** If $(X, A, \tau)$ be a soft topological space (an enriched soft topological space) and if $\tau^* = \{ (G, A) \in S(X) : G(\alpha) \in \tau^\alpha, \ \forall \ \alpha \in A \}$ then $\tau^*$ is an enriched soft topology on $X$ such that $\tau \subseteq \tau^*$ and $[\tau^*]^\alpha = \tau^\alpha, \ \forall \alpha \in A$.

**Definition 2.14.** Let $(X, A, \tau)$ and $(Y, A, \nu)$ be soft topological spaces. The mapping $f : (X, A, \tau) \rightarrow (Y, A, \nu)$ is said to be soft continuous if $f^{-1}([F, A]) \subseteq \tau$, $\forall (F, A) \in \nu$.

**Proposition 2.15 ([18]).** Let $(X, A, \tau), (Y, A, \nu)$ and $(Z, A, \omega)$ be soft topological spaces. If $f : (X, A, \tau) \rightarrow (Y, A, \nu)$ and $g : (Y, A, \nu) \rightarrow (Z, A, \omega)$ are soft continuous and $f(X) \subseteq Y$, then the mapping $gf : (X, A, \tau) \rightarrow (Z, A, \omega)$ is soft continuous.

**Proposition 2.16 ([18]).** Let $(X, A, \tau)$ and $(Y, A, \nu)$ be two soft topological spaces. Then $\mathcal{F} = \{ (F, A) \times (G, A) : (F, A) \in \tau, (G, A) \in \nu \}$ forms an open base of the soft topology on $X \times Y$.

**Definition 2.17 ([18]).** The soft topology in $X \times Y$ induced by the open base $\mathcal{F}$ is said to be the product soft topology of the soft topologies $\tau$ and $\nu$. It is denoted by $\tau \times \nu$. The soft topological space $[X \times Y, A, \tau \times \nu]$ is said to be the soft topological product of the soft topological spaces $(X, A, \tau)$ and $(Y, A, \nu)$.

**Proposition 2.18 ([18]).** Let $(X, A, \tau)$ and $(Y, A, \nu)$ be two soft topological spaces. Then the projection mappings $\pi_X : (X \times Y, A, \tau \times \nu) \rightarrow (X, A, \tau)$ and $\pi_Y : (X \times Y, A, \tau \times \nu) \rightarrow (Y, A, \nu)$. 

Y, A, τXν) → (Y, A, ν) are soft continuous and soft open. Also τXν is the smallest soft topology in X × Y for which the projection mappings are soft continuous.

**Proposition 2.19** ([18]). Let (X, A, τ) be the product space of two soft topological spaces (X1, A, τ1) and (X2, A, τ2) respectively and πi : (X, A, τ) → (Xi, A, τi), i = 1, 2 be the projection mappings. If (Y, A, ν) be any soft topological space then the mapping f : (Y, A, ν) → (X, A, τ) is soft continuous iff the mappings πjf : (Y, A, ν) → (Xi, A, τi), i = 1, 2 are soft continuous.

**Proposition 2.20** ([18]). Let (X, A, τ) be a soft topological space. Then for each α ∈ A, (τXτ)α = τα × τα.

**Proposition 2.21** ([18]). Let (X, A, τ) be a soft topological space and define T* = \{(F, A) ∈ S(X × X) such that F(α) ∈ (τα × τα), ∀ α ∈ A\}. Then T* is a soft topology over X × X and T* = ττ × ττ, where ττ is as in Proposition 2.13.

**Definition 2.22** ([21]). Let τ be a soft topology on X. Then a soft set (F, A) is said to be a τ-soft neighbourhood (shortly soft nbd) of the soft element Eα in (X, A, τ) if there exists a soft set (G, A) ∈ τ such that Eα ∈ (G, A) ⊆ (F, A).

The soft nbd system of a soft element Eα in (X, A, τ), denoted by Nτ(Eα), is the family of all its soft nbds.

**Proposition 2.23** ([21]). If \{Nτ(Eα) : Eα ∈ E\} be the system of soft nbds then

(i) Nτ(Eα) ≠ φ, ∀ Eα ∈ E

(ii) Eα ∈ (F, A), ∀ (F, A) ∈ Nτ(Eα)

(iii) (F, A) ∈ Nτ(Eα) and (F, A) ⊆ (G, A) ⇒ (G, A) ∈ Nτ(Eα)

(iv) (F, A), (G, A) ∈ Nτ(Eα) ⇒ (F, A) ∩ (G, A) ∈ Nτ(Eα)

(v) (F, A) ∈ Nτ(Eα) ⇒ ∃ (G, A) ∈ Nτ(Eα) such that (G, A) ⊆ (F, A) and (G, A) ∈ Nτ(Eα), ∀ Eβ ∈ (G, A).

**Proposition 2.24** ([18]). Let (X, A, τ) be a soft topological space and G ⊆ X. Then τG = \{(F, A) ∩ (G, A) : (F, A) ∈ τ\}, where G(α) = G, ∀ α ∈ A is a soft topology on G.

**Definition 2.25** ([18]). A mapping f : (X, A, τ) → (Y, A, ν) is said to be soft homeomorphism if f is bijective and f, f⁻¹ are soft continuous.

**Definition 2.26** ([21]). Let (X, A, τ) and (Y, A, ν) be two soft topological spaces. A mapping f : (X, A, τ) → (Y, A, ν) is said to be

(i) soft open if (F, A) ∈ τ ⇒ f[(F, A)] ∈ ν;

(ii) soft closed if (F, A) is soft closed in (X, A, τ) ⇒ f[(F, A)] is soft closed in (Y, A, ν).

**Proposition 2.27** ([21]). Let (X, A, τ) and (Y, A, ν) be two soft topological spaces. For a bijective mapping f : (X, A, τ) → (Y, A, ν), the following statements are equivalent.

(i) f : (X, A, τ) → (Y, A, ν) is soft homeomorphism;

(ii) f : (X, A, τ) → (Y, A, ν) and f⁻¹ : (Y, A, ν) → (X, A, τ) are soft continuous;

(iii) f : (X, A, τ) → (Y, A, ν) is both soft continuous and soft open;

(iv) f : (X, A, τ) → (Y, A, ν) is both soft continuous and soft closed;
Let \( (F, A) \) be a soft topology over \( X \) such that \( x \neq y \). If there exist soft open sets \((F, A)\) and \((G, A)\) such that \( E_x \in (F, A) \) and \( E_y \notin (F, A) \) or \( E_y \in (G, A) \) and \( E_x \notin (G, A) \), then \((X, A, \tau)\) is called a soft \( T_0 \)-space.

**Definition 2.29** ([23]). Let \((X, A, \tau)\) be a soft topology over \( X \) such that \( x \neq y \). If there exist soft open sets \((F, A)\) and \((G, A)\) such that \( E_x \in (F, A) \) and \( E_y \notin (F, A) \) or \( E_y \in (G, A) \) and \( E_x \notin (G, A) \), then \((X, A, \tau)\) is called a soft \( T_1 \)-space.

**Definition 2.30** ([23]). Let \((X, A, \tau)\) be a soft topology over \( X \) such that \( x \neq y \). If there exist soft open sets \((F, A)\) and \((G, A)\) such that \( E_x \in (F, A) \) and \( E_y \in (G, A) \) and \((F, A) \cap (G, A) = (\bar{\phi}, A)\), then \((X, A, \tau)\) is called a soft \( T_2 \)-space.

### 3. Some Properties of Soft Topological Spaces

In this section we study some continuity properties of soft topological spaces; the soft topology that we have considered is in the sense of Shabir and Naz [23]. Unless otherwise stated, \( X \) is an initial universal set, \( A \) is the nonempty set of parameters and \( S(X, A) \) denotes the collection of all soft sets over \( X \) under the parameter set \( A \).

**Proposition 3.1.** Let \( X \) be an infinite set and \( \tau \) consist of \((\bar{\phi}, A)\) and all those soft sets \((F, A)\) over \( X \) such that \([(\bar{X}, A) - (F, A)] \) i.e. \((F^c, A)\) are finite soft sets. Then \((X, A, \tau)\) is a soft topological space.

**Proof.** Since \((\bar{X}^c, A) = (\bar{\phi}, A)\) is a finite soft set, it follows that \((\bar{X}, A) \in \tau\). Again let \((F_1, A), (F_2, A) \in \tau\). If any one of \((F_1, A), (F_2, A)\) be \((\bar{\phi}, A)\), then \([(F_1, A) \cap (F_2, A)] \in \tau\). Now let \((F_1, A), (F_2, A) \neq (\bar{\phi}, A)\). So, \((F_1, A), (F_2, A)\) are finite soft sets and hence \([(F_1, A) \cap (F_2, A)]^c = (F_1^c, A) \cup (F_2^c, A)\) is a finite soft set and therefore \((F_1, A) \cap (F_2, A) \in \tau\). Next let \(\{F_i, A\}, i \in \Delta\) be any collection of members of \(\tau\) and without any loss of generality we assume that \((F_i, A) \neq (\bar{\phi}, A), \forall i \in \Delta\). So, \((F_i^c, A)\) is finite soft set, \(\forall i \in \Delta\). Thus \([\bigcup_{i \in \Delta}(F_i, A)]^c = \bigcap_{i \in \Delta}[(F_i^c, A)]\) is finite soft set and hence \(\bigcup_{i \in \Delta}(F_i, A) \in \tau\). Therefore \((X, A, \tau)\) is a soft topological space. \(\square\)

**Definition 3.2.** The soft topology \(\tau\) of Proposition 3.1, is called the cofinite soft topology over \(X\).

**Definition 3.3.** Let \((X, A, \tau)\) be a soft topological space. A soft element \(E^x_\alpha \in \mathcal{E}\) is said to be a limiting soft element of a soft set \((F, A)\) over \(X\) if every open soft set containing \(E^x_\alpha\) contains at least one soft element \(E^x_\beta\) of \((F, A)\) other than \(E^x_\alpha\), i.e. if \(\forall (G, A) \in \tau\) with \(E^x_\alpha \in (G, A)\), \(F(a) \cap [G(a) - \{x\}] \neq \phi\).

The union of all limiting soft elements of \((F, A)\) is a soft set over \(X\), called the derived soft set of \((F, A)\) and is denoted by \((F, A)'\) or \((F', A)\).

**Example 3.4.** Let \(X = \{x, y, z\}\). \(A = \{\alpha, \beta\}\) and \(\tau = \{(\bar{\phi}, A) = \{\phi, \alpha\}\}, (F_1, A) = \{\{x\}, \{y\}\}, (F_2, A) = \{\{y\}, \{z\}\}, (F_3, A) = \{\{x, y\}, \{y, z\}\}\).
Let \( \{x, y, z\}\). Now \( E^x_\alpha \subseteq (X, A) \). Therefore, \( E^x_\alpha \) is not a limiting soft element of \((F, A)\). The open soft sets containing \( E^x_\alpha \) are \((F_2, A), (F_3, A) \) and \( (\bar{X}, A) \). But \( F(\alpha) \cap \{F_2(\alpha) - \{y\}\} = \phi \). Therefore, \( E^y_\alpha \) is not a limiting soft element of \((F, A)\). The only open soft set containing \( E^z_\alpha \) is \((X, A)\) and \( F(\alpha) \cap \{X(\alpha) - \{z\}\} = \{x, y\} \neq \phi \). Therefore, \( E^z_\alpha \) is a limiting soft element of \((F, A)\).

Similarly, we can show that \( E^y_\beta \) is a limiting soft element of \((F, A)\) and \( E^y_\beta \) are not the limiting soft elements of \((F, A)\). Therefore, \( (F', A) = \{\{z\}, \{x\}\} \).

**Proposition 3.5.** Let \( \tau \) be a soft topology over \( X \). A soft set \((F, A)\) over \( X \) is said to be an \( \tau \)-open soft set iff \( \forall E^x_\alpha \subseteq (F, A), \exists (G, A) \in \tau \) such that \( E^x_\alpha \subseteq (G, A) \subseteq (F, A) \).

**Proposition 3.6.** If the soft set \((F, A)\) of a soft topological space \((X, A, \tau)\) is closed soft set then \((F, A)\) contains all its limiting soft elements.

**Proof.** Let \((F, A)\) be a closed soft set and \( E^x_\alpha \notin (F, A) \). Since \((F, A)\) is closed soft set, \((F^c, A)\) is open soft set and \( E^x_\alpha \subseteq (F^c, A) \). Since \( F(\alpha) \cap F^c(\alpha) = \phi \), \( E^x_\alpha \) cannot be a limiting soft element of \((F, A)\). Therefore, \((F, A)\) contains all its limiting soft elements. \(\square\)

**Remark 3.7.** The converse of the above Proposition is not always true which is shown in the following example:

**Example 3.8.** Let \( X, A, \tau \) be the same as in Example 3.4. If \((F, A) = \{\{x, y, z\}, \{x, y\}\}\) then \((F', A) = \{\{z\}, \{x\}\}\). Therefore, \((F, A)\) contains all its limiting soft element. But \((F, A)\) is not closed soft set.

**Proposition 3.9.** If \((X, A, \tau)\) is an enriched soft topological space and \((F, A)\) contains all its limiting soft elements, then \((F, A)\) is a closed soft set.

**Proof.** Let \( E^x_\alpha \subseteq (F^c, A) \). Then \( E^x_\alpha \notin (F, A) \) and hence \( E^x_\alpha \) is not a limiting soft element of \((F, A)\). So \( \exists (G, A) \in \tau \) such that \( E^x_\alpha \subseteq (G, A) \) and \( F(\alpha) \cap G(\alpha) = \{x\} \). Thus \( F(\alpha) \cap G(\alpha) = \phi \), since \( E^x_\alpha \notin (F, A) \). Consider the soft set \((H, A)\) where \( H(\alpha) = X \) and \( H(\alpha) = \phi \). Since \( \tau \) is enriched, \((H, A), A \in \tau \). So, \( E^x_\alpha \subseteq [(G, A) \cap (H, A)] \subseteq (F, A) \). Therefore \((F^c, A)\) is open soft set and hence \((F, A)\) is closed soft set. \(\square\)

**Remark 3.10.** A soft set \((F, A)\) of an enriched soft topological space \((X, A, \tau)\) is closed iff \((F, A)\) contains all of its limiting soft elements.

**Proposition 3.11.** For any two soft sets \((F, A), (G, A)\) of a soft topological space \((X, A, \tau)\), the following conditions hold.

(i) \( (\bar{\sigma}, A) = (\bar{\bar{\sigma}}, A) \);

(ii) \( (F, A) \subseteq (G, A) \Rightarrow (F', A) \subseteq (G', A) \);

(iii) \( ([F \cup G]', A) = ([F' \cup G'], A) \);

(iv) \( ([F \cup F]', A) \subseteq ([F \cup F'], A) \).

**Proof.** (i) and (ii) follow from definition.

(iii) From (ii), we have \((F', A) \subseteq ([F \cup G]', A)\) and hence \( (F' \cup G') A \subseteq ([F \cup G]', A) \).

Next, let \( E^x_\alpha \notin (F', A) \cup (G', A) \). Then \( \exists (H, A), (K, A) \in \tau \) such that...
The closure of a soft set \((F, A)\), \(E_{\alpha}^\ast \subset (K, A)\), \(F(\alpha) \cap [H(\alpha) - \{x\}] = \phi\)

and \(G(\alpha) \cap [K(\alpha) - \{x\}] = \phi\). Let \((V, A) = (H, A) \cap (K, A)\). Then \(E_{\alpha}^\ast \subset (V, A) \in \tau\).

Also \(V(\alpha) \cap [(F(\alpha) \cup G(\alpha) - \{x\}) = \phi \cap [V(\alpha) \cap (G(\alpha) - \{x\})]] \subseteq [H(\alpha) \cap [(F(\alpha) - \{x\})]] \cup [K(\alpha) \cap (G(\alpha) - \{x\})] = \phi\). Therefore, \(E_{\alpha}^\ast\) is not a limiting soft element of \((F, A) \cup (G, A)\). Therefore, \(E_{\alpha}^\ast \not\subset [(F \cup G)', A]\) and hence \([(F \cup G)', A] \subset (F', A) \cup (G', A)\). ……(2).

From (1) and (2) we have \([F \cup G]', A] = (F', A) \cup (G', A)\).

(iv) Let \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]'\), we have to show that \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]'\).

Case - I: If \(E_{\alpha}^\ast \subset (F, A)\) then \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]'\).

Case - II: If \(E_{\alpha}^\ast \not\subset (F, A)\), then \(x \notin F(\alpha)\).

Also since \([F, A) \cup (F, A)]' = (F, A)' \cup [(F, A)'\) and hence \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]'\) and hence \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]'\) or \(E_{\alpha}^\ast \subset [(F, A)'] \).

Subcase II (a): If \(E_{\alpha}^\ast \subset (F, A)')\), then \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)']\).

Subcase II (b): If \(E_{\alpha}^\ast \not\subset (F, A)'\), then \(x \notin F'(\alpha)\). Also, \(E_{\alpha}^\ast \subset [(F, A)']\).

Let \((G, A)\) be any soft open set containing \(E_{\alpha}^\ast\). Since \(E_{\alpha}^\ast\) is a limiting soft element of \((F, A)'\), we have \(G(\alpha) \cap [(F(\alpha) - \{x\}) = \phi\).

Let \(y \in G(\alpha) \cap [F(\alpha) - \{x\}]\). Then \(E_{\alpha}^\ast \subset (G, A) \cap [(F, A)' - E_{\alpha}^\ast]\) and \((G, A)\) is an open soft set containing \(E_{\alpha}^\ast\), which is a limiting soft element of \((F, A)\).

Therefore, \(G(\alpha) \cap [F(\alpha) - \{x\}]\). Let \(z \in G(\alpha) \cap [F(\alpha) - \{x\}]\). Then \(E_{\alpha}^\ast \subset (G, A) \cap [(F, A) - E_{\alpha}]\).

Now \(z \neq x\) as \(E_{\alpha}^\ast \not\subset (F, A)\) from choice of \(E_{\alpha}^\ast\) and \(z \in [G(\alpha) \cap [F(\alpha) - \{x\}]\).

Therefore, \(E_{\alpha}^\ast\) is a limiting soft element of \((F, A)\) i.e. \(E_{\alpha}^\ast \subset (F, A)'\).

Thus in all Cases \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]' \) then \(E_{\alpha}^\ast \subset [(F, A) \cup (F, A)]'\).

Hence, \([F, A) \cup (F, A)]' \subseteq (F, A) \cup (F, A)'\).

Let us now define the closure of a soft set in terms of limiting soft element.

**Definition 3.12.** The closure of a soft set \((F, A)\) over \(X\) denoted by \((\overline{F, A}) = (\overline{F}, A)\)

is defined by \((\overline{F, A}) = (F, A) \cup (F', A)\).

**Example 3.13.** Let \(X, A, \tau\) be the same as in Example 3.4.

If \((F, A) = \{\{x, y\}, \{x, z\}\}\) then \((F', A) = \{\{z\}, \{x\}\}\) and \((\overline{F, A}) = \{\{x, y, z\}, \{x, z\}\}\).

**Proposition 3.14.** Let \((X, A, \tau)\) be a soft topological space. If \((F, A)\) be a closed soft set then \((\overline{F, A}) = (\overline{F}, A)\). But if \((X, A, \tau)\) is an enriched soft topological space and \((F, A) = (\overline{F}, A)\), then \((F, A)\) is a closed soft set.

**Proof.** Proof follows from Proposition 3.6 and Proposition 3.9.

**Proposition 3.15.** For any soft set \((F, A)\), \(\overline{F(\alpha)'} = \overline{F(\alpha)'}\), \(\forall \alpha \in A\), where \(\overline{F(\alpha)'}\)

is the closure of \(F(\alpha)\) with respect to the topology \(\tau\).

**Proof.** Let \(x \in \overline{F(\alpha)}\). Then \(x \in F(\alpha)\) or \(E_{\alpha}^\ast\) is a limiting soft element of \((F, A)\). If \(x \in F(\alpha)\), then \(x \in \overline{F(\alpha)'}\) and hence \(\overline{F(\alpha)} \subseteq \overline{F(\alpha)'}\). Now let \(x \notin F(\alpha)\) and \(E_{\alpha}^\ast\) is a limiting soft element of \((F, A)\). Let \(G \in \tau\) with \(x \in G\). Then \(\exists (H, A) \in \tau\) such that \(H(\alpha) = G\) and, since \(E_{\alpha}^\ast\) is a limiting soft element of \((F, A)\), we have \(F(\alpha) \cap [H(\alpha) - \{x\}] = \phi\). Hence \(x\) is a limit point of \(F(\alpha)\) with respect to \(\tau\). So, \(x \in \overline{F(\alpha)'}\). Therefore \(\overline{F(\alpha)} \subseteq \overline{F(\alpha)'}\).
Conversely let \( x \in \overline{F(\alpha)}^\alpha \). Then \( x \in F(\alpha) \) or \( x \) is a limit point of \( F(\alpha) \) with respect to \( \tau^\alpha \). If \( x \in F(\alpha) \), then \( x \in \overline{F(\alpha)} \). If \( x \notin F(\alpha) \) then \( x \) is a limit point of \( F(\alpha) \) with respect to \( \tau^\alpha \). Let \((K, A) \in \tau\) such that \( E^\alpha_a \in (K, A) \). Then \( F(\alpha) \cap (K(\alpha) - \{x\}) \neq \emptyset \). Thus \( E^\alpha_a \) is a limiting soft element of \((F, A)\) and hence \( x \in \overline{F(\alpha)} \). Therefore \( \overline{F(\alpha)}^\alpha \subseteq \overline{F(\alpha)} \) and hence, \( \overline{F(\alpha)} = \overline{F(\alpha)}^\alpha \), \( \forall \alpha \in A \). □

**Proposition 3.16.** For any two soft sets \((F, A), (G, A)\) of a soft topological space \((X, A, \tau)\), the following conditions hold.

(i) \( (\overline{\phi}, A) = (\overline{\phi}, A) \);

(ii) \( (F, A) \subseteq (\overline{F}, A) \);

(iii) \( [(F, A) \cup (G, A)] = (\overline{F}, A) \cup (G, A) \);

(iv) \( [(F, A)] = (F, A) \).

**Proof.** Proposition follows from Proposition 3.11. □

**Proposition 3.17.** The closure of a soft set \((F, A)\) w.r.t an enriched soft topological space \((X, A, \tau)\) is the smallest closed soft set containing \((F, A)\), i.e. it is the meet of all closed soft sets containing \((F, A)\).

**Proof.** Since \( [\overline{F, A}] = (\overline{F}, A) \) and \( \tau \) is enriched, therefore \((\overline{F}, A)\) is closed. Also \((F, A) \subseteq (\overline{F}, A)\), therefore \((\overline{F}, A)\) is a closed soft set containing \((F, A)\). Let \((G, A)\) be any closed soft set containing \((F, A)\). Then \((F, A) \subseteq (G, A) \Rightarrow (\overline{F}, A) \subseteq (G, A) = (G, A)\). Thus \((\overline{F}, A)\) is the smallest closed soft set containing \((F, A)\). □

**Definition 3.18.** Let \((X, A, \tau)\) be a soft topological space. If for \( E^\alpha_a, E^\beta_b \in \mathcal{E} \) with \( E^\alpha_a \neq E^\beta_b \), there exists \((F, A) \in \tau\) such that \([E^\alpha_a] \in (F, A) \) and \([E^\beta_b] \notin (F, A)\) or \([E^\beta_b] \in (F, A) \) and \([E^\alpha_a] \notin (F, A)\), then \((X, A, \tau)\) is called a soft \( T_0 \)-space.

**Proposition 3.19.** An enriched soft topological space \((X, A, \tau)\) is soft \( T_0 \)-space iff \( \forall E^\alpha_a, E^\beta_b \in \mathcal{E} \), \( E^\alpha_a \neq E^\beta_b \Rightarrow \overline{E^\alpha_a} \neq \overline{E^\beta_b} \).

**Proof.** Let \((X, A, \tau)\) be a soft \( T_0 \)-space and \( E^\alpha_a \neq E^\beta_b \). Then \( \exists (F, A) \in \tau \) such that one of \( E^\alpha_a \), \( E^\beta_b \) belongs to \((F, A)\) but not the other. Let \( E^\alpha_a \in (F, A) \) and \( E^\beta_b \notin (F, A) \). Then \((\overline{F}, A)\) is a soft closed set containing \( E^\beta_b \) but \( E^\alpha_a \notin (\overline{F}, A)\). Since \( \tau \) is enriched, \( \overline{F^\gamma} \) is the meet of all soft closed set containing \( E^\beta_b \), it follows that \( E^\gamma \notin \overline{F^\beta} \). But \( E^\beta_b \notin \overline{F^\beta} \). Therefore, \( \overline{E^\alpha_a} \neq \overline{E^\beta_b} \).

Conversely let the given condition be satisfied. Let \( E^\alpha_a, E^\beta_b \in \mathcal{E} \) such that \( E^\alpha_a \neq E^\beta_b \). So, by the given condition \( \overline{E^\alpha_a} \neq \overline{E^\beta_b} \). Then \( \exists E^\gamma \in \mathcal{E} \) such that \( E^\gamma \) belongs to one of the soft sets \( \overline{E^\alpha_a} \) and \( \overline{E^\beta_b} \) but not the other. Let \( E^\gamma \in \overline{E^\alpha_a} \) but \( E^\gamma \notin \overline{E^\beta_b} \). If \( E^\gamma \in \overline{E^\beta_b} \), then \( E^\alpha_a \subseteq \overline{E^\beta_b} \) and hence \( E^\gamma \subseteq \overline{E^\gamma} \subseteq \overline{E^\beta_b} = \overline{E^\beta_b} \). But \( E^\gamma \notin \overline{E^\beta_b} \). So, \( E^\alpha_a \notin \overline{E^\beta_b} \). Let \((U, A) = (\overline{X}, A) - \overline{E^\beta_b} \). Since \( \tau \) is enriched, \((U, A) \in \tau \) such that \( E^\alpha_a \subseteq (U, A) \) but \( E^\beta_b \notin (U, A) \). Therefore, \((X, A, \tau)\) is a soft \( T_0 \)-space. □

**Example 3.20.** Let \( X = \{x, y, z\}, A = \{\alpha, \beta\} \) and \( \tau = \{\{\phi, A\}, \{X, \phi\}, \{\phi, X\}, \{F_1, A\}, \{F_2, A\}, \{F_3, A\}, \{F_4, A\}, \{\overline{X}, A\}\} \).
where \((F_1, A) = \{(x), \phi\}, (F_2, A) = \{(x), \{y\}\}, (F_3, A) = \{\{y\}, \{z\}\}, (F_4, A) = \{(x, y), \{y, z\}\}\).

Now \(\overline{E^y_\alpha} = \{(x, z), \phi\}, \overline{E^y_\beta} = \{(y, z), \phi\}, \overline{E^y_\alpha} = \{\{z\}, \phi\}, \overline{E^y_\beta} = \{\phi, \{x\}\}\). \(\overline{E^y_\alpha} \neq \overline{E^y_\beta} \neq \overline{E^y_\alpha} \neq \overline{E^y_\beta} \neq \overline{E^y_\alpha}\). Thus \((X, A, \tau)\) is a soft \(T_0\)-space.

**Definition 3.21.** Let \((X, A, \tau)\) be a soft topological space over \(X\). If for any \(E^y_\alpha, E^y_\beta \in E\) with \(E^y_\alpha \neq E^y_\beta, \exists (F, A), (G, A) \in \tau\) such that \([E^y_\alpha] \subset (F, A), E^y_\beta \not\subset (F, A)]\) and \([E^y_\alpha] \subset (G, A), E^y_\beta \not\subset (G, A)\) then \((X, A, \tau)\) is called a soft \(T_1\)-space.

**Proposition 3.22.** A soft topological space \((X, A, \tau)\) is soft \(T_1\)-space iff \(\forall E^y_\alpha \in E, \{E^y_\alpha\}\) is soft closed.

**Proof.** Let \((X, A, \tau)\) be a soft topological space and \(E^y_\alpha \in E\). Let \(E^y_\beta \in [(\bar{X}, A) - E^y_\alpha]\). Then \(E^y_\alpha \neq E^y_\beta\). Since \((X, A, \tau)\) is a \(T_1\)-space, \(\exists (U_y, A) \in \tau\) such that \(E^y_\beta \not\subset (U_y, A)\) but \(E^y_\beta \not\subset (U_y, A)\). Thus \((U_y, A) \subset [(\bar{X}, A) - E^y_\alpha]\) and hence \(E^y_\beta \subset [(\bar{X}, A) - E^y_\alpha]\). Similarly \(E^y_\beta \subset [(\bar{X}, A) - E^y_\alpha]\). So,

\[
[(\bar{X}, A) - E^y_\alpha] = \overline{\bigcup_{E^y_\beta \in [(\bar{X}, A) - E^y_\alpha]} E^y_\beta} \subset \bigcup_{E^y_\beta \in [(\bar{X}, A) - E^y_\alpha]} (U_y, A) \subset [(\bar{X}, A) - E^y_\alpha].
\]

Therefore, \([\bar{X}, A) - E^y_\alpha] = \bigcup_{E^y_\beta \in [(\bar{X}, A) - E^y_\alpha]} (U_y, A)\). Since each \((U_y, A)\) is open soft set, \([\bar{X}, A) - E^y_\alpha] \subset \bigcup_{E^y_\beta \in [(\bar{X}, A) - E^y_\alpha]} (U_y, A)\) and hence \(E^y_\alpha\) is closed soft set.

Conversely let the given condition be satisfied. Let \(E^y_\alpha, E^y_\beta \in E\) such that \(E^y_\alpha \neq E^y_\beta\). Let \((U, A) = [(\bar{X}, A) - E^y_\alpha]\) and \((V, A) = [(\bar{X}, A) - E^y_\beta]\). Since \(E^y_\alpha\) and \(E^y_\beta\) are closed soft sets, it follows that \((U, A), (V, A)\) are open soft sets. Also \(E^y_\beta \not\subset (U, A), E^y_\alpha \not\subset (U, A)\) and \(E^y_\alpha \not\subset (V, A), E^y_\beta \not\subset (V, A)\). Therefore, \((X, A, \tau)\) is a soft \(T_1\)-space.

**Remark 3.23.** Every soft \(T_1\)-space is obviously a soft \(T_0\)-space. The following example shows that the converse is not always true.

**Example 3.24.** Let \(X, A\) and \(\tau\) be the same as in Example 3.20. Then \((X, A, \tau)\) is a soft \(T_0\)-space. But \(\{E^y_\alpha\}\) is not closed soft set since \(\{(y, z), \{x, y, z\}\} \not\subset \tau\) and hence \((X, A, \tau)\) is not a soft \(T_1\)-space.

**Example 3.25.** Let \(X\) be an infinite set and \(\tau\) be the soft cofinite topology over \(X\). Then for each \(E^y_\alpha \in E, (\bar{X}, A) - [(\bar{X}, A) - E^y_\alpha]\) is a finite soft set and hence \([\bar{X}, A) - E^y_\alpha]\) \(\in \tau\). Thus, \(\{E^y_\alpha\}\) is soft closed \(\forall E^y_\alpha \in E\). Therefore, \((X, A, \tau)\) is a soft \(T_1\)-space.

**Definition 3.26.** Let \((X, A, \tau)\) be a soft topological space over \(X\). If for any \(E^y_\alpha, E^y_\beta \in E\) with \(E^y_\alpha \neq E^y_\beta\), \(\exists (F, A), (G, A) \in \tau\) such that \(E^y_\alpha \subset (F, A), E^y_\beta \subset (G, A)\) and \((F, A) \cap (G, A) = (\phi, A)\), then \((X, A, \tau)\) is called a soft \(T_2\)-space.

**Remark 3.27.** Every soft \(T_2\)-space is obviously a soft \(T_1\)-space. The following example shows that the converse is not always true.

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Example 3.28. Let $X$, $A$ and $\tau$ be the same as in Example 3.25. Then $(X, A, \tau)$ is a soft $T_1$-space.

Let $(U, A)$, $(V, A) \in \tau$ such that $(U, A)$, $(V, A) \neq (\emptyset, A)$ and $(U, A) \cap (V, A) = (\emptyset, A)$. Then $(U, A) \subseteq [(\bar{X}, A) - (V, A)]$. This is not possible since $(U, A)$ is infinite soft set and $[(\bar{X}, A) - (V, A)]$ is a finite soft set. Therefore, $(X, A, \tau)$ is not a soft $T_2$-space.

Example 3.29. Let $R$ be the real number space and $A$ be a non empty parameter set. Let for each $\alpha \in A$, $\tau^\alpha$ be the usual topology on $R$. Then $(R, A, \tau^*)$ where $\tau^*$ be the soft topology generated by $\{\tau^\alpha, \alpha \in A\}$ as in Proposition 2.13, is a soft $T_2$-space.

Definition 3.30. A soft topological space $(X, A, \tau)$ is said to be soft regular space if for any soft closed set $(F, A)$ and any soft element $E^x_\alpha$ such that $E^x_\alpha \notin (F, A)$, $\exists$ open soft sets $(U, A)$, $(V, A)$ such that $E^x_\alpha \in (U, A)$, $(F, A) \subseteq (V, A)$ and $(U, A) \cap (V, A) = (\emptyset, A)$.

Proposition 3.31. If a soft topological space $(X, A, \tau)$ is soft regular then $\forall E^x_\alpha \in \mathcal{E}$ and $\forall (U, A) \in \tau$ such that $E^x_\alpha \in (U, A)$, $\exists (V, A) \in \tau$ such that $E^x_\alpha \in (V, A)$ and $(V, A) \subseteq (U, A)$.

Proof. Let $(X, A, \tau)$ be a soft regular space. Let $E^x_\alpha \in \mathcal{E}$ and $(U, A) \in \tau$ such that $E^x_\alpha \in (U, A)$. Consider $(F, A) = (\bar{X}, A) - (U, A)$. Then $(F, A)$ is soft closed and $E^x_\alpha \notin (F, A)$. Since $(X, A, \tau)$ is soft regular space, $\exists (V, A)$, $(W, A) \in \tau$ such that $E^x_\alpha \in (V, A)$, $(F, A) \subseteq (W, A)$ and $(V, A) \cap (W, A) = (\emptyset, A)$. Thus $(V, A) \subseteq [(\bar{X}, A) - (W, A)]$ and hence $(V, A) \subseteq [(\bar{X}, A) - (W, A)] = [(\bar{X}, A) - (W, A)] \subseteq [(\bar{X}, A) - (F, A)] = (U, A)$. Therefore, the given condition is satisfied.

Conversely, let the given condition is satisfied. Let $E^x_\alpha \in \mathcal{E}$ and $(F, A)$ be soft closed set such that $E^x_\alpha \notin (F, A)$. Consider $(U, A) = [(\bar{X}, A) - (F, A)]$. Then $(U, A) \in \tau$ such that $E^x_\alpha \subseteq (U, A)$. So, by the given condition $\exists (V, A) \in \tau$ such that $E^x_\alpha \in (V, A)$ and $(V, A) \subseteq (U, A)$. Let $(W, A) = [(\bar{X}, A) - (V, A)]$. Since $\tau$ is enriched, $(W, A) \in \tau$ and $(W, A) = [(\bar{X}, A) - (V, A)] \subseteq [(\bar{X}, A) - (U, A)] = (F, A)$, also $(V, A) \cap (W, A) = (V, A) \cap [(\bar{X}, A) - (V, A)] = (\emptyset, A)$. Therefore, $(X, A, \tau)$ is a soft regular space.

Definition 3.32. A soft topological space $(X, A, \tau)$ is said to be soft normal space if for any two disjoint soft closed sets $(F_1, A)$ and $(F_2, A)$ $\exists$ soft open sets $(U, A)$, $(V, A)$ such that $(F_1, A) \subseteq (U, A)$, $(F_2, A) \subseteq (V, A)$ and $(U, A) \cap (V, A) = (\emptyset, A)$.

Proposition 3.33. If a soft topological space $(X, A, \tau)$ is soft normal, then $\forall$ soft closed set $(F, A)$ and $\forall (U, A) \in \tau$ such that $(F, A) \subseteq (U, A)$, $\exists (V, A) \in \tau$ such that $(F, A) \subseteq (V, A)$ and $(V, A) \subseteq (U, A)$. The converse is true if $\tau$ is enriched.

Proof. Let $(X, A, \tau)$ be a soft normal space. Let $(F, A)$ be a soft closed set and $(U, A) \in \tau$ such that $(F, A) \subseteq (U, A)$.

Consider $(F_1, A) = (\bar{X}, A) - (U, A)$.

Then $(F_1, A)$ is soft closed set and $(F, A) \cap (F_1, A) = (\emptyset, A)$.

Since $(X, A, \tau)$ is soft normal space, $\exists (V, A)$, $(W, A) \in \tau$ such that $(F, A) \subseteq (V, A)$, $(F_1, A) \subseteq (W, A)$ and $(V, A) \cap (W, A) = (\emptyset, A)$.  

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Thus $(V, A) \subseteq [(X, A) - (W, A)]$ and hence
\[
(V, A) \subseteq [(X, A) - (W, A)] = [(X, A) - (W, A)] \subseteq [(X, A) - (F_1, A)] = (U, A).
\]
Therefore, the given condition is satisfied. Conversely let the given condition be satisfied.

Let $(F_1, A)$, $(F_2, A)$ be two disjoint soft closed sets.

Consider $(U, A) = [(X, A) - (F_2, A)]$. Then $(U, A) \in \tau$ such that $(F_1, A) \subseteq (U, A)$.

So, by the given condition $\exists (V, A) \in \tau$ such that $(F_1, A) \subseteq (V, A)$ and $(V, A) \subseteq (U, A)$.

Let $(W, A) = [(X, A) - (V, A)]$. Since $\tau$ is enriched, $(W, A) \in \tau$ and
\[
(W, A) = [(X, A) - (V, A)] \subseteq [(X, A) - (U, A)] = (F_2, A),
\]
also
\[
(V, A) \cap (W, A) = (V, A) \cap [(X, A) - (V, A)] = (\phi, A)
\]

Therefore, $(X, A, \tau)$ is a soft normal space. \[\square\]

4. SOME PROPERTIES OF NEIGHBOURHOOD SYSTEM OF IDENTITY SOFT ELEMENTS OF GROUP SOFT TOPOLOGY

Let us now turn our attention to the identity element of a group (with a soft topology defined on it) and look into basic facets of the system of neighbourhood of the identity soft elements of group soft topology. In this section, $\tau$ is a group soft topology over a group $G$ and $E_{\alpha}$, $\alpha \in A$ are the identity soft elements of $(G, A, \tau)$ where $e$ is the identity element of the group $G.$

**Definition 4.1** ([18]). Let $G$ be a group and $\tau$ be a soft topology on $G$. Then $\tau$ is said to be a group soft topology on $G$ if the mappings

(i) $f : (G \times G, A, \tau \times \tau) \rightarrow (G, A, \tau)$, defined by $f(x, y) = xy$ and

(ii) $g : (G, A, \tau) \rightarrow (G, A, \tau)$ defined by $g(x) = x^{-1}$

are soft continuous.

**Proposition 4.2** ([18]). Let $\tau$ be a group soft topology on $G$. Then
\[
\tau^\alpha = \{F(\alpha) : (F, A) \in \tau\}
\]
is a group topology on $G$, $\forall \alpha \in A$.

**Proposition 4.3** ([18]). Let $\tau^\alpha$ be a group topology on $G$, $\forall \alpha \in A$. Then $\tau^* = \{(F, A) \in S(X, A) : F(\alpha) \in \tau^\alpha, \forall \alpha \in A\}$ is a group soft topology on $G$.

**Definition 4.4** ([18]). Let $(F, A)$, $(H, A)$ be two soft sets over a group $(G, \circ)$. Then

(i) $(F, A) \circ (H, A) = (F \circ H, A)$ where $(F \circ H)(\alpha) = F(\alpha) \circ H(\alpha)$, $\forall \alpha \in A$.

(ii) $(F, A)^{-1} = (F^{-1}, A)$ where $F^{-1}(\alpha) = [F(\alpha)]^{-1}$, $\forall \alpha \in A$.

**Proposition 4.5** ([18]). Let $\tau$ be a group soft topology on a group $G$ and $\tau$ contains all those soft sets $(F, A)$ such that $F(\alpha) = \phi$ or $G$, $\forall \alpha \in A$ i.e. $\tau$ is an enriched soft topology. Then the mappings

(i) $R_a : (G, A, \tau) \rightarrow (G, A, \tau)$ defined by $R_a(x) = xa$

(ii) $L_a : (G, A, \tau) \rightarrow (G, A, \tau)$ defined by $L_a(x) = ax$

(iii) $\alpha : (G, A, \tau) \rightarrow (G, A, \tau)$ defined by $\alpha(x) = axa^{-1}$

are all are soft homeomorphisms.

**Proposition 4.6** ([18]). Let $\tau$ be a group soft topology on a group $G$ and $\tau$ contains all those soft sets $(F, A)$ such that $F(\alpha) = \phi$ or $G$, $\forall \alpha \in A$ i.e. $\tau$ is an
enriched soft topology. If \((P,A) \in \tau\), \((H,A)\) is closed soft set and \(a \in G\) then \((P \overset{H}{\circ} E_a,A)\), \((E_a \overset{P}{\circ} P,A)\), \((P,A)^{-1}\) are open soft sets and \((E_a \overset{H}{\circ} H,A)\), \((H, \overset{E_a}{\circ} A)\), \((H^{-1},A)\) are closed soft sets.

**Definition 4.7** ([18]). A soft nbd system \(B\) of a soft element \(E^a\) is said to be a fundamental soft nbd system or soft nbd base of \(E^a\) if for any soft nbd \((F,A)\) of \(E^a\), \(\exists (H,A) \in B\) such that \((H,A) \sqsubseteq (F,A)\).

**Proposition 4.8** ([18]). Let \(B\) be a fundamental soft nbd system of the soft element \(E^a\) in \((G,A,\tau)\). Then \(\forall (F,A) \in B\), \((F,A)^{-1}\) is also a soft nbd of \(E^a\).

**Definition 4.9** ([18]). A soft set \((F,A)\) over a group \(G\) is said to be symmetric if \((F,A)^{-1} = (F,A)\).

**Definition 4.10** ([18]). A soft nbd system is said to be a symmetric soft nbd system if all the members of that system are symmetric. Also a fundamental soft nbd system \(B\), every member of which is symmetric, is said to be a fundamental symmetric soft nbd system.

**Proposition 4.11** ([18]). Let \(\tau\) be a group soft topology over a group \(G\) and \(c\) be the identity element of \(G\). Then there exists a fundamental symmetric soft nbd system of the soft element \(E^c\) in \((G,A,\tau)\).

**Proposition 4.12** ([18]). For each soft nbd \((W,A)\) of \(E^c\) in \((G,A,\tau)\) where \(\tau\) is an enriched soft topology and for each finite set \(\{k_1, k_2, \ldots, k_n\}\) where \(k_i = 1\) or \(-1\), \(i = 1, 2, \ldots, n\), \(\exists\) a symmetric soft nbd \((U,A)\) of \(E^c\) such that \((U,A)^k \overset{c}{\circ} (U,A)^{k_2} \overset{c}{\circ} \ldots \overset{c}{\circ} (U,A)^{k_n} \sqsubseteq (W,A)\).

**Proposition 4.13.** If \(B\) be a soft nbd base of \(E^c\) in \((G,A,\tau)\) where \(\tau\) is an enriched soft topology then \(B' = \{E_x \overset{c}{\circ} (F,A) : (F,A) \in B\}\) is a soft nbd base of \(E^c\).

**Proof.** Let \((G,A)\) be any soft nbd of \(E^c\). Then \(L_{x^{-1}}[(G,A)]\) is a soft nbd of \(L_{x^{-1}}(E^c)\) i.e. \(E_{x^{-1}} \overset{c}{\circ} (G,A)\) is a soft nbd of \(E_{x^{-1}} \overset{c}{\circ} E^c = E^c\). So \(\exists\) \((F,A) \in B\) such that \(E^c \sqsubseteq (F,A) \sqsubseteq E_{x^{-1}} \overset{c}{\circ} (G,A)\). Hence \(L_{x}[(F,A)]\) is a soft nbd of \(L_{x}(E^c)\) such that

\[L_{x}(E^c) \sqsubseteq L_{x}[(F,A)] \sqsubseteq L_{x}(E_{x^{-1}} \overset{c}{\circ} (G,A)).\]

Thus \(E^c \sqsubseteq E_x \overset{c}{\circ} (F,A) \sqsubseteq E_x \overset{c}{\circ} (E_{x^{-1}} \overset{c}{\circ} (G,A)) = (G,A)\). Therefore \(B'\) is a soft nbd base of \(E^c\). \(\Box\)

**Proposition 4.14.** If \(B\) be a soft nbd base of \(E^c\) in \((G,A,\tau)\) where \(\tau\) is an enriched soft topology then \(B' = \{E_{x^{-1}} \overset{c}{\circ} (F,A) : (F,A) \in B\}\) is a soft nbd base of \(E^c\).

**Proof.** Proof is similar to that of Proposition 4.13. \(\Box\)

**Proposition 4.15.** If \(B\) be the system of all soft nbds of \(E^c\), in \((G,A,\tau)\), \(\alpha \in A\) where \(\tau\) is an enriched soft topology. Then \(B_a = \{U(\alpha) : (U,A) \in B\}\) is the system of all nbds of \(c\) in \((G,\tau^\alpha)\).

**Proof.** Let \(B\) be the system of all soft nbds of \(E^c\), \(\alpha \in A\) in \((G,A,\tau)\) and \(B_a = \{U(\alpha) : (U,A) \in B\}\).

Let \(U\) be any nbd of \(c\) in \((G,\tau^\alpha)\). Then \(\exists V \in \tau^\alpha\) such that \(c \in V \subseteq U\).
Now construct a soft set \((U, A)\), where \(U(\alpha) = U\) and \(U(\beta) = \phi, \forall \beta \neq \alpha\).
Also since \(V \in \tau^\alpha\) and \(\tau\) is enriched, we have a soft set \((V, A) \in \tau\) such that \(V(\alpha) = V\) and \(V(\beta) = \phi, \beta \neq \alpha\). Clearly \(E_\alpha^\circ \subseteq (U, A) \subseteq (V, A)\).
Thus, \((U, A) \in \mathcal{B}\) and hence \(U(\alpha) = U \in B_\alpha\).
Therefore, \(B_\alpha = \{U(\alpha) : (U, A) \in \mathcal{B}\}\) is the system of all nbds of \(\epsilon\) in \((G, \tau^\alpha)\) .

**Proposition 4.16.** Let \(\mathcal{B}\) be the system of all soft nbds of \(E_\alpha^\circ\) in \((G, A, \tau)\) and \((F, A)\) be any soft set over \(G\). Then \(F(\alpha) = \bigcap_{(U, A) \in \mathcal{B}} [F(\alpha) \circ U(\alpha)] = \bigcap_{(U, A) \in \mathcal{B}} [U(\alpha) \circ F(\alpha)]\).

**Proof.** Since \(\mathcal{B}\) is the system of all soft nbds of \(E_\alpha^\circ\) in \((G, A, \tau)\), it follows that \(\mathcal{B}_\alpha\) is the system of all nbds of \(\epsilon\) in \((G, \tau^\alpha)\). Also \((F, A)\) is a soft set over \(G\).
So \(F(\alpha)\) is a subset of \(G\). Then from the topological group theory we get 
\[F(\alpha) = \bigcap_{(U, A) \in \mathcal{B}} [F(\alpha) \circ U(\alpha)] = \bigcap_{(U, A) \in \mathcal{B}} [U(\alpha) \circ F(\alpha)].\]
Similarly we get 
\[F(\alpha) = \bigcap_{(U, A) \in \mathcal{B}} [U(\alpha) \circ F(\alpha)].\]

**Proposition 4.17.** For any soft nbd \((U, A)\) of \(E_\alpha^\circ\) in \((G, A, \tau)\) where \(\tau\) is an enriched soft topology, \(\exists\) a soft nbd \((V, A)\) of \(E_\alpha^\circ\) such that \((V, A) \subseteq (U, A)\).

**Proof.** Let \((U, A)\) be any nbd of \(E_\alpha^\circ\). Then by Proposition 4.12, \(\exists\) a nbd \((W, A)\) of \(E_\alpha^\circ\) such that \((W, A) \subseteq (U, A)\).
Also, by the Proposition 4.16, \(\overline{W(\alpha)} = \overline{W(\alpha)} \subseteq [W(\alpha) \circ W(\alpha)]\).
Now construct a soft set \((V, A)\) where \(V(\alpha) = W(\alpha)\) and \(V(\beta) = \phi, \forall \beta \neq \alpha\).
Then \((V, A) \in \tau\) and \(E_\alpha^\circ \subseteq (V, A) \subseteq (W, A)\).
So \((V, A)\) is a soft nbd of \(E_\alpha^\circ\) such that \(\overline{V(\alpha)} \subseteq \overline{W(\alpha)} \subseteq [W(\alpha) \circ W(\alpha)]\) and \(\phi = \overline{V(\beta)} \subseteq [W(\beta) \circ W(\beta)]\), \(\forall \beta \neq \alpha\) and hence \((V, A) \subseteq [(W, A) \circ (W, A)] \subseteq (U, A)\).

**Proposition 4.18.** If an enriched soft topology \(\tau\) be a group topology over a group \(G\), then the soft topological space \((G, A, \tau)\) is soft regular.

**Proof.** Let \(E_\alpha^\circ \in \mathcal{E}\) and \((U, A) \in \tau\) such that \(E_\alpha^\circ \subseteq (U, A)\).
Then \((U, A)\) is a soft nbd of \(E_\alpha^\circ\) and hence by Proposition 4.13, \(E_{\alpha^{-1}} \circ (U, A) = (W, A)(\text{say})\) is a soft nbd of \(E_\alpha^\circ\).
Then \(\exists\) soft nbd \((V, A)\) of \(E_\alpha^\circ\) such that \((V, A) \subseteq (W, A)\).
So, \(E_{\alpha^{-1}} \circ (V, A)\) is a soft nbd of \(E_\alpha^\circ\) and hence \((P, A) \in \tau\) such that \(E_\alpha^\circ \subseteq (P, A) \subseteq E_{\alpha^{-1}} \circ (V, A)\).
Thus, \((P, A) \subseteq E_{\alpha^{-1}} \circ (V, A) = L_x([V, A]) = L_x([V, A])\) (since \(L_x\) is soft homeomorphism) \(E_{\alpha^{-1}} \circ (V, A) \subseteq E_{\alpha^{-1}} \circ (U, A) = (U, A)\).
Therefore, the soft topological space \((G, A, \tau)\) is soft regular.

**Proposition 4.19.** Let \(\tau\) be a group soft topology over a group \(G\) where \(\tau\) is an enriched soft topology. Then there exists a fundamental soft nbd system \(\mathcal{B}\) of closed soft nbds of \(E_\alpha^\circ\) such that
(i) each \((U, A) \in \mathcal{B}\) is symmetric;
(ii) \(\forall\ (U, A) \in \mathcal{B}, \exists\ (V, A) \in \mathcal{B}\) such that \((V, A) \circ (V, A) \subseteq (U, A)\);
(iii) \(\forall\ (U, A) \in \mathcal{B}\) and \(\forall\ a \in G, \exists\ (V, A) \in \mathcal{B}\) such that \((V, A) \circ E_{\alpha^{-1}} \circ (U, A) \circ E_{\alpha^{-1}} \circ (V, A)\)
\(E_{\alpha^{-1}} \circ (V, A) \circ E_{\alpha^{-1}} \circ (U, A) \circ E_{\alpha^{-1}} \circ (V, A)\)
\(\circ\ i.e.\ E_{\alpha^{-1}} \circ (V, A) \circ E_{\alpha^{-1}} \circ (U, A) \circ E_{\alpha^{-1}} \circ (V, A)\)
(iv) \(\forall\ (V, A) \in \mathcal{B}\) and \(\forall\ E_{\alpha^{-1}} \circ (V, A) \in \mathcal{B}\), \(\exists\ (U, A) \in \mathcal{B}\) such that \(E_{\alpha^{-1}} \circ (U, A) \circ E_{\alpha^{-1}} \circ (V, A)\).

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We know that a fundamental system $B'$ of symmetric nbds of $E^c_\alpha$.

Let $B = \{(U, A) : (U, A) \in B'\}$. Since $(U, A) \in B' \Rightarrow (U, A)$ is symmetric and hence $(U, A)^{-1} = (U, A)$. Again as the inverse mapping is soft homeomorphism and hence $\forall \alpha \in A, [U^{-1}(\alpha)] = [U(\alpha)]^{-1} = [U(\alpha)]^{-1} = U^{-1}(\alpha)$. So $(U, A) = (U, A)^{-1} = [(U, A)]^{-1}$. Thus $(U, A)$ is a symmetric closed nbhd of $E^c_\alpha$. We shall now show that $B$ is a fundamental system of nbds of $E^c_\alpha$. Let $(W, A)$ be any nbhd of $E^c_\alpha$. Then by Proposition 4.17, there exists a nbhd $(V, A)$ of $E^c_\alpha$ such that $(V, A) \subseteq (W, A)$. Since $B'$ is a fundamental system of nbds of $E^c_\alpha$, $\exists (U, A) \in B'$ such that $(U, A) \subseteq (V, A)$. Then $(U, A) \in B$ and $(U, A) \subseteq (V, A) \subseteq (W, A)$. Therefore $B$ is a fundamental system of closed nbds of $E^c_\alpha$ such that each member of $B$ is symmetric.

(ii) Proof follows from Proposition 4.12.

(iii) Let $(U, A) \in B$. Since $E_\alpha\circ E^c_\alpha\circ E_{\alpha^{-1}} = E^c_\alpha$, $(U, A)$ is a soft nbhd of $E^c_\alpha$ and translation mapping is soft homeomorphism, $\exists$ a soft nbhd $(V_1, A)$ of $E^c_\alpha$ such that $E_\alpha\circ (V_1, A)\circ E_{\alpha^{-1}} \subseteq (U, A)$. Since $B$ is a fundamental soft nbhd system of $E^c_\alpha$, $\exists (V, A) \in B$ such that $(V, A) \subseteq (V_1, A)$. Therefore

$$E^c_\alpha \circ (V, A) \circ E_{\alpha^{-1}} = E^c_\alpha \circ (V, A) \circ E_{\alpha^{-1}} \subseteq E_\alpha \circ (V_1, A) \circ E_{\alpha^{-1}} \subseteq (U, A).$$

(iv) Let $(V, A) \in B$ and $E^c_\alpha \subseteq (V, A)$.

Since $E_\alpha\circ E^c_\alpha = E^c_\alpha$ i.e. $L_a(E^c_\alpha) = E^c_\alpha$, $(V, A)$ is a soft nbhd of $E^c_\alpha$ and left translation mapping is soft homeomorphism, there exists a soft nbhd $(U_1, A)$ of $E^c_\alpha$ such that $L_a[(U_1, A)] \subseteq (V, A)$ i.e. $E_\alpha \circ (U_1, A) \subseteq (V, A)$. Since $B$ is a fundamental soft nbhd system of $E^c_\alpha$, $\exists (U, A) \in B$ such that $(U, A) \subseteq (U_1, A)$. Therefore

$$E^c_\alpha \circ (U, A) \subseteq E_\alpha \circ (U, A) \subseteq E_\alpha \circ (U_1, A) \subseteq (V, A).$$

\[
\text{Proposition 4.20. Let } \tau \text{ be a group soft topology over a group } G \text{ where } \tau \text{ is an enriched soft topology and } e \text{ as the identity element of } G. \text{ Then the following statements are equivalent.}
\]

(i) $(G, A, \tau)$ is a soft $T_0$-space.

(ii) $(G, A, \tau)$ is a soft $T_1$-space.

(iii) $(G, A, \tau)$ is a soft $T_2$-space.

(iv) $\bigcap\{(U, A) : (U, A) \in B\} = \{E^c_\alpha\}$, where $B$ is a fundamental system of nbds of $E^c_\alpha$, $\alpha \in A$.

\[
\text{Proof. Let } (G, A, \tau) \text{ be a soft } T_0\text{-space and } E^c_\alpha, E^y_\beta \in \mathcal{E} \text{ such that } E^c_\alpha \neq E^y_\beta. \text{ If } \alpha \neq \beta \text{ and since } \tau \text{ is enriched there exists open soft sets } (F, A), (G, A) \text{ where } F(\alpha) = X, F(\beta) = \phi, \forall \beta \neq \alpha \text{ and } G(\beta) = X, G(\alpha) = \phi, \forall \alpha \neq \beta. \text{ So } E^c_\alpha \subseteq (F, A), E^y_\beta \not\subseteq (F, A), E^c_\alpha \not\subseteq (G, A), E^y_\beta \subseteq (G, A). \text{ Thus we are to prove the case when } \alpha = \beta \text{ and } x \neq y. \text{ Since } (G, A, \tau) \text{ is a soft } T_0\text{-space, it follows that there exists an open soft set } (F, A) \text{ such that one of } E^c_\alpha \text{ and } E^y_\beta \text{ belong to } (F, A) \text{ but the other does not belong to } (F, A). \text{ Let } E^c_\alpha \subseteq (F, A) \text{ but } E^y_\beta \not\subseteq (F, A). \text{ Consider } (V, A) = E^c_{x^{-1}} \circ (F, A). \text{ Then } (V, A) \text{ is an open soft set containing } E^c_\alpha \text{ and } (U, A) = (V, A) \cap (V, A)^{-1} \text{ is a symmetric open soft set containing } E^c_\alpha. \text{ So } E^c_{x^{-1}} \circ (U, A) \text{ is an open soft set containing } E^c_\alpha. \text{ We shall now show that } E^c_{x^{-1}} \not\subseteq E^y_\beta \circ (U, A). \text{ Now if possible, } E^c_{x^{-1}} \subseteq E^y_\beta \circ (U, A), \text{ then}
\]

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$E^x_\alpha \subseteq (U, A)^{-1} \circ E_y^{-1} = (U, A) \circ E_y^{-1} \subseteq (V, A) \circ E_y^{-1} = E_{x^{-1}} \circ (F, A) \circ E_y^{-1}$. Thus $E^x_\alpha = E^x_\alpha \circ E^x_\beta \subseteq E^x_\beta \circ E_x^{-1} \circ (F, A) \circ E_y^{-1} = E^x_\alpha \circ (F, A) \circ E_y^{-1}$. Therefore $E^y_\alpha \subseteq (F, A)$, which is a contradiction. Therefore $(G, A, \tau)$ is a $T_1$-space. This shows that $(i) \Rightarrow (ii)$.

Next assume that $(G, A, \tau)$ be $T_1$–space. Let $E^x_\alpha, E^y_\alpha \in \mathcal{E}$ such that $E^x_\alpha \neq E^y_\alpha$. Since $\tau$ is $T_1$–space, it follows that $\{E^x_\alpha\}$ is closed soft set and hence $(P, A) = (G, A) - E^x_\alpha$ is open soft set containing $E^y_\alpha$. So $E_y^{-1} \circ (P, A)$ is a nbd of $E^y_\alpha$.

Then by Proposition 4.12, there exists an open soft nbd $(V, A)$ of $E^x_\alpha$ such that $(V, A) \circ (V, A)^{-1} \subseteq E_y^{-1} \circ (P, A)$. So $E_y \circ (V, A)$ is an open soft nbd of $E^y_\alpha$ and $(Q, A) = (G, A) - [E_y \circ (V, A)]$ is an open soft set. If $E^x_\alpha \subseteq [E_y \circ (V, A)]$, then $[E_y \circ (V, A)] \cap [E_y \circ (V, A)] = (\emptyset, A)$ and

$E^x_\alpha \subseteq E_y \circ (V, A) \circ (V, A)^{-1} \subseteq E_y \circ E_y^{-1} \circ (P, A) = (P, A),$

which is a contradiction because $E^x_\alpha \not\subseteq (P, A)$. Thus $(Q, A)$ is a soft open containing $E^x_\alpha$. Clearly $E^x_\alpha \subseteq (Q, A)$, $E^y_\alpha \subseteq [E_y \circ (V, A)]$ and $(Q, A) \nmid [E_y \circ (V, A)] = (\emptyset, A)$. Therefore $(ii) \Rightarrow (iii)$.

Next let $(G, A, \tau)$ be a soft $T_2$-space and $B$ be a fundamental system of nbds of $E^x_\alpha$. Let $E^x_\alpha \subseteq \bigcap\{(U, A) : (U, A) \in B\}$. If possible let $E^x_\alpha \neq E^\gamma_\alpha$. Since $(G, A, \tau)$ is a $T_2$-space, there exists a soft open set $(F, A)$ such that $E^x_\alpha \subseteq (F, A)$ but $E^x_\alpha \not\subseteq (F, A)$. Since $B$ is a fundamental system of nbds of $E^x_\alpha$, $\exists (U, A) \in B$ such that $(U, A) \not\subseteq (F, A)$. So $E^x_\alpha \not\subseteq (U, A)$, which contradicts our assumption $E^x_\alpha \subseteq (U, A)$, $\forall (U, A) \in B$. Thus $E^x_\alpha = E^\gamma_\alpha$ and $\bigcap\{(U, A) : (U, A) \in B\} = \{E^x_\alpha\}$. This shows that $(iii) \Rightarrow (iv)$. Again let $\bigcap\{(U, A) : (U, A) \in B\} = \{E^x_\alpha\}$ and $E^x_\alpha, E^y_\alpha \in \mathcal{E}$ such that $E^x_\alpha \neq E^x_\alpha$. Then $E^y_\alpha^{-1} \circ E^x_\alpha \neq E^y_\alpha$ and $\exists (U, A) \in B$ such that $E^y_\alpha^{-1} \circ E^x_\alpha \not\subseteq (U, A)$.

So $E^y_\alpha \not\subseteq E_y \circ (U, A)$. Thus $E_y \circ (U, A)$ is a soft open set containing $E^y_\alpha$ but $E^x_\alpha \not\subseteq E_y \circ (U, A)$. Therefore $(G, A, \tau)$ is a soft $T_0$-space and $(iv) \Rightarrow (i)$. Therefore the statements $(i)$, $(ii)$, $(iii)$ and $(iv)$ are equivalent. \hfill $\Box$

5. Conclusion

In this paper, we have studied separation properties of group soft topology [18]. In this context the neighbourhood systems of soft elements play important role because of the homeomorphism property of the translation operations. It is just a beginning of this study. There is an ample scope of its generalization where the underlying mappings are soft mappings instead of ordinary mappings and parallel soft product is replaced by usual soft product.

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