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Fuzzy orbit* continuous mappings

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Abstract. The purpose of this paper is to introduce the concepts of fuzzy orbit open set, fuzzy orbit continuous, almost*-fuzzy orbit continuous, weakly*-fuzzy orbit continuous, fuzzy orbit* continuous with some interesting properties.

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Keywords: Fuzzy orbit open set, Fuzzy orbit continuous, Almost*-fuzzy orbit continuous, Weakly*-fuzzy orbit continuous, Fuzzy orbit* continuous.

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1. Introduction

Fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh[11]. Fuzzy sets have applications in many fields such as information [8] and control[10]. The theory of fuzzy topological spaces was introduced and developed by Chang[4] and since then various notions in classical topology have been extended to fuzzy topological spaces[1, 2, 3]. The concept of slightly continuous mappings was introduced by Singal and Jain[7]. The concept of orbit function in general metric space was introduced by R. L. Devaney[5]. In this paper the concepts of fuzzy orbit open set, almost*-fuzzy orbit continuous, weakly*-fuzzy orbit continuous, fuzzy orbit* continuous are introduced. Some interesting properties and characterizations of fuzzy orbit* continuous mappings are discussed with necessary examples.

2. Preliminaries

Throughout this paper, the symbol I will denote the unit interval and X will denote a nonempty set.

Definition 2.1 ([11]). A fuzzy set in X is a function with domain X and values in I, i.e., is an element of I^X.

Definition 2.2 ([4]). A fuzzy set U in a fuzzy topological space (X, T) is a neighborhood (nbhd for short) of a fuzzy set A, if there exists an open fuzzy set O such that $A \subset O \subset U$.

Definition 2.3 ([9]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be almost*-fuzzy continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq int(cl(\mu))$.

Definition 2.4 ([9]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be fuzzy weakly* continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq cl(\mu)$.

Definition 2.5 ([9]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be slightly fuzzy continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy clopen set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mu$.

Definition 2.6 ([6]). Let $(D, \geqslant)$ be directed set. Let X be an ordinary set. Let $\mathfrak{f}$ be the collection of all fuzzy points in X. The function $S : D \to \mathfrak{f}$ is called a fuzzy net in X. In otherwords, a fuzzy net is a pair $(S, \geqslant)$ such that S is a function : $D \to \mathfrak{f}$ and $\geqslant$ directs the domain of S. For $n \in D$, $S(n)$ is often denoted by S_n and hence a net S is often denoted by $\{S_n, n \in D\}$.

Definition 2.7 ([4]). A sequence of fuzzy sets, say $\{A_n; n = 1, 2, ...\}$, is eventually contained in a fuzzy set A, if there is an integer m such that, if $n \geqslant m$, then $A_n \subset A$.

Definition 2.8 ([1]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. For a mapping $f : (X, T) \to (Y, S)$, the graph $g : (X, T) \to (X, T) \times (Y, S)$ of f is defined by $g(x) = (x, f(x))$, for each $x \in X$.

Definition 2.9 ([1]). A mapping $f : X \to Y$ from a fuzzy topological space$X$ to another fuzzy topological space Y is called a fuzzy weakly continuous mapping, if for each fuzzy open set μ of Y, $f^{-1}(\mu) \leq int(f^{-1}(cl\mu))$.

Definition 2.10 ([5]). Orbit of a point x in X under the mapping f is

$$O_f(x) = \{x, f(x), f^2(x), ...\}.$$

Theorem 2.11 ([4]). *Let f be a function from X to Y.*

(1) $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$, *for any fuzzy set λinY.*
(2) $1 - f(\lambda) \subset f(1 - \lambda)$, *for any fuzzy set λinX.*
(3) $\mu_1 \subset \mu_2 \Rightarrow f^{-1}(\mu_1) \subset f^{-1}(\mu_2)$, *where μ_1 and μ_2 are fuzzy sets in Y.*
(4) $\lambda_1 \subset \lambda_2 \Rightarrow f(\lambda_1) \subset f(\lambda_2)$, *where λ_1 and λ_2 are fuzzy sets in X.*
(5) $ff^{-1}(\mu) \subset \mu$, *for any fuzzy set μinY.*
(6) $\lambda \subset f^{-1}(f(\lambda))$, *for any fuzzy set λinX.*
(7) *Let f be a function from X to Y and g be a function from Y to Z. Then $(g \circ f)^{-1}\eta = f^{-1}(g^{-1}(\eta))$ for any fuzzy set ηinZ, where $g \circ f$ is the composition of g and f.*

3. Properties and characterization of fuzzy orbit* continuous mappings

Definition 3.1. Let X be a nonempty set and let f : X→ X be any mapping. Let λ be any fuzzy set in X. The fuzzy orbit $O_f(\lambda)$ of λ under the mapping f is defined as $O_f(\lambda) = \{\lambda, f(\lambda), f^2(\lambda), ...\}$.

Definition 3.2. Let X be a nonempty set and let f : X→ X be any mapping. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \{\lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge ...$ $\}$the intersection of all members of $O_f(\lambda)$.

Example 3.3. Let X = $\{a, b, c\}$. Define a fuzzy set λ: X → [0,1] as follows $\lambda(a)$ = 0.5, $\lambda(b)$ = 0.6, $\lambda(c)$ = 0.7. Define f : X → X as f (a) = b, f(b) = c, f(c) = a. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge ....$ $FO_f(\lambda)$(a) = 0.5, $FO_f(\lambda)$(b) = 0.5, $FO_f(\lambda)$(c) = 0.5.

Definition 3.4. Let (X, T) be a fuzzy topological space. Let f : X → X be any mapping. The fuzzy orbit set under the mapping f which is in fuzzy topology T is called fuzzy orbit open set under the mapping f. Its complement is called a fuzzy orbit closed set under the mapping f.

Example 3.5. Let X = $\{a, b, c\}$. Define T = {0, 1, λ, γ} where λ, γ: X → [0,1] are defined as $\lambda(a)$ = 0.3, $\lambda(b)$ = 0.3, $\lambda(c)$ = 0.1, $\gamma(a)$ = 0.3, $\gamma(b)$ = 0, $\gamma(c)$ =0. Define f : X → X as f(a) = a, f(b) = a, f(c) = a. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge ...$. $FO_f(\lambda) = \gamma$. Then γ is a fuzzy orbit open set under the mapping f.

Definition 3.6. Let (X, T) be a fuzzy topological space. Let f : X → X be any mapping. The fuzzy orbit under the mapping f in a fuzzy topological space (X, T) is said to be fuzzy orbit clopen set under the mapping f, if it is both fuzzy orbit open and fuzzy orbit closed under the mapping f.

Definition 3.7. A fuzzy set λ in a fuzzy topological space (X, T) is a fuzzy orbit neighborhood, or FOnbhd for short, of a fuzzy set μ, if there exists a fuzzy orbit open set α such that $\mu \subset \alpha \subset \lambda$.

Definition 3.8. Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f : X → X be a mapping. A mapping g : (X, T) → (Y, S) is said to be fuzzy orbit continuous, if the inverse image of every fuzzy open set in (Y, S) is fuzzy orbit open set under the mapping f in (X, T).

Definition 3.9. Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f_1 : X → X and f_2 : Y → Y be any two mappings. A mapping g : (X, T) → (Y, S) is said to be almost*-fuzzy orbit continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy orbit open set μ under the mapping f_2 with $g(\alpha) \leq \mu$, there exists a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that $g(\lambda) \leq$ int (cl(μ)).

Definition 3.10. Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f_1 : X → X and f_2 : Y → Y be any two mappings. A mapping g : (X, T) → (Y, S) is said to be weakly*-fuzzy orbit continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy orbit open set μ under the mapping f_2 with $g(\alpha) \leq \mu$, there exists a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that $g(\lambda) \leq$ cl(μ).

Definition 3.11. Let (X, T)and (Y, S) be any two fuzzy topological spaces. Let f_1 : X → X and f_2 : Y → Y be any two mappings. A mapping g : (X, T) → (Y, S) is said to be fuzzy orbit* continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy orbit open set μ under the mapping f_2 with $g(\alpha) \leq \mu$, there exists a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that $g(\lambda) \leq \mu$.

Definition 3.12. Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let λ and μ be fuzzy orbit open sets under the mapping f_1 and f_2 respectively. The product fuzzy orbit open set $(\lambda \times \mu)$: $X \times Y$ → I is defined by $(\lambda \times \mu)$ (x, y) = M (λ(x), (μ(y)) ∀ (x, y) ∈ $X \times Y$.(where M denote the minimum).

Remark 3.13. **Fuzzy orbit continuous** ⟶ **Fuzzy orbit* continuous**

↘ ↙

Almost*-fuzzy orbit continuous

↓

Weakly*-fuzzy orbitcontinuous

Proposition 3.14. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If g : (X, T) → (Y,S) is fuzzy orbit continuous, then g is almost*-fuzzy orbit continuous.*

Proof. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let $\alpha \in I^X$ be any fuzzy set and μ be any fuzzy orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. By Theorem 2.11, $\alpha \leq g^{-1}(g(\alpha)) \leq g^{-1}(\mu)$. Then $\alpha \leq g^{-1}(\mu)$. Since g is fuzzy orbit continuous, $\alpha \leq g^{-1}(\mu) = \lambda$, λ is a fuzzy orbit open set under the mapping f_1. By Theorem 2.11, $gg^{-1}(\mu) \leq \mu$. Thus $g(\lambda) = gg^{-1}(\mu) \leq \mu$. Since μ is fuzzy orbit open, μ is fuzzy open and hence $\mu \leq$ int(cl(μ)) which implies that $g(\lambda) \leq$ int(cl(μ)). So g is almost*-fuzzy orbit continuous. □

Remark 3.15. The converse of the Proposition 3.14 need not be true as shown in the following example.

Example 3.16. Let X = { a, b, c } = Y. Define T = { 0, 1, λ, λ_1 } and S = { 0, 1, μ, μ_1, μ_2, μ_3 } where λ, λ_1, μ, μ_1, μ_2, μ_3 : X → [0, 1] are such that λ (a) = 0, λ (b) = 0, λ (c) = 0.3, λ_1 (a) = 0.7, λ_1 (b) = 0.6, λ_1 (c) = 0.3,μ (a) = 0.6, μ (b) = 0.6, μ (c) = 0.6, μ_1 (a) = 0.6, μ_1 (b) = 0.7, μ_1 (c) = 0.8, μ_2 (a) = 0.3, μ_2 (b) = 0.3, μ_2 (c) = 0.3 μ_3(a) = 0.3, μ_3 (b) = 0.4, μ_3 (c) = 0.5. Clearly (X, T) and (Y, S) are fuzzy topological spaces.

Define g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = c, f_1(b) =c, f_1(c) = c and f_2(a) = b, f_2(b) =c, f_2(c) = a. Let α : X → [0,1] be any fuzzy set such that α (a) = 0, α (b) = 0, α (c) = 0.2. For the fuzzy orbit open set μ under the mapping f_2 in (Y, S) with $g(\alpha) \leqslant \mu$.

Now the fuzzy orbit open set λ under the mapping f_1 in (X, T) with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant$ int(cl (μ)). Then g is almost*-fuzzy orbit continuous.

Now the fuzzy open sets μ_1, μ_3 in (Y, S), but $g^{-1}(\mu_1)$ and $g^{-1}(\mu_3)$ are not fuzzy orbit open under the mapping f_1 in (X, T). Thus g is not fuzzy orbit continuous.

Proposition 3.17. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If g : (X, T) → (Y,S) is fuzzy orbit continuous, then g is weakly*-fuzzy orbit continuous.*

Proof. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let $\alpha \in I^X$ be any fuzzy set and μ be any fuzzy orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. By Theorem 2.11, $\alpha \leq g^{-1}(g(\alpha)) \leq g^{-1}(\mu)$. Then $\alpha \leq g^{-1}(\mu)$. Since g is fuzzy orbit continuous, $\alpha \leq g^{-1}(\mu) = \lambda$, λ is a fuzzy orbit open set under the mapping f_1. By Theorem 2.11, $gg^{-1}(\mu) \leq \mu$. Thus $g(\lambda) = gg^{-1}(\mu) \leq \mu \leq \text{cl}(\mu)$. So g is weakly*-fuzzy orbit continuous. □

Remark 3.18. The converse of the Proposition 3.17 need not be true as shown in the following example.

Example 3.19. Let X = { a, b, c } = Y. Define T = { 0, 1, λ, λ_1 } and S = { 0, 1, μ, μ_1, μ_2, μ_3 } where λ, λ_1, μ, μ_1, μ_2, μ_3 : X → [0, 1] are such that λ (a) = 0, λ (b) = 0.4, λ (c) = 0, λ_1 (a) = 0.7, λ_1 (b) = 0.5, λ_1 (c) = 0.4,μ (a) = 0.3, μ (b) = 0.3, μ (c) = 0.3, μ_1 (a) = 0.6, μ_1 (b) = 0.8, μ_1 (c) = 0.9, μ_2 (a) = 0.6, μ_2 (b) = 0.6, μ_2 (c) = 0.6 μ_3(a) = 0.3, μ_3 (b) = 0.4, μ_3 (c) = 0.5. Clearly (X, T) and (Y, S) are fuzzy topological spaces.

Define g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = b, f_1(b) =b, f_1(c) = b and f_2(a) = b, f_2(b) =c, f_2(c) = a. Let α : X → [0,1] be any fuzzy set such that α (a) = 0, α (b) = 0.2, α (c) = 0. For the fuzzy orbit open set μ under the mapping f_2 in (Y, S) with $g(\alpha) \leqslant \mu$.

Now the fuzzy orbit open set λ under the mapping f_1 in (X, T) with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant$ cl (μ). Then g is weakly*-fuzzy orbit continuous.

Now the fuzzy open sets μ_1, μ_3 in (Y, S), but $g^{-1}(\mu_1)$ and $g^{-1}(\mu_3)$ are not fuzzy orbit open under the mapping f_1 in (X, T). Thus g is not fuzzy orbit continuous.

Proposition 3.20. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If g : (X, T) → (Y,S) is almost*-fuzzy orbit continuous, then g is weakly*-fuzzy orbit continuous.*

Proof. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let $\alpha \in I^X$ be any fuzzy set and μ be any fuzzy orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. Since g is almost*-fuzzy orbit continuous, there exists a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that $g(\lambda) \leq \text{int}(\text{cl}(\mu))$, which implies that $g(\lambda) \leq \text{cl}(\mu)$. Then g is weakly*-fuzzy orbit continuous. □

Remark 3.21. The converse of the Proposition 3.20 need not be true as shown in the following example.

Example 3.22. Let X = { a, b, c } = Y. Define T = { 0, 1, λ, λ_1 } and S = { 0, 1, μ, μ_1, μ_2, μ_3 }where λ, λ_1, μ, μ_1, μ_2, μ_3 : X → [0, 1] are such that λ (a) = 0, λ (b) = 0.4, λ (c) = 0, λ_1 (a) = 0.6, γ (b) = 0.5, γ (c) = 0.4, μ (a) = 0.3, μ (b) = 0.3, μ (c) = 0.3, μ_1 (a) = 0.3, μ_1 (b) = 0.4,μ_1 (c) = 0.5, μ_2(a) = 0.6, μ_2 (b) = 0.6, μ_2 (c) = 0.6, μ_3 (a) = 0.6, μ_3 (b) = 0.8, μ_3 (c) = 0.9. Clearly (X, T) and (Y, S) are fuzzy topological spaces.

Define g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = b, f_1(b) = b, f_1(c) = b and f_2(a) = b, f_2(b) =c, f_2(c) = a. Let α : X → [0,1] be any fuzzy set such that α (a) = 0, α (b) = 0.3, α (c) = 0. For the fuzzy orbit open set μ under the mapping f_2 in (Y, S)with $g(\alpha) \leqslant \mu$.

Now the fuzzy orbit open set λ under the mapping f_1 in (X, T) with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant$ cl μ. Then g is weakly*fuzzy orbit continuous.

Now, $g(\alpha) \leqslant \mu$. But there is no fuzzy orbit open set λ under the mapping f_1 with $\alpha \leqslant \lambda$ such that $g(\lambda) \nleq$ int(cl(μ)). Thus g is not almost*-fuzzy orbit continuous.

Proposition 3.23. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If g : (X, T) → (Y,S) is fuzzy orbit* continuous, then g is almost*-fuzzy orbit continuous.*

Proof. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let $\alpha \in I^X$ be any fuzzy set and μ be any fuzzy orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. Since g is fuzzy orbit* continuous, there exists a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that $g(\lambda) \leq \mu$. Since μ is fuzzy opeen, which implies that $g(\lambda) \leq$ int(cl(μ)). Then g is almost*-fuzzy orbit continuous. □

Remark 3.24. The converse of the Proposition 3.23 need not be true as shown in the following example.

Example 3.25. Let X = { a, b, c } = Y. Define T = { 0, 1, λ, λ_1 } and S = { 0, 1, μ, μ_1, μ_2, μ_3 } where λ, λ_1, μ, μ_1, μ_2, μ_3 : X → [0, 1] are such that λ (a) = 0.4, λ (b) = 0.4, λ (c) = 0.4, λ_1 (a) = 0.4, γ (b) = 0.5, γ (c) = 0.6, μ (a) = 0.3, μ (b) = 0.3, μ (c) = 0.3, μ_1 (a) = 0.3, μ_1 (b) = 0.4,μ_1 (c) = 0.5, μ_2 (a) = 0.4, μ_2 (b) = 0.4, μ_2 (c) = 0.5, μ_3 (a) = 0.6, μ_3 (b) = 0.7, μ_3 (c) = 0.8. Clearly (X, T) and (Y, S) are fuzzy topological spaces.

Define g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = b, f_1(b) = c, f_1(c) = a and f_2(a) = b, f_2(b) =c, f_2(c) = a. Let α : X → [0,1] be any fuzzy set such that α (a) = 0.2, α (b) = 0.2, α (c) = 0.2. For the fuzzy orbit open set μ under the mapping f_2 in (Y, S)with $g(\alpha) \leqslant \mu$.

Now the fuzzy orbit open set λ under the mapping f_1 in (X, T) with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant$ int(cl(μ)). Then g is almost*fuzzy orbit continuous.

Now, $g(\alpha) \leqslant \mu$. But there is no fuzzy orbit open set λ under the mapping f_1 with $\alpha \leqslant \lambda$ such that $g(\lambda) \nleq \mu$. Thus g is not fuzzy orbit* continuous.

Proposition 3.26. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If g : (X, T) → (Y,S) is fuzzy orbit* continuous, then g is weakly*-fuzzy orbit continuous.*

Proof. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let $\alpha \in I^X$ be any fuzzy set and μ be any fuzzy orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. Since g is fuzzy orbit* continuous, there exists a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that $g(\lambda) \leq \mu$, which implies that $g(\lambda) \leq$ cl(μ). Then g is weakly*-fuzzy orbit continuous. □

Remark 3.27. The converse of the Proposition 3.26 need not be true as shown in the following example.

Example 3.28. Let X = { a, b, c } = Y. Define T = { 0, 1, λ, λ_1 } and S = { 0, 1, μ, μ_1, μ_2 } where λ, λ_1, μ, μ_1, μ_2 : X → [0, 1] are such that λ (a) = 0.4, λ (b) = 0, λ (c) = 0, λ_1 (a) = 0.4, γ (b) = 0.5, γ (c) = 0.6, μ (a) = 0.3, μ (b) = 0.3, μ (c) = 0.3, μ_1 (a) = 0.3, μ_1 (b) = 0.4,μ_1 (c) = 0.4, μ_2(a) = 0.4, μ_2 (b) = 0.5, μ_2 (c) = 0.4. Clearly (X, T) and (Y, S) are fuzzy topological spaces.

Define g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = a, f_1(b) = a, f_1(c) = a and f_2(a) = b, f_2(b) =c, f_2(c) = a. Let α : X → [0,1] be any fuzzy set such that α (a) = 0.2, α (b) = 0, α (c) = 0. For the fuzzy orbit open set μ under the mapping f_2 in (Y, S)with $g(\alpha) \leqslant \mu$.

Now the fuzzy orbit open set λ under the mapping f_1 in (X, T) with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant$ cl (μ). Then g is weakly*fuzzy orbit continuous.

Now, $g(\alpha) \leqslant \mu$. But there is no fuzzy orbit open set λ under the mapping f_1 with $\alpha \leqslant \lambda$ such that $g(\lambda) \nleq \mu$. Thus g is not fuzzy orbit* continuous.

Proposition 3.29. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. If g : (X, T) → (Y,S) is fuzzy orbit continuous, then g is fuzzy orbit* continuous.*

Proof. Let f_1 : X→ X and f_2 : Y→ Y be any two mappings. Let $\alpha \in \mathrm{I}^X$ be any fuzzy set and μ be any fuzzy orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. By Theorem 2.11, $\alpha \leq g^{-1}(g(\alpha)) \leq g^{-1}(\mu)$. Then $\alpha \leq g^{-1}(\mu)$. Since g is fuzzy orbit continuous, $\alpha \leq g^{-1}(\mu) = \lambda$, λ is a fuzzy orbit open set under the mapping f_1. By Theorem 2.11, $gg^{-1}(\mu) \leq \mu$. Therefore $g(\lambda) = gg^{-1}(\mu) \leq \mu$ which implies that $g(\lambda) \leq \mu$. Then g is fuzzy orbit* continuous. □

Remark 3.30. The converse of the Proposition 3.29 need not be true as shown in the following example.

Example 3.31. Let X = { a, b, c } = Y. Define T = { 0, 1, λ, λ_1 } and S = { 0, 1, μ, μ_1 } where λ, λ_1, μ, μ_1 : X → [0, 1] are such that λ (a) = 0, λ (b) = 0, λ (c) = 0.4, λ_1 (a) = 0.4, λ_1 (b) = 0.6, λ_1 (c) = 0.7,μ (a) = 0.6, μ (b) = 0.6, μ (c) = 0.6, μ_1 (a) = 0.6, μ_1 (b) = 0.7, μ_1 (c) = 0.8. Clearly (X, T) and (Y, S) are fuzzy topological spaces.

Define g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = c, f_1(b) =c, f_1(c) = c and f_2(a) = b, f_2(b) =c, f_2(c) = a. Let α :X→ [0,1] be any fuzzy set such that α (a) = 0, α (b) = 0, α (c) = 0.2.

Now the fuzzy orbit open set λ under the mapping f_1 in (X, T) with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant \mu$. Then g is fuzzy orbit* continuous.

Now the fuzzy open set μ_1 in (Y, S), but $g^{-1}(\mu_1)$ is not fuzzy orbit open under the mapping f_1 in (X, T). Thus g is not fuzzy orbit continuous.

Proposition 3.32. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let g : (X, T) → (Y, S) and f_1 : X → X be any two mappings. Then the following are equivalent:*

(1) *g is fuzzy orbit continuous mapping,*

(2) *inverse image of every fuzzy closed set in (Y, S) is a fuzzy orbit closed set under the mapping f_1 in (X, T).*

Proof. (1) ⇒ (2): Assume that g is a fuzzy orbit continuous mapping. Letλ be any fuzzy closed set in (Y, S). Then $(1 - \lambda)$ is a fuzzy open set in (Y,S). Thus by assumption, $g^{-1}(1 - \lambda)$ is a fuzzy orbit open set under the mapping f_1 in (X, T). Now, $g^{-1}(1 - \lambda) = 1 - g^{-1}(\lambda)$. So, $g^{-1}(\lambda)$ is a fuzzy orbit closed set under the mapping f_1 in (X, T).

(2) ⇒ (1): The proof is similar to (1) ⇒ (2). □

Proposition 3.33. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let g : (X, T) → (Y, S), f_1 : X → X and f_2 : Y → Y be any two mappings. Then the following are equivalent:*

(1) *g is a fuzzy orbit continuous mapping,*

(2) *for each fuzzy set λ of X and every fuzzy neighborhood λ of $g(\lambda)$, $g^{-1}(\lambda)$ is a fuzzy orbit neighborhood of γ,*

(3) *for each fuzzy set γ of X and every fuzzy neighborhood λ of $g(\gamma)$, there exists a fuzzy orbit neighborhood μ of γ such that $g(\mu) \leq \lambda$.*

Proof. (1) ⇒ (2): Let γ be a fuzzy set of X. Let λ be a fuzzy neighborhood of $g(\gamma)$. Then there exists a fuzzy open set μ such that $g(\gamma) \leq \mu \leq \lambda$. Now $g^{-1}(g(\gamma)) \leq g^{-1}(\mu) \leq g^{-1}(\lambda)$. By hypothesis, $g^{-1}(\mu)$ is a fuzzy orbit open set under the mapping f_1 in (X, T). But, $\gamma \leq g^{-1}(g(\gamma))$. Thus $g^{-1}(\lambda)$ is a fuzzy orbit neighborhood of γ.

(2) ⇒ (3): Let γ be a fuzzy set of X. Let λ be a fuzzy neighborhood of $g(\gamma)$. By hypothesis, $g^{-1}(\lambda)$ is a fuzzy orbit neighborhood of γ in (X, T) such that $g(g^{-1}(\lambda)) \leq \lambda$.

(3) ⇒ (1): Let γ be a fuzzy set of X such that $\gamma \leq g^{-1}(\lambda)$. Let λ be a fuzzy orbit open set under the mapping f_2 in (Y, S). Since every fuzzy orbit open set is a fuzzy neighborhood, λ is a fuzzy neighborhood of $g(\delta)$ in (Y, S). Then by hypothesis, $g^{-1}(\lambda)$ is a fuzzy orbit neighborhood of γ in (X, T). Since every fuzzy orbit neighborhood set is a fuzzy orbit open set, $g^{-1}(\lambda)$ is a fuzzy orbit open set under the mapping f_1 in (X, T). Thus g is fuzzy orbit continuous. □

Proposition 3.34. *Let (X, T_1) and (Y, T_2) be any two fuzzy topological spaces. Let $f_1 : X \to X$ and $f_2 : Y \to Y$ be any mappings. For a mapping g : (X, T_1) → (Y, T_2). The following conditions are equivalent :*

(1) *g is fuzzy orbit* continuous,*

(2) *inverse image of every fuzzy orbit clopen set under the mapping f_2 of (Y, T_2) is a fuzzy orbit open set under the mapping f_1 of (X, T_1),*

(3) *inverse image of every fuzzy orbit clopen set under the mapping f_2 of (Y, T_2) is a fuzzy orbit clopen set under the mapping f_1 of (X, T_1).*

(4) *for each fuzzy set $\alpha \in I^X$ and for every fuzzy net $\{ S_n, n \in D \}$ which converges to α, the fuzzy net $\{ f(S_n), n \in D \}$ is eventually in each fuzzy orbit clopen set λ under the mapping f_1 with $g(\alpha) \leqslant \lambda$.*

Proof. (1) ⇒ (2): Let σ be a fuzzy orbit clopen set under the mapping f_2 of (Y, T_2). Let $\lambda \in I^X$ be any fuzzy set such that $\lambda \leqslant g^{-1}(\sigma)$. Then $g(\lambda) \leqslant \sigma$. Now σ is a fuzzy orbit clopen set under the mapping f_2 with $g(\lambda) \leqslant \sigma$. Then by the condition (1), there exists a fuzzy orbit open set μ under the mapping f_1 of (X, T_1) with $\lambda \leqslant \mu$ such that $g(\mu) \leqslant \sigma$. Thus $g^{-1}(\sigma)$ is a fuzzy orbit open set under the mapping f_1 of (X, T_1).

(2) ⇒ (3): Let β be a fuzzy orbit clopen set under the mapping f_2 of (Y, T_2). Now, $1 - \beta$ is a fuzzy orbit clopen set under the mapping f_2 of (Y, T_2). Then by the condition (2), $g^{-1}(1 - \beta) = 1 - g^{-1}(\beta)$ is a fuzzy orbit open set under the mapping f_1 of (X, T_1). That is, $g^{-1}(\beta)$ is a fuzzy orbit closed set under the mapping f_1 of (X, T_1). Thus by the condition (2), $g^{-1}(\beta)$ is a fuzzy orbit open set under the mapping f_1 of (X, T_1). So $g^{-1}(\beta)$ is a fuzzy orbit clopen set under the mapping f_1 of (X, T_1).

(3) ⇒ (4): Let $\{ g(S_n), n \in D \}$ be a fuzzy net covering to a fuzzy set α and let λ be a fuzzy orbit clopen set under the mapping f_1 with $g(\alpha) \leqslant \lambda$. Then by the condition (3), there exists a fuzzy orbit open set μ under the mapping f_1 with $\alpha \leqslant$

μ such that $g(\mu) \leqslant \lambda$. Since the net { $g(\mathrm{S}_n)$, n $\in$ D } coverges to α implies $\mathrm{S}_n \leqslant \alpha$. Now, $\mathrm{S}_n \leqslant \alpha \leqslant \mu$. Thus $g(\mathrm{S}_n) \leqslant g(\alpha) \leqslant \lambda$. So { g (S_n), n $\in$ D } is eventually in λ.

(4) $\Rightarrow$ (1): Suppose that g is not fuzzy orbit* continuous. Then there does not exist a fuzzy orbit open set λ under the mapping f_1 with $\alpha \leqslant \lambda$ such that $g(\lambda) \leqslant \mu$ and thus $g(\mathrm{S}_n) \nleqslant \mu$, which is a contradiction. So g is fuzzy orbit* continuous. □

Proposition 3.35. *Let (X, T),(Y,S) and (Z, R) be any three fuzzy topological spaces. Letf_1 : X → X be any mapping. Let g : (X, T) → (Y, S) be fuzzy orbit continuous and h: (Y, S)→ (Z, R) be fuzzy continuous mappings, then their composition h o g is fuzzy orbit continuous.*

Proof. Let λ be a open set of (Z, R). By Def, h^{-1} (λ) is a fuzzy open set of (Y, S). Since f is fuzzy orbit continuous, g^{-1} (h^{-1} (λ)) is a fuzzy orbit open set under the mapping f_1 of (X, T). But g^{-1} (h^{-1} (λ)) = $(h\ o\ g)^{-1}$ (λ). Then h o g is fuzzy orbit continuous. □

Proposition 3.36. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let g : (X, T) → (Y, S) be any mapping. Then the graph of g, h : X → X × Y is fuzzy orbit* continuous.*

Proof. Let f_1 : X → X and f_2 : Y → Y be any two mappings. Suppose that g : (X, T) → (Y, S) is fuzzy orbit* continuous and let h : X → X × Y be the graph of g. Let $\lambda \times \mu$ be a fuzzy orbit clopen set of X × Y. Then

$$\begin{aligned} h^{-1}(\lambda \times \mu) &= (\lambda \times \mu)(h(x)) \\ &= (\lambda \times \mu)(x, g(x)) \\ &= (\lambda(x), \mu(g(x))) \\ . &= \lambda \wedge g^{-1}(\mu(x)). \end{aligned}$$

Thus $h^{-1}(\lambda \times \mu) = \lambda \wedge g^{-1}$ (μ (x)). Since $h^{-1}(\lambda \times \mu)$ is a fuzzy orbit open set under the mapping f_1 of (X, T), by Proposition 3.34, g is fuzzy orbit* continuous.

Conversely, let λ be fuzzy orbit clopen set under the mapping f_2 in (Y, S). Then $1 \times \lambda$ is fuzzy orbit clopen in X × Y. Since h is fuzzy orbit* continuous, h^{-1} (1× λ) is fuzzy orbit open set under the mapping f_1 in (X, T). Also, h^{-1} (1× λ) = g^{-1} (λ). Thus g^{-1} (λ) is fuzzy orbit open set under the mapping f_1 in (X, T). So g is fuzzy orbit* continuous. □

Proposition 3.37. *Let (X, T),(Y,S) and (Z, R) be any three fuzzy topological spaces. Let g : (X, T) → (Y, S) be fuzzy orbit* continuous. Then the mapping h : (X, T) → (Z, R), where R = S / Z is fuzzy orbit* continuous.*

Proof. Let f_1 : X → X, f_2 : Y → Y and f_3 : Z → Z be any three mappings. Let μ be any fuzzy orbit clopen set under the mapping f_3 in (Z, R). Then $\mu = \lambda$ / Z, for some fuzzy orbit clopen set λ under the mapping f_2 of (Y, S). Thus by hypothesis, g^{-1} (λ) = h^{-1} (μ). So by Propostion 3.32, g^{-1} (λ) is fuzzy orbit open set under the mapping f_1 in (X, T). Hence h^{-1} (μ) is fuzzy orbit open set under the mapping f_1 in (X, T). Therefore h is fuzzy orbit* continuous. □

Proposition 3.38. *Let (X, T), (X_1, T_1) and (X_2, T_2) be any three fuzzy topological spaces. Let p_j : ($X_1 * X_2$) → X_j, (j = 1, 2) be the projections of ($X_1 * X_2$) onto X_j.*
If g : X → ($X_1 * X_2$) is a fuzzy orbit continuous mapping, then $p_j \circ g$ is also fuzzy orbit* continuous.*

Proof. The projection p_j (j = 1, 2) are fuzzy continuous and then are fuzzy orbit* continuous. Thus the proof follows from Proposition 3.36. □

Proposition 3.39. *Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let X = A ∪ B, where A and B are subsets of X such that ψ_A, $\psi_B \in$ T. Let g : (X, T) → (Y, S) be such that g / A and g / B are fuzzy orbit* continuous. Then g is fuzzy orbit* continuous.*

Proof. Let f_1 : X → X and f_2 : Y → Y be any two mappings. Let λ be a fuzzy orbit open set under the mapping f_2 of (Y, S). Since g / A is fuzzy orbit* continuous, $(g$ / A$)^{-1}$ (λ) is fuzzy orbit open set under the mapping f_1 in A and similarly (g / B$)^{-1}$ (λ) is fuzzy orbit open set under the mapping f_2 in B. Then there exists μ / A ∈ T / A and μ / B ∈ T / B such that μ / A = (g / A$)^{-1}$ (λ) and μ / B = (g / B$)^{-1}$ (λ). Then $\mu = g^{-1}$ (λ) and $\mu \in$ T. Thus by Proposition 3.34, f is fuzzy orbit* continuous. □

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