

## *npn*-soft sets theory and their applications

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**ABSTRACT.** In this paper, we firstly defined neutrosophic parameterized neutrosophic soft sets (*npn*-soft sets) which is combination of a neutrosophic sets and a soft sets. Our *npn*-soft sets generalizes the concept of the other soft sets such as; fuzzy soft sets, intuitionistic fuzzy soft sets, neutrosophic soft sets, fuzzy parameterized soft sets, intuitionistic fuzzy parameterized soft sets, neutrosophic parameterized soft sets and so on. Then, we introduce some definitions and operations on *npn*-soft sets and some properties of the sets which are connected to operations have been established. Also, we have introduced the concept of *npn*-soft matrix and their operators which are more functional to make theoretical studies in the *npn*-soft set theory. Finally, we proposed the decision making method on the *npn*-soft set theory which can be applied to problems of many fields that contain uncertainty and provided an example that demonstrated that this method can be successfully worked.

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### 1. INTRODUCTION

In literature, a number of theories have been proposed which can be applied in many real applications to handle uncertainty, vagueness and indeterminacy. Theory of fuzzy set theory [60], theory of intuitionistic fuzzy sets [6], theory of neutrosophic theory [54, 55] are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system.

The concept of soft sets was introduced by Molodtsov [46] for the inadequacy of the parameterization tool of the theories. Later on, many interesting results of soft set theory have been obtained by embedding the idea of fuzzy set, intuitionistic fuzzy set, neutrosophic set and so on. For example, on fuzzy soft set [38], on fuzzy

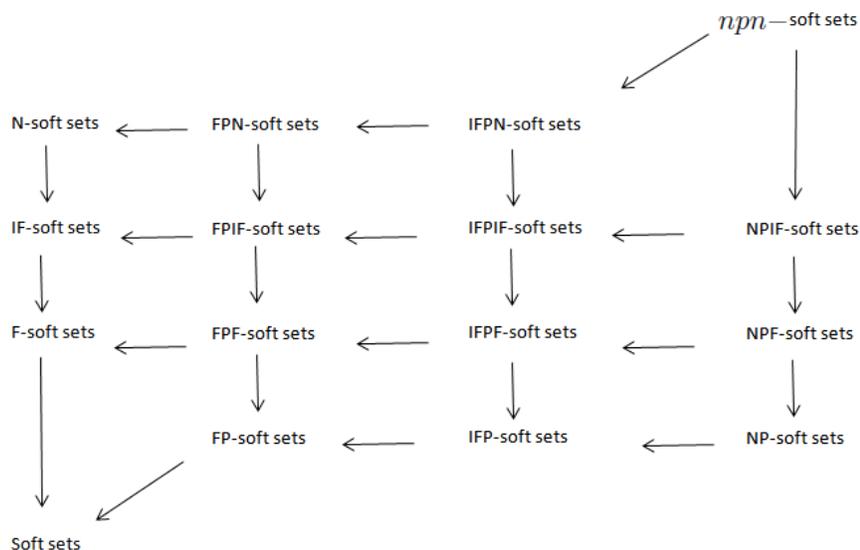
parameterized soft set [17, 25], on fuzzy parameterized fuzzy soft set [21], on intuitionistic fuzzy soft set [18, 40], on intuitionistic fuzzy parameterized soft set [24], on intuitionistic fuzzy parameterized fuzzy soft set [59], on neutrosophic soft set [22, 23, 36], on neutrosophic parameterized soft set [12], on generalized neutrosophic soft set [9], on intuitionistic neutrosophic soft set [10] and so on. The theories has developed in many directions and applied to wide variety of fields such as; on soft set [1, 2, 3, 13, 30, 31, 33, 34, 52], on fuzzy soft set [28, 29, 35, 42, 56, 57], on fuzzy parameterized soft set [19, 20], on intuitionistic fuzzy soft set [26, 32, 37, 49, 50], on neutrosophic soft set [22, 39] and so on. Recently Cagman et al [14] proposed soft matrices and applied it in decision making problem. Then, they defined fuzzy soft matrices [16]. Mondal and Roy [43, 45] defined intuitionistic fuzzy soft matrices. Deli and Broumi [22] proposed neutrosophic soft matrices with some desired propositions. The matrices has differently developed in many directions and applied to wide variety of fields in [8, 11, 41, 43, 44, 45, 48, 51, 53].

In this paper, we introduced  $n\text{pn}$ -soft sets which is a combination of a neutrosophic sets [54] and a soft sets [46] by using [7, 14, 16, 23, 36, 41, 43, 44, 45, 48, 51, 55, 53, 58]. The neutrosophic parameterized neutrosophic soft sets( $n\text{pn}$ -soft sets) generalizes the following sets:

- (1) Soft sets,
- (2) fuzzy soft sets,
- (3) intuitionistic fuzzy soft sets,
- (4) neutrosophic soft sets,
- (5) fuzzy parameterized soft sets,
- (6) fuzzy parameterized fuzzy soft sets,
- (7) fuzzy parameterized intuitionistic fuzzy soft sets,
- (8) fuzzy parameterized neutrosophic soft sets,
- (9) intuitionistic fuzzy parameterized soft sets,
- (10) intuitionistic fuzzy parameterized fuzzy soft sets,
- (11) intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets,
- (12) intuitionistic fuzzy parameterized neutrosophic soft sets,
- (13) neutrosophic parameterized soft sets,
- (14) neutrosophic parameterized fuzzy soft sets,
- (15) neutrosophic parameterized intuitionistic fuzzy soft sets,

The relationship among  $n\text{pn}$ -soft sets and other soft sets is showed in Figure 1.

Our objective is to present the concept of neutrosophic parameterized neutrosophic soft sets(or  $n\text{pn}$ -soft sets) and its applications in decision making problem. The remaining part of this paper is organized as follows. In section 2, we give basic definitions and notations that are used in the remaining parts of the paper. In section 3, we defined neutrosophic parameterized neutrosophic soft sets( $n\text{pn}$ -soft sets) which is a combination of a neutrosophic sets [54] and a soft sets [46]. Then we introduce some definitions and operations on  $n\text{pn}$ -soft sets and some properties of the sets which are connected to operations have been established. In section 4, we have introduced the concept of  $n\text{pn}$ -soft matrix and their operators which are more functional to make theoretical studies in the  $n\text{pn}$ -soft set theory. In section 5, we proposed the decision making method on the  $n\text{pn}$ -soft set theory which can be applied to problems of many fields that contain uncertainty and we provided an



N: Neutrosophic P: Parameterized I: Intuitionistic F: Fuzzy

FIGURE 1. Relationship among npn-soft set and other soft sets

example that demonstrated that this method can be successfully worked. In section 7, conclusion is made.

## 2. PRELIMINARY

In this section, we give the basic definitions and results of neutrosophic set theory [54] and soft set theory [46] that are useful for subsequent discussions.

For more details, the reader could refer to [4, 5, 13, 15, 47, 55, 58].

**Definition 2.1.** [54] Let  $U$  be a space of points (objects), with a generic element in  $U$  denoted by  $u$ . A neutrosophic sets(N-sets)  $A$  in  $U$  is characterized by a truth-membership function  $T_A$ , a indeterminacy-membership function  $I_A$  and a falsity-membership function  $F_A$ .  $T_A(u)$ ;  $I_A(u)$  and  $F_A(u)$  are real standard or nonstandard subsets of  $[0, 1]$ . It can be written as

$$A = \{ \langle u, (T_A(u), I_A(u), F_A(u)) \rangle : u \in U, T_A(u), I_A(u), F_A(u) \in [0, 1] \}.$$

There is no restriction on the sum of  $T_A(u)$ ;  $I_A(u)$  and  $F_A(u)$ , so  $0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3$ .

**Definition 2.2.** [46] Let  $U$  be an initial universe,  $P(U)$  be the power set of  $U$ ,  $E$  be a set of all parameters and  $X \subseteq E$ . Then a soft set  $F_X$  over  $U$  is a set defined by a function representing a mapping

$$f_X : E \rightarrow P(U) \text{ such that } f_X(x) = \emptyset \text{ if } x \notin X$$

Here,  $f_X$  is called approximate function of the soft set  $F_X$ , and the value  $f_X(x)$  is a set called  $x$ -element of the soft set for all  $x \in E$ . It is worth noting that the sets  $f_X(x)$  may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set over  $U$  can be represented by the set of ordered pairs

$$F_X = \{(x, f_X(x)) : x \in E, f_X(x) \in P(U)\}$$

**Definition 2.3.** [27]  $t$ -norms are associative, monotonic and commutative two valued functions  $t$  that map from  $[0, 1] \times [0, 1]$  into  $[0, 1]$ . These properties are formulated with the following conditions:  $\forall a, b, c, d \in [0, 1]$ ,

- (1)  $t(0, 0) = 0$  and  $t(a, 1) = t(1, a) = a$ ,
- (2) If  $a \leq c$  and  $b \leq d$ , then  $t(a, b) \leq t(c, d)$
- (3)  $t(a, b) = t(b, a)$
- (4)  $t(a, t(b, c)) = t(t(a, b), c)$

**Definition 2.4.** [27]  $t$ -conorms ( $s$ -norm) are associative, monotonic and commutative two placed functions  $s$  which map from  $[0, 1] \times [0, 1]$  into  $[0, 1]$ . These properties are formulated with the following conditions:  $\forall a, b, c, d \in [0, 1]$ ,

- (1)  $s(1, 1) = 1$  and  $s(a, 0) = s(0, a) = a$ ,
- (2) if  $a \leq c$  and  $b \leq d$ , then  $s(a, b) \leq s(c, d)$
- (3)  $s(a, b) = s(b, a)$
- (4)  $s(a, s(b, c)) = s(s(a, b), c)$

$t$ -norm and  $t$ -conorm are related in a sense of logical duality. Typical dual pairs of non parametrized  $t$ -norm and  $t$ -conorm are compiled below:

- (1) Drastic product:

$$t_w(a, b) = \begin{cases} \min\{a, b\}, & \max\{ab\} = 1 \\ 0, & \text{otherwise} \end{cases}$$

- (2) Drastic sum:

$$s_w(a, b) = \begin{cases} \max\{a, b\}, & \min\{ab\} = 0 \\ 1, & \text{otherwise} \end{cases}$$

- (3) Bounded product:

$$t_1(a, b) = \max\{0, a + b - 1\}$$

- (4) Bounded sum:

$$s_1(a, b) = \min\{1, a + b\}$$

- (5) Einstein product:

$$t_{1.5}(a, b) = \frac{a \cdot b}{2 - [a + b - a \cdot b]}$$

- (6) Einstein sum:

$$s_{1.5}(a, b) = \frac{a + b}{1 + a \cdot b}$$

- (7) Algebraic product:

$$t_2(a, b) = a \cdot b$$

(8) Algebraic sum:

$$s_2(a, b) = a + b - a.b$$

(9) Hamacher product:

$$t_{2.5}(a, b) = \frac{a.b}{a + b - a.b}$$

(10) Hamacher sum:

$$s_{2.5}(a, b) = \frac{a + b - 2.a.b}{1 - a.b}$$

(11) Minumum:

$$t_3(a, b) = \min\{a, b\}$$

(12) Maximum:

$$s_3(a, b) = \max\{a, b\}$$

### 3. *n*pn–SOFT SETS

In this section, we present neutrosophic parameterized neutrosophic soft sets which is generalized the concept of the sets by given Figure 1. Then, we introduce some definitions and operations on neutrosophic parameterized neutrosophic soft sets and some properties of the sets which are connected to operations have been established. The method and application on neutrosophic soft set defined in [23] are extended to the case of neutrosophic parameterized neutrosophic soft sets.

**Definition 3.1.** Let  $U$  be a universe,  $N(U)$  be the set of all neutrosophic sets on  $U$ ,  $E$  be a set of parameters that are describe the elements of  $U$  and  $K$  be a neutrosophic set over  $E$ . Then, a neutrosophic parameterized neutrosophic soft set(*n*pn–soft set)  $N$  over  $U$  is a set defined by a set valued function  $f_N$  representing a mapping

$$f_N : K \rightarrow N(U)$$

where  $f_N$  is called approximate function of the *n*pn–soft set  $N$ . For  $x \in E$ , the set  $f_N(x)$  is called  $x$ -approximation of the *n*pn–soft set  $N$  which may be arbitrary, some of them may be empty and some may have a nonempty intersection. It can be written a set of ordered pairs,

$$N = \left\{ \left( \langle x, T_N(x), I_N(x), F_N(x) \rangle, \langle u, T_{f_N(x)}(u), I_{f_N(x)}(u), F_{f_N(x)}(u) \rangle : x \in U \right) : x \in E \right\}$$

where

$$F_N(x), I_N(x), T_N(x), T_{f_N(x)}(u), I_{f_N(x)}(u), F_{f_N(x)}(u) \in [0, 1]$$

**Definition 3.2.** Let  $N$  be an *n*pn–soft sets. Then, the complement of an *n*pn–soft set  $N$  denoted by  $N^c$  and is defined by

$$N^c = \left\{ \left( \langle x, F_N(x), 1 - I_N(x), T_N(x) \rangle, \langle u, F_{f_N(x)}(u), 1 - I_{f_N(x)}(u), T_{f_N(x)}(u) \rangle : x \in U \right) : x \in E \right\}$$

**Definition 3.3.** Let  $N_1$  and  $N_2$  be two  $npn$ -soft sets. Then, the union of  $N_1$  and  $N_2$  is denoted by  $N_3 = N_1 \tilde{\cup} N_2$  and is defined by

$$N_3 = \left\{ \begin{aligned} & \langle x, T_{N_3}(x), I_{N_3}(x), F_{N_3}(x) \rangle, \\ & \{ \langle u, T_{f_{N_3(x)}}(u), I_{f_{N_3(x)}}(u), F_{f_{N_3(x)}}(u) \rangle : x \in U \} \end{aligned} : x \in E \right\}$$

where

$$\begin{aligned} T_{N_3}(x) &= s(T_{N_1}(x), T_{N_2}(x)), & T_{f_{N_3(x)}}(u) &= s(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)), \\ I_{N_3}(x) &= t(I_{N_1}(x), I_{N_2}(x)), & I_{f_{N_3(x)}}(u) &= t(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)), \\ F_{N_3}(x) &= t(F_{N_1}(x), F_{N_2}(x)), & F_{f_{N_3(x)}}(u) &= t(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)) \end{aligned}$$

**Definition 3.4.** Let  $N_1$  and  $N_2$  be two  $npn$ -soft sets. Then, the intersection of  $N_1$  and  $N_2$  is denoted by  $N_4 = N_1 \tilde{\cap} N_2$  and is defined by

$$N_4 = \left\{ \begin{aligned} & \langle x, T_{N_4}(x), I_{N_4}(x), F_{N_4}(x) \rangle, \\ & \{ \langle u, T_{f_{N_4(x)}}(u), I_{f_{N_4(x)}}(u), F_{f_{N_4(x)}}(u) \rangle : x \in U \} \end{aligned} : x \in E \right\}$$

where

$$\begin{aligned} T_{N_4}(x) &= t(T_{N_1}(x), T_{N_2}(x)), & T_{f_{N_4(x)}}(u) &= t(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)), \\ I_{N_4}(x) &= s(I_{N_1}(x), I_{N_2}(x)), & I_{f_{N_4(x)}}(u) &= s(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)), \\ F_{N_4}(x) &= s(F_{N_1}(x), F_{N_2}(x)), & F_{f_{N_4(x)}}(u) &= s(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)) \end{aligned}$$

**Example 3.5.** Let  $U = \{u_1, u_2, u_3\}$ ,  $E = \{x_1, x_2, x_3\}$ .  $N_1$  and  $N_2$  be two  $npn$ -soft sets as

$$N_1 = \left\{ \begin{aligned} & \langle x_1, (0.6, 0.7, 0.8) \rangle, \{ \langle u_1, (0.4, 0.5, 0.6) \rangle, \langle u_2, (0.6, 0.1, 0.1) \rangle, \\ & \langle u_3, (0.3, 0.4, 0.4) \rangle \}, \langle x_2, (0.4, 0.1, 0.2) \rangle, \langle u_1, (0.5, 0.8, 0.5) \rangle, \\ & \langle u_2, (0.5, 0.6, 0.8) \rangle, \langle u_3, (0.6, 0.6, 0.9) \rangle \}, \langle x_3, (0.4, 0.1, 0.5) \rangle, \\ & \{ \langle u_1, (0.5, 0.4, 0.6) \rangle, \langle u_2, (0.6, 0.6, 0.7) \rangle, \langle u_3, (0.2, 0.1, 0.8) \rangle \} \end{aligned} \right\}$$

and

$$N_2 = \left\{ \begin{aligned} & \langle x_1, (0.9, 0.7, 0.8) \rangle, \{ \langle u_1, (0.3, 0.6, 0.8) \rangle, \langle u_2, (0.2, 0.3, 0.1) \rangle, \\ & \langle u_3, (0.9, 0.7, 0.4) \rangle \}, \langle x_2, (0.4, 0.5, 0.8) \rangle, \langle u_1, (0.5, 0.7, 0.1) \rangle, \\ & \langle u_2, (0.1, 0.6, 0.3) \rangle, \langle u_3, (0.1, 0.7, 0.5) \rangle \}, \langle x_3, (0.2, 0.8, 0.9) \rangle, \\ & \{ \langle u_1, (0.7, 0.9, 0.6) \rangle, \langle u_2, (0.5, 0.6, 0.3) \rangle, \langle u_3, (0.7, 0.5, 0.8) \rangle \} \end{aligned} \right\}$$

here;

$$N_1^c = \left\{ \begin{aligned} & \langle x_1, (0.8, 0.3, 0.6) \rangle, \{ \langle u_1, (0.6, 0.5, 0.4) \rangle, \langle u_2, (0.1, 0.9, 0.6) \rangle, \\ & \langle u_3, (0.4, 0.6, 0.3) \rangle \}, \langle x_2, (0.2, 0.9, 0.4) \rangle, \langle u_1, (0.5, 0.2, 0.5) \rangle, \\ & \langle u_2, (0.8, 0.4, 0.5) \rangle, \langle u_3, (0.9, 0.4, 0.6) \rangle \}, \langle x_3, (0.5, 0.9, 0.4) \rangle, \\ & \{ \langle u_1, (0.6, 0.6, 0.5) \rangle, \langle u_2, (0.7, 0.4, 0.6) \rangle, \langle u_3, (0.8, 0.9, 0.2) \rangle \} \end{aligned} \right\}$$

Let us consider the t-norm  $\min\{a, b\}$  and s-norm  $\max\{a, b\}$ . Then,

$$N_1 \tilde{\cup} N_2 = \left\{ \left( \langle x_1, (0.9, 0.7, 0.8) \rangle, \{ \langle u_1, (0.4, 0.5, 0.6) \rangle, \langle u_2, (0.6, 0.1, 0.1) \rangle, \langle u_3, (0.9, 0.4, 0.4) \rangle \} \right), \left( \langle x_2, (0.4, 0.1, 0.2) \rangle, \langle u_1, (0.5, 0.7, 0.1) \rangle, \langle u_2, (0.5, 0.6, 0.3) \rangle, \langle u_3, (0.6, 0.6, 0.5) \rangle \right), \left( \langle x_3, (0.4, 0.1, 0.5) \rangle, \langle u_1, (0.7, 0.4, 0.6) \rangle, \langle u_2, (0.6, 0.6, 0.3) \rangle, \langle u_3, (0.7, 0.1, 0.8) \rangle \right) \right\}$$

and

$$N_1 \tilde{\cap} N_2 = \left\{ \left( \langle x_1, (0.6, 0.7, 0.8) \rangle, \{ \langle u_1, (0.3, 0.6, 0.8) \rangle, \langle u_2, (0.2, 0.3, 0.1) \rangle, \langle u_3, (0.4, 0.7, 0.4) \rangle \} \right), \left( \langle x_2, (0.4, 0.5, 0.8) \rangle, \langle u_1, (0.5, 0.8, 0.5) \rangle, \langle u_2, (0.1, 0.6, 0.8) \rangle, \langle u_3, (0.1, 0.7, 0.9) \rangle \right), \left( \langle x_3, (0.2, 0.8, 0.9) \rangle, \langle u_1, (0.5, 0.9, 0.6) \rangle, \langle u_2, (0.5, 0.6, 0.7) \rangle, \langle u_3, (0.2, 0.5, 0.8) \rangle \right) \right\}$$

**Proposition 3.6.** Let  $N_1, N_2$  and  $N_3$  be any three  $n$ pn–soft sets. Then,

- (1)  $N_1 \tilde{\cup} N_2 = N_2 \tilde{\cup} N_1$
- (2)  $N_1 \tilde{\cap} N_2 = N_2 \tilde{\cap} N_1$
- (3)  $N_1 \tilde{\cup} (N_2 \tilde{\cup} N_3) = (N_1 \tilde{\cup} N_2) \tilde{\cup} N_3$
- (4)  $N_1 \tilde{\cap} (N_2 \tilde{\cap} N_3) = (N_1 \tilde{\cap} N_2) \tilde{\cap} N_3$

**Proof:** The proofs can be easily obtained since the t-norm function and s-norm functions are commutative and associative.

#### 4. MORE ON $n$ pn–SOFT SETS WITH $n$ pn–SOFT MATRICES

In this section, we presented  $n$ pn–soft matrices which are representative of the  $n$ pn–soft sets. Some of it is quoted from [7, 14, 16, 23, 36, 41, 43, 44, 45, 48, 51, 55, 53, 58]

**Definition 4.1.** Let  $U = \{u_1, u_2, \dots, u_m\}$ ,  $E = \{x_1, x_2, \dots, x_n\}$  and  $N$  be an  $n$ pn–soft set over  $N(U)$  as;

$$N = \left\{ \left( \langle x_i, T_N(x_i), I_N(x_i), F_N(x_i) \rangle, \{ \langle u_j, T_{f_N(x_i)}(u_j), I_{f_N(x_i)}(u_j), F_{f_N(x_i)}(u_j) \rangle : u_j \in U \} \right) : x_i \in E \right\}$$

If  $k_i = \langle x_i, T_N(x_i), I_N(x_i), F_N(x_i) \rangle$  and  $a_{ij} = \langle u_j, T_{f_N(x_i)}(u_j), I_{f_N(x_i)}(u_j), F_{f_N(x_i)}(u_j) \rangle$ , then we can define a matrix

$$[k_i | a_{ij}] = \left[ \begin{array}{c|cccc} k_1 & a_{11} & a_{12} & \cdots & a_{1m} \\ k_2 & a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_n & a_{n1} & a_{n2} & \cdots & a_{nm} \end{array} \right]$$

such that  $k_i = (T_N(x_i), I_N(x_i), F_N(x_i)) = (T_i^a, I_i^a, F_i^a)$  and  $a_{ij} = (T_{f_N(x_j)}(u_i), I_{f_N(x_j)}(u_i), F_{f_N(x_j)}(u_i)) = (T_{ij}^a, I_{ij}^a, F_{ij}^a)$ , which is called an  $n \times m$   $n$ pn–soft matrix (or namely NPNS-matrix) of the  $n$ pn–soft set  $N$  over  $U$ .

According to this definition, an  $npn$ -soft set  $N$  is uniquely characterized by matrix  $[k_i|a_{ij}]_{n \times m}$ . Therefore, we shall identify any  $npn$ -soft set with its NPNS-matrix and use these two concepts as interchangeable. The set of all  $n \times m$  NPNS-matrix over  $U$  will be denoted by  $\tilde{N}_{n \times m}$ . From now on we shall delete the subscripts  $n \times m$  of  $[k_i|a_{ij}]_{n \times m}$ , we use  $[k_i|a_{ij}]$  instead of  $[k_i|a_{ij}]_{n \times m}$ , since  $[k_i|a_{ij}] \in \tilde{N}_{n \times m}$  means that  $[k_i|a_{ij}]$  is an  $n \times m$  NPNS-matrix for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

**Example 4.2.** Let  $U = \{u_1, u_2, u_3\}$ ,  $E = \{x_1, x_2, x_3\}$ .  $N$  be an  $npn$ -soft sets over  $U$  as

$$N = \left\{ \left( \langle x_1, (0.5, 0.3, 0.8) \rangle, \{ \langle u_1, (0.2, 0.5, 0.1) \rangle, \langle u_2, (0.7, 0.1, 0.6) \rangle, \langle u_3, (0.8, 0.4, 0.1) \rangle \} \right), \left( \langle x_2, (0.1, 0.6, 0.2) \rangle, \langle u_1, (0.5, 0.8, 0.5) \rangle, \langle u_2, (0.7, 0.2, 0.8) \rangle, \langle u_3, (0.7, 0.6, 0.9) \rangle \right), \left( \langle x_3, (0.5, 0.1, 0.3) \rangle, \langle u_1, (0.6, 0.4, 0.2) \rangle, \langle u_2, (0.9, 0.6, 0.4) \rangle, \langle u_3, (0.7, 0.1, 0.6) \rangle \right) \right\}$$

Then, the NPNS-matrix  $[k_i|a_{ij}]$  is written by

$$[k_i|a_{ij}] = \left[ \begin{array}{c|ccc} (0.5, 0.3, 0.8) & (0.2, 0.5, 0.1) & (0.7, 0.1, 0.6) & (0.8, 0.4, 0.1) \\ (0.1, 0.6, 0.2) & (0.5, 0.8, 0.5) & (0.7, 0.2, 0.8) & (0.7, 0.6, 0.9) \\ (0.5, 0.1, 0.3) & (0.6, 0.4, 0.2) & (0.9, 0.6, 0.4) & (0.7, 0.1, 0.6) \end{array} \right]$$

**Definition 4.3.** An  $npn$ -soft matrix of order  $m \times n$  is said to be a square  $npn$ -soft matrix if  $m = n$  i.e., the number of rows and the number of columns are equal. That means a square- $npn$ -soft matrix is formally equal to an  $npn$ -soft set having the same number of objects and parameters.

**Example 4.4.** Consider the Example 4.2. Here since the  $npn$ -soft matrix contains three rows and three columns, so it is a square- $npn$ -soft matrix.

**Definition 4.5.** The transpose of a square  $npn$ -soft matrix  $[k_i|a_{ij}]$  of order  $m \times n$  is another square  $npn$ -soft matrix of order  $n \times m$  obtained from  $[k_i|a_{ij}]$  by interchanging its rows and columns. It is denoted by  $[k_i|a_{ij}^T]$ . Therefore the  $npn$ -soft set associated with  $[k_i|a_{ij}^T]$  becomes a new  $npn$ -soft set over the same universe and over the same set of parameters.

**Example 4.6.** Consider the Example 4.2. If the NPNS-matrix  $[k_i|a_{ij}]$  is written by

$$[k_i|a_{ij}] = \left[ \begin{array}{c|ccc} (0.5, 0.3, 0.8) & (0.2, 0.5, 0.1) & (0.7, 0.1, 0.6) & (0.8, 0.4, 0.1) \\ (0.1, 0.6, 0.2) & (0.5, 0.8, 0.5) & (0.7, 0.2, 0.8) & (0.7, 0.6, 0.9) \\ (0.5, 0.1, 0.3) & (0.6, 0.4, 0.2) & (0.9, 0.6, 0.4) & (0.7, 0.1, 0.6) \end{array} \right]$$

then, its transpose  $npn$ -soft matrix as;

$$[k_i|a_{ij}^T] = \left[ \begin{array}{c|ccc} (0.5, 0.3, 0.8) & (0.2, 0.5, 0.1) & (0.5, 0.8, 0.5) & (0.6, 0.4, 0.2) \\ (0.1, 0.6, 0.2) & (0.7, 0.1, 0.6) & (0.7, 0.2, 0.8) & (0.9, 0.6, 0.4) \\ (0.5, 0.1, 0.3) & (0.8, 0.4, 0.1) & (0.7, 0.6, 0.9) & (0.7, 0.1, 0.6) \end{array} \right]$$

**Definition 4.7.** A square  $npn$ -soft matrix  $[k_i|a_{ij}]$  of order  $n \times n$  is said to be a symmetric  $npn$ -soft matrix, if its transpose be equal to it, i.e., if  $[k_i|a_{ij}^T] = [k_i|a_{ij}]$ . Hence the  $npn$ -soft matrix  $[k_i|a_{ij}]$  is symmetric, if  $[k_i|a_{ij}] = [k_i|a_{ji}] \forall i, j$ .

**Example 4.8.** Let  $U = \{u_1, u_2, u_3\}$ ,  $E = \{x_1, x_2, x_3\}$ .  $N$  be an  $n$ pn–soft sets as

$$N = \left\{ \left( \langle x_1, (0.5, 0.3, 0.8) \rangle, \{ \langle u_1, (0.2, 0.5, 0.1) \rangle, \langle u_2, (0.5, 0.8, 0.5) \rangle, \langle u_3, (0.6, 0.4, 0.2) \rangle \} \right), \left( \langle x_2, (0.1, 0.6, 0.2) \rangle, \{ \langle u_1, (0.5, 0.8, 0.5) \rangle, \langle u_2, (0.7, 0.2, 0.8) \rangle, \langle u_3, (0.9, 0.6, 0.4) \rangle \} \right), \left( \langle x_3, (0.5, 0.1, 0.3) \rangle, \{ \langle u_1, (0.6, 0.4, 0.2) \rangle, \langle u_2, (0.9, 0.6, 0.4) \rangle, \langle u_3, (0.7, 0.1, 0.6) \rangle \} \right) \right\}$$

Then, the symmetric neutrosophic matrix  $[k_i|a_{ij}]$  is written by

$$[k_i|a_{ij}] = \left[ \begin{array}{c|ccc} (0.5, 0.3, 0.8) & (0.2, 0.5, 0.1) & (0.5, 0.8, 0.5) & (0.6, 0.4, 0.2) \\ (0.1, 0.6, 0.2) & (0.5, 0.8, 0.5) & (0.7, 0.2, 0.8) & (0.9, 0.6, 0.4) \\ (0.5, 0.1, 0.3) & (0.6, 0.4, 0.2) & (0.9, 0.6, 0.4) & (0.7, 0.1, 0.6) \end{array} \right]$$

**Definition 4.9.** Let  $[k_i|a_{ij}] \in \tilde{N}_{n \times m}$ . Then  $[k_i|a_{ij}]$  is called

- (1) A zero  $n$ pn–soft matrix, denoted by  $[\tilde{0}]$ , if  $k_i = (0, 1, 1)$  and  $a_{ij} = (0, 1, 1)$  for all  $i$  and  $j$ .
- (2) A universal  $n$ pn–soft matrix, denoted by  $[\tilde{1}]$ , if  $k_i = (1, 0, 0)$  and  $a_{ij} = (1, 0, 0)$  for all  $i$  and  $j$ .

**Example 4.10.** Let  $U = \{u_1, u_2, u_3\}$ ,  $E = \{x_1, x_2, x_3\}$ . Then, a zero  $n$ pn–soft matrix  $[k_i|a_{ij}]$  is written by

$$[\tilde{0}] = \left[ \begin{array}{c|ccc} (0, 1, 1) & (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \\ (0, 1, 1) & (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \\ (0, 1, 1) & (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \end{array} \right]$$

and a universal  $n$ pn–soft matrix  $[k_i|a_{ij}]$  is written by

$$[\tilde{1}] = \left[ \begin{array}{c|ccc} (1, 0, 0) & (1, 0, 0) & (1, 0, 0) & (1, 0, 0) \\ (1, 0, 0) & (1, 0, 0) & (1, 0, 0) & (1, 0, 0) \\ (1, 0, 0) & (1, 0, 0) & (1, 0, 0) & (1, 0, 0) \end{array} \right]$$

**Definition 4.11.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $[k_i|a_{ij}]$  is an NS-submatrix of  $[\acute{k}_i|b_{ij}]$ , denoted,  $[k_i|a_{ij}] \tilde{\subseteq} [\acute{k}_i|b_{ij}]$ , if  $T_i^b \geq T_i^a$ ,  $I_i^a \geq I_i^b$ ,  $F_i^a \geq F_i^b$ ,  $T_{ij}^b \geq T_{ij}^a$ ,  $I_{ij}^a \geq I_{ij}^b$  and  $F_{ij}^a \geq F_{ij}^b$ , for all  $i$  and  $j$ .
- (2)  $[k_i|a_{ij}]$  is a proper NS-submatrix of  $[\acute{k}_i|b_{ij}]$ , denoted,  $[k_i|a_{ij}] \tilde{\subset} [\acute{k}_i|b_{ij}]$ , if  $T_i^a \geq T_i^b$ ,  $I_i^a \leq I_i^b$ ,  $F_i^a \leq F_i^b$ ,  $T_{ij}^a \geq T_{ij}^b$ ,  $I_{ij}^a \leq I_{ij}^b$  and  $F_{ij}^a \leq F_{ij}^b$  for at least  $T_i^a > T_i^b$  and  $I_i^a < I_i^b$ ,  $F_i^a < F_i^b$ ,  $T_{ij}^a > T_{ij}^b$  and  $I_{ij}^a < I_{ij}^b$  and  $F_{ij}^a < F_{ij}^b$  for all  $i$  and  $j$ .
- (3)  $[k_i|a_{ij}]$  and  $[\acute{k}_i|b_{ij}]$  are IFS equal matrices, denoted by  $[k_i|a_{ij}] = [\acute{k}_i|b_{ij}]$ , if  $k_i = \acute{k}_i$  and  $a_{ij} = b_{ij}$  for all  $i$  and  $j$ .

**Definition 4.12.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1) Union of  $[k_i|a_{ij}]$  and  $[\acute{k}_i|b_{ij}]$ , denoted,  $[\acute{k}_i|c_{ij}] = [k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}]$ , such that  $([(T_i^c, I_i^c, F_i^c)] \wedge [(T_{ij}^c, I_{ij}^c, F_{ij}^c)])$  where  $T_i^c = s\{T_i^a, T_i^b\}$ ,  $I_i^c = t\{I_i^a, I_i^b\}$ ,  $F_i^c = \min\{F_i^a, F_i^b\}$ ,  $T_{ij}^c = s\{T_{ij}^a, T_{ij}^b\}$ ,  $I_{ij}^c = t\{I_{ij}^a, I_{ij}^b\}$  and  $F_{ij}^c = t\{F_{ij}^a, F_{ij}^b\}$  for all  $i$  and  $j$ .

- (2) Intersection of  $[k_i|a_{ij}]$  and  $[\acute{k}_i|b_{ij}]$ , denoted,  $[\acute{\acute{k}}_i|d_{ij}] = [k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}]$ , such that  $([(T_i^d, I_i^d, F_i^d)] \wedge [(T_{ij}^d, I_{ij}^d, F_{ij}^d)])$  where  $T_i^d = t\{T_i^a, T_i^b\}$ ,  $I_i^d = s\{I_i^a, I_i^b\}$ ,  $F_i^d = s\{F_i^a, F_i^b\}$ ,  $T_{ij}^d = t\{T_{ij}^a, T_{ij}^b\}$ ,  $I_{ij}^d = s\{I_{ij}^a, I_{ij}^b\}$  and  $F_{ij}^d = s\{F_{ij}^a, F_{ij}^b\}$  for all  $i$  and  $j$ .
- (3) Complement of  $[k_i|a_{ij}]$ , denoted  $[\acute{\acute{k}}_i|e_{ij}] = [k_i|a_{ij}]^c$ , such that  $[(T_i^e, I_i^e, F_i^e)] \wedge [(T_{ij}^e, I_{ij}^e, F_{ij}^e)]$  where  $T_i^e = F_i^a$ ,  $I_i^e = 1 - I_i^a$ ,  $F_i^e = T_i^a$ ,  $T_{ij}^e = F_{ij}^a$ ,  $I_{ij}^e = 1 - I_{ij}^a$  and  $F_{ij}^e = T_{ij}^a$  for all  $i$  and  $j$ .

**Example 4.13.** Consider the Example 3.5 and the t-norm  $\min\{a, b\}$  and s-norm  $\max\{a, b\}$ . Then,

$$[k_i|a_{ij}] \tilde{\cup} [k_i|b_{ij}] = \left[ \begin{array}{c|ccc} (0.9, 0.7, 0.8) & (0.4, 0.5, 0.6) & (0.6, 0.1, 0.1) & (0.9, 0.4, 0.4) \\ (0.4, 0.1, 0.2) & (0.5, 0.7, 0.1) & (0.5, 0.6, 0.3) & (0.6, 0.6, 0.5) \\ (0.4, 0.1, 0.5) & (0.7, 0.4, 0.6) & (0.6, 0.6, 0.3) & (0.7, 0.1, 0.8) \end{array} \right]$$

$$[k_i|a_{ij}] \tilde{\cap} [k_i|b_{ij}] = \left[ \begin{array}{c|ccc} (0.6, 0.7, 0.8) & (0.3, 0.6, 0.8) & (0.2, 0.3, 0.1) & (0.4, 0.7, 0.4) \\ (0.4, 0.5, 0.8) & (0.5, 0.8, 0.5) & (0.1, 0.6, 0.8) & (0.1, 0.7, 0.9) \\ (0.2, 0.8, 0.9) & (0.5, 0.9, 0.6) & (0.5, 0.6, 0.7) & (0.2, 0.5, 0.8) \end{array} \right]$$

and

$$[k_i|a_{ij}]^c = \left[ \begin{array}{c|ccc} (0.8, 0.3, 0.6) & (0.6, 0.5, 0.4) & (0.1, 0.9, 0.6) & (0.4, 0.6, 0.3) \\ (0.2, 0.9, 0.4) & (0.5, 0.2, 0.5) & (0.8, 0.4, 0.5) & (0.9, 0.4, 0.6) \\ (0.5, 0.9, 0.4) & (0.6, 0.6, 0.5) & (0.7, 0.4, 0.6) & (0.8, 0.9, 0.2) \end{array} \right].$$

**Definition 4.14.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}] \in \tilde{N}_{n \times m}$ . Then  $[k_i|a_{ij}]$  and  $[\acute{k}_i|b_{ij}]$  are disjoint, if  $[k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|b_{ij}] = [\tilde{0}]$  for all  $i$  and  $j$ .

**Proposition 4.15.** Let  $[k_i|a_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $([k_i|a_{ij}]^c)^c = [k_i|a_{ij}]$
- (2)  $[\tilde{0}]^c = [\tilde{1}]$ .

**Proposition 4.16.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $[k_i|a_{ij}] \subseteq [\tilde{1}]$
- (2)  $[\tilde{0}] \tilde{\subseteq} [k_i|a_{ij}]$
- (3)  $[k_i|a_{ij}] \tilde{\subseteq} [k_i|a_{ij}]$
- (4)  $[k_i|a_{ij}] \tilde{\subseteq} [\acute{k}_i|b_{ij}] \wedge [\acute{k}_i|b_{ij}] \tilde{\subseteq} [\acute{k}_i|c_{ij}] \Rightarrow [k_i|a_{ij}] \tilde{\subseteq} [\acute{k}_i|c_{ij}]$

**Proposition 4.17.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}], [\acute{k}_i|c_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $[k_i|a_{ij}] = [\acute{k}_i|b_{ij}]$  and  $[\acute{k}_i|b_{ij}] = [\acute{k}_i|c_{ij}] \Leftrightarrow [k_i|a_{ij}] = [\acute{k}_i|c_{ij}]$
- (2)  $[k_i|a_{ij}] \tilde{\subseteq} [\acute{k}_i|b_{ij}]$  and  $[\acute{k}_i|b_{ij}] \tilde{\subseteq} [k_i|a_{ij}] \Leftrightarrow [k_i|a_{ij}] = [\acute{k}_i|b_{ij}]$

**Proposition 4.18.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}], [\acute{k}_i|c_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $[k_i|a_{ij}] \tilde{\cup} [k_i|a_{ij}] = [k_i|a_{ij}]$
- (2)  $[k_i|a_{ij}] \tilde{\cup} [\tilde{0}] = [k_i|a_{ij}]$
- (3)  $[k_i|a_{ij}] \tilde{\cup} [\tilde{1}] = [\tilde{1}]$
- (4)  $[k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}] = [\acute{k}_i|b_{ij}] \tilde{\cup} [k_i|a_{ij}]$

$$(5) ([k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}]) \tilde{\cup} [\acute{k}_i|c_{ij}] = [k_i|a_{ij}] \tilde{\cup} ([\acute{k}_i|b_{ij}] \tilde{\cup} [\acute{k}_i|c_{ij}])$$

**Proposition 4.19.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}], [\acute{k}_i|c_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $[k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|a_{ij}] = [k_i|a_{ij}]$
- (2)  $[k_i|a_{ij}] \tilde{\cap} [\tilde{0}] = [\tilde{0}]$
- (3)  $[k_i|a_{ij}] \tilde{\cap} [\tilde{1}] = [k_i|a_{ij}]$
- (4)  $[k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|b_{ij}] = [\acute{k}_i|b_{ij}] \tilde{\cap} [k_i|a_{ij}]$
- (5)  $([k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|b_{ij}]) \tilde{\cap} [\acute{k}_i|c_{ij}] = [k_i|a_{ij}] \tilde{\cap} ([\acute{k}_i|b_{ij}] \tilde{\cap} [\acute{k}_i|c_{ij}])$

**Proposition 4.20.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}] \in \tilde{N}_{n \times m}$ . Then De Morgan's laws are valid

- (1)  $([k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}])^c = [k_i|a_{ij}]^c \tilde{\cap} [\acute{k}_i|b_{ij}]^c$
- (2)  $([k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|b_{ij}])^c = [k_i|a_{ij}]^c \tilde{\cup} [\acute{k}_i|b_{ij}]^c$

**Proof:** *i.*

$$\begin{aligned} ([k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}])^c &= (([T_i^d, I_i^d, F_i^d])^c \wedge [(T_{ij}^c, I_{ij}^c, F_{ij}^c)])^c \\ &= [(s\{T_i^a, T_i^b\}, t\{I_i^a, I_i^b\}, t\{F_{ij}^a, F_{ij}^b\})]^c \\ &\quad \wedge [(s\{T_{ij}^a, T_{ij}^b\}, t\{I_{ij}^a, I_{ij}^b\}, t\{F_{ij}^a, F_{ij}^b\})]^c \\ &= [(s\{F_i^a, F_i^b\}, 1 - t\{I_i^a, I_i^b\}, t\{T_{ij}^a, T_{ij}^b\})] \\ &\quad \wedge [(s\{F_{ij}^a, F_{ij}^b\}, 1 - t\{I_{ij}^a, I_{ij}^b\}, t\{T_{ij}^a, T_{ij}^b\})] \\ &= [(t\{F_i^a, F_i^b\}, s\{1 - I_i^a, 1 - I_i^b\}, s\{T_i^a, T_i^b\})] \\ &\quad \wedge [(t\{F_{ij}^a, F_{ij}^b\}, s\{1 - I_{ij}^a, 1 - I_{ij}^b\}, s\{T_{ij}^a, T_{ij}^b\})] \\ &= (([T_i^c, I_i^c, F_i^c])^c \wedge [(T_{ij}^c, I_{ij}^c, F_{ij}^c)])^c \\ &= [k_i|a_{ij}]^c \tilde{\cap} [\acute{k}_i|b_{ij}]^c \end{aligned}$$

*i.*

$$\begin{aligned} ([k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|b_{ij}])^c &= (([T_i^d, I_i^d, F_i^d])^c \wedge [(T_{ij}^d, I_{ij}^d, F_{ij}^d)])^c \\ &= [(t\{T_i^a, T_i^b\}, s\{I_i^a, I_i^b\}, s\{F_{ij}^a, F_{ij}^b\})]^c \\ &\quad \wedge [(t\{T_{ij}^a, T_{ij}^b\}, s\{I_{ij}^a, I_{ij}^b\}, s\{F_{ij}^a, F_{ij}^b\})]^c \\ &= [(t\{F_i^a, F_i^b\}, 1 - s\{I_i^a, I_i^b\}, s\{T_{ij}^a, T_{ij}^b\})] \\ &\quad \wedge [(t\{F_{ij}^a, F_{ij}^b\}, 1 - s\{I_{ij}^a, I_{ij}^b\}, s\{T_{ij}^a, T_{ij}^b\})] \\ &= [(s\{F_i^a, F_i^b\}, t\{1 - I_i^a, 1 - I_i^b\}, t\{T_i^a, T_i^b\})] \\ &\quad \wedge [(s\{F_{ij}^a, F_{ij}^b\}, t\{1 - I_{ij}^a, 1 - I_{ij}^b\}, t\{T_{ij}^a, T_{ij}^b\})] \\ &= (([T_i^d, I_i^d, F_i^d])^c \wedge [(T_{ij}^d, I_{ij}^d, F_{ij}^d)])^c \\ &= [k_i|a_{ij}]^c \tilde{\cup} [\acute{k}_i|b_{ij}]^c \end{aligned}$$

**Proposition 4.21.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ij}], [\acute{k}_i|c_{ij}] \in \tilde{N}_{n \times m}$ . Then

- (1)  $[k_i|a_{ij}] \tilde{\cap} ([\acute{k}_i|b_{ij}] \tilde{\cup} [\acute{k}_i|c_{ij}]) = ([k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|b_{ij}]) \tilde{\cup} ([k_i|a_{ij}] \tilde{\cap} [\acute{k}_i|c_{ij}])$
- (2)  $[k_i|a_{ij}] \tilde{\cup} ([\acute{k}_i|b_{ij}] \tilde{\cap} [\acute{k}_i|c_{ij}]) = ([k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|b_{ij}]) \tilde{\cap} ([k_i|a_{ij}] \tilde{\cup} [\acute{k}_i|c_{ij}])$

**Definition 4.22.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ik}] \in \tilde{N}_{m \times n}$ . Then *And*-product of  $[k_i|a_{ij}]$  and  $[\acute{k}_i|b_{ij}]$ , denoted by  $[k_i|a_{ij}] \wedge [\acute{k}_i|b_{ik}]$ , is defined by

$$\wedge : \tilde{N}_{m \times n} \times \tilde{N}_{m \times n} \rightarrow \tilde{N}_{m \times n^2}$$

$$[k_i|a_{ij}] \wedge [\acute{k}_i|b_{ik}] = [\acute{k}_i|c_{ip}] = [ \langle T_i^c, I_i^c, F_i^c \rangle | \langle T_{ip}^c, I_{ip}^c, F_{ip}^c \rangle ]$$

where

$$T_i^c = t(T_i^a, T_i^b), I_i^c = s(I_i^a, I_i^b) \text{ and } F_i^c = s(F_i^a, F_i^b)$$

and where

$$T_{ip}^c = t(T_{ij}^a, T_{jk}^b), I_{ip}^c = s(I_{ij}^a, I_{jk}^b) \text{ and } F_{ip}^c = s(F_{ij}^a, F_{jk}^b) \text{ such that } p = n(j-1) + k$$

**Definition 4.23.** Let  $[k_i|a_{ij}], [\acute{k}_i|b_{ik}] \in \tilde{N}_{m \times n}$ . Then *Or*-product of  $[k_i|a_{ij}]$  and  $[\acute{k}_i|b_{ij}]$ , denoted by  $[k_i|a_{ij}] \vee [\acute{k}_i|b_{ik}]$ , is defined by

$$\vee : \tilde{N}_{m \times n} \times \tilde{N}_{m \times n} \rightarrow \tilde{N}_{m \times n^2}$$

$$[k_i|a_{ij}] \vee [\acute{k}_i|b_{ik}] = [\acute{k}_i|c_{ip}] = [ \langle T_i^c, I_i^c, F_i^c \rangle | \langle T_{ip}^c, I_{ip}^c, F_{ip}^c \rangle ]$$

where

$$T_i^c = s(T_i^a, T_i^b), I_i^c = t(I_i^a, I_i^b) \text{ and } F_i^c = t(F_i^a, F_i^b)$$

and where

$$T_{ip}^c = s(T_{ij}^a, T_{jk}^b), I_{ip}^c = t(I_{ij}^a, I_{jk}^b) \text{ and } F_{ip}^c = t(F_{ij}^a, F_{jk}^b) \text{ such that } p = n(j-1) + k$$

**Proposition 4.24.** Let  $[a_{ij}], [b_{ij}], [c_{ij}] \in \tilde{N}_{m \times n}$ . Then the De Morgan's types of results are true.

- (1)  $([a_{ij}] \vee [b_{ij}])^c = [a_{ij}]^c \wedge [b_{ij}]^c$
- (2)  $([a_{ij}] \wedge [b_{ij}])^c = [a_{ij}]^c \vee [b_{ij}]^c$

### 5. NPNSS-AGGREGATION OPERATOR

In this section, we propose an aggregate fuzzy set of an  $npn$ -soft set. We also define NPNSS-aggregation operator that produce an aggregate fuzzy set from an  $npn$ -soft set and its neutrosophic parameter set. Some of it is quoted from [14, 16, 23, 36]

**Definition 5.1.** Let  $N_1$  be any an  $npn$ -soft sets. Then NPNSS-aggregation operator, denoted by  $NPNSS_{agg}$ , is defined by

$$NPNSS_{agg} : N(E) \times NPNS(U) \rightarrow F(U)$$

$$NPNSS_{agg}(X, N_1) = N_1^*$$

where

$$N_1^* = \{ \mu_{N_1^*}(u) / u : u \in U \}$$

which is a fuzzy set over  $U$ . The value  $N_1^*$  is called aggregate fuzzy set of the  $N_1$ .

Here, the membership degree  $\mu_{N_1^*}(u)$  of  $u$  is defined as follows

$$\mu_{N_1^*}(u) = \frac{1}{|E|} \sum_{x \in E} (|T_{N_1}(x) + I_{N_1}(x) - F_{N_1}(x)|) (|T_{f_N(x)}(u) + I_{f_N(x)}(u) - F_{f_N(x)}(u)|)$$

where  $|E|$  is the cardinality of  $E$ .

**Algorithm**

The algorithm for the solution is given below

**Step 1:** Choose feasible an *nfn*–soft set  $N_1$  over  $U$ ,

**Step 2:** Find the aggregate fuzzy set  $N_1^*$  of  $N_1$ ,

**Step 3:** Find the largest membership grade  $\max\{\mu_{N_1^*}(u)\}$ .

**Case study:** In this study, we have proposed a numerical application for the method. Let  $U = \{u_1, u_2, u_3\}$ ,  $E = \{x_1, x_2, x_3\}$ . Then,

**Step 1:** We chosen feasible an *nfn*–soft set  $N_1$  over  $U$  as,

$$N_1 = \left\{ \left( \langle x_1, (0.6, 0.7, 0.8) \rangle, \{ \langle u_1, (0.4, 0.5, 0.6) \rangle, \langle u_2, (0.6, 0.1, 0.1) \rangle, \langle u_3, (0.3, 0.4, 0.4) \rangle \} \right), \left( \langle x_2, (0.4, 0.1, 0.2) \rangle, \{ \langle u_1, (0.5, 0.8, 0.5) \rangle, \langle u_2, (0.5, 0.6, 0.8) \rangle, \langle u_3, (0.6, 0.6, 0.9) \rangle \} \right), \left( \langle x_3, (0.4, 0.7, 0.5) \rangle, \{ \langle u_1, (0.5, 0.4, 0.6) \rangle, \langle u_2, (0.6, 0.6, 0.7) \rangle, \langle u_3, (0.2, 0.1, 0.8) \rangle \} \right) \right\}$$

**Step 2:**We found the aggregate fuzzy set  $N_1^*$  of  $N_1$  as,

$$N_1^* = \{u_1/0.13, u_2/0.13, u_3/0.16\}$$

**Step 3:** Finally, the largest membership grade can be chosen by  $\max\{\mu_{N_1^*}(u)\}$  which means that the  $u_3$  has the largest membership grade, hence it is selected for decision making.

6. CONCLUSION

In this paper we define the notion of soft sets, is called *nfn*–soft sets, in a new way by using neutrosophic sets. The *nfn*–soft sets generalizes the concept of the other soft sets such as; fuzzy soft sets, intuitionistic fuzzy soft sets, neutrosophic soft sets, fuzzy parameterized soft sets, intuitionistic fuzzy parameterized soft sets, neutrosophic parameterized soft sets,.... Then, we introduce some definitions and operations on *nfn*–soft sets and propose the concept of *nfn*–soft matrix and their operators which are more functional to make theoretical studies in the *nfn*–soft set theory. Finally, we proposed the decision making method on the *nfn*–soft set theory and provided an example that demonstrated that this method can be successfully worked. The approach should be more comprehensive in the future to solve the problems that contain uncertainty. Researchers, can be just study on *nfn*–soft sets instead of similar work on separately other soft sets that is in Figure 1. *nfn*–soft set can be expanding with new research subjects as; neutrosophic metric spaces and smooth topological spaces, neutrosophic numbers and arithmetical operations, relational structures, relational equations, similarity relations, orderings, probability, logical operations, implicators, multi-valued mappings, algebraic structures and models, cognitive maps, matrix, graph, fusion rules, relational maps, relational databases, image processing, linguistic variables, decision making and preference structures, expert systems, reliability theory, soft computing techniques and so on.

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