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Neutrsophic complex $\mathcal{N}$-continuity

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Abstract. In this paper, the concept of $\mathcal{N}$-open set in neutrosophic complex topological space is introduced. Some of the interesting properties of neutrosophic complex $\mathcal{N}$-open sets are studied. The idea of neutrosophic complex $\mathcal{N}$-continuous function and its characterization are discussed. Also the interrelation among the sets and continuity are established.

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1. Introduction

The concept of fuzzy sets was introduced by L. A. Zadeh in 1965[18]. After the introduction of fuzzy sets, and fuzzy logic have been applied in many real applications to handle uncertainty. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy set, interval-valued fuzzy sets, etc. After the introduction of intuitionistic fuzzy sets by Atanassov[4], the concept of imprecise data called neutrosophic sets was introduced by Smarandache[17]. The concept of neutrosophic topological space was introduced in [12, 13, 14, 16]. In classical topological space, the idea of b-open and semi open sets and their properties were introduced in [2, 3]. The extention of α-open, semi-open sets respectivley in neutrosophic crisp topological space was discussed in [15]. Also S. Broumi[5, 6, 7, 8, 9, 10] introduced some interesting concepts like interval valued neutrosophic graphs, single valued neutrosophic graphs and its applications. Moreover the concept of complex fuzzy set was introduced in [11] and it was generalized to complex neutrosophic set by Ali[1].

2. Preliminaries

Definition 2.1 ([16])**.** Let X be a nonempty set. A neutrosophic set A in X is defined as an object of the form $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$ such that $T_A, I_A, F_A : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.2 ([11])**.** A complex fuzzy set S defined on a universe of discourse X is characterized by a membership function $\eta_S(x)$ that assigns any element $x \in X$ a complex-valued grade of membership in S. The value $\eta_S(x)$ all lie within the unit circle in the coplex plane and thus all of the form $p_S(x)e^{j\mu_S(x)}$ where $p_S(x)$ and $\mu_S(x)$ are both real valued and $p_S(x) \in [0,1]$. Here $p_S(x)$ is termed as amplitude term and $e^{j\mu_S(x)}$ is termed as phase term. The complex fuzzy set may be represented in the form as $S = \{(x, \eta_S(x)) : x \in X\}$.

Definition 2.3 ([11])**.** Let S be a complex fuzzy set S on X. the complement of S denoted as $c(S)$ and is specified by a function $\eta_{c(S)}(x) = (1 - p_S(x))e^{j(2\pi - \mu_S(x))}$.

Definition 2.4 ([11])**.** Let A and B be two complex fuzzy sets on X and $\eta_A(x) = r_A(x)e^{j\mu_A(x)}$ and $\eta_B(x) = r_B(x)e^{j\mu_B(x)}$ be their membership functions respectively. The union of A and B is denoted as $A \cup B$ which is specified by a function

$$\eta_{A\cup B}(x) = r_{A\cup B}(x)e^{j\mu_{A\cup B}(x)} = (r_A(x) \vee r_B(x))e^{j(\mu_A(x)\vee\mu_B(x))},$$

where $\vee$ denote the max operator.

Definition 2.5 ([11])**.** Let A and B be two complex fuzzy sets on X and $\eta_A(x) = r_A(x)e^{j\mu_A(x)}$ and $\eta_B(x) = r_B(x)e^{j\mu_B(x)}$ be their membership functions respectively. The intersection of A and B is denoted as $A \cap B$ which is specified by a function

$$\eta_{A\cap B}(x) = r_{A\cap B}(x)e^{j\mu_{A\cap B}(x)} = (r_A(x) \wedge r_B(x))e^{j(\mu_A(x)\wedge\mu_B(x))},$$

where $\wedge$ denote the min operator.

3. Neutrosophic complex set theory

Definition 3.1. Let X be a nonempty set. A neutrosophic complex set

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

is defined on the universe of discourse X which is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A that assigns a complex values grade of $T_A(x), I_A(x), F_A(x)$ in A for any $x \in X$. The values $T_A(x), I_A(x)$ and $F_A(x)$ and their sum may all within the unit circle in the complex plane and so is of the following form:

$$T_A(x) = p_A(x).e^{j\mu_A(x)},\ I_A(x) = q_A(x).e^{j\nu_A(x)} \text{ and } F_A(x) = r_A(x).e^{j\omega_A(x)},$$

where $p_A(x), q_A(x), r_A(x)$ and $\mu_A(x), \nu_A(x), \omega_A(x)$ are respectively real valued and $p_A(x), q_A(x), r_A(x) \in [0,1]$ such that $0 \le p_A(x) + q_A(x) + r_A(x) \le 3$.

Definition 3.2. Let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be any two neutrosophic complex sets in X. Then the intersection of A and B is denoted and defined as $A \Cap B = \langle x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)\rangle$, where

$$T_{A\cap B}(x) = [p_A(x) \wedge P_B(x)]e^{j[\mu_A(x)\wedge\mu_B(x)]},$$

$$I_{A\cap B}(x) = [q_A(x) \wedge q_B(x)]e^{j[\nu_A(x)\wedge\nu_B(x)]}$$

and

$$F_{A\cap B}(x) = [r_A(x) \vee r_B(x)]e^{j[\omega_A(x)\vee\omega_B(x)]}.$$

Definition 3.3. Let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be any two neutrosophic complex sets in X. Then the union of A and B is denoted and defined as $A \Cup B = \langle x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)\rangle$, where

$$T_{A\cup B}(x) = [p_A(x) \vee P_B(x)]e^{j[\mu_A(x)\vee\mu_B(x)]},$$

$$I_{A\cup B}(x) = [q_A(x) \vee q_B(x)]e^{j[\nu_A(x)\vee\nu_B(x)]}$$

and

$$F_{A\cup B}(x) = [r_A(x) \wedge r_B(x)]e^{j[\omega_A(x)\wedge\omega_B(x)]}.$$

Definition 3.4. Let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be any two neutrosophic complex sets in X. Then $A \subseteq B$ if

$$T_A(x) \leq T_B(x), I_A(x) \leq I_B(x) \text{ and } F_A(x) \geq F_B(x), \text{ for all } x \in X.$$

That is,

$p_A(x) \leq P_B(x)$, $\mu_A(x) \leq \mu_B(x)$, $q_A(x) \leq q_B(x)$, $\nu_A(x) \leq \nu_B(x)$, $r_A(x) \geq r_B(x)$ and $\omega_A(x) \geq \omega_B(x)$, for all $x \in X$.

Example 3.5. Let $X = \{a\}$ be a nonempty set. Let

$A = \langle a, 0.3e^{j0.5}, 0.3e^{j0.2}, 0.5e^{j0.5}\rangle$ and $B = \langle a, 0.6e^{j0.8}, 0.4e^{j0.4}, 0.4e^{j0.3}\rangle$

be any two neutrosophic complex sets in X. Then $A \subseteq B$.

Example 3.6. Let $X = \{a\}$ be a nonempty set. Let

$A = \langle a, 0.3e^{j0.8}, 0.4e^{j0.2}, 0.5e^{j0.5}\rangle$ and $B = \langle a, 0.6e^{j0.5}, 0.3e^{j0.4}, 0.4e^{j0.3}\rangle$

be any two neutrosophic complex sets in X. Then

(1) $A \Cup B = \langle a, 0.6e^{j0.8}, 0.4e^{j0.4}, 0.4e^{j0.3}\rangle$,

(2) $A \Cap B = \langle a, 0.3e^{j0.5}, 0.3e^{j0.2}, 0.5e^{j0.5}\rangle$.

Definition 3.7. Let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be any neutrosophic complex set in X. Then the complement of A is denoted and defined as

$A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x)\rangle$, where

$$T_{A^c}(x) = [1 - p_A(x)]e^{j[1-\mu_A(x)]},$$

$$I_{A^c}(x) = [1 - q_A(x)]e^{j[1-\nu_A(x)]}$$

and

$$F_{A^c}(x) = [1 - r_A(x)]e^{j[1-\omega_A(x)]}.$$

Definition 3.8. The neutrosophic complex sets $1^\blacktriangle$ and $0^\blacktriangle$ are defined by

$1^\blacktriangle = \{\langle x, 1e^{j0}, 1e^{j0}, 0e^{j1}\rangle : x \in X\}$ and $0^\blacktriangle = \{\langle x, 0e^{j1}, 0e^{j1}, 1e^{j0}\rangle : x \in X\}$.

Remark 3.9. Let A and B be any two neutrosophic complex sets in X. Then

(1) $A^{cc} = A$,

(2) $A \Cap B^c = A^c \Cup B^c$,

(3) $A \Cup B^c = A^c \Cap B^c$,

(4) $0^{\blacktriangle c} = 1^\blacktriangle$,

(5) $1^{\blacktriangle c} = 0^\blacktriangle$.

Definition 3.10. A neutrosophic complex topology (NCT for short) on a nonempty set X is a family $\mathscr{T}$ of IFSs in X satisfying the following axioms:

(i) $0^{\blacktriangle}, 1^{\blacktriangle} \in \mathscr{T}$,
(ii) $G_1 \Cap G_2 \in \mathscr{T}$ for any $G_1, G_2 \in \mathscr{T}$,
(iii) $\Cup G_i \in \mathscr{T}$, for arbitrary family $\{G_i \mid i \in I\} \subseteq \mathscr{T}$.

Then $(X, \mathscr{T})$ is called a neutrosophic complex topological space ($NCTS$, for short) on X and each neutrosophic complex set in $\mathscr{T}$ is called a neutrosophic complex open set ($NCOS$, for short).

The complement A^c of a neutrosophic complex open set $NCOS$ A in a neutrosophic complex topological spaces $(X, \mathscr{T})$ is called an neutrosophic complex closed set ($NCCS$, for short) in X.

Example 3.11. Let $X = \{a\}$ be a nonempty set and let $A = \langle a, 0.6e^{j0.8}, 0.3e^{j0.4}, 0.5e^{j0.3}\rangle$ be a neutrosophic complex set in X. Then $\mathscr{T} = \{0^{\blacktriangle}, A, 1^{\blacktriangle}\}$ is a neutrosophic complex topology and the pair $(X, \mathscr{T})$ is a neutrosophic complex topological space. Also, A is a neutrosophic complex open set in X and its complement $A^c = \langle a, 0.4e^{j0.2}, 0.7e^{j0.6}, 0.5e^{j0.7}\rangle$ is a neutrosophic complex closed set in X.

Definition 3.12. Let $(X, \mathscr{T})$ be any $NCTS$ and let A be a neutrosophic complex set in X. Then the neutrosophic complex interior and neutrosophic complex closure of A are defined by

(i) $NCint(A) = \Cup\{G \mid G$ is a $NCOS$ in X and $G \subseteq A\}$,
(ii) $NCcl(A) = \Cap\{G \mid G$ is a $NCCS$ in X and $G \supseteq A\}$.

Remark 3.13. For any neutrosophic complex set A in $(X, \mathscr{T})$, we have

(1) $NCcl(A^c) = NCint(A)^c$,
(2) $NCint(A^c) = NCcl(A)^c$,
(3) A is a $NCCS$ iff $NCcl(A) = A$,
(4) A is a $NCOS$ iff $NCint(A) = A$,
(5) $NCcl(A)$ is a $NCCS$ in X,
(6) $NCint(A)$ is a $NCOS$ in X.

Proposition 3.14. *Let $(X, \mathscr{T})$ be any neutrosophic complex topological space $NCTS$ and A, B be neutrosophic complex sets in X. Then the following properties hold:*

(1) $A \subseteq NCcl(A)$,
(2) $A \subseteq B \Rightarrow NCcl(A) \subseteq NCcl(B)$,
(3) $NCcl(NCcl(A)) = NCcl(A)$,
(4) $NCcl(A \Cup B) = NCcl(A) \Cup NCcl(B)$,
(5) $NCcl(0^{\blacktriangle}) = 0^{\blacktriangle}$,
(6) $NCcl(1^{\blacktriangle}) = 1^{\blacktriangle}$.

Proposition 3.15. *Let $(X, \mathscr{T})$ be any neutrosophic complex topological space $NCTS$ and A, B be neutrosophic complex sets in X. Then the following properties hold:*

(1) $NCint(A) \subseteq A$,
(2) $A \subseteq B \Rightarrow NCint(A) \subseteq NCint(B)$,
(3) $NCint(NCint(A)) = NCint(A)$,
(4) $NCint(A \Cap B) = NCint(A) \Cap NCint(B)$,

(5) $NCint(0^{\blacktriangle}) = 0^{\blacktriangle}$,
(6) $NCint(1^{\blacktriangle}) = 1^{\blacktriangle}$.

Definition 3.16. Let A be any neutrosophic complex set of a neutrosophic complex topological space $(X, \mathscr{T})$. Then A is called a neutrosophic complex

(i) regular open ($NCROS$) set, if $A = NCint(NCcl(A))$,
(ii) preopen ($NCPOS$) set, if $A \subseteq NCint(NCcl(A))$,
(iii) semiopen ($NCSOS$) set, if $A \subseteq NCcl(NCint(A))$,
(iv) α-open ($NC\alpha OS$) set, if $A \subseteq NCint(NCcl(NCint(A)))$,
(v) β-open ($NC\beta OS$) set, if $A \subseteq NCcl(NCint(NCcl(A)))$,
(vi) b-open ($NCbOS$) set, if $A \subseteq NCint(NCcl(A)) \Cup NCcl(NCint(A))$.

Definition 3.17. Let A be any neutrosophic complex set of a neutrosophic complex topological space $(X, \mathscr{T})$. Then A is called a neutrosophic complex

(i) regular closed ($NCRCS$) set, if A^c is a $NCROS$,
(ii) preclosed ($NCPCS$) set, if A^c is a $NCPOS$,
(iii) semiopen ($NCSCS$) set, if A^c is a $NCSOS$,
(iv) α-open ($NC\alpha CS$) set, if A^c is a $NC\alpha OS$,
(v) β-open ($NC\beta CS$) set, if A^c is a $NC\beta OS$,
(vi) b-closed ($NCbCS$) set if A^c is a $NCbOS$.

Definition 3.18. Let $(X, \mathscr{T})$ be any $NCTS$ and let A be a neutrosophic complex set in X. Then the neutrosophic complex b-interior and neutrosophic complex semi-closure of A are defined by

(i) $NCbint(A) = \Cup\{G \mid G \text{ is a } NCbOS \text{ in } X \text{ and } G \subseteq A\}$,
(ii) $NCScl(A) = \Cap\{G \mid G \text{ is a } NCSCS \text{ in } X \text{ and } G \supseteq A\}$.

Definition 3.19. Let $(X, \mathscr{T})$ be any $NCTS$ and let A be a neutrosophic complex set in X. Then A is called a neutrosophic complex $\mathcal{N}$-open set ($NC\mathcal{N}OS$), if

$$A \subseteq NCScl(NCbint(A)) \Cup NCcl(NCint(A)).$$

The complement of a neutrosophic complex $\mathcal{N}$-open set is a neutrosophic complex $\mathcal{N}$-closed set($NC\mathcal{N}CS$)

Proposition 3.20. *Let A be any neutrosophic complex open set in $(X, \mathscr{T})$.*

(1) *If A is a neutrosophic complex $\mathcal{N}$-open set and $NCint(A) = 0^{\blacktriangle}$, then $A \subseteq NCScl(NCbint(A))$.*

(2) *If A is a neutrosophic complex β-open set and a neutrosophic complex closed set, then A is a neutrosophic complex $\mathcal{N}$-open set.*

Proof. (1) Let A be any neutrosophic complex $\mathcal{N}$-open set in X such that $NCint(A) = 0^{\blacktriangle}$. Then $A \subseteq NCScl(NCbint(A)) \Cup NCcl(NCint(A)) \subseteq NCScl(NCbint(A))$.

(2) Let A be any neutrosophic complex β-open set and a neutrosophic complex closed set in X. Since $A \subseteq NCbint(A)$ and A is a neutrosophic complex closed set in X, $NCScl(A) \subseteq NCScl(NCbint(A))$. Then $A \subseteq NCScl(NCbint(A))$.
Also, we have $A \subseteq NCcl(NCint(NCcl(A))) \subseteq NCcl(NCint(A))$. Thus

$$A \subseteq NCScl(NCbint(A)) \Cup NCcl(NCint(A)).$$

So A is a neutrosophic complex $\mathcal{N}$-open set in X. □

Proposition 3.21. *Let $(X, \mathscr{T})$ be any neutrosophic complex topological space. Then the union of any family of neutrosophic complex $\mathcal{N}$-open sets is a neutrosophic complex $\mathcal{N}$-open set in X.*

Proof. Suppose that A_1 and A_2 be any two neutrosophic complex $\mathcal{N}$-open sets in X. Then

$$A_1 \subseteq NCScl(NCbint(A_1)) \Cup NCcl(NCint(A_1))$$

and

$$A_2 \subseteq NCScl(NCbint(A_2)) \Cup NCcl(NCint(A_2)).$$

Thus

$$\begin{aligned} A_1 \Cup A_2 &\subseteq [NCScl(NCbint(A_1)) \Cup NCcl(NCint(A_1))] \\ &\qquad \Cup [NCScl(NCbint(A_2)) \Cup NCcl(NCint(A_2))] \\ &\subseteq [NCScl(NCbint(A_1)) \Cup NCScl(NCbint(A_2))] \\ &\qquad \Cup [NCcl(NCint(A_1)) \Cup NCcl(NCint(A_2))] \\ &\subseteq [NCScl(NCbint(A_1) \Cup NCbint(A_2))] \\ &\qquad \Cup [NCcl(NCint(A_1) \Cup NCint(A_2))] \\ &\subseteq [NCScl(NCbint(A_1 \Cup A_2))] \\ &\qquad \Cup [NCcl(NCint(A_1 \Cup A_2))]. \end{aligned}$$

So $A_1 \Cup A_2$ is a neutrosophic complex $\mathcal{N}$-open set in X. By using induction method, we have the union of any family of neutrosophic complex $\mathcal{N}$-open sets is a neutrosophic complex $\mathcal{N}$-open set in X. □

Proposition 3.22. *If A is a neutrosophic complex $\mathcal{N}$-open set and neutrosophic complex α-closed set in $(X, \mathscr{T})$, then A is a neutrosophic complex regular closed set in X.*

Proof. Let A be neutrosophic complex $\mathcal{N}$-open set and neutrosophic complex α-closed set in X. Then

$$\begin{aligned} NCcl(NCint(NCcl(A))) &\subseteq A \\ &\subseteq NCScl(NCbint(A)) \cup NCcl(NCint(A)) \\ &\subseteq NCcl(NCint(NCcl(A))). \end{aligned}$$

Thus $NCcl(NCint(NCcl(A))) = A$. Clearly, we have A is a neutrosophic complex closed set in X. So $NCcl(NCint(A)) = A$. Hence A is a neutrosophic complex regular closed set in X. □

Proposition 3.23. *If A is a neutrosophic complex $\mathcal{N}$-open set and neutrosophic complex α-closed set in $(X, \mathscr{T})$, then $A = NCScl(NCbint(A)) \Cup NCcl(NCint(A))$.*

Proof. Let A be neutrosophic complex $\mathcal{N}$-open set in X. Then

$$A \subseteq NCScl(NCbint(A)) \cup NCcl(NCint(A)). \tag{3.1}$$

Since A is a neutrosophic complex α-closed set in X, A is a neutrosophic complex semiclosed set and neutrosophic complex preclosed set in X. Thus

$$NCcl(NCint(A)) \subseteq A \text{ and } NCint(NCcl(A)) \subseteq A.$$

So $NCcl(NCint(A)) \Cup NCint(NCcl(A)) \subseteq A$.

Also A is a neutrosophic complex semiopen set in X. Then $NCSint(A) = A$. Thus $NCSint(A) \Cup NCint(NCcl(A)) \Cup NCcl(NCint(A)) \subseteq A$. So

$$NCScl(NCbint(A)) \Cup NCcl(NCint(A)) \subseteq A \tag{3.2}$$

Hence $A = NCScl(NCbint(A)) \Cup NCcl(NCint(A))$. □

Proposition 3.24. *Let $(X, \mathscr{T})$ be any $NCTS$. Then*
(1) *every neutrosophic complex open set is a neutrosophic complex $\mathcal{N}$-open set,*
(2) *every neutrosophic complex regular open set is a neutrosophic complex $\mathcal{N}$-open set,*
(3) *every neutrosophic complex α-open set is a neutrosophic complex $\mathcal{N}$-open set,*
(4) *every neutrosophic complex semi open set is a neutrosophic complex $\mathcal{N}$-open set,*
(5) *every neutrosophic complex preopen set is a neutrosophic complex $\mathcal{N}$-open set,*
(6) *every neutrosophic complex $\mathcal{N}$-open set is a neutrosophic complex β-open set.*

Remark 3.25. The converse of Proposition 3.7 need not be true as shown in the following examples.

Example 3.26. Let $X = \{a, b\}$ be a nonempty set. Let

$$A = \{\langle a, 0.9e^{j0.7}, 0.9e^{j0.8}, 0.1e^{j0.1}\rangle, \langle b, 0.7e^{j0.7}, 0.7e^{j0.8}, 0.2e^{j0.2}\rangle\}$$

and

$$B = \{\langle a, 0.9e^{j0.8}, 0.9e^{j0.9}, 0.1e^{j0.1}\rangle, \langle b, 0.8e^{j0.8}, 0.8e^{j0.8}, 0.2e^{j0.2}\rangle\}$$

be any two neutrosophic complex sets in X. Then $\mathscr{T} = \{0^\blacktriangle, A, 1^\blacktriangle\}$ is a neutrosophic complex topology and the pair $(X, \mathscr{T})$ is a neutrosophic complex topological space. Since $B \subseteq NCScl(NCbint(B)) \Cup NCcl(NCint(B)) = 1^\blacktriangle$, B is a neutrosophic complex $\mathcal{N}$-open set in X. Since $NCint(B) \neq B$ and $NCint(NCcl(B)) = 1^\blacktriangle \neq B$, B is not a neutrosophic complex regular open set and also not neutrosophic complex open set in X.

Example 3.27. Let $X = \{a, b\}$ be a nonempty set. Let

$$A = \{\langle a, 0.2e^{j0.2}, 0.2e^{j0.2}, 0.7e^{j0.7}\rangle, \langle b, 0.1e^{j0.1}, 0.1e^{j0.2}, 0.5e^{j0.5}\rangle\}$$

and

$$B = \{\langle a, 0.3e^{j0.3}, 0.3e^{j0.3}, 0.7e^{j0.7}\rangle, \langle b, 0.2e^{j0.2}, 0.2e^{j0.2}, 0.5e^{j0.5}\rangle\}$$

be any two neutrosophic complex sets in X. Then $\mathscr{T} = \{0^\blacktriangle, A, 1^\blacktriangle\}$ is a neutrosophic complex topology and the pair $(X, \mathscr{T})$ is a neutrosophic complex topological space. Since $B \subseteq NCScl(NCbint(B)) \Cup NCcl(NCint(B)) = A^c$, B is a neutrosophic complex $\mathcal{N}$-open set in X. Since $B \nsubseteq NCint(NCcl(B)) = A$ and $B \nsubseteq NCint(NCcl(NCcl((B)) = A$, B is not a neutrosophic complex pre open set and also not neutrosophic complex α-open set in X.

Example 3.28. Let $X = \{a, b\}$ be a nonempty set. Let

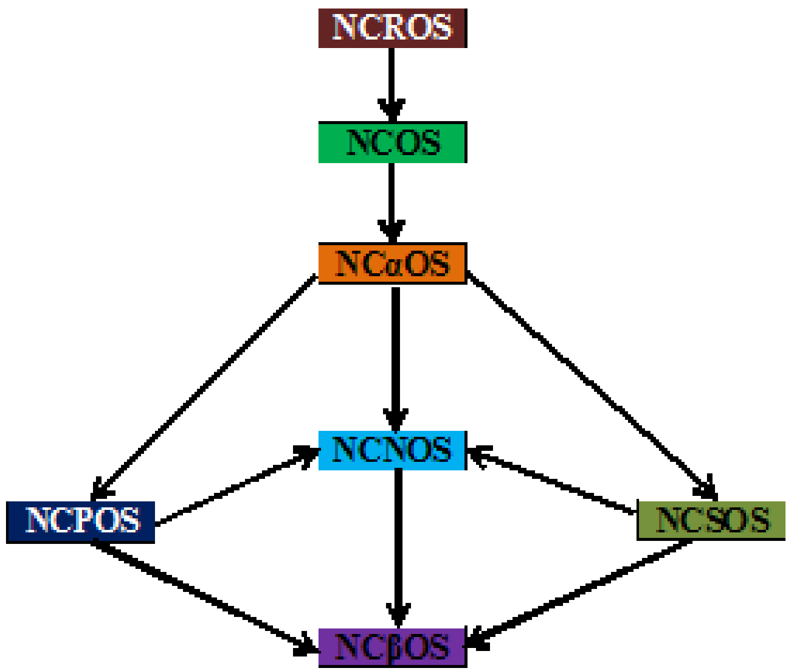
$$A = \{\langle a, 0.5e^{j0.5}, 0.5e^{j0.5}, 0.3e^{j0.3}\rangle, \langle b, 0.5e^{j0.5}, 0.5e^{j0.5}, 0.2e^{j0.2}\rangle\}$$

and

$$B = \{\langle a, 0.4e^{j0.4}, 0.5e^{j0.5}, 0.4e^{j0.4}\rangle, \langle b, 0.5e^{j0.5}, 0.5e^{j0.5}, 0.4e^{j0.4}\rangle\}$$

be any two neutrosophic complex sets in X. Then $\mathscr{T} = \{0^{\blacktriangle}, A, 1^{\blacktriangle}\}$ is a neutrosophic complex topology and the pair $(X, \mathscr{T})$ is a neutrosophic complex topological space. Since $B \subseteq NCScl(NCbint(B)) \Cup NCcl(NCint(B)) = 1^{\blacktriangle}$, B is a neutrosophic complex $\mathcal{N}$-open set in X. Since $B \nsubseteq NCcl(NCint(B)) = 0^{\blacktriangle}$, B is not a neutrosophic complex semi open set in X.

Remark 3.29. Clearly, the following diagram holds.



NCROS	neutrosophic complex regular open set
NCOS	neutrosophic complex open set
NCαOS	neutrosophic complex α-open set
NCβOS	neutrosophic complex β-open set
NC$\mathcal{N}$OS	neutrosophic complex $\mathcal{N}$-open set
NCPOS	neutrosophic complex preopen set
NCSOS	neutrosophic complex semi open set

Definition 3.30. Let $(X, \mathscr{T})$ be any $NCTS$ and let A be a neutrosophic complex set in X. Then the neutrosophic complex $\mathcal{N}$-interior and neutrosophic complex $\mathcal{N}$-closure of A are defined by

(i) $NC\mathcal{N}int(A) = \Cup\{G \mid G \text{ is a } NC\mathcal{N}OS \text{ in } X \text{ and } G \subseteq A\}$,

(ii) $NC\mathcal{N}cl(A) = \Cap\{G \mid G \text{ is a } NC\mathcal{N}CS \text{ in } X \text{ and } G \supseteq A\}$.

Proposition 3.31. *Let $(X, \mathscr{T})$ be any neutrosophic complex topological space $NCTS$ and A, B be neutrosophic complex sets in X. Then the following properties hold:*
(1) $NC\mathcal{N}int(A) \subseteq A \subseteq NC\mathcal{N}cl(A)$,
(2) *if* $A \subseteq B$, *then* $NC\mathcal{N}cl(A) \subseteq NC\mathcal{N}cl(B)$ *and* $NC\mathcal{N}int(A) \subseteq NC\mathcal{N}int(B)$,
(3) $NC\mathcal{N}cl(NC\mathcal{N}cl(A)) = NC\mathcal{N}cl(A)$ *and* $NC\mathcal{N}int(NC\mathcal{N}int(A)) = NC\mathcal{N}int(A)$,
(4) $NC\mathcal{N}cl(A \Cup B) = NC\mathcal{N}cl(A) \Cup NC\mathcal{N}cl(B)$
and $NC\mathcal{N}int(A \Cap B) = NC\mathcal{N}int(A) \Cap NC\mathcal{N}int(B)$,
(5) $NC\mathcal{N}cl(0^{\blacktriangle}) = 0^{\blacktriangle} = NC\mathcal{N}int(0^{\blacktriangle})$,
(6) $NC\mathcal{N}cl(1^{\blacktriangle}) = 1^{\blacktriangle} = NC\mathcal{N}int(1^{\blacktriangle})$.

Proposition 3.32. *Let $(X, \mathscr{T})$ be any neutrosophic complex topological space $NCTS$ and A be neutrosophic complex sets in X. Then we have*
(1) $NC\mathcal{N}cl(A) \supseteq NCSint(NCbcl(A)) \Cap NCint(NCcl(A))$,
(2) $NC\mathcal{N}int(A) \subseteq NCScl(NCbint(A)) \Cup NCcl(NCint(A))$

Proof. (1) Let A be any neutrosophic complex sets in X. Then $NC\mathcal{N}cl(A)$ is a neutrosophic complex $\mathcal{N}$-closed set in X and $A \subseteq NC\mathcal{N}cl(A)$. Thus

$$\begin{aligned} NC\mathcal{N}cl(A) &\supseteq NCSint(NCbcl(NC\mathcal{N}cl(A))) \Cap NCint(NCcl(NC\mathcal{N}cl(A))) \\ &\supseteq NCSint(NCbcl(A)) \Cap NCint(NCcl(A)). \end{aligned}$$

(2) The proof of (2) follows from (1) by taking complementation. □

4. Characterizations of neutrosophic complex $\mathcal{N}$-continuous functions

Definition 4.1. Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be any two neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function. Then f is said to be a neutrosophic complex $\mathcal{N}$-continuous function, if for each netrosophic complex open set A in $(Y, \mathscr{S})$, $f^{-1}(A)$ is a neutrosophic complex $\mathcal{N}$-open set in $(X, \mathscr{T})$.

Proposition 4.2. *Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be any two neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function. Then the following statements are equivalent:*
(1) *f is a neutrosophic complex $\mathcal{N}$-continuous function,*
(2) *$f^{-1}(A)$ is a neutrosophic complex closed set in $(X, \mathscr{T})$, for each neutrosophic complex closed set A in $(Y, \mathscr{S})$,*
(3) *$f(NC\mathcal{N}cl(A)) \subseteq NCcl(f(A))$, for each neutrosophic complex set A in $(X, \mathscr{T})$,*
(4) *$NC\mathcal{N}cl(f^{-1}(A)) \subseteq f^{-1}(NCcl(A))$, for each neutrosophic complex set A in $(Y, \mathscr{S})$.*

Proof. (1) ⇔ (2): The proof is Obvious.

(2) ⇒ (3): Let A be any neutrosophic complex closed set in Y. Then $NCcl(f(A))$ is a neutrosophic complex closed set in Y. By (2), $f^{-1}(NCcl(f(A)))$ is a neutrosophic complex $\mathcal{N}$-closed set in X. Also we know that $f(A) \subseteq NCcl(f(A))$. Then

$$f^{-1}(f(A)) \subseteq f^{-1}(NCcl(f(A)))$$

. Thus

$$NC\mathcal{N}cl(f^{-1}(f(A))) = NC\mathcal{N}cl(A) \subseteq NC\mathcal{N}cl(f^{-1}(NCcl(f(A)))).$$

So $f(NC\mathcal{N}cl(A)) \subseteq NCcl(f(A))$.

(3) $\Rightarrow$ (4): Let A be any neutrosophic complex open set in Y. By (3),

$$f(NC\mathcal{N}cl(f^{-1}(A))) \subseteq NCcl(f(f^{-1}(A))).$$

Thus $NC\mathcal{N}cl(f^{-1}(A)) \subseteq f^{-1}(NCcl(A))$.

(4) $\Rightarrow$ (1): Let A be any neutrosophic complex open set in Y. Then A^c is a neutrosophic complex closed set in Y. By (4), $NC\mathcal{N}cl(f^{-1}(A^c)) \subseteq f^{-1}(NCcl(A^c)) = f^{-1}(A^c)$.

Also we know that $NC\mathcal{N}cl(f^{-1}(A^c)) \supseteq f^{-1}(A^c)$. Then $NC\mathcal{N}cl(f^{-1}(A^c)) = f^{-1}(A^c)$. Thus A^c is a neutrosophic complex $\mathcal{N}$-closed set in X. So A is a neutrosophic complex $\mathcal{N}$-open set in X. Hence f is a neutrosophic complex $\mathcal{N}$-continuous function. □

Proposition 4.3. *Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be any two neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function. Then f is a neutrosophic complex $\mathcal{N}$-continuous function if and only if $f^{-1}(NCint(A)) \subseteq NC\mathcal{N}int(f^{-1}(A))$, for each neutrosophic complex set A in $(Y, \mathscr{S})$.*

Proof. Assume that f is a neutrosophic complex $\mathcal{N}$-continuous function. Let A be any neutrosophic complex set in Y. Then $NCint(A)$ is a neutrosophic complex open set in Y. Since f is a neutrosophic complex $\mathcal{N}$-continuous function, $f^{-1}(NCint(A))$ is a neutrosophic complex $\mathcal{N}$-open set in X.

Also, we know that $NCint(A) \subseteq A$. Then $f^{-1}(NCint(A)) \subseteq f^{-1}(A)$. Thus

$$NC\mathcal{N}int(f^{-1}(NCint(A))) = f^{-1}(NCint(A) \subseteq NC\mathcal{N}int(f^{-1}(A)).$$

So $f^{-1}(NCint(A)) \subseteq NC\mathcal{N}int(f^{-1}(A))$.

Conversely, suppose $f^{-1}(NCint(A)) \subseteq NC\mathcal{N}int(f^{-1}(A))$, for each neutrosophic complex set A in $(Y, \mathscr{S})$. If A is a neutrosophic complex open set in Y, then by hypothesis, $f^{-1}(A) \subseteq NC\mathcal{N}int(f^{-1}(A))$. We know that $f^{-1}(A) \supseteq NC\mathcal{N}int(f^{-1}(A))$. Then $f^{-1}(A) = NC\mathcal{N}int(f^{-1}(A))$. Thus $f^{-1}(A)$ is a neutrosophic complex $\mathcal{N}$-open set in X. So f is a neutrosophic complex $\mathcal{N}$-continuous function. □

Proposition 4.4. *Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be any two neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function. Then the following statements are equivalent:*

(1) *f is a neutrosophic complex $\mathcal{N}$-continuous function,*

(2) *$NCSint(NCbcl(f^{-1}(B))) \Cap Ncint(NCcl(f^{-1}(B))) \subseteq f^{-1}(NCcl(B))$, for each neutrosophic complex set B in $(Y, \mathscr{S})$.*

Proof. (1) $\Leftrightarrow$ (2): Let B be any neutrosophic complex set in Y. Then $NCcl(B)$ is a neutrosophic complex closed set in Y. Since f is a neutrosophic complex $\mathcal{N}$-continuous function, $f^{-1}(NCcl(B))$ is a neutrosophic complex $\mathcal{N}$-closed set in X. By Proposition 3.9,

$$\begin{aligned} &NC\mathcal{N}cl(f^{-1}(NCcl(B))) \\ &= f^{-1}(NCcl(B)) \\ &\supseteq NCSint(NCbcl(f^{-1}(NCcl(B)))) \Cap NCint(NCcl(f^{-1}(NCcl(B)))). \end{aligned}$$

Thus $f^{-1}(NCcl(B)) \supseteq NCSint(NCbcl(f^{-1}(B))) \Cap NCint(NCcl(f^{-1}(B)))$.

(2) $\Rightarrow$ (1): Let B be any neutrosophic complex closed set in Y. Then by 2),

$$f^{-1}(NCcl(B)) \supseteq NCSint(NCbcl(f^{-1}(B))) \Cap NCint(NCcl(f^{-1}(B))).$$

Thus $f^{-1}(B)$ is a neutrosophic complex $\mathcal{N}$-closed set in X. So f is a neutrosophic complex $\mathcal{N}$-continuous function. □

5. Interrelationship of neutrosophic complex $\mathcal{N}$-continuity

Definition 5.1. Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be any two neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function. Then f is said to be a neutrosophic complex

(i) continuous function, if for each netrosophic complex open set A in $(Y, \mathscr{S})$, $f^{-1}(A)$ is a neutrosophic complex open set in $(X, \mathscr{T})$,

(ii) regular (resp. semi, pre, α, β)-continuous function, if for each netrosophic complex open set A in $(Y, \mathscr{S})$, $f^{-1}(A)$ is a neutrosophic complex regular (resp. semi, pre, α, β)-open set in $(X, \mathscr{T})$.

Proposition 5.2. *Let $(X, \mathscr{T})$, $(Y, \mathscr{S})$ and $(Z, \mathscr{R})$ be any three neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ and $g : (Y, \mathscr{S}) \to (Z, \mathscr{R})$ be any two functions.*

(1) *If f is a neutrosophic complex $\mathcal{N}$-continuous function and g is a neutrosophic complex continuous function, then $g \circ f$ is a a neutrosophic complex $\mathcal{N}$-continuous function.*

(2) *If f is a neutrosophic complex $\mathcal{N}$-continuous function and g is a neutrosophic complex regular continuous function, then $g \circ f$ is a a neutrosophic complex $\mathcal{N}$-continuous function.*

Proposition 5.3. *Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be any two neutrosophic complex topological spaces. Let $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function.*

(1) *Every neutrosophic complex continuous function is a neutrosophic complex $\mathcal{N}$-continuous function.*

(2) *Every neutrosophic complex regular continuous function is a neutrosophic complex $\mathcal{N}$-continuous function.*

(3) *Every neutrosophic complex α-continuous function is a neutrosophic complex $\mathcal{N}$-continuous function.*

(4) *Every neutrosophic complex semi-continuous function is a neutrosophic complex $\mathcal{N}$-continuous function.*

(5) *Every neutrosophic complex pre-continuous function is a neutrosophic complex $\mathcal{N}$-continuous function.*

(6) *Every neutrosophic complex $\mathcal{N}$-continuous function is a neutrosophic complex β-continuous function.*

Remark 5.4. The converse of Proposition 5.2 need not be true as shown in the following examples.

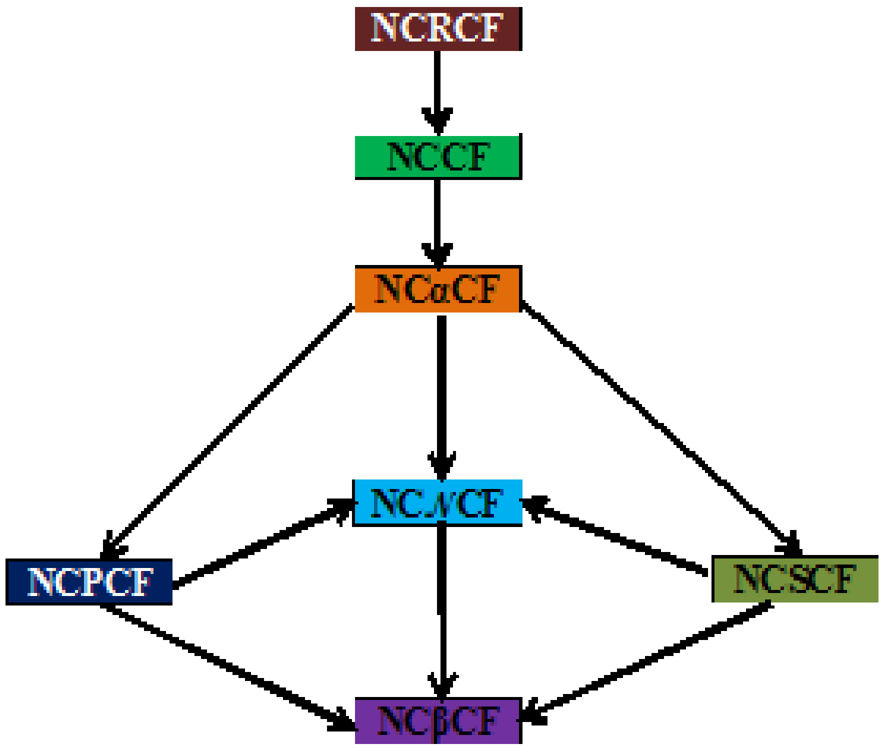
Example 5.5. Let $X = \{a\}$ and $Y = \{b\}$ be any two nonempty sets. Let $A = \{\langle a, 0.9e^{j0.7}, 0.9e^{j0.8}, 0.1e^{j0.1}\rangle$ and $B = \langle b, 0.9e^{j0.8}, 0.9e^{j0.9}, 0.1e^{j0.1}\rangle\}$ be any two neutrosophic complex sets in X and Y respectively. Then the family $\mathscr{T} =$

$\{0^\blacktriangle, 1^\blacktriangle, A\}$ and $\mathscr{S} = \{0^\blacktriangle, 1^\blacktriangle, B\}$are the neutrosophic complex topologies on X and Y respectively. Define a function $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function such that $f(a) = b$. Thus $f^{-1}(B) = B$ is a neutrosophic complex $\mathcal{N}$-open set in X. But $f^{-1}(B)$ is not a neutrosophic complex open set and not a neutrosophic complex regular open set in X. So, f is a neutrosophic complex $\mathcal{N}$-continuous function. But f is not neutrosophic complex continuous function and not a neutrosophic complex regular continuous function.

Example 5.6. Let $X = \{a\}$ and $Y = \{b\}$ be any two nonempty sets. Let $A = \{\langle a, 0.2e^{j0.2}, 0.2e^{j0.2}, 0.7e^{j0.7}\rangle$ and $B = \langle b, 0.2e^{j0.3}, 0.2e^{j0.3}, 0.5e^{j0.5}\rangle\}$ be any two neutrosophic complex sets in X and Y respectively. Then the family $\mathscr{T} = \{0^\blacktriangle, 1^\blacktriangle, A\}$ and $\mathscr{S} = \{0^\blacktriangle, 1^\blacktriangle, B\}$are the neutrosophic complex topologies on X and Y respectively. Define a function $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function such that $f(a) = b$. Thus $f^{-1}(B) = B$ is a neutrosophic complex $\mathcal{N}$-open set in X. But $f^{-1}(B)$ is not a neutrosophic complex preopen set and not a neutrosophic complex α-open set in X. So, f is a neutrosophic complex $\mathcal{N}$-continuous function. But f is not neutrosophic complex pre-continuous function and not a neutrosophic complex α-continuous function.

Example 5.7. Let $X = \{a\}$ and $Y = \{b\}$ be any two nonempty sets. Let $A = \{\langle a, 0.5e^{j0.5}, 0.5e^{j0.5}, 0.3e^{j0.3}\rangle$ and $B = \langle b, 0.4e^{j0.4}, 0.5e^{j0.5}, 0.4e^{j0.4}\rangle\}$ be any two neutrosophic complex sets in X and Y respectively. Then the family $\mathscr{T} = \{0^\blacktriangle, 1^\blacktriangle, A\}$ and $\mathscr{S} = \{0^\blacktriangle, 1^\blacktriangle, B\}$are the neutrosophic complex topologies on X and Y respectively. Define a function $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ be a function such that $f(a) = b$. Thus $f^{-1}(B) = B$ is a neutrosophic complex $\mathcal{N}$-open set in X. But $f^{-1}(B)$ is not a neutrosophic complex semiopen set in X. So,f is a neutrosophic complex $\mathcal{N}$-continuous function. But f is not neutrosophic complex semi-continuous function.

Remark 5.8. Clearly, the following diagram holds.



NCRCF	neutrosophic complex regular continuous function
NCCF	neutrosophic complex continuous function
NCαCF	neutrosophic complex α continuous function
NCβCF	neutrosophic complex β continuous function
NC$\mathcal{N}$ CF	neutrosophic complex $\mathcal{N}$ continuous function
NCPCF	neutrosophic complex precontinuous function
NCSCF	neutrosophic complex semicontinuous function

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