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Some properties of fuzzy soft groups

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Abstract. The aim of this paper is two folded. Firstly, we have defined the order of an element of a fuzzy soft group and the order of an e-approximate element of a fuzzy soft group with respect to some parameters and secondly, we have introduced the notion of a fuzzy soft cyclic group with respect to some parameters. Some theorems related to these concepts have been developed. To justify the existence of these definitions and theorems some examples have been also provided.

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Keywords: Soft Set, Fuzzy Soft Set, Fuzzy Group, Soft Group, Fuzzy Soft Group, Order of a Fuzzy Soft Group, Fuzzy Soft Cyclic Group.

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1. Introduction

The real world is full of uncertain impressions and vagueness. In ancient age, the concept of uncertainty was full of mysteries. But now-a-days, uncertainty or vagueness has become more transparent, more apprehensible. In daily life we have to face with some uncertainties in different fields such as economics, engineering, social science, medical science, environmental science etc.. In such type of fields we cannot always use classical methods which are based on two-valued-logic. i.e., yes or no type rather than more or less type. To deal with such type of uncertainties, Zadeh [30] first apprised us about the prosperous idea, fuzzy set theory. After that many theories have come out such as, Intuitionistic fuzzy sets [8], Rough sets [25], theory of Vague sets [12], theory of Interval mathematics, theory of Probability etc. But Molodtsov observed that all of these theories have their own inherent difficulties. Then in 1999, Molodtsov [24] proposed a new mathematical concept, soft set theory for modeling uncertainties. In soft set theory, we do not need to introduce the notion of an exact solution, so we can use soft set in many different fields.

For the time being, experiments on soft sets have been very efficacious and many exigent emanations have been followed by many researchers. After Molodtsov's work, many researchers have worked on soft set theory. Maji et al. [18], [19] have done further research on soft set theory. They have also developed the concept of fuzzy soft set [17],[20] and introduced a decision making method for finding the optimal choice object. But Kong et al.[14] have shown that Maji's algorithm [20] is not right. After that, Feng et al.[11] have found the limitations of Maji's method [20] as well as the Kong's method[14] and then proposed the new notion, level soft set. By using the concept of level soft set, they have also developed an algorithm to solve a fuzzy soft set based decision making problem. But, Basu et al.[9] have found the limitations of Feng's method and then introduced the new approach, mean potentiality approach(MPA) to solve a fuzzy soft set based decision making problem. Kong et al.[15] have also introduced a decision making method based on grey relational analysis by using different single evaluation basis such as choice value, score value etc..

The algebraic structure in fuzzy environment has already dwelled. First in 1971, Rosenfield[26] encountered the algebraic structure in fuzzy environment and introduced the concept of fuzzy group. After that Anthony et al.[6] revised the concept of fuzzy group, Yuan et al.[29] defined the fuzzy group based on fuzzy binary operations, Das et al.[10] initiated fuzzy groups and level sub-groups, Aktas et al.[3] discussed the generalized product of fuzzy subgroups and level subgroups, Abraham et al.[2] gave the idea to fuzzify the Cayley's and Lagrange's Theorems, Kamble et al.[13] also studied algebraic structures on fuzzy sets. Afterwards Aktas et al.[4] have introduced the concept of soft groups and their properties which extend the notion of a group to include the algebraic structures of soft sets. After that Ulucay et al.[28] have defined the soft representation of soft groups, Aslam et al.[7] also have worked on soft groups, Moinuddin et al.[23] have defined a new approach of soft group.Then the notion of fuzzy soft group have been presented by Abdulkadir[1]. Sarala et al.[27] have also given some properties of fuzzy soft groups. Manemaran[21] have studied on fuzzy soft groups. After that the concept of cyclic soft groups and their properties have been developed by Aktas et al.[5]. Liu et al.[16] have w0orked on general fuzzy soft groups, Mattam et al.[22] have defined the factors group of a fuzzy soft group.

In this paper, first we have discussed the power of an element of a fuzzy soft group corresponding to some parameters and then we have described the order of an element of a fuzzy soft group and order of an e-approximate element of a fuzzy soft group. We have also developed some properties and related theorems. Finally we have introduced the concept of fuzzy soft cyclic group. Some theorems have been also put on which are related to fuzzy soft cyclic group. More over, to verify these theorems we have also discussed some examples.

This paper has organized as follows. In section 1, the introduction has given. In section 2, some preliminaries such as soft set, fuzzy soft set, soft group, fuzzy soft group etc. have been discussed. In section 3, we have defined the order of an element of a fuzzy soft group and the order of an e-approximate element of a fuzzy soft group with respect to some parameters. In this section some theorems has been also developed. In section 4, we have introduced the concept of fuzzy soft cyclic group with some of their properties and related theorems.

2. Preliminaries

In this section we will recall some basic definitions which have been used in our subsequent discussions. In the whole paper we have selected X as an initial universal set and $I = [0, 1]$ be the unit closed interval.

Definition 2.1 ([9, 18, 19, 24])**.** Let E be the set of parameters and $A \subset E$. Then a pair (F, A) is called a soft set over X if and only if F is a mapping given by, $F : A \to P(X)$, where $P(X)$ denotes the power set of X.

In other words, a soft set is a parameterized family of subsets of the universe X. For, $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2 ([9, 17, 20])**.** Let E be the set of parameters and let $A \subset E$. Let I^X be the set of all fuzzy sub sets on X. Then a pair (f, A) is called a fuzzy soft set over X, where f is a mapping given by $f : A \to I^X$, which is defined as, $f(a) = f_a : X \to [0, 1]$.

For each $e \in A$, $f(e)$ is called the e-approximate elements of the fuzzy soft set (f, A).

Definition 2.3 ([9, 17, 20])**.** The union of two fuzzy soft sets (f, A) and (g, B) over a common universe X is a fuzzy soft set (h, C), where $C = A \cup B$. Then $\forall c \in C$, we can defined it as:

$$h_c(x) = \begin{cases} f_c(x) & ,\text{if } c \in A - B \\ g_c(x) & ,\text{if } c \in B - A \\ f_c(x) \cup g_c(x) & ,\text{if } c \in A \cap B. \end{cases}$$

We can expressed it as, $(f, A)\tilde{\cup}(g, B) = (h, C)$.

Definition 2.4 ([9, 17, 20])**.** The intersection of two fuzzy soft sets (f, A) and (g, B) over a common universe X is a fuzzy soft set (h, C), where $C = A \cap B$. Then $\forall c \in C$ and $\forall x \in X$, we can defined it as, $h_c(x) = f_c(x) \cap g_c(x)$ and expressed it as $(f, A)\tilde{\cap}(g, B) = (h, C)$.

Definition 2.5 ([4, 7, 23, 28])**.** Let $(X, .)$ be a group and A be a non-empty parameterized set. R will be refer to an arbitrary binary relation between the elements of A and the elements of G. Let (F, A) be a soft set over X where F is a mapping given by $F : A \to P(X)$, which can be defined as

$$F(x)=\{y \in X\text{: } (x, y) \in R,\ x \in X \text{ and } y \in X\},$$

where $R = \{(x, y) \in A \times G : y \in F(x)\}$. Then a soft set (F, A) is said to be a soft group over X if and only if $F(x) < X$, i.e., for each $x \in A$, $F(x)$ gives us a collection of subgroups of X.

Definition 2.6 ([1, 16, 21, 27])**.** Let $(X, .)$ be a group and let (f, A) be a fuzzy soft set over X where A be the parameterized set. Then the pair (f, A) is said to be a fuzzy soft group over X if and only for each $a \in A$ and $x, y \in X$,
(i) $f_a(x.y) \geq min\{f_a(x), f_a(y)\}$,
(ii) $f_a(x^{-1}) \geq f_a(x)$.
Here each of the subsets f_a or $f(a)$ of X is a fuzzy subgroup of X in Rosenfeld's sense.

Example 2.7. Let $X = \{1, -1, i, -i\}$ be a set which forms a group under multiplication and let $A = \{e_1, e_2\}$ be the parameterized set. Where
e_1 stands for the parameter 'square elements of X' and e_2 stands for the parameter 'cubic elements of X'.
It is noted that we do not take any element more than one time.
Now, let us consider a mapping $f : A \to I^X$ such that $f_a : X \to [0, 1]$, which is defined as:

$$f_a(x) = \{\tfrac{1}{n} : x^n = 1,\, a \in A,\, x \in X\}.$$

Then the pair (f, A) is a fuzzy soft set over X, which is written as:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

This fuzzy soft set has fulfilled all the conditions of a fuzzy soft group. Thus (f, A) forms a fuzzy soft group over X.

Definition 2.8 ([1])**.** Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X, where A be the parameterized set. Then the fuzzy soft group (f, A) is said to be a fuzzy soft abelian group over X, if $\forall a \in A$, $f_a(x.y) = f_a(y.x)$, $x, y \in X$.

Lemma 2.9 ([1])**.** *Let $(X, .)$ be group and let (f, A) be a fuzzy soft group over X, where A be the parameterized set. Then for each $a \in A$,*
(1) $f_a(x^{-1}) = f_a(x)$,
(2) $f_a(e) \geq f_a(x)$, *where* $x \in X$.

Theorem 2.10 ([1, 27])**.** (Necessary and Sufficient Condition)
Let $(X, .)$ be a group and let (f, A) be a fuzzy soft set over X. Then (f, A) is a fuzzy soft group over X if and only if for each $a \in A$, $f_a(x.y^{-1}) \geq min\{f_a(x), f_a(y)\}$, where $x, y \in X$.

Theorem 2.11 ([1, 21, 27])**.** *Let $(X, .)$ be a group and let (f, A), (g, B) be two fuzzy soft groups over X. Then their intersection $(f, A)\tilde{\cap}(g, B)$ is also a fuzzy soft group over X.*

Theorem 2.12 ([1, 21, 27])**.** *Let $(X, .)$ be a group and let (f, A), (g, B) be two fuzzy soft groups over X. If $A \cap B = \phi$, then their union $(f, A)\tilde{\cup}(g, B)$ is also a fuzzy soft group over X.*

Theorem 2.13 ([1, 21, 27])**.** *Let $(X, .)$ be a group and let (f, A), (g, B) be two fuzzy soft groups over X. If $A \cap B \neq \phi$, then their union $(f, A)\tilde{\cup}(g, B)$ is also a fuzzy group over X.*

3. Order of a fuzzy soft group

Definition 3.1. Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X. Then $\{(x^n/f_a(x^n)) : a \in A, x \in X\}$ is called the nth power of the element x with respect to the parameter $a \in A$, where $n \in Z$(the set of integers).

Example 3.2. Let us consider the example 2.7. The fuzzy soft group (f, A) defined as,

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Now, with respect to the parameter e_2, the third power of the element i is,

$$i^3/f_{e_2}(i^3) = -i/f_{e_2}(-i) = -i/0.25.$$

Note: If $(X, .)$ be an additive group, then the nth power of the element x with respect to the parameter $a \in A$ is denoted by, $\{nx/f_a(nx) : a \in A, x \in X\}$, $n \in Z$(the set of integers).

Definition 3.3. Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X. Then $\forall a \in A$, $\{f(a)\}^n = \{x^n/f_a(x^n) : x \in X,\ a \in A$ and n is an integer$\}$ is called the nth power of $f(a)$(a-approximate element of the fuzzy soft set (f, A)).

Example 3.4. Let us consider the set of all permutations $X = \{\rho_0, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$ on the set $\{1, 2, 3\}$ which forms a group under multiplication and let $A = X = \{\rho_0, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$ be the parameterized set. Now let us consider a fuzzy soft set (f, A) over X as follows.

$$\begin{aligned}&(f, A)\\ &= \{(\rho_0, \{\rho_0/1\}), (\rho_1, \{\rho_0/1, \rho_1/0.33, \rho_2/0.33\}), (\rho_2, \{\rho_0/1, \rho_1/0.33, \rho_2/0.33\}),\\ &\qquad (\rho_3, \{\rho_0/1, \rho_3/0.5\}), (\rho_4, \{\rho_0/1, \rho_4/0.5\}), (\rho_5, \{\rho_0/1, \rho_5/0.5\})\}.\end{aligned}$$

Then this fuzzy soft set satisfied all the conditions of a fuzzy soft group. Thus (f, A) forms a fuzzy soft group over X.

$$\begin{aligned}\text{Now, } \{f(\rho_1)\}^2 &= \{(\rho_0)^2/f_{\rho_1}({\rho_0}^2), (\rho_1)^2/f_{\rho_1}({\rho_1}^2), (\rho_2)^2/f_{\rho_1}({\rho_2}^2)\}\\ &= \{\rho_0/f_{\rho_1}(\rho_0), \rho_2/f_{\rho_1}(\rho_2), \rho_1/f_{\rho_1}(\rho_1)\}.\\ &= \{\rho_0/1, \rho_2/0.33, \rho_1/0.33\}.\end{aligned}$$

So the 2nd-power of $f(\rho_1) = \{\rho_0/1, \rho_2/0.33, \rho_1/0.33\}$.

Theorem 3.5. *Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X. Let $f(a), f(b) \in (f, A)$, $a, b \in A$. Then for all $n \in Z$(the set of integers),*
(1) $\{f(a) \cap f(b)\}^n \subseteq \{f(a)\}^n \cap \{f(b)\}^n$,
(2) $\{f(a) \cup f(b)\}^n = \{f(a)\}^n \cup \{f(b)\}^n$.

Proof. (1) Let

$$a^n \in \{f(a) \cap f(b)\}^n. \tag{3.1}$$

Then $a \in f(a) \cap f(b)$. Thus $a \in f(a)$ and $a \in f(b)$. So $a^n \in \{f(a)\}^n$ and $a^n \in \{f(b)\}^n$. These imply that

$$a^n \in \{f(a)\}^n \cap \{f(b)\}^n. \tag{3.2}$$

Hence, from (3.1) and (3.2), $\{f(a) \cap f(b)\}^n \subseteq \{f(a)\}^n \cap \{f(b)\}^n$.

(2). The proof of this part is same as given above. But here converse part is holds.

□

Remark 3.6. In the Theorem 3.5 (1), the equality does not hold always.

Now we provides some examples to explain this.

Example 3.7. Let us consider the group of unit quaternions $H = \{I, -I, J, -J, K, -K, L, -L\}$ which forms a group under multiplication, where

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $-I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $J = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$, $-J = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$,

$K = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $-K = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $L = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, $-L = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$.

Let $A = \{e_1, e_2, e_3\}$ be the parameterized set. Now we choose a fuzzy soft set over X as follows:

$$(f, A)$$
$$= \{(e_1, \{I/1, -I/1\}), (e_2, \{I/1, -I/1, J/0.5, -J/0.5\}),$$
$$(e_3, \{I/1, -I/1, K/0.5, -K/0.5\}), (e_4, \{I/1, -I/1, L/0.5, -L/0.5\})\}.$$

This fuzzy soft set satisfied all the conditions of a fuzzy soft group over H.

Now $f(e_2) = \{I/1, -I/1, J/0.5, -J/0.5\}$, $f(e_3) = \{I/1, -I/1, K/0.5, -K/0.5\}$. Then $f(e_2) \cap f(e_3) = \{I/1, -I/1\}$. Thus $\{f(e_2) \cap f(e_3)\}^2 = \{I/1\}$.

On the other hand, $\{f(e_2)\}^2 = \{I/1, -I/1\}$, $\{f(e_3)\}^2 = \{I/1, -I/1\}$, and $\{f(e_2)\}^2 \cap \{f(e_3)\}^2 = \{I/1, -I/1\}$. So,$\{f(e_2) \cap f(e_3)\}^2 \subset \{f(e_2)\}^2 \cap \{f(e_3)\}^2$. Hence we have seen that here the equality does not holds.

Now we have given an example where equality holds.

Example 3.8. Consider the fuzzy soft group (f, A) defined in Example 3.4 :

$$(f, A)$$
$$= \{(\rho_0, \{\rho_0/1\}), (\rho_1, \{\rho_0/1, \rho_1/0.33, \rho_2/0.33\}), (\rho_2, \{\rho_0/1, \rho_1/0.33, \rho_2/0.33\}),$$
$$(\rho_3, \{\rho_0/1, \rho_3/0.5\}), (\rho_4, \{\rho_0/1, \rho_4/0.5\}), (\rho_5, \{\rho_0/1, \rho_5/0.5\})\}.$$

Then $f(\rho_1) \cap f(\rho_3) = \rho_0/1$. Thus $\{f(\rho_1) \cap f(\rho_3)\}^2 = \rho_0/1$.

On one hand,

$$\{f(\rho_1)\}^2 = \{\rho_0/1, \rho_2/0.33, \rho_1/0.33\}$$

and

$$\{f(\rho_3)\}^2 = \{\rho_0/1, \rho_0/1\} = \rho_0/1.$$

So $\{f(\rho_1)\}^2 \cap \{f(\rho_3)\}^2 = \rho_0/1$

Hence, $\{f(\rho_1) \cap f(\rho_3)\}^2 = \{f(\rho_1)\}^2 \cap \{f(\rho_3)\}^2$.

Example 3.9. Consider Example 3.8. Here,

$$f(\rho_1) \cup f(\rho_3) = \{\rho_0/1, \rho_1/0.33, \rho_2/0.33, \rho_3/0.5\}.$$

Then,

$$\{f(\rho_1) \cup f(\rho_3)\}^2 = \{\rho_0/1, \rho_2/0.33, \rho_1/0.33, \rho_0/1\} = \{\rho_0/1, \rho_2/0.33, \rho_1/0.33\}$$

and

$$\{f(\rho_1)\}^2 \cup \{f(\rho_3)\}^2 = \{\rho_0/1, \rho_2/0.33, \rho_1/0.33\}.$$

Thus we have seen that $\{f(\rho_1) \cup f(\rho_3)\}^2 = \{f(\rho_1)\}^2 \cup \{f(\rho_3)\}^2$.

Definition 3.10. Let $(X, .)$ be a group with the identity element e and let (f, A) be a fuzzy soft group over X.

If there exists a least positive integer n such that

$$\{f_a(x^n) = f_a(e) : a \in A, x \in X\},$$

then the least positive integer n is called the order of x with respect to the parameter $a \in A$ and is denoted by $|f_a(x)|$ or $O(f_a(x))$.

If no such n exists then x is said to have an infinite order corresponding to the parameter $a \in A$.

Example 3.11. Consider the fuzzy soft group (f, A) defined in Example 2.7:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Then $f_{e_1}((-1)^2) = f_{e_1}(1)$. Thus corresponding to the parameter e_1, $O(f_{e_1}(-1)) = 2$. Again $O(f_{e_2}(-i)^4) = f_{e_2}(1)$. So corresponding to the parameter e_2, $O(f_{e_2}(-i)) = 4$.

Definition 3.12. Let $(X, .)$ be a group of finite order and let (f, A) be a fuzzy soft group over X. Then $\forall a \in A$, the order of $f(a)$ is defined as:

$$O(f(a)) = lcm\{O(f_a(x)) \colon a \in A, \text{ and } x \in X\}.$$

Example 3.13. Consider the fuzzy soft group (f, A) defined in Example 2.7:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Then $O(f_{e_2}(1)) = 1$, $O(f_{e_2}(-1)) = 2$, $O(f_{e_2}(i)) = 4$, $O(f_{e_2}(-i)) = 4$. Thus

$$\begin{aligned} &O(f(e_2)) \\ &= lcm\{O(f_{e_2}(1)), O(f_{e_2}(-1)), O(f_{e_2}(i)), O(f_{e_2}(-i))\} = lcm\{1, 2, 4, 4\} \\ &= 4. \end{aligned}$$

Theorem 3.14. *Let $(X, .)$ be a group of finite order and (f, A) be a fuzzy soft group over X. Let $f(a), f(b)$ be two elements of (f, A). Then $\forall a, b \in A$ the followings are hold:*

(1) $O(f(a) \cap f(b)) \leq gcd\{O(f(a)), O(f(b))\}$, $\forall a, b \in A$.
(2) $O(f(a) \cup f(b)) = lcm\{O(f(a)), O(f(b))\}$, $\forall a, b \in A$.

To verify this theorem let us consider the examples given below.

Example 3.15. Consider the fuzzy soft group (f, A) defined in Example 2.7:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Then, $O(f(e_1)) = lcm\{O(f_{e_1}(1)), O(f_{e_1}(-1))\} = lcm\{1, 2\} = 2$
and

$O(f(e_2)) = lcm\{O(f_{e_2}(1)), O(f_{e_2}(-1)), O(f_{e_2}(i)), O(f_{e_2}(-i))\} = lcm\{1, 2, 4, 4\} = 4$.

Thus, $gcd(O(f(e_1)), O(f(e_2))) = gcd\{2, 4\} = 2$
and

$$lcm(O(f(e_1)), O(f(e_2))) = lcm\{2, 4\} = 4.$$

On one hand, $f(e_1) \cap f(e_2) = \{1/1, -1/0.5\}$
and

$$f(e_1) \cup f(e_2) = \{1/1, -1/0.5, i/0.25, -i/0.25\}.$$

So, $O(f(e_1) \cap f(e_2)) = lcm\{1, 2\} = 2$
and

$O(f(e_1) \cup f(e_2)) = lcm\{1, 2, 4, 4\} = 4$.
Hence we see that, $O(f(e_1) \cap f(e_2)) = gcd(O(f(e_1)), O(f(e_2)))$
and

$$O(f(e_1) \cup f(e_2)) = lcm(O(f(e_1)), O(f(e_2))).$$

Now we have discussed an example where the equality does not holds.

Example 3.16. Consider the fuzzy soft group (f, A) defined in Example 3.7:

(f, A)
$= \{(e_1, \{I/1, -I/1\}), (e_2, \{I/1, -I/1, J/0.5, -J/0.5\}),$
$(e_3, \{I/1, -I/1, K/0.5, -K/0.5\}), (e_4, \{I/1, -I/1, L/0.5, -L/0.5\})\}.$
where I, $-I$, J, $-J$, K, $-K$,L, $-L$ are given in Example 3.7.
Then,$f(e_2) \cap f(e_3) = \{I/1, -I/1\}$. Thus $O(f(e_2) \cap f(e_3)) = 2$.
Again,$O(f(e_2)) = 4$ and $O(f(e_3)) = 4$. So $gcd(O(f(e_2)), O(f(e_3))) = 4$. Hence,

$$O(f(e_2) \cap f(e_3)) < gcd(O(f(e_2)), O(f(e_3))).$$

Hence from these examples we have seen that for this theorem the equality sign does not holds aiways.

Theorem 3.17. *Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X. Then corresponding to a parameter $a \in A$, $O(f_a(x)) = O(f_a(x^{-1}))$ where $x \in X$.*

Proof. Case (i): Suppose $O(f_a(x)) = n$ (finite). Then $f_a(x^n) = f_a(e)$. Thus, by Lemma 2.9,

$$f_a(x^{-1})^n = f_a((x^n)^{-1}) = f_a(x^n) = f_a(e).$$

Thus $f_a(x^{-1})^n = f_a(e)$. So, $O(f_a(x)) = O(f_a(x^{-1}))$.

Case (ii): Suppose $O(f_a(x))$ is infinite. Then we are to prove that $O(f_a(x^{-1}))$ is infinite. Assume that $O(f_a(x^{-1}))$ is finite, say $O(f_a(x^{-1})) = m$, a finite number. Then $f_a((x^{-1})^m) = f_a(e)$. Thus $f_a((x^m)^{-1}) = f_a(e)$. Then by the Lemma 2.9, $f_a(x^m) = f_a(e)$. So we get $O(f_a(x)) = m$ a finite number which is a contradiction. Hence
$O(f_a(x^{-1}))$ is infinite. Therefore we see that the theorem is true for both cases. □

Example 3.18. Consider the fuzzy soft group (f, A) defined in Example 2.7:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Then $O(f_{e_1}(1)) = O(f_{e_1}(1)^{-1})$,
$O(f_{e_1}(-1)) = O(f_{e_1}(-1)^{-1})$,
$O(f_{e_2}(i)) = O(f_{e_2}(i)^{-1})$,
$O(f_{e_2}(-i)) = O(f_{e_2}(-i)^{-1})$.
Thus we that $\forall a \in A$, $O(f_a(x)) = O(f_a(x^{-1}))$, where $x \in X$.

Theorem 3.19. *Let $(X, .)$ be a group with the identity element e and let (f, A) be a fuzzy soft group over X. If corresponding to some parameter $a \in A$, there exists an element $x \in X$ such that $f_a(x^m) = f_a(e)$. Then the order of $f_a(x)$ is a divisor of m. i.e., $O(f_a(x)) \mid m$.*

Proof. Corresponding to the parameter $a \in A$, let $O(f_a(x)) = n$. then $f_a(x^n) = f_a(e)$(n is the least positive integer). Now let n does not divides m. then, by the divisor algorithm, there exists integers s and t such that $m = ns+t$, where $0 \leq t < n$.

On one hand,

$$\begin{aligned} f_a(x^t) &= f_a(x^{m-ns}) = f_a(x^m.x^{-ns}) \\ &\geq min\{f_a(x^m), f_a(x^{-ns})\} \text{ [by the definition of fuzzy soft group]} \\ &= min\{f_a(e), f_a(x^{ns})^{-1}\}\text{[by the given condition]} \\ &= f_a(x^{ns})^{-1}\text{[by Lemma 2.9]} \\ &= f_a(x^{ns}){=}f_a(x^n)^s \\ &\geq f_a(x^n) = f_a(e). \end{aligned}$$

Thus we get $f_a(x^t) \geq f_a(e)$. Again it is obvious that $f_a(x^t) \leq f_a(e)$. So we get, $f_a(x^t) = f_a(e)$. But this contradicts the minimality of n. Hence n is a divisor of m, i.e., $O(f_a(x)) \mid m$ holds. □

Example 3.20. Consider the fuzzy soft group (f, A) defined in Example 2.7:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Then, $O(f_{e_2}(i)) = 4$ and $f_{e_2}(i^8) = f_{e_2}(1)$. Thus we see that $O(f_{e_2}(i))$ is a divisor of 8.

Theorem 3.21. *let $(X, .)$ be a group of finite order and let (f, A) be a fuzzy soft group over X. If $O(f_a(x)) = n$ for some $a \in A$ and $x \in X$, then for a positive integer m, $O(f_a(x^m)) = \frac{n}{gcd(m,n)}$.*

Proof. Let $gcd(m, n) = d$, where m and n are integers. Then there exists elements u and v such that $d = mu + nv$. Now let $O(f_a(x^m)) = t$. Then for some $a \in A$ and $x \in X$,

$$f_a(x^{mt}) = f_a(e). \tag{3.3}$$

Since $O(f_a(x)) = n$,

$$f_a(x^n) = f_a(e). \tag{3.4}$$

On one hand,

$$\begin{aligned} f_a(x^m)^{\frac{n}{d}} &= f_a(x^n)^{\frac{m}{d}}) = f_a((x^n)^k) \\ &\geq f_a(x^n) \text{ (we consider } \tfrac{m}{d} = k) \\ &= f_a(e), \text{ by (3.4)}. \end{aligned}$$

It is also obvious that $f_a(x^m)^{\frac{n}{d}} \leq f_a(e)$ (by Lemma 2.9). Thus we get, $f_a(x^m)^{\frac{n}{d}} = f_a(e)$. So, by the above theorem,

$$t \mid (n/d). \tag{3.5}$$

Again,

$$\begin{aligned} f_a(x^{td}) &= f_a(x^{t(mu+nv)}) = f_a(x^{tmu}.x^{tnv}) \\ &\geq min\{f_a(x^{tmu}), f_a(x^{tnv})\} \\ &= min\{f_a((x^{mt})^u), f_a((x^n)^{vt})\} \\ &\geq min\{f_a(x^{mt}), f_a(x^n)\} \\ &= min\{f_a(e), f_a(e)\}, \text{ by (3.3) and (3.4)}. \end{aligned}$$

Then $f_a(x^{td}) \geq f_a(e)$. Also it is obvious that $f_a(e) \geq f_a(x^{td})$.
Thus we get, $f_a(x^{td}) = f_a(e)$. By the above theorem, $n \mid td$. So

$$(n/d) \mid t. \tag{3.6}$$

Hence, from (3.5) and (3.6), we get $t = \frac{n}{d}$. This proves the theorem. □

Example 3.22. Consider the fuzzy soft group (f, A) defined in Example 2.7:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

Then, with respect to the parameter e_2, $O(f_{e_2}(i)) = 4$. Thus

$$O(f_{e_2}(i^7)) = \frac{4}{gcd(4, 7)} = 4.$$

Theorem 3.23. *Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X. If (f, A) be a fuzzy soft abelian group over X and for some parameter $a \in A$, $x, y \in X$, $gcd(O(f_a(x)), O(f_a(y))) = 1$, then $O(f_a(xy)) = O(f_a(x))O(f_a(y))$.*

Proof. Let $O(f_a(x)) = m$, $O(f_a(y)) = n$ and $O(f_a(xy)) = k$ such that $gcd(m, n) = 1$. We are now prove that $k = mn$.

Since $O(f_a(x)) = m$, $f_a(x^m) = f_a(e)$. Then

$$f_a(x^{mk}) = f_a(e). \tag{3.7}$$

Since $O(f_a(y)) = n$, $f_a(y^n) = f_a(e)$. Then

$$f_a(y^{nk}) = f_a(e). \tag{3.8}$$

Since $O(f_a(xy)) = k$,

$$f_a((xy)^k) = f_a(e). \tag{3.9}$$

On one hand, $f_a((xy)^{mn}) = f_a(x^{mn}.y^{mn})$
$\geq min\{f_a(x^{mn}), f_a(y^{mn})\}$
$= min\{f_a((x^m)^n), f_a((y^n)^m)\}$
$\geq min\{f_a(x^m), f_a(y^n)\}$
$= min\{f_a(e), f_a(e)\} = f_a(e)$.
Again it is obvious that $f_a(e) \geq f_a((xy)^{mn})$. Thus,

$$f_a((xy)^{mn}) = f_a(e). \tag{3.10}$$

So, by Theorem 3.18,

$$k \mid mn. \tag{3.11}$$

Now we are to prove that $mn \mid t$.
Now, $f_a(x^k) = f_a(x^k.y^k.y^{-k}) = f_a((xy)^k.y^{-k})$
$\geq min\{f_a(xy)^k, f_a(y^{-k})\}$
$\geq min\{f_a(xy)^k, f_a(y)^k\}$(by (3.9))
$= min\{f_a(e), f_a(y^k)\}$
$= f_a(y^k)$(by Lemma 2.9).
Again similarly, $f_a(y^k) \geq f_a(x^k)$. Then $f_a(x^k) = f_a(y^k)$.
Thus, $f_a(x^{nk}) = f_a(y^{nk})$. From (3.8), we get $f_a(x^{nk}) = f_a(e)$. So, from (3.7), we see that $m \mid nk$. Since $(m, n) = 1$,

$$m \mid k. \tag{3.12}$$

Similarly,$f_a(x^{mk}) = f_a(y^{mk})$. Then, from (3.7), we get $f_a(y^{mk}) = f_a(e)$. Thus, from (3.8), we get $n \mid mk$. Since $gcd(m, n) = 1$,

$$m \mid k. \tag{3.13}$$

So from (3.12) and (3.13), we get

$$mn \mid k. \tag{3.14}$$

Hence, form (3.11) and (3.14), we get $k = mn$. □

Theorem 3.24. *Corresponding to some parameter $a \in A$ and $x \in X$, let $O(f_a(x)) = n$. Then $O(f_a(x^p)) = n$ if and only if p is prime to n.*

Proof. Let $O(f_a(x^p)) = n$. We are to prove that $gcd(p, n) = 1$.
Suppose $O(f_a(x)) = n$. Then $f_a(x^n) = f_a(e)$. By Theorem 3.20,
$O(f_a(x^p)) = \frac{n}{gcd(p,n)}$. Thus $gcd(p, n) = \frac{n}{O(f_a(x^p))}$. So $gcd(p, n) = \frac{n}{n} = 1$.

The proof of the converse part can also prove, by Theorem 3.20. □

Theorem 3.25. *Let X be an finite group and (f, A) be a fuzzy soft group over X then the orders of the elements of (f, A) are finite.*

Proof. The proof is straightforward. □

Definition 3.26. Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X.Then for $a \in A$, the set $Z(f(a)) = \{x \in X : f_a(xy) = f_a(yx), \forall y \in X\}$ is called the center of $f(a)$ in (f, A) over X.

Definition 3.27. Let $(X, .)$ be a group and (f, A) be a fuzzy soft group over X. Then the set
$C(f(a))$=$\{x \in X$: $y, z \in X$, $f_a(xy) = f_a(yx)$ and $f_a(xyz) = f_a(yxz)$, $a \in A\}$
is called the centralizer of $f(a)$ over X.

Theorem 3.28. *Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X, then $x \in C(f(a))$ if and only if with respect to the parameter $a \in A$,*

$$f_a(xy_1y_2...y_n) = f_a(y_1xy_2...y_n) = ... = f_a(y_1y_2....y_nx),$$

for all $y_1, y_2, ..., y_n \in X$.

Proof. We proof it by the method of induction. Suppose,$x \in C(f(a))$ and $y_1, y_2 \in X$. Then, by the definition of $C(f(a))$,

$$f_a(xy_1y_2) = f_a(y_1xy_2. \tag{3.15}$$

Again let, $y_1, y_2, y_3 \in X$. Then, by using (3.15), we get,

$$f_a(xy_1y_2y_3) = f_a(x(y_1y_2)y_3) = f_a(y_1y_2xy_3).$$

Thus we see that the theorem is true for $k = 2, 3$.

Let us assume that the theorem is true for $k = n$. Then $\forall y_1, y_2, ..., y_n \in X$,

$$f_a(xy_1y_2.....y_n) = f_a(y_1xy_2.....y_n) = ... = f_a(xy_1y_2.....y_n). \tag{3.16}$$

Now let, $y_1, y_2, .., y_n, y_{n+1} \in X$. Then, by (3.16), we get

$$f_a(xy_1y_2...y_ny_{n+1}) = f_a(y_1xy_2...y_ny_{n+1}) = ... = f_a(y_1y_2...y_nxy_{n+1}).$$

Thus, by the method induction, the theorem is proved. □

Theorem 3.29. *Prove that $\forall a \in A$, $C(f(a))$ is a subgroup of X.*

Proof. Corresponding to a parameter $a \in A$, let $u, v \in C(f(a))$. Then, by the above Theorem, $f_a((uv)y) = f_a(y(uv))$. Thus $uv \in C(f(a))$.

Again let $u \in C(f(a))$. Then

$$f_a(u^{-1}y) = f_a(u^{-1}yu^{-1}u) = f_a(uu^{-1}yu^{-1}) = f_a(yu^{-1}).$$

Thus $u^{-1} \in C(f(a))$. So, $C(f(a))$ is a subgroup of X. □

4. Fuzzy soft cyclic group

Definition 4.1. Let $(X, .)$ be a group and let (f, A) be a fuzzy soft group over X. If $\forall a \in A$, there exists an element $x \in X$ such that $f(a) =< x/f_a(x) >$, i.e., $f(a) = \{(x^n/f_a(x^n)) : n \in Z\}$, then (f, A) is a fuzzy soft cyclic group over X.
Here $(x/f_a(x))$ is called the fuzzy soft generator with respect to the parameter $a \in A$.

Note. Each $f(a)$ be a fuzzy cyclic group over X.

Example 4.2. Let $(X, .)$ be a cyclic group and let (f, A) be a soft cyclic group over X, then obviously (f, A) be a fuzzy soft cyclic group over X. Since every subgroup of a cyclic group is cyclic and each soft group is consider as a fuzzy soft group.

Example 4.3. Let $X = \{1, -1, i, -i\}$ be a set which forms a group under multiplication and let $A = \{e_1, e_2\}$ be the parameterized set, where

e_1 stands for the parameter 'all real roots' e_2 stands for the parameter 'all roots'.

Now let consider mapping $f : A \to I^X$ such that $f_a : X \to [0, 1]$ which can be defined as:

$$f_a(x) = \{\tfrac{1}{n} : x^n = 1\}.$$

Then the fuzzy soft set (f, A) can be written as:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/0.5, i/0.25, -i/0.25))\}.$$

This fuzzy set forms a fuzzy soft group over X.

Now, with respect to the parameter e_1,

$$< 1/1 >= \{1/1\} \text{ and } < -1/0.5 >= \{1/1, -1/0.5\}.$$

Then $< -1/0.5 >$ is the generator of $f(e_1)$.

Similarly, $< i/0.25 >$ and $< -i/0.25 >$ are two generators of $f(e_2)$. So (f, A) be a fuzzy soft cyclic group over X.

Example 4.4. Let $X = \{e, a, b, c\}$ be the initial universal set which is not a cyclic group and let $A = \{e, a, b, c\}$ be the parameterized set. We now construct a mapping $f : A \to I^X$ such that $f(a) = f_a : X \to [0.1]$ which is defined as:

$$f_a(x) = \{\tfrac{1}{n} : a^n = x, \forall a \in A \text{ and } x \in X\}.$$

Then the fuzzy soft set (f, A) is defined as:

$$(f, A) = \{(e, \{e/1\}), (a, \{e/1, a/0.5\}), (b, \{e/1, b/0.5\}), (c, \{e/1, c/0.5\})\}.$$

Clearly, this fuzzy soft set satisfies the conditions of a fuzzy soft group. Thus (f, A) is a fuzzy soft group over X.

Now we see that $< e/1 >, < a/0.5 >, < b/0.5 >, < c/0.5 >$ are the fuzzy soft generator with respect to the parameter e, a, b, c, respectively. So (f, A) is a fuzzy soft cyclic group over X, but here X is not a cyclic group.

Theorem 4.5. *Let $(X, .)$ be a group and let (f, A) and (g, B) be two fuzzy soft cyclic groups over X. If $A \cap B = \phi$, then $(f, A)\tilde{\cup}(g, B)$ is a fuzzy soft cyclic group over X.*

Proof. Let $(f, A)\tilde{\cup}(g, B) = (h, C)$, where $C = A \cup B$. Since $A \cap B = \phi$, $\forall c \in C$, $h(c) = f_c$, if $c \in A - B$ and $h(c) = g_c$, if $c \in B - A$. Since (f, A) and (g, B) are both fuzzy soft cyclic groups over X, (h, C) is a fuzzy soft cyclic group over X. □

Theorem 4.6. *Let $(X, .)$ be a group and let (f, A) and (g, B) be two fuzzy soft cyclic groups over X. Then their intersection $(f, A)\tilde{\cap}(g, B)$ is also a fuzzy soft cyclic group over X.*

Proof. It is trivial. □

Theorem 4.7. *Let $(X, .)$ be a group of finite order and let (f, A) be a fuzzy soft cyclic group over X. Let for some $a \in A$, there exists an element $x \in X$ such that $f(a) =< x/f_a(x) >$, i.e., $f(a) = \{(x^n/f_a(x^n)) : n$ be any integer$\}$, where $(x/f_a(x))$ be a fuzzy generator of $f(a)$. Then $(x^k/f_a(x^k))$ is also a fuzzy soft generator of $f(a)$ if and only if $gcd(k, n) = 1$, where k be a positive integer.*

Proof. Since $x/f_a(x)$ is a fuzzy generator of $f(a)$ such that $O(f(a)) = n$, $O(f_a(x)) = n$. Now let, $O(f_a(x^k)) = n$. Then, by Theorem 3.20, $O(f_a(x^k)) = \frac{n}{gcd(k,n)}$. Thus we get $gcd(k, n) = 1$.

Conversely, let $gcd(k, n) = 1$. Then we can also get $O(f_a(x^k)) = \frac{n}{gcd(k,n)} = n$. Thus $x^k/f_a(x^k)$ is a fuzzy soft generator of $f(a)$. □

Example 4.8. Consider the fuzzy soft group (f, A) defined in Example 4.3:

$$(f, A) = \{(e_1, (1/1, -1/0.5)), (e_2, (1/1, -1/1, i/0.25, -i/0.25))\}.$$

Then, with respect to the parameter e_2, we see that $i, -i$ are two generators such that $O(f_{e_2}(i)) = 4$ and $O(f_{e_2}(-i)) = 4$.

Now take $k = 5$ such that $gcd(k, n) = 1$. Then we see that $O(f_{e_2}(i^5)) = O(f_{e_2}(i))$ and clearly $< i^5/f_{e_2}(i^5) >$ is a generator of $f(e_2)$.

Again let $k = 7$ such that $gcd(k, n) = 1$. Then we see that $O(f_{e_2}(i^7)) = O(f_{e_2}(-i))$ and so $< i^7/f_{e_2}(i^7) >$ is also a generator of $f(e_2)$.

Theorem 4.9. *Every fuzzy soft cyclic group is also fuzzy soft abelian group.*

Proof. Let (f, A) be a fuzzy soft cyclic group over the group $(X, .)$, i.e., $\forall a \in A$, there exists an element $x \in X$ such that $f(a) =< x/f_a(x) >$. Now we choose, with respect to a parameter $a \in A$, $u, v \in X$. Since x is a generator, let $u = x^r$ and $v = x^s$, where r, s be any two integers. Then,

$x^r/f_a(x^r).x^s/f_a(x^s)$

$= x^{r+s}/f_a(x^r).f_a(x^s)$

$= x^{s+r}/f_a(x^s).f_a(x^r)$

$= x^s/f_a(x^s).x^r/f_a(x^r)$ (since $r, s \in Z$). This proves that, every fuzzy soft cyclic group is an fuzzy soft abelian group. □

Theorem 4.10. *In a finite fuzzy soft cyclic group, the inverse element of a generator with respect to a parameter is also a generator with respect to the same parameter.*

Proof. Let (f, A) be a finite fuzzy soft cyclic group over a finite group $(X, .)$, where A is the parameterized set. Then, with respect to some parameter $a \in A$, there exists an element$x \in X$ such that $f(a) =< x/f_a(x) >$, i.e., $f(a) = \{(x^n/f_a(x^n)) : n$ is an integer$\}$. Let us consider $x^i = u$, where i is an integer and $u \in X$. Now we can write $u = (x^{-1})^{-i}$, since $-i$ is also an integer. Thus we can write $u = (x^{-1})^m$, $m \in Z$. So we see that $< (x^{-1})/f_a(x^{-1}) >$ is also a generator. To verify this theorem let us consider the following example. □

Example 4.11. Consider the fuzzy soft group (f, A) defined in Example 3.7:

$$(f, A) = \{(e_1, \{I/1, -I/1\}), (e_2, \{I/1, -I/1, J/0.5, -J/0.5\}), (e_3, \{I/1, -I/1, K/0.5, -K/0.5\}), (e_4, \{I/1, -I/1, L/0.5, -L/0.5\})\}.$$

In this example, we see that with respect to each parameter inverse element of each element is also a generator.

5. Conclusion

In this paper we have developed fuzzy soft group. First we have introduced the order of an element of a fuzzy soft group with respect to a parameter then we describe some new theorems and to verify these theorems some examples are given. Finally we introduce the concept of fuzzy soft cyclic group and some related theorems. To extend this work one could study the properties of fuzzy soft cyclic groups and other algebraic structures such as rings and field etc..

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