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Reduction of an intuitionistic fuzzy matrix to fuzzy matrix with some algebraic properties

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Abstract. In this paper we define two operators namely conjunction($\odot$) and disjunction($\oplus$) from Lukasiewicz's type over Intuitionistic Fuzzy Matrix and several algebraic properties are investigated when the above said operators combined with other well known operators on IFM. Also some reduction operators which reduce an Intuitionistic Fuzzy Matrix to Fuzzy Matrix are introduced. Finally we obtain addition law on probability which contains reduction, conjunction and disjunction operators.

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1. Introduction

There have been theories evolved over the years to deal with the various types of uncertainties. These evolved theories are put into practice and when found to be wanting are improved upon, paving the way for new theories to handle the tricky uncertainties. The Probability theory is one such important theory concerned with the analysis of random phenomena. In 1965, Zadeh [24] came out with the concept of Fuzzy Set which is indeed an extension of the classical notion of set. Fuzzy Set has been found to be an effective tool to deal with fuzziness. However, it often falls short of the expected standard when describing the neutral state. As a result, a new concept namely Intuitionistic Fuzzy Set(IFS) was worked out and the same was introduced in 1983 by Atanassov [1, 2]. Using the concept of IFS, Im et.al [7, 8] studied Intuitionistic Fuzzy Matrix(IFM).

IFM generalizes the Fuzzy Matrix introduced by Thomson [21] and has been useful in dealing with areas such as decision making, relational equations, clustering analysis etc,. IFM is also very useful in the discussion of Intuitionistic fuzzy relation

[6, 11]. Z.S.Xu [22, 26] studied Intuitionistic Fuzzy Value and also IFMs. He defined intuitionistic fuzzy similarity relation and also utilise it in clustering analysis.

Since then different have contributed significantly for the development of IFMs in [10, 13, 14, 20, 23]. Permanant of interval valued triangular fuzzy numbers and a new method for system of linear equations based on certain decomposition of its coefficient matrix is studied in [5] and [15] respectively. The period of powers of Square IFMs is discussed at length along with some of the results for the equivalence IFM by Jeong and Park [9] while Pal et.al. [16] made a comprehensive study and neatly developed IFM in 2002. Another researcher namely Mondal [12] studied similarity relations, invertibility and eigenvalues of IFM.

In [18, 25] some new operators are introduced and several algebraic properties are discussed on FMs. Monoids on IFMs are studied in [19]. Here we introduce Lukasiewicz's conjunction($\odot$) and disjunction($\oplus$) operators on IFMs which are different from the operators introduced in [19]. In [4] Zadeh's conjunction and disjuntion properties are studied. Atanassov, Tcvetkov [3] and Riecan [17] introduced the operations conjunction and disjunction from Lukasiewicz's type over IFSs and studied its algebraic properties. We extend it to IFM and studied some of the basic properties of these operations with other predefined operators. Also some operators called reduction operators are introduced which give a FM from an IFM. Finally we obtain addition law on probability which connects all the operators defined above.

2. Preliminaries

In this section let us recall some basic concepts about IFMs for a better understanding of the main body of the paper.

Definition 2.1 ([1, 2])**.** Let a set $X = \{x_1, x_2, ...x_n\}$ be fixed, then an IFS A is defined as an object of the following form $A = \{(x, \mu_A(x_i), \gamma_A(x_i))/x_i \in X\}$, where the functions $\mu_A(x_i) : X \to [0, 1]$ and $\gamma_A(x_i) : X \to [0, 1]$ define the membership and non membership function of the element $x_i \in X$ respectively and for every $x_i \in X$, $0 \leq \mu_A(x_i) + \gamma_A(x_i) \leq 1$.

Definition 2.2 ([22, 26])**.** The two tuple $\alpha(x_i) = (\mu_\alpha(x_i), \gamma_\alpha(x_i))$ is called an Intuitionistic fuzzy value, if $\mu_\alpha(x_i) \in [0, 1]$, $\gamma_\alpha(x_i) \in [0, 1]$ such that

$$\mu_\alpha(x_i) + \gamma_\alpha(x_i) \leq 1.$$

For our convenience, we write $(\mu_A(x_i), \gamma_A(x_i)) = (x, x')$

Definition 2.3 ([1, 2])**.** For $(x, x'), (y, y') \in$ IFS, define:
(i) $(x, x') \vee (y, y') = (\max\{x, y\}, \min\{x', y'\})$,
(ii) $(x, x') \wedge (y, y') = (\min\{x, y\}, \max\{x', y'\})$,
(iii) $(x, x')^c = (x', x)$.

Definition 2.4 ([3, 17])**.** For any two elements $(x, x'), (y, y') \in$ IFS, Lukasiewicz type disjunction and conjunction operators denoted by $\oplus$ and $\odot$ respectively and defined as follows:
(i) $(x, x') \oplus (y, y') = \{(x + y) \wedge 1, (x' + y' - 1) \vee 0\}$,
(ii) '$(x, x') \odot (y, y') = \{(x + y - 1) \vee 0, (x' + y') \wedge 1\}$.

Definition 2.5 ([10]). A fuzzy matrix is a matrix with elements having values in the closed interval $[0, 1]$.

Definition 2.6 ([26]). Let $A = [(a_{ij})]_{m\times n}$ be a matrix of order $m \times n$. If the value $a_{ij}(i = 1, 2, ...m, j = 1, 2, ...n)$ are intuitionistic fuzzy values, then A is called an IFM. F_{mn} denotes set of all IFMs of order $m \times n$

Definition 2.7 ([7, 13, 16]). For any two elements $A = [(a_{ij}, a'_{ij})], B = [(b_{ij}, b'_{ij})] \in F_{mn}$ define:
- (i) $A \vee B = [(a_{ij}, a'_{ij}) \vee (b_{ij}, b'_{ij})]$.
- (ii) $A \wedge B = [(a_{ij}, a'_{ij}) \wedge (b_{ij}, b'_{ij})]$.
- (iii) $A^c = (a'_{ij}, a_{ij})$.
- (iv) $A \leq B$, if $a_{ij} \leq b_{ij}$ and $a'_{ij} > b'_{ij}$ for all i, j in which A and B are comparable.
- (v) If $A^2 \geq A$, then A is said to be compact.
- (vi) If $A^2 \leq A$, then A is said to be transitive.

3. Algebraic properties of conjuntion and disjunction operators on IFMs

Now let us define Lukasiewicz's type disjunction operator on IFM denoted by $(\oplus)$ and conjunction operator by $(\odot)$ and some algebraic laws over the above operators with other predefined operators.

Definition 3.1. For any two IFMs $A, B \in F_{mn}$, we define Lukasiewicz disjunction and conjunction operators on IFMs as follows:
- (i) $A \oplus B = \{(a_{ij} + b_{ij}) \wedge 1, (a'_{ij} + b'_{ij} - 1) \vee 0\}$,
- (ii) $A \odot B = \{(a_{ij} + b_{ij} - 1) \vee 1, (a'_{ij} + b'_{ij}) \wedge 1\}$

Proposition 3.2. *For any two IFMs $A, B \in F_{mn}$, we have the following:*
- (1) *$\oplus$ and $\odot$ are commutative,*
- (2) *$\oplus$ and $\odot$ are monotonically increasing operators,*
- (3) $A \odot B \leq A \wedge B \leq A \vee B \leq A \oplus B$.

Proof. (1) From the Definition, it is clear that $A \oplus B = B \oplus A$ and $A \odot B = B \odot A$.

(2) To prove monotonically increasing property, it is enough prove the following:
$$If A \leq B, \text{ then } A \oplus C \leq B \oplus C \text{ and } A \odot C \leq B \odot C.$$
For that consider
$$A \oplus C = [((a_{ij} + c_{ij}) \wedge 1, (a'_{ij} + c'_{ij} - 1) \vee 0)]$$
and
$$B \oplus C = [((b_{ij} + c_{ij}) \wedge 1, (b'_{ij} + c'_{ij} - 1) \vee 0)].$$
Since $a_{ij} \leq b_{ij}$ and $a'_{ij} \geq b'_{ij}$, we have
$$a_{ij} + c_{ij} \leq b_{ij} + c_{ij} \text{ and } a'_{ij} + c'_{ij} \geq b'_{ij} + c'_{ij}.$$
Then
$$(a_{ij} + c_{ij}) \wedge 1 \geq (b_{ij} + c_{ij}) \wedge 1 \tag{3.1}$$
Clearly, $(a'_{ij} + c'_{ij} - 1) \geq (b'_{ij} + c'_{ij} - 1)$implies
$$(a'_{ij} + c'_{ij} - 1) \vee 0 \geq (b'_{ij} + c'_{ij} - 1) \vee 0 \tag{3.2}$$
From (3.1) and (3.2), $A \oplus C \leq B \oplus C$.

Similarly, we can prove $A \odot C \leq B \odot C$.

(3) $A \odot B = [((a_{ij} + b_{ij} - 1) \vee 0, (a'_{ij} + b'_{ij}) \wedge 1)]$. Then

$a_{ij} + b_{ij} - 1 = a_{ij} + (b_{ij} - 1) \leq a_{ij}$ and $a_{ij} + b_{ij} - 1 = b_{ij} + (a_{ij} - 1) \leq b_{ij}$.
Thus $a_{ij} + b_{ij} - 1 \leq (a_{ij} \wedge b_{ij})$. So $(a_{ij} + b_{ij} - 1) \vee 0 \leq (a_{ij} \wedge b_{ij})$. (3.3)
Similarly, $(a'_{ij} + b'_{ij}) \wedge 1 \geq (a'_{ij} \vee b'_{ij})$. (3.4)
From (3.3) and (3.4), $A \odot B \leq A \wedge B$.
Since $a_{ij} \leq a_{ij} + b_{ij}, b_{ij} \leq a_{ij} + b_{ij}, a_{ij} \vee b_{ij} \leq a_{ij} + b_{ij}$,

$$a_{ij} \vee b_{ij} \leq (a_{ij} + b_{ij}) \wedge 1 \tag{3.5}$$

Also $a'_{ij} + b'_{ij} - 1 = a'_{ij} + (b'_{ij} - 1) \leq a'_{ij}$ and $a'_{ij} + b'_{ij} - 1 = b'_{ij} + (a'_{ij} - 1) \leq b'_{ij}$.
Thus $a'_{ij} + b'_{ij} - 1 \leq a'_{ij} \wedge b'_{ij}$. So

$$a'_{ij} \wedge b'_{ij} \geq (a'_{ij} + b'_{ij} - 1) \vee 0 \tag{3.6}$$

From (3.5) and (3.6), $A \vee B \leq A \oplus B$.
Since $A \wedge B \leq A \vee B$ is obvious, we have the following
$A \odot B \leq A \wedge B \leq A \vee B \leq A \oplus B$.

□

Proposition 3.3. *For any IFM A, we have*
(1) $\oplus$ *is compact,*
(2) $\odot$ *is transitive.*

Proof. (1) The ij^{th} element of $A \oplus A$ is $[(2a_{ij} \wedge 1, (2a'_{ij} - 1) \vee 0)]$. It is enough to prove that $2a_{ij} \wedge 1 > a_{ij}$ and $(2a'_{ij} - 1) \vee 0 < a'_{ij}$.
Since $2a_{ij} \geq a_{ij}$, $2a_{ij} \wedge 1 > a_{ij}$. (3.7)
Since $2a'_{ij} - 1 = a'_{ij} + (a'_{ij} - 1) \leq a'_{ij}$, $(2a'_{ij} - 1) \vee 0 < a'_{ij}$. (3.8)
From (3.7) and (3.8), $A \oplus A \geq A$. Then from the Definition 2.7, $\oplus$ is compact.
(2) In dual of the above, we can easily prove $A \odot A \leq A$, gives $\odot$ is transitive □

The following statements are trivial from the Definition 3.1 and Proposition 3.3.

Proposition 3.4. *(Absorption Laws) For any* $A, B \in F_{mn}$,
(1) $A \wedge (A \oplus B) = A$,
(2) $A \vee (A \odot B) = A$.

Proposition 3.5. *(Demorgan's Laws) If* $A, B \in F_{mn}$, *then we have*
(1) $(A \oplus B)^c = A^c \odot B^c$,
(2) $(A \odot B)^c = A^c \oplus B^c$.

Proof. (1) It is clear that $(A \oplus B)^c = [(a'_{ij} + b'_{ij} - 1) \vee 0, (a_{ij} + b_{ij}) \wedge 1]$.
On one hand, $A^c \odot B^c = [(a'_{ij}, a_{ij})] \odot [(b'_{ij}, b_{ij})] = [(a'_{ij} + b'_{ij} - 1) \vee 0, (a_{ij} + b_{ij}) \wedge 1]$.
Then from the above two equations, Demorgan's law (1) holds.
(2) the proof is similar to that of (1). □

Proposition 3.6. *(Distributive Laws) For any three IFMs* $A, B, C \in F_{mn}$, *we have*
(1) $A \oplus (B \vee C) = (A \oplus B) \vee (A \oplus C)$ *($\oplus$ is left distributive over $\vee$),*
(2) $A \oplus (B \wedge C) = (A \oplus B) \wedge (A \oplus C)$ *($\oplus$ is left distributive over $\wedge$),*
(3) $A \odot (B \vee C) = (A \odot B) \vee (A \odot C)$ *($\odot$ is left distributive over $\vee$)*
(4) $A \odot (B \wedge C) = (A \odot B) \wedge (A \odot C)$ *($\odot$ is left distributive over $\wedge$).*

Proof. (1) Let $(d_{ij}, d'_{ij}), (e_{ij}, e'_{ij}), (f_{ij}, f'_{ij}), (g_{ij}, g'_{ij})$ and (h_{ij}, h'_{ij}) are the ij^{th} elements of $B \vee C, A \oplus B, A \oplus C, A \oplus (B \vee C)$ and $(A \oplus B) \vee (A \oplus C)$, respectively. Then

$$H = (h_{ij}, h'_{ij}) = [e_{ij} \vee f_{ij}, e'_{ij} \wedge f'_{ij}]$$

$$= \begin{cases} (e_{ij}, e'_{ij}) & \text{if } e_{ij} \geq f_{ij}, e'_{ij} \leq f'_{ij} \\ (f_{ij}, f'_{ij}) & \text{if } f_{ij} \geq e_{ij}, f'_{ij} \leq e'_{ij} \\ (e_{ij}, f'_{ij}) & \text{if } e_{ij} \geq f_{ij}, f'_{ij} \leq e'_{ij} \\ (f_{ij}, e'_{ij}) & \text{if } f_{ij} \geq e_{ij}, e'_{ij} \leq f'_{ij} \end{cases}$$

$$= \begin{cases} [(a_{ij} + b_{ij}) \wedge 1, (a'_{ij} + b'_{ij} - 1) \vee 0] & \text{if } e_{ij} \geq f_{ij}, e'_{ij} \leq f'_{ij} \\ [(a_{ij} + c_{ij}) \wedge 1, (a'_{ij} + c'_{ij} - 1) \vee 0] & \text{if } e_{ij} \leq f_{ij}, e'_{ij} \geq f'_{ij} \\ [(a_{ij} + b_{ij}) \wedge 1, (a'_{ij} + c'_{ij} - 1) \vee 0] & \text{if } e_{ij} \geq f_{ij}, e'_{ij} \geq f'_{ij} \\ [(a_{ij} + c_{ij}) \wedge 1, (a'_{ij} + b'_{ij} - 1) \vee 0] & \text{if } e_{ij} \leq f_{ij}, e'_{ij} \leq f'_{ij}. \end{cases}$$

Case (i): If $e_{ij} \geq f_{ij}, e'_{ij} \leq f'_{ij}$, then

$(a_{ij} + b_{ij}) \wedge 1 \geq (a_{ij} + c_{ij}) \wedge 1$ and $(a'_{ij} + b'_{ij} - 1) \vee 0 \leq (a'_{ij} + c'_{ij} - 1) \vee 0$. Thus $(a_{ij} + b_{ij}) \geq (a_{ij} + c_{ij})$ and $(a'_{ij} + b'_{ij} - 1) \leq (a'_{ij} + c'_{ij} - 1)$. So $b_{ij} \geq c_{ij}$ and $b'_{ij} \leq c'_{ij}$. Hence $(d_{ij}, d'_{ij}) = (b_{ij}, b'_{ij})$. Therefore $(g_{ij}, g'_{ij}) = (e_{ij}, e'_{ij}) = (f_{ij}, f'_{ij})$ gives $A \oplus (B \vee C) = (A \oplus B) \vee (A \oplus C)$.

Similarly, we can prove the other three cases.

Proofs of (2), (3) and (4) are similar to (1). □

Proposition 3.7. *For any three IFMs $A, B, C \in F_{mn}$, we have*

(1) $(A \vee B) \oplus C = (A \oplus C) \vee (B \oplus C)$ *($\oplus$ is right distributive over $\vee$),*
(2) $(A \wedge B) \oplus C = (A \oplus C) \wedge (B \oplus C)$ *($\oplus$ is right distributive over $\wedge$),*
(3) $(A \vee B) \odot C = (A \odot C) \vee (B \odot C)$ *($\odot$ is right distributive over $\vee$),*
(4) $(A \wedge B) \odot C = (A \odot C) \wedge (B \odot C)$ *($\odot$ is right distributive over $\wedge$).*

Proof. (1) Let $(d_{ij}, d'_{ij}), (e_{ij}, e'_{ij}), (f_{ij}, f'_{ij}), (g_{ij}, g'_{ij})$ and (h_{ij}, h'_{ij}) are the ij^{th} elements of $A \vee B, A \oplus C, B \oplus C, (A \vee B) \oplus C$ and $(A \oplus C) \vee (B \oplus C)$, respectively. Then

$$H = (h_{ij}, h'_{ij}) = [e_{ij} \vee f_{ij}, e'_{ij} \wedge f'_{ij}]$$

$$= \begin{cases} (e_{ij}, e'_{ij}) & \text{if } e_{ij} \geq f_{ij}, e'_{ij} \leq f'_{ij} \\ (f_{ij}, f'_{ij}) & \text{if } f_{ij} \geq e_{ij}, f'_{ij} \leq e'_{ij} \\ (e_{ij}, f'_{ij}) & \text{if } e_{ij} \geq f_{ij}, f'_{ij} \leq e'_{ij} \\ (f_{ij}, e'_{ij}) & \text{if } f_{ij} \geq e_{ij}, e'_{ij} \leq f'_{ij} \end{cases}$$

$$= \begin{cases} [(a_{ij} + c_{ij}) \wedge 1, (a'_{ij} + c'_{ij} - 1) \vee 0] & \text{if } e_{ij} \geq f_{ij}, e'_{ij} \leq f'_{ij} \\ [(b_{ij} + c_{ij}) \wedge 1, (b'_{ij} + c'_{ij} - 1) \vee 0] & \text{if } e_{ij} \leq f_{ij}, e'_{ij} \geq f'_{ij} \\ [(a_{ij} + c_{ij}) \wedge 1, (b'_{ij} + c'_{ij} - 1) \vee 0] & \text{if } e_{ij} \geq f_{ij}, e'_{ij} \geq f'_{ij} \\ [(b_{ij} + c_{ij}) \wedge 1, (a'_{ij} + c'_{ij} - 1) \vee 0] & \text{if } e_{ij} \leq f_{ij}, e'_{ij} \leq f'_{ij} \end{cases}$$

Case (i): If $e_{ij} \geq f_{ij}, e'_{ij} \leq f'_{ij}$, then

$(h_{ij}, h'_{ij}) = (a_{ij} + c_{ij}) \wedge 1, (a'_{ij} + c'_{ij} - 1) \vee 0$ and $e_{ij} \geq f_{ij}$, $e'_{ij} \leq f'_{ij}$. Thus $(a_{ij} + c_{ij}) \wedge 1 \geq (b_{ij} + c_{ij}) \wedge 1$ and $(a'_{ij} + c'_{ij} - 1) \vee 0 \leq (b'_{ij} + c'_{ij} - 1) \vee 0$. So $(a_{ij} + c_{ij}) \geq (b_{ij} + c_{ij})$ and $(a'_{ij} + c'_{ij} - 1) \leq (b'_{ij} + c'_{ij} - 1)$. Hence $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$, and thus $(d_{ij}, d'_{ij}) = (a_{ij}, a'_{ij})$. Therefore $(g_{ij}, g'_{ij}) = (d_{ij}, d'_{ij}) \oplus (c_{ij}, c'_{ij}) = (a_{ij}, a'_{ij}) \oplus (c_{ij}, c'_{ij}) = (h_{ij}, h'_{ij})$ gives $(A \vee B) \oplus C = (A \oplus C) \vee (B \oplus C)$.

Similarly we can prove the other three cases.

Proofs of (2), (3) and (4) are similar to (1). □

4. Properties of reduction operators

In this section we introduce four operators called reduction operators on IFM which reduce an IFM to a FM. Also we relate conjunction and disjunction operators with reduction operators.

Definition 4.1. Consider $A = [(a_{ij}, a'_{ij})] \in F_{mn}$ be an IFM and $r \in [0, 1]$, define the reduction operators $r_1, r_2, r_3, r_4 : IFM \to FM$ as follows:

(i) $r_1[A] = r_1[(a_{ij}, a'_{ij})] = ra_{ij} + (1 - r)(1 - a'_{ij})$,
(ii) $r_2[A] = r_2[(a_{ij}, a'_{ij})] = 1 - r_1[(a_{ij}, a'_{ij})]$,
(iii) $r_3[A] = r_3[(a_{ij}, a'_{ij})] = (1 - r)a_{ij} + r(1 - a'_{ij})$,
(iv) $r_4[A] = r_4[(a_{ij}, a'_{ij})] = 1 - r_3[(a_{ij}, a'_{ij})]$.

Proposition 4.2. *For any IFM $A \in F_{mn}$ and $r \in [0, 1]$, we have the following statements:*

(1) *when $r = 0.5$, $r_1[A] = r_3[A]$ and $r_2[A] = r_4[A]$,*
(2) $r_1(1, 0) = r_3(1, 0) = r_2(0, 1) = r_4(0, 1) = 1$
and
$$r_2(1, 0) = r_4(1, 0) = r_1(0, 1) = r_2(0, 1) = 0,$$
(3) $r_1[A^c] = r_4[A]$ *and* $r_2[A^c] = r_3$.

Proof. (1) When $r = 0.5$, then $1 - r = 0.5$. Thus from Definition 4.1, $r_1[A] = r_3[A]$ and $r_2[A] = r_4[A]$.

(2) It is straightforward from Definition 4.1.

(3) $r_1[A^c] = r_1[(a'_{ij}, a_{ij})] = ra'_{ij} + (1 - r)(1 - a_{ij})$
$= 1 - [a_{ij} - ra_{ij} + r - ra'_{ij}] = 1 - [(1 - r))a_{ij} + r(1 - a'_{ij})$
$= 1 - r_3[A] = r_4[A]$.

Similarly, we can prove $r_2[A^c] = r_3[A]$. □

Proposition 4.3. *If A and B are two comparable IFMs with same order such that $A \leq B$, then for some $r \in [0, 1]$, we have*

(1) $r_1[A] \leq r_1[B]$,
(2) $r_2[A] \geq r_2[B]$
(3) $r_3[A] \geq r_3[B]$
(4) $r_4[A] \leq r_4[B]$.

Proof. (1) From the Definition of reduction operators, we have
$$r_1[A] = r_1[(a_{ij}, a'_{ij})] = ra_{ij} + (1 - r)(1 - a'_{ij})$$
and
$$r_1[B] = r_1[(b_{ij}, b'_{ij})] = rb_{ij} + (1 - r)(1 - b'_{ij}).$$
Since it is given that $A \leq B$ means $a_{ij} \leq b_{ij}$, $ra_{ij} \leq rb_{ij}$ and $a'_{ij} \geq b'_{ij}$. Thus $(1-r)(1-a'_{ij}) \leq (1-r)(1-b'_{ij})$. So $ra_{ij} + (1-r)(1-a'_{ij}) \leq rb_{ij} + (1-r)(1-b'_{ij})$. Hence $r_1[A] \leq r_1[B]$.

(2) From definition 2.9, $r_2[A] = 1 - r_1[A]$ and from (1), we have $r_1[A] \leq r_1[B]$. Then $1 - r_1[A] \geq 1 - r_1[B]$. Thus $r_2[A] \geq r_2[B]$.

(3) The proof of (3) similar to (1).

(4) The proof of (4) similar to (2). □

Theorem 4.4. *For any two IFMs $A, B \in F_{mn}$ and some $r \in [0,1]$, all the reduction operators satisfies addition law on probability with conjunction and disjunction operators as follows:*
(1) $r_1[A \oplus B] = r_1[A] + r_1[B] - r_1[A \odot B]$,
(2) $r_2[A \oplus B] = r_2[A] + r_2[B] - r_2[A \odot B]$,
(3) $r_3[A \oplus B] = r_3[A] + r_3[B] - r_3[A \odot B]$,
(4) $r_4[A \oplus B] = r_4[A] + r_4[B] - r_4[A \odot B]$.

Proof. (1) From Definitions 2.8 and 2.9, we have
$$r_1[(A \oplus B)] = r_1[(a_{ij} + b_{ij}) \wedge 1, (a'_{ij} + b'_{ij} - 1) \vee 0]$$
$$= r[(a_{ij} + b_{ij}) \wedge 1] + (1-r)[1 - (a'_{ij} + b'_{ij} - 1) \vee 0].$$
Similarly, $r_1[A \odot B] = r[(a'_{ij} + b'_{ij} - 1) \vee 0] + (1-r)[1 - (a'_{ij} + b'_{ij} \wedge 1]$
and
$$r_1[A] + r_1[B] = r[a_{ij} + b_{ij}] + (1-r)[(1 - a'_{ij}) + (1 - b'_{ij})].$$
Case(i): If $(a_{ij} + b_{ij}) \geq 1$ and $(a'_{ij} + b'_{ij} - 1) \leq 0$, then
$$r_1[A \oplus B] = r + (1-r) = 1$$
and
$$r_1[A \odot B] = r(a'_{ij} + b'_{ij} - 1) + (1-r)(1 - a'_{ij} - b'_{ij}).$$
Adding the above two equations, we get
$$r_1[(A \oplus B)] + r_1[A \odot B] = r[a_{ij} + b_{ij}] + (1-r)[(1 - a'_{ij}) + (1 - b'_{ij})]$$
$$= r_1[A] + r_1[B].$$
Thus $r_1[A \oplus B] = r_1[A] + r_1[B] - r_1[A \odot B]$.
Case(ii): If $(a_{ij} + b_{ij}) \leq 1$ and $(a'_{ij} + b'_{ij} - 1) \leq 0$, then
$$r_1[(A \oplus B)] = r[a_{ij} + b_{ij}] + (1-r)$$
and
$$r_1[A \odot B] = (1-r)(1 - a'_{ij} - b'_{ij}).$$
Thus $r_1[(A \oplus B)] + r_1[A \odot B] = r[a_{ij} + b_{ij}] + (1-r) + (1-r)(1 - a'_{ij} - b'_{ij})$
$$= r[a_{ij} + b_{ij}] + (1-r)[(1 - a'_{ij}) + (1 - b'_{ij})]$$
$$= r_1[A] + r_1[B].$$
So $r_1[A \oplus B] = r_1[A] + r_1[B] - r_1[A \odot B]$.
Case(iii): If $(a_{ij} + b_{ij}) \leq 1$ and $(a'_{ij} + b'_{ij} - 1) \geq 0$, then
$r_1[(A \oplus B)] = r[a_{ij} + b_{ij}] + (1-r)[(1 - a'_{ij}) + (1 - b'_{ij})]$ and $r_1[A \odot B] = 0$.
In this case, also $r_1[A \oplus B] = r_1[A] + r_1[B] - r_1[A \odot B]$.
Case(iv): If $(a_{ij} + b_{ij}) \geq 1$ and $(a'_{ij} + b'_{ij} - 1) \geq 0$, then A and B are not IFMs. Thus this case not possible. From all the above, four cases we conclude that $r_1[A \oplus B] = r_1[A] + r_1[B] - r_1[A \odot B]$.
(2) From Definition 2.9,
$$r_2[A \oplus B] + r_2[A \odot B] = 1 - r_1[A \oplus B] + 1 - r_1[A \odot B]$$
$$= 1 - [r_1[A] + r_1[B] - r_1[A \odot B]] + 1 - r_1[A \odot B]$$
$$= 1 - r_[A] + 1 - r_1[B] = r_2[A] + r_2[B].$$
Then $r_2[A \oplus B] = r_2[A] + r_2[B] - r_2[A \odot B]$.
(3) Since $r_2[A^c] = r_3[A]$, $r_3[A \oplus B] + r_3[A \odot B] = r_2[(A \oplus B)^c] + r_2[(A \odot B)^c]$.
From demorgan's law of conjunction and disjunction operators, we can write the above as follows:
$$r_3[A \oplus B] + r_3[A \odot B] = r_2[B^c \odot A^c)] + r_2[B^c \oplus A^c]$$

$$= r_2[B^c] + r_2[A^c] = r_3[B] + r_3[A].$$

Then $r_3[A \oplus B] = r_3[A] + r_3[B] - r_3[A \odot B]$.

(4) The proof is similar to (2) or (3). □

5. Conclusions

In this paper we introduce some operators on IFMs. Several algebraic laws like commutative, absorption, distributive, demorgan's etc., are studied. Also we know that every Fuzzy Matrix $A = [(a_{ij})]$is an Intuitionistic Fuzzy Matrix in the form $[(a_{ij}, 1 - a_{ij})]$ and the converse need not be true. Here using reduction we reduce an IFM to Fuzzy Matrix. Finally the relation between conjunction, disjunction and reduction operators gives addition law on probability. Using the algebraic laws various algebraic structures can be formed in future.

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