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A new approach to soft belonging

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Abstract. The development of concept of belonging to the soft sets still needs to be improved, since there are some of associated difficulties such as being the soft point neither belong to soft set nor its complement. In this work, the concept of soft element is introduced and studied by modifying the concept of soft point to be free from the associated problems related to the soft point. Moreover, the concept of two distinct soft elements is also defined. By applying these concepts, the interior and closure of a soft set are studied in addition to the ability to investigate some properties of soft separation axioms.

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1. Introduction

The concept of soft set theory has been introduced, in 1999, by Molodtsov [3]. It has been applied to several branches of mathematics such as operation research, game theory and among others. The applications of soft set theory increase to several researchers, especially in the recent years. This is because of the general nature of parameterizations expressed by a soft set. Therefore, several special sets have been introduced in the soft set theory and their properties have been studied, within the soft topological space. The notion of soft topological spaces was formulated by Shabir and Naz [5]. Hussain and Ahmad [1] introduced and studied several notions of soft topological spaces. Zorlutuna et al. [7] introduced the concept of soft point. In the present study, we introduce a new modification of soft point that is called soft element; also we study some new concepts such as soft interior point, soft closure point and soft separation axioms in the soft topological spaces. Applying our modification, some examples such as (3.7, 3.8, 3.14 and 3.26 in [6], 7, 8 and 9 in [5]) are not as they described. Moreover, a good feedback of this modification on some well-known results will be discussed in this paper.

2. Preliminaries

Molodtsov [3] defined soft sets in the following manner. Let X be an initial Universe set and E be a set of parameters. Let $P(X)$ denote the power set of X, and $A \sqsubseteq E$.

Definition 2.1 ([3]). A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

For illustration, Molodtsov Provided several examples in [3]. The set of all soft sets over X is denoted by $SS(X)$.

Definition 2.2 ([4]). For two soft sets (F, A) and (G, B) over a common universe X, (F, A) is a soft subset of (G, B), denoted by $(F, A)\widetilde{\sqsubseteq}(G, B)$, if $A \sqsubseteq B$ and $\forall e \in A$, $F(e)\widetilde{\sqsubseteq}G(e)$

Definition 2.3 ([2]). Two soft sets (F, A) and (G, B) over a common universe X, are said to be soft equal, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4 ([2]). A soft set (F, A) over X is said to be a null soft set, denoted by Φ, if $\forall e \in A$, $F(e) = \phi$.

Definition 2.5 ([2]). A soft set (F, A) over X is said to be an absolute soft set, denoted by $\widetilde{X}$, if $\forall e \in A$, $F(e) = X$.

Definition 2.6 ([5]). The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$, is the soft set (H, E) where $\forall e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.7 ([2]). The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, for all $e \in A$, and F^c is the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.8 ([2]). The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \sqcup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & If \;\; e \in A - B, \\ G(e) & if \;\; e \in B - A, \\ F(e) \sqcup G(e) & if \;\; e \in A \sqcup B \end{cases}$$

This relationship is written as $(H, C) = (F, A)\widetilde{\sqcup}(G, B)$.

Definition 2.9 ([2]). The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \sqcap B$ and for all $e \in C, H(e) = F(e) \sqcap G(e)$. This relationship is written as $(H, C) = (F, A)\widetilde{\sqcap}(G, B)$.

Definition 2.10 ([7]). The soft set $(F, A)\widetilde{\in}SS(X)$ is called a soft point in $\widetilde{X}$, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e^{'}) = \phi$, for all $e^{'} \in A - \{e\}$.

Definition 2.11 ([7]). The soft point e_F is said to be in the soft set (G, A), denoted by $e_F\widetilde{\in}(G, A)$, if for the element $e \in A$, $F(e)\widetilde{\sqsubseteq}G(e)$.

Definition 2.12 ([5]). Let τ be the collection of soft sets over X with the fixed set of parameters E. Then τ is said to be a soft topology on X, if

(i) Φ, $\widetilde{X}$ belong to τ,

(ii) the union of any number of soft sets in τ belongs to τ,

(iii) the intersection of any two soft sets in τ belongs to τ.

The triplet $(\widetilde{X}, \tau, E)$ is called a soft topological space over X.

Definition 2.13 ([5]). Let $(\widetilde{X}, \tau, E)$ be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 2.14 ([5]). Let $(\widetilde{X}, \tau, E)$ be a soft space over X. A soft set (F,E) over X is said to be a soft closed set in X, if its relative complement $(F, E)^c$ belongs to τ.

Definition 2.15 ([5]). Let X be an initial universe set and E be the set of parameters. A soft topology $\tau = \{\Phi, \widetilde{X}\}$. is called a soft indiscrete topology on X and $(\widetilde{X}, \tau, E)$ is called a soft indiscrete space over X.

Definition 2.16 ([5]). Let X be an initial universe set, E the set of parameters and τ the collection of all soft sets which can be defined over X. τ is called a soft discrete topology on X and $(\widetilde{X}, \tau, E)$ is called a soft discrete space over X.

3. Soft elements and distinguished of two soft elements

Previously, some modifications to the soft point has been done considering that e/x and e/y are distinct soft points whenever $x \neq y$, in spite of being e/x and e/y images for the same parameter. Actually, for more generalization of the concept, e_i/x and e_j/y with $e_i, e_j \in E$ are more distinguished ($x \neq y$) since two parameters e_i and e_j may be different. Also there is a good feedback of this modification on some properties that will be discussed later.

Definition 3.1. A soft set (F, E) over X is called a soft element, denoted by e/x or e_F, if $F(e) = \{x\}$, $F(e^{'}) = \phi$, for all $e^{'} \in E - \{e\}$. We shall say that

(i) $e/x \widetilde{\in} (G, B)$, read as e/x belongs to the soft set (G, B), if $F(e) \widetilde{\sqsubseteq} G(e)$,

(ii) e_i/x and e_j/y are two distinct soft elements, if $x \neq y$.

Remark 3.2. Similar trial of the above modification was previously reported by Tantawy et al.[6], The application of our modification shows that some examples in [6] are not as they described.

Remark 3.3. It is clear that

(1) each member of a soft set (F, A) can be expressed as a union of all its soft elments,

(2) if $e/x \widetilde{\in} (G, A)$, then $e/x \widetilde{\notin} (G, A)^c$,

(3) if $e/x \widetilde{\notin} (G, A)$, then $e/x \widetilde{\in} (G, A)^c$.

Example 3.4. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$, $(F, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_2, h_3\})\}$.
Clearly the soft elements $(F_1, E) = \{(e_1, h_1), (e_2, \phi)\} = e_1/h_1, (F_2, E) = \{(e_1, h_2), (e_2, \phi)\} = e_1/h_2, (F_3, E) = \{(e_1, \phi), (e_2, h_2)\} = e_2/h_2$ and $(F_4, E) = \{(e_1, \phi), (e_2, h_3)\} = e_2/h_3$ belong to the soft set (F, E) while the soft elements $(F_5, E) = \{(e_1, h_3), (e_2, \phi)\} = e_1/h_3$ and $(F_6, E) = \{(e_1, \phi), (e_2/h_1)\} = e_2/h_1$ are not. Also $(e_2/h_1)^c = X \setminus e_2/h_1 = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_2, h_3\})\}$ and $(F, E) \setminus e_1/h_1 = \{(e_1, \{h_2\}), (e_2, \{h_2, h_3\})\}$.

Remark 3.5. It is clear that a soft element is a soft point [7] but generally the reverse need not be true as shown in the following example

Example 3.6. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$, $(F, E) = \{(e_1, \{h_1, h_2\}), (e_2, X)\}$. Clearly $(G, E) = \{(e_1, \{h_1, h_2\}), (e_2, \phi)\}$ is a soft point, but not a soft element in spite of being $G(e_2) = \phi$, because $G(e_1) = \{h_1, h_2\}$ is not singleton.

Theorem 3.7. *Let P be the collection of soft sets over X containing the particular soft element e/p, then P is a soft topology on X.*

Proof. It is obvious. □

The soft topology P in the above theorem is called particular soft element topology and the triple $(\widetilde{X}, P, E)$ is called a particular soft element space

Theorem 3.8. *Let ξ be the collection of soft sets over X not containing the particular soft element e/p, then ξ is a soft topology on X.*

Proof. It is obvious. □

The topology ξ in the above theorem is called excluding soft element topology and the triple $(\widetilde{X}, \xi, E)$ is called an excluding soft element space.

Definition 3.9. A soft set (F, E) over an infinite universe X is finite, if $\forall e \in A, F(e)$ is a finite subset of X. Otherwise (F, E) is an infinte soft set.

Theorem 3.10. *Let C be the collection of all soft sets $(F_i, A)_{i\in I}$ over an infinite intial universe X such that $C = \{(F_i, E)_{i\in I} \ \widetilde{\sqsubseteq} \ \widetilde{X} : (F_i, E)^c{}_{i\in I}$ is a finite soft set$\}$. Then C is a soft topology on X.*

Proof. It is obvious. □

The topology C in the above theorem is called cofinite soft topology and the triple $(\widetilde{X}, C, E)$ is called a cofinite soft topological space.

Definition 3.11. Let $(\widetilde{X}, \tau, E)$ be a soft topological space.

(i) A soft element e/x is called an interior soft element of a soft set of (F, E), if there exists an open soft set $(G, E) : e/x \widetilde{\in} (G, E) \widetilde{\sqsubseteq} (F, E)$. (F, E) is called a neighborhood of a soft element e/x, whenever e/x is an interior soft element of (F, E). The set of all interior soft element of a soft set (F, E) is denoted by $(F, E)^\circ$.

Clearly $(F, E)^\circ$ is a open soft set and $(F, E)^\circ = \widetilde{\sqcup}\{(G, E) : (G, E)$ is an open soft set and $(G, E) \widetilde{\sqsubseteq} (F, E)\}$.

(ii) A soft element e/x is called a closure soft element of a soft set (F, E), if for each open soft set $(G, E) : e/x \widetilde{\in} (G, E), (G, E) \widetilde{\sqcap} (F, E) \neq \Phi$. The set of all closure soft elements of a soft set (F, E) denoted by $\overline{(F, E)}$. Clearly $\overline{(F, E)}$ is a closed soft set, since $\overline{(F, E)} = \widetilde{\sqcap}\{(G, E) \ \forall$ closed soft set (G, E) whenever $(F, E) \widetilde{\sqsubseteq} (G, E)\}$.

(iii) A soft element e/x is called a derived soft element of a soft set (F, E), if for each open soft set $(G, E) : e/x \widetilde{\in} (G, E), (G, E) - e/x \widetilde{\sqcap} (F, E) \neq \Phi$. The set of all derived soft elements of a soft set (F, E) denoted by $(F, E)^{'}$.

Example 3.12. Let $X = \{a, b\}$, $E = \{e_1, e_2\}, (F_1, E) = \{(e_1, \{a\}), (e_2, \phi)\}$,
$(F_2, E) = \{(e_1, \{b\}), (e_2, \{a\})\}, (F_3, E) = \{(e_1, \{a, b\}), (e_2, \{a\})\}$,
$(F_4, E) = \{(e_1, \{b\}), (e_2, \{a, b\})\}, (F_5, E) = \{(e_1, \{a, b\}), (e_2, \{b\})\}$,
$(F_6, E) = \{(e_1, \{b\}), (e_2, \phi)\}, (F_7, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$,and

$(F_8, E) = \{(e_1, \{a, b\}), (e_2, \phi)\}$. Then the collection

$$\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E)\}$$

is a soft topology over X.

If $(F, E) = \{(e_1, \{a\}), (e_2, \{a, b\})\}$, then $(F, E)^\circ = e_1/a$, $\overline{(F, E)} = (F, E)$ and $(F, E)^{'} = \phi$.

Theorem 3.13. *Let $(\widetilde{X}, \tau, E)$ be a soft topological space, $(F, E), (G, E) \in SS(X)$. Then*
(1) $\Phi^\circ = \Phi$, $(\widetilde{X})^\circ = \widetilde{X}$,
(2) $(F, E)^\circ \ \widetilde{\sqsubseteq}\ (F, E)$,
(3) (F, E) *is an open soft set iff* $(F, E)^\circ = (F, E)$,
(4) $(F, E)^\circ \ \widetilde{\sqcap}\ (G, E)^\circ = ((F, E) \ \widetilde{\sqcap}\ (G, E))^\circ$,
(5) $(F, E)^\circ \ \widetilde{\sqcup}\ (G, E)^\circ \ \widetilde{\sqsubseteq}\ ((F, E) \ \widetilde{\sqcup}\ (G, E))^\circ$.

Proof. we only give a proof to (4), the other cases can be proved easily.

(4) Let $e/x \widetilde{\in} (F, E)^\circ \widetilde{\sqcap} (G, E)^\circ$. Then $e/x \widetilde{\in} (F, E)^\circ$ and $e/x \widetilde{\in} (G, E)^\circ$. Thus $\exists$ two open soft sets $(F_1, E), (G_1, E)$ such that

$$e/x \widetilde{\in} (F_1, E) \widetilde{\sqsubseteq} (F, E) \text{ and } e/x \widetilde{\in} (G_1, E) \widetilde{\sqsubseteq} (G, E).$$

So $\exists$ a non empty open soft set $(F_1, E) \widetilde{\sqcap} (G_1, E) = (H, E)$ such that

$$e/x \widetilde{\in} (H, E) \widetilde{\sqsubseteq} (F, E) \widetilde{\sqcap} (G, E).$$

Hence $e/x \widetilde{\in} ((F, E) \widetilde{\sqcap} (G, E))^\circ$. Therefore

$$(F, E)^\circ \widetilde{\sqcap} (G, E)^\circ \widetilde{\sqsubseteq} ((F, E) \widetilde{\sqcap} (G, E))^\circ. \tag{3.1}$$

By the same way, we can prove

$$((F, E) \widetilde{\sqcap} (G, E))^\circ \widetilde{\sqsubseteq} (F, E)^\circ \widetilde{\sqcap} (G, E)^\circ \tag{3.2}$$

From (3.1) and (3.2), we have $(F, E)^\circ \widetilde{\sqcap} (G, E)^\circ = ((F, E) \widetilde{\sqcap} (G, E))^\circ$. □

The following example shows that generally the equality in (5) of the above theorem need not be true.

Example 3.14. In Example 3.12 $(F, E)^\circ = \{e_1/a\}$, where $(F, E) = \{(e_1, \{a\}), (e_2, \{a, b\})\}$ and $(F_2, E)^\circ = \{(e_1, \{b\}), (e_2, \{a\})\}$. Then clearly

$$((F, E) \widetilde{\sqcup} (F_2, E))^\circ = \widetilde{X}$$
$$\widetilde{\not\sqsubseteq}\ (F, E)^\circ \widetilde{\sqcup} (G, E)^\circ = \{(e_1, \{a, b\}), (e_2, \{a\})\}.$$

Theorem 3.15. *Let $(\widetilde{X}, \tau, E)$ be a soft topological space, $(F, E), (G, E) \in SS(X)$. Then*

(1) $\overline{\Phi} = \Phi$, $\overline{(\widetilde{X})} = \widetilde{X}$,
(2) $(F, E) \ \widetilde{\sqsubseteq}\ \overline{(F, E)}$,
(3) (F, E) *is a closed soft set iff* $\overline{(F, E)} = (F, E)$,
(4) $\overline{\overline{(F, E)}} = \overline{(F, E)}$,
(5) $\overline{(F, E)} \ \widetilde{\sqcup}\ \overline{(G, E)} = \overline{((F, E) \widetilde{\sqcup} (G, E))}$,
(6) $\overline{(F, E)} \ \widetilde{\sqcap}\ \overline{(G, E)} \ \widetilde{\sqsubseteq}\ \overline{((F, E) \widetilde{\sqcap} (G, E))}$.

Proof. It is obvious □

The following example shows that generally the equality in (6) of the above theorem need not be true.

Example 3.16. In Example 3.12, if $(F, E) = \{(e_1, \{a\}), (e_2, \{b\})\}$ and $(G, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$, then $\overline{(F, E)} = \{(e_1, \{a\}), (e_2, \{b\})\}, \overline{(G, E)} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$. Thus

$$\overline{(F, E)} \,\widetilde{\sqcap}\, \overline{(G, E)} = \{(e_1, \phi), (e_2, \{b\})\} \not\sqsubseteq \overline{((F, E) \,\widetilde{\sqcap}\, (G, E))} = \Phi.$$

Example 3.17. Let $(\widetilde{X}, P, E)$ be a particular soft element space, $(F, E)\widetilde{\in}SS(X)$. Then

$$\overline{(F, E)} = \begin{cases} \widetilde{X} & \text{if } e/p\widetilde{\in}(F, E) \\ (F, E) & \text{if } e/p\widetilde{\notin}(F, E), \end{cases}$$

$$(F, E)^{'} = \begin{cases} \widetilde{X} - e/p & \text{if } e/p\widetilde{\in}(F, E) \\ \Phi & \text{if } e/p\widetilde{\notin}(F, E), \end{cases}$$

$$(F, E)^{\circ} = \begin{cases} (F, E) & \text{if } e/p\widetilde{\in}(F, E) \\ \Phi & \text{if } e/p\widetilde{\notin}(F, E). \end{cases}$$

Example 3.18. Let $(\widetilde{X}, \xi, E)$ be an excluding soft element space,$(F, E)\widetilde{\in}SS(X)$. Then

$$\overline{(F, E)} = \begin{cases} (F, E) & \text{if } e/p\widetilde{\in}(F, E) \\ (F, E)\widetilde{\sqcup}e/p & \text{if } e/p\widetilde{\notin}(F, E), \end{cases}$$

$$(F, E)^{'} = \begin{cases} \Phi & \text{if } (F, E) = e/p \\ e/p & \text{if } (F, E) \neq e/p, (F, E) \neq \Phi, \end{cases}$$

$$(F, E)^{\circ} = \begin{cases} (F, E) - e/p & \text{if } e/p\widetilde{\in}(F, E), (F, E) \neq \widetilde{X} \\ (F, E) & \text{if } e/p\widetilde{\notin}(F, E) or (F, E) = \widetilde{X}. \end{cases}$$

Definition 3.19. A soft topological space $(\widetilde{X}, \tau, E)$ is called a soft T_0-space, if for each two distinct soft elements e_i/x and e_j/y, there exists an open soft set (F, E) : $e_i/x\widetilde{\in}(F, E)$ and $e_j/y\widetilde{\notin}(F, E)$ or $e_j/y\widetilde{\in}(F, E)$ and $e_i/x\widetilde{\notin}(F, E)$.

Example 3.20. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where

$$F_1(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \{x\} & \text{if } \alpha = e_2, \end{cases} \qquad F_2(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \widetilde{X} & \text{if } \alpha = e_2, \end{cases}$$

$$F_3(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \{y\} & \text{if } \alpha = e_2, \end{cases} \quad F_4(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \Phi & \text{if } \alpha = e_2. \end{cases}$$

Then $(\widetilde{X}, \tau, E)$ is a soft topological space. It is also a soft T_0-space since for each two distinct soft elements e_i/x and e_j/y, $i, j \in \{1, 2\}$, there exists an open soft set containing one but not the other.

Example 3.21. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$, $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where

$$F_1(\alpha) = \begin{cases} \widetilde{X} & \text{if } \alpha = e_1 \\ \{y\} & \text{if } \alpha = e_2, \end{cases}$$

$$F_2(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \widetilde{X} & \text{if } \alpha = e_2, \end{cases}$$

$$F_3(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \{y\} & \text{if } \alpha = e_2. \end{cases}$$

Then clearly a soft topological space $(\widetilde{X}, \tau, E)$ is not soft T_0-space since e_1/x and e_2/y are two distinct soft elements while each open soft set containing one of them containing the other. This result is in contrast to what reported previously of describing the same example as a soft T_0-space Example 3.2 in [6].

Definition 3.22. A soft topological space $(\widetilde{X}, \tau, E)$ is called a soft T_1-space, if for each two distinct soft elements e_i/x and e_j/y, there exist two open soft set (F, E) and (G, E) such that

$$e_i/x\widetilde{\in}(F, E),\ e_j/y\widetilde{\notin}(F, E) \text{ and } e_j/y\widetilde{\in}(G, E),\ e_i/x\widetilde{\notin}(G, E).$$

Example 3.23. Let $X = \{x, y, z\}$, $E = \{e\}$. The collection $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ is a soft topology, where $F_1(e) = \{x\}$, $F_2(e) = \{y\}$, $F_3(e) = \{z\}$, $F_4(e) = \{x, y\}$, $F_5(e) = \{x, z\}$ and $F_6(e) = \{x, y\}$. Then $(\widetilde{X}, \tau, E)$ is a soft T_1-space.

Example 3.24. The soft space in Example 3.20 is a soft T_0-space but notT_1-space.

Theorem 3.25. *$(\widetilde{X}, \tau, E)$ is a soft T_1-space if and only if every soft element is a soft closed set.*

Proof. Assume that every soft element is a closed soft set and that e_i/x and e_j/y are two distinct soft elements. Then there exist two open soft sets $(F, E) = \widetilde{X} - e_i/x$, $(G, E) = \widetilde{X} - e_j/y$ such that

$$e_i/x\widetilde{\in}(G, E),\ e_j/y\widetilde{\notin}(G, E) \text{ and } e_j/y\widetilde{\in}(F, E),\ e_i/x\widetilde{\notin}(F, E).$$

Thus $(\widetilde{X}, \tau, E)$ is T_1-space.

Conversely, let $(\widetilde{X}, \tau, E)$ be a soft T_1-space and e_i/x be a soft element and $e_j/y\widetilde{\in}\ \widetilde{X} - e_i/x$. Since e_j/y and e_i/x are two distinct soft elements and $(\widetilde{X}, \tau, E)$ is a soft T_1-space, there exists an open soft set (F, E) such that

$$e_j/y\widetilde{\in}(F, E) \text{ and } e_i/x\widetilde{\notin}(F, E).$$

Clearly $e_j/y\widetilde{\in}(\widetilde{X} - e_i/x)^{\circ}$. So $\widetilde{X} - e_i/x$ is a soft open set. Hence e_i/x is a closed soft set. □

Corollary 3.26. *Every finite soft T_1-space has to be a discrete soft space.*

Remark 3.27. The previous theorem supports the success of our modification compared to previously reported one that prescribed Example 3.7 in [6] and Example 7 in [5] as a soft T_1-space, while not every soft element is closed so, this examples are not soft T_1-spaces as we will show.

Example 3.28. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$, $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E)\}$, where

$$F_1(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \{y\} & \text{if } \alpha = e_2, \end{cases} \qquad F_2(\alpha) = \begin{cases} \{x\} & \text{if } \alpha = e_1 \\ \{y\} & \text{if } \alpha = e_2. \end{cases}$$

Then clearly the soft topology $(\widetilde{X}, \tau, E)$ is not soft T_1-space as shown in Example 3.7 that is explained by Shabir and Naz [6] since there exist two distinct soft elements e_1/x and e_2/y while each open soft set containing one of them contains also the other . Of course $(\widetilde{X}, \tau, E)$ is not a soft T_2-space as shown in Example 3.14 that is reported in [6].

Remark 3.29. Clearly

(1) Example 3.8 in [6] is not a soft T_1-space as they described,
(2) Example 3.26 is not a soft T_4-space as shown in [6], since it is not soft T_1-space.

Definition 3.30. A soft topological space $(\widetilde{X}, \tau, E)$ is called a soft T_2-space, if for each two distinct soft elements e_i/x and e_j/y, there exist two disjoint open soft sets (F, E) and (G, E) such that

$$e_i/x\widetilde{\in}(F, E),\ e_j/y\widetilde{\in}(G, E) \text{ and } (F, E)\widetilde{\sqcap}(G, E) = \Phi.$$

Remark 3.31. The soft space given in Example 3.23 is a soft T_2-space

Remark 3.32. The cofinite soft space is a soft T_1-space but not a soft T_2-space.

Remark 3.33. It is clear from above definitions, Example 3.20 and Remark 3.32 that the following implications are true but their reverse generally need not be true.

Soft T_2-space $\rightarrow$ Soft T_1-space$\rightarrow$ Soft T_0-space.

4. Conclusion

This paper is just beginning of the application of the new approach of the concept of soft belonging and the concept of two distinct soft elements to study some concepts in soft topological spaces.

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