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Soft topological vector space

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Abstract. In the present paper a notion of soft topological vector spaces has been presented and some basic properties of such spaces are studied. Soft seminorm, soft Minkowski functional on a soft linear space are defined and the problem of soft normability of soft topological vector spaces has been addressed to.

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1. Introduction

Most of the practical problems in economics, engineering, environmental science, social science, medical science and so forth cannot be dealt with classical methods because of various types of uncertainties present in these problems. Several theories, for example, probability theory, fuzzy set theory, rough set theory, interval analysis etc. are evolved to address the different kinds of uncertainty problems. But each of these theories has their own difficulties and limitations, perhaps, due to the lack of the parametrization tools of the theory as it was indicated by Molodtsov in [24]. Molodtsov initiated a new mathematical tool, namely, soft set as a generalization of fuzzy set, for dealing with uncertainties and applied it in many different fields such as smoothness of functions, game theory, Riemann integration, Perron integration, probability theory etc. Maji et al. [20, 21] dealt with the algebraic operations over soft sets. Recently investigations are going on in developing different mathematical structures such as algebraic, topological, algebraico-topological etc. over soft sets. To mention some of them, Aktas and Cagman [1] have introduced soft groups; Jun [17, 18] applied soft sets to the theory of BCK/BCI algebras and introduced the concept of soft BCK/BCI-algebras; Feng et al. [13] defined soft semi-rings; Shabir and Ali [29] studied soft semi-groups and soft ideals; Babitha and Sunil [3] studied soft set relations and functions; Kharal and Ahmed [19] as well as Majumdar and

Samanta [22] studied soft mappings. Shabir and Naz [30] came up with an idea of soft topological spaces. Afterwords Zorlutuna et al. [32], Cagman et al. [5], Hussain and Ahmed [16], Hazra et al. [15], Georgiou et al.[14], Aygunoglu et al. [2], Babitha et al. [4], Mondal et al. [25], M. Chiney et al. [8] and many other authors studied on soft topological spaces. Recently metric space, linear space, topological group, Banach algebra, topological vector space structures in soft setting are also studied [6, 9, 10, 11, 12, 26, 28, 31]. In this paper, we introduce a notion of soft topological vector space. For doing this we consider the vector space to be a soft vector space and the underlying topology is taken to be a new type of soft topology which is defined and developed by using the concepts of elementary union, intersection and complement of soft sets although, interestingly, with respect to these operations the relevant distributive properties and the law of excluded middle do not hold. Also in this paper, we study some basic properties of this space and finally the problem of soft normability of soft topological vector space is addressed to.
This paper is organized as follows: In Section 2, we briefly review some basic notions and facts on soft sets which are used to prove or illustrate results in subsequence sections. Section 3 is devoted to study some properties of balanced, convex and absorbing soft sets. The concept of a soft topological vector space is introduced in section 4 along with some basic properties of such spaces. In section 5, we introduce soft seminorm, soft Minkowski functional and study the problem of soft normability of soft topological vector spaces. Finally, in Section 6, we present the conclusion.

2. Preliminaries

Definition 2.1 ([24])**.** Let X be a universal set and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For $\lambda \in A$, $F(\lambda)$ may be considered as the set of λ - approximate elements of the soft set (F, A).

In [23] the soft sets are redefined as follows:
Let E be the set of parameters and $A \subseteq E$. Then for each soft set (F, A) over X a soft set (H, E) is constructed over X, where for every $\lambda \in E$,

$$H(\lambda) = \begin{cases} F(\lambda) & if\ \lambda \in A \\ \phi & if\ \lambda \in E \setminus A. \end{cases}$$

Thus the soft sets (F, A) and (H, E) are equivalent to each other and the usual set operations of the soft sets $(F_i, A_i), i \in \Delta$ is the same as those of the soft sets $(H_i, E), i \in \Delta$. For this reason, in this paper, we have considered our soft sets over the same parameter set A.

Definition 2.2 ([21])**.** A soft set (F, A) over X is said to be
(i) an absolute soft set denoted by $(\tilde{X}, A)$, if for all $\lambda \in A$, $F(\lambda) = X$,
(ii) a null soft set, denoted by $(\tilde{\Phi}, A)$ if for all $\lambda \in A$, $F(\lambda) = \phi$.

Following Molodtsov and Maji et al. [20, 21, 24] definitions of soft subset, complement, arbitrary union, arbitrary intersection of soft sets etc. are presented in [27] considering the same parameter set.

Definition 2.3 ([9])**.** Let X be a non-empty set and A be a non-empty parameter set. Then a function $e : A \to X$ is said to be a soft element of X. A soft element e of X is said to belong to a soft set (F, A) of X , which is denoted by $e\tilde{\in}(F, A)$, if $e(\lambda) \in F(\lambda)$, for all $\lambda \in A$. Thus for a soft set (F, A) over X with respect to the index set A with $F(\lambda) \neq \phi$, for all $\lambda \in A$, we have $F(\lambda) = \{e(\lambda) : e\tilde{\in}(F, A)\}$, for all $\lambda \in A$. In general, soft elements are denoted by $\tilde{x}, \tilde{e}$ etc.

Definition 2.4 ([9])**.** Let $\mathbb{R}$ be the set of real numbers and $\mathscr{B}(\mathbb{R})$, the collection of all non-empty bounded subsets of $\mathbb{R}$ and A be taken as a set of parameters. Then a mapping $F : A \to \mathscr{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F, A). If specifically (F, A) is a singleton soft set, then after identifying (F, A) with the corresponding soft element, it will be called a soft real number.
The set of all soft real numbers is denoted by $\mathbb{R}(A)$ and set of all non-negative soft real numbers by $\mathbb{R}(A)^*$.

Definition 2.5 ([10])**.** Let $\mathbb{C}$ be the set of complex numbers and $\wp(\mathbb{C})$, the collection of all non-empty bounded subsets of complex numbers and A be taken as a set of parameters. Then a mapping $F : A \to \wp(\mathbb{C})$ is called a soft complex set. It is denoted by (F, A). If in particular (F, A) is a singleton soft set, then after identifying (F, A) with the corresponding soft element, it will be called a soft complex number.
The set of all soft complex numbers is denoted by $\mathbb{C}(A)$.

To avoid ambiguity of notations we use $\hat{r}$,$\hat{s}$,$\hat{t}$ to denote soft real numbers or soft complex numbers whereas $\bar{r}$,$\bar{s}$,$\bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc. For example $\bar{0}$ is the soft real number where $\bar{0}(\lambda) = 0, \bar{1}(\lambda) = 1$ for all $\lambda \in A$.

Definition 2.6 ([9])**.** For two soft real numbers $\hat{r}$,$\hat{s}$ we define
(i) $\hat{r}\tilde{\leq}\hat{s}$ if $\hat{r}(\lambda) \leq \hat{s}(\lambda)$, for all $\lambda \in A$,
(ii) $\hat{r}\tilde{\geq}\hat{s}$, if $\hat{r}(\lambda) \geq \hat{s}(\lambda)$, for all $\lambda \in A$,
(iii) $\hat{r}\tilde{<}\hat{s}$, if $\hat{r}(\lambda) < \hat{s}(\lambda)$, for all $\lambda \in A$,
(iv) $\hat{r}\tilde{>}\hat{s}$, if $\hat{r}(\lambda) > \hat{s}(\lambda)$, for all $\lambda \in A$.

Definition 2.7. The inverse of any soft real or soft complex number $\hat{r}$, denoted by $\hat{r}^{-1}$, defined by $\hat{r}^{-1}(\lambda) = (\hat{r}(\lambda))^{-1}$, for each $\lambda \in A$.

Proposition 2.8 ([11])**.** *Any collection of soft elements of a soft set can generate a soft subset of that soft set.*

Remark 2.9 ([11])**.** Let X be any non-empty set. By $S(\tilde{X})$ we denote the collection of all soft sets (F, A) over X for which $F(\lambda) \neq \phi$, for all $\lambda \in A$ together with the null soft set $(\tilde{\Phi}, A)$. For any soft set $(F, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$, the collection of all soft elements of (F, A) is denoted by $SE(F, A)$. For a collection $\mathscr{B}$ of soft elements of $(\tilde{X}, A)$, the soft set generated by $\mathscr{B}$ is denoted by $SS(\mathscr{B})$.

Proposition 2.10 ([11])**.** *For any soft sets* $(F, A), (G, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$, $(F, A)\tilde{\subseteq}(G, A)$ *iff every soft element of* (F, A) *is also a soft element of* (G, A).

Definition 2.11 ([11])**.** For any two soft sets $(F,A),(G,A)(\neq(\tilde{\Phi},A))\in S(\tilde{X})$,

(i) elementary union of (F,A) and (G,A) is denoted by $(F,A)\Cup(G,A)$ and is defined by $(F,A)\Cup(G,A)=SS(\mathscr{B})$, where $\mathscr{B}=\{\tilde{x}\tilde{\in}(\tilde{X},A):\tilde{x}\tilde{\in}(F,A)\ or\ \tilde{x}\tilde{\in}(G,A)\}$,

(ii) elementary intersection of (F,A) and (G,A) is denoted by $(F,A)\Cap(G,A)$ and is defined by $(F,A)\Cap(G,A)=SS(\mathscr{B})$, where
$\mathscr{B}=\{\tilde{x}\tilde{\in}(\tilde{X},A):\tilde{x}\tilde{\in}(F,A)\ and\ \tilde{x}\tilde{\in}(G,A)\}$,

(iii) the elementary complement of (F,A) is denoted by $(F,A)^{\mathbb{C}}$ and is defined by $(F,A)^{\mathbb{C}}=SS(\mathscr{B})$, where $\mathscr{B}=\{\tilde{x}\tilde{\in}(\tilde{X},A):\tilde{x}\tilde{\in}(F,A)^{C}\}$ and $(F,A)^{C}$ is the complement of (F,A).

Remark 2.12. (1) The operations elementary union and elementary intersection are not distributive over $S(\tilde{X})$ (See [7]).

(2) In general,$(F,A)\Cup(F,A)^{\mathbb{C}}\tilde{\subseteq}(\tilde{X},A)$, $(F,A)\Cup(F,A)^{\mathbb{C}}\neq(\tilde{X},A)$. However if $(F,A)^{\mathbb{C}}\neq(\tilde{\Phi},A)$, then $(F,A)\Cup(F,A)^{\mathbb{C}}=(\tilde{X},A)$ (See [11]).

(3) In general, $\big((F,A)^{\mathbb{C}}\big)^{\mathbb{C}}\neq(F,A)$. However if $(F,A)^{\mathbb{C}}\neq(\tilde{\Phi},A)$, then $\big((F,A)^{\mathbb{C}}\big)^{\mathbb{C}}$ $=(F,A)$ (See [11]).

Proposition 2.13 ([11])**.** *For any two soft sets $(F,A),(G,A)\in(\neq(\tilde{\Phi},A))S(\tilde{X})$,*

(1) $(F,A)\Cup(G,A)=(F,A)\tilde{\cup}(G,A)$,

(2) $(F,A)\Cap(G,A)=(F,A)\tilde{\cap}(G,A)$, *if* $(F,A)\Cap(G,A)\neq(\tilde{\Phi},A)$,

where the notations $\tilde{\cup}$ and $\tilde{\cap}$ are used for usual soft set union and soft set intersection.

Definition 2.14 ([10])**.** Let $(F,A),(G,A)\in\mathbb{C}(A)\,or\,\mathbb{R}(A)$. Then the sum, difference, product and division are defined by

$(F+G)(\lambda)=z+w$; $z\in F(\lambda)$, $w\in G(\lambda)$, for each $\lambda\in A$,

$(F-G)(\lambda)=z-w$; $z\in F(\lambda)$, $w\in G(\lambda)$, for each $\lambda\in A$,

$(F\cdot G)(\lambda)=zw$; $z\in F(\lambda)$, $w\in G(\lambda)$, for each $\lambda\in A$,

$(F/G)(\lambda)=z/w$; $z\in F(\lambda)$, $w(\neq 0)\in G(\lambda)$, for each $\lambda\in A$,provided $G(\lambda)\neq\phi$ for each $\lambda\in A$.

Definition 2.15 ([11])**.** A mapping $d:SE(\tilde{X})\times SE(\tilde{X})\to\mathbb{R}(A)^{*}$, is said to be a soft mapping on the soft set $(\tilde{X},A)$ if d satisfies the following conditions:

$(M1)$ $d(\tilde{x},\tilde{y})\tilde{\geq}\bar{0}$, for all $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$,

$(M2)$ $d(\tilde{x},\tilde{y})=\bar{0}$ if and only if $\tilde{x}=\tilde{y}$,

$(M3)$ $d(\tilde{x},\tilde{y})=d(\tilde{y},\tilde{x})$, for all $\tilde{x},\tilde{y}\tilde{\in}(\tilde{X},A)$,

$(M4)$ For all $\tilde{x},\tilde{y},\tilde{z}\tilde{\in}(\tilde{X},A)$, $d(\tilde{x},\tilde{z})\tilde{\leq}d(\tilde{x},\tilde{y})+d(\tilde{y},\tilde{z})$.

The soft set $(\tilde{X},A)$ with a soft metric d on $(\tilde{X},A)$ is said to be a soft metric space and is denoted by $(\tilde{X},d,A)$ or $(\tilde{X},d)$. $(M1)$, $(M2)$, $(M3)$ and $(M4)$ are said to be soft metric axioms.

Theorem 2.16 ([11], Decomposition Theorem)**.** *Let a soft metric d satisfy the condition:*

$(M5)$ *For $(\xi,\eta)\in X\times X$, and $\lambda\in A$, $\{d(\tilde{x},\tilde{y})(\lambda):\tilde{x}(\lambda)=\xi,\tilde{y}(\lambda)=\eta\}$ is a singleton set and if for $\lambda\in A$, $d_\lambda:X\times X\to\mathbb{R}^{+}$ is defined by*

$$d_\lambda(\tilde{x}(\lambda),\tilde{y}(\lambda))=d(\tilde{x},\tilde{y})(\lambda),\ \tilde{x},\tilde{y}\tilde{\in}\tilde{X}.$$

Then d_λ is a metric on X.

Remark 2.17 ([11])**.** Every crisp metric d on a crisp set X can be extended to a soft metric on the soft set $(\tilde{X}, A)$. The soft metric defined using the crisp metric d, is said to be the soft metric generated by d. This type of soft metric always satisfies $(M5)$.

Definition 2.18 ([11])**.** Let $(\tilde{X}, d, A)$ be a soft metric space, $\hat{r}$ be a non-negative soft real number and $\tilde{a}\tilde{\in}\tilde{X}$. By an open ball with centre $\tilde{a}$ and radius $\hat{r}$, we mean the collection of soft elements of $(\tilde{X}, A)$ satisfying $d(\tilde{x}, \tilde{a})\tilde{\leq}\hat{r}$. The open ball centered at $\tilde{a}$ and radius $\hat{r}$ is denoted by $B(\tilde{a}, \hat{r})$. Thus

$$B(\tilde{a}, \hat{r}) = \{\tilde{x}\tilde{\in}\tilde{X} : d(\tilde{x}, \tilde{a})\tilde{\leq}\hat{r}\} \subset SE(\tilde{X}).$$

$SS(B(\tilde{a}, \hat{r}))$ will be called a soft open ball centered $\tilde{a}$ and radius $\hat{r}$.

Proposition 2.19 ([11])**.** *Let $(\tilde{X}, d, A)$ be a soft metric space satisfying $(M5)$. Then for every open ball $B(\tilde{a}, \hat{r})$ in $(\tilde{X}, d)$, $SS(B(\tilde{a}, \hat{r}))(\lambda) = B(\tilde{a}(\lambda), \hat{r}(\lambda))$, an open ball in (X, d_λ), for each $\lambda \in A$.*

Definition 2.20 ([11])**.** Let $\mathscr{B}$ be a collection of soft elements of $(\tilde{X}, A)$ in a soft metric space $(\tilde{X}, d, A)$. Then a soft element $\tilde{a}$ is said to be an interior soft element of $\mathscr{B}$, if there exists a positive soft real number $\hat{r}$ such that $\tilde{a} \in B(\tilde{a}, \hat{r}) \subseteq \mathscr{B}$.

Definition 2.21 ([11])**.** Let $(\tilde{X}, d, A)$ be a soft metric space and $\mathscr{B}$ be a non-null collection of soft elements of $(\tilde{X}, A)$. Then $\mathscr{B}$ is said to be open in $(\tilde{X}, A)$ with respect to d or open in $(\tilde{X}, d, A)$, if all elements of $\mathscr{B}$ are interior element of $\mathscr{B}$.

Definition 2.22 ([11])**.** Let $(\tilde{X}, d, A)$ be a soft metric space and (Y, A) be a non-null soft subset of $S(\tilde{X})$ in $(\tilde{X}, d, A)$. Then (Y, A) is said to be 'soft open in $(\tilde{X}, A)$ with respect to d' or 'soft open in $(\tilde{X}, d, A)$', if there is a collection $\mathscr{B}$ of soft elements of (Y, A) such that $\mathscr{B}$ is open in $(\tilde{X}, A)$ with respect to d and $(Y, A) = SS(\mathscr{B})$.

Definition 2.23 ([11])**.** Let $(\tilde{X}, d, A)$ be a soft metric space satisfying $(M5)$. Then the collection τ of all soft open sets form a topology on $(\tilde{X}, A)$ with respect to elementary union and elementary intersection of soft sets. This topology will be called "soft metric topology" on $(\tilde{X}, A)$.

Definition 2.24 ([12])**.** Let X be a vector space over a field K and A be the parameter set. Let (F_1, A), (F_2, A), ..., (F_n, A) be n soft sets in $(\tilde{X}, A)$. Then $(F, A) = (F_1, A) + (F_2, A) + ... + (F_n, A)$ is a soft set over $(\tilde{X}, A)$ and is defined as $F(\lambda) = \{x_1 + x_2 + ... + x_n : x_i \in F_i(\lambda), i = 1, 2, ..., n\}$, for each $\lambda \in A$. Let $\alpha \in K$ be any scalar and (F, A) be a soft set over $(\tilde{X}, A)$, then αF is a soft set over (X, A) and is defined as follows: $\alpha(F, A) = (G, A)$, $G(\lambda) = \{\alpha x : x \in F(\lambda)\}$, for each $\lambda \in A$.

Definition 2.25 ([12])**.** Let X be a vector space over a field K and A be the parameter set. Let (G, A) be a soft set over X. Then G is said to be a soft vector space or a soft linear space of X over K, if $G(\lambda)$ is a vector subspace of X, for each $\lambda \in A$.

Definition 2.26 ([12])**.** Let (F, A) be a soft vector space of X over K.

(i) (F, A) is said to be null soft vector space, if $F(\lambda) = \{\theta\}$, for each $\lambda \in A$, where θ is the null element of X.

(ii) (F, A) is said to be absolute soft vector space, if $F(\lambda) = X$, for each $\lambda \in A$.

Definition 2.27 ([12]). Let (F, A) be a soft vector space of X over K. Let (G, A) be a soft set over (X, A). Then (G, A) is said to be a soft vector subspace of (F, A) if it satisfies the following conditions:

(i) for each $\lambda \in A$, $G(\lambda)$ is a vector subspace of X over K,

(ii)$F(\lambda) \supseteq G(\lambda)$, for each $\lambda \in A$.

Definition 2.28 ([12]). Let (F, A) be a soft vector space of X over K. Then a soft element of (F, A) is said to be a soft vector of (F, A). In a similar manner a soft element of the soft set (K, A) is said to be a soft scalar, K being the scalar field.

Definition 2.29 ([12]). A soft vector $\tilde{x}$ in a soft vector space (F, A) is said to be the null soft vector, if $\tilde{x}(\lambda) = \theta$, for each $\lambda \in A$, θ being the zero element of X. It will be denoted by Θ.

Definition 2.30 ([12]). Let $\tilde{x}, \tilde{y}$ be soft vectors of (F, A) and $\hat{k}$ be a soft scalar. Then the addition $\tilde{x} + \tilde{y}$ of $\tilde{x}, \tilde{y}$ and scalar multiplication $\hat{k}.\tilde{x}$ of $\hat{k}$ and $\tilde{x}$ are defined by

$$(\tilde{x} + \tilde{y})(\lambda) = \tilde{x}(\lambda) + \tilde{y}(\lambda), \quad (\hat{k}.\tilde{x})(\lambda) = \hat{k}(\lambda).\tilde{x}(\lambda), \text{ for each } \lambda \in A.$$

Obviously, $\tilde{x} + \tilde{y}$ and $\hat{k}.\tilde{x}$ are soft vectors of (F, A).

Remark 2.31 ([12]). However $\hat{k}.\tilde{x} = \Theta$ does not necessarily imply that either $\hat{k} = \bar{0}$ or $\tilde{x} = \Theta$.

Definition 2.32 ([12]). Let $(\tilde{X}, A)$ be the absolute soft vector space i.e. $\tilde{X}(\lambda) = X$, for all $\lambda \in A$. Then a mapping $\|.\| : SE(\tilde{X}) \to \mathbb{R}(A)^*$ is said to be a soft norm on the soft vector space $\tilde{X}$ if $\|.\|$ satisfies the following conditions:

$(N1)$ $\|\tilde{x}\| \tilde{\geq} \bar{0}$, for all $\tilde{x} \tilde{\in} \tilde{X}$,

$(N2)$ $\|\tilde{x}\| = \bar{0}$ if and only if $\tilde{x} = \Theta$,

$(N3)$ $\|\hat{\alpha}\tilde{x}\| = |\ \hat{\alpha}\ |\ \|\tilde{x}\|$, for all $\tilde{x} \in \tilde{X}$ and for every soft scalar $\hat{\alpha}$,

$(N4)$ For all $\tilde{x}, \tilde{y} \in (\tilde{X}, A)$, $\|\tilde{x} + \tilde{y}\| \tilde{\leq} \|\tilde{x}\| + \|\tilde{y}\|$.

The soft vector space $(\tilde{X}, A)$ with a soft norm $\|.\|$ on $(\tilde{X}, A)$ is said to be a soft normed linear space and is denoted by $(\tilde{X}, \|.\|, A)$ or $(\tilde{X}, \|.\|)$. $(N1)$, $(N2)$, $(N3)$ and $(N4)$ are said to be soft norm axioms.

Theorem 2.33 ([12], Decomposition Theorem). *Every soft norm $\|.\|$ satisfies the condition*

$(N5)$ *For $\xi \in X$ and $\lambda \in A$, $\{\|\tilde{x}\|(\lambda) : \tilde{x}(\lambda) = \xi\}$ is a singleton set.*

And hence each soft norm $\|\|$ can be decomposed into a family of crisp norms $\{\|.\|_\lambda, \lambda \in A\}$, where $\|.\|_\lambda : X \to \mathbb{R}^+$ is defined by the following:

$$\textit{for each } \xi \in X,\ \|\xi\|_\lambda = \|\tilde{x}\|(\lambda), \textit{ with } \tilde{x} \tilde{\in} \tilde{X} \textit{ such that } \tilde{x}(\lambda) = \xi.$$

Proposition 2.34 ([12]). *Let $(\tilde{X}, \|.\|, A)$ be a soft normed linear space. Let us define $d : SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(A)^*$ by $d(\tilde{x}, \tilde{y}) = \|\tilde{x} - \tilde{y}\|$ for all $\tilde{x}, \tilde{y} \in (\tilde{X}, A)$. Then d is a soft metric on $(\tilde{X}, A)$.*

Definition 2.35 ([12])**.** Let $(\tilde{X}, A)$ be an absolute soft vector space. A soft set $(F, A) \in S(\tilde{X})$ is said to be convex soft set, if for all $\tilde{x}, \tilde{y} \tilde{\in} (F, A)$ and for all soft real numbers $\hat{t}$ with $\hat{t}(\lambda) \in [0, 1]$, for all $\lambda \in A$, $\hat{t}\tilde{x} + (\bar{1} - \hat{t})\tilde{y} \tilde{\in} (F, A)$.

Proposition 2.36 ([12])**.** *A soft open or closed ball in a soft normed linear space, is a convex soft set.*

Proposition 2.37 ([12])**.** *The elementary intersection of arbitrary number of convex soft sets is a convex soft set.*

Definition 2.38 ([7])**.** Let τ be the collection of soft sets of $S(\tilde{X})$. Then τ is said to be a soft topology on $(\tilde{X}, A)$, if it satisfies the following axioms:
(i) $(\tilde{\Phi}, A)$ and $(\tilde{X}, A)$ belong to τ,
(ii) The elementary union of any number of soft sets in τ belongs to τ,
(iii) The elementary intersection of two soft sets in τ belongs to τ.
The triplet $(\tilde{X}, \tau, A)$ is called a soft topological space and the members of τ are called soft open sets.

Definition 2.39 ([7])**.** Let $(\tilde{X}, \tau, A)$ be a soft topological space. Then a subcollection $\mathscr{B}$ of τ containing $(\tilde{\Phi}, A)$, is said to be an open base τ, if for all $\tilde{x} \tilde{\in} (\tilde{X}, A)$ and for any soft open set (F, A) containing the soft element $\tilde{x}$, there exists $(G, A) \in \mathscr{B}$ such that $\tilde{x} \tilde{\in} (G, A) \tilde{\subseteq} (F, A)$. The members of $\mathscr{B}$ are called soft basic open sets in $(\tilde{X}, \tau, A)$.

Definition 2.40 ([7])**.** Let $(\tilde{X}, \tau, A)$ be a soft topological space and $(F, A) \in S(\tilde{X})$. The interior of the soft set (F, A), denoted by $Int(F, A)$, is defined by

$$Int(F, A) = \{\tilde{x} \tilde{\in} (F, A) : \tilde{x} \tilde{\in} (G, A) \tilde{\subseteq} (F, A) \; for \; some \; (G, A) \in \tau\}.$$

$SS[Int(F, A)]$ is said to be soft interior of (F, A) and denoted by $(F, A)^{\circ}$.

Proposition 2.41 ([7])**.** *Let $(\tilde{X}, \tau, A)$ be a soft topological space and $(F, A) \in S(\tilde{X})$. Then $(F, A)^{\circ}$ is the elementary union of all soft open sets contained in (F, A). It is the largest soft open set in $(\tilde{X}, \tau, A)$ contained in (F, A).*

Definition 2.42 ([7])**.** Let $(\tilde{X}, \tau, A)$ be a soft topological space. Then $(F, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$ is a soft neighbourhood (soft nbd) of the soft element $\tilde{x}$, if there exists a soft set $(G, A) \in \tau$ such that $\tilde{x} \tilde{\in} (G, A) \tilde{\subseteq} (F, A)$.

Proposition 2.43 ([7])**.** *Let $(\tilde{X}, \tau, A)$ be a soft topological space. Then $(F, A)(\neq (\tilde{\Phi}, A)) \in S(\tilde{X})$ is soft open if and only if (F, A) is soft nbd of all of its soft elements.*

Definition 2.44 ([7])**.** Let X and Y be two non-empty sets and $\{f_\lambda : X \to Y, \lambda \in A\}$ be a collection of functions. Then a function $f : SE(\tilde{X}) \to SE(\tilde{Y})$, associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$, defined by $[f(\tilde{x})](\lambda) = f_\lambda(\tilde{x}(\lambda))$, for each $\lambda \in A$, is called a soft function.

Definition 2.45 ([7])**.** Let $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$.

(i) The image of a soft set (F, A) over X under the soft function f, denoted by $f[(F, A)]$, is defined by $f[(F, A)] = SS\{f(SE(F, A))\}$.

(ii) The inverse image of a soft set (G, A) over Y under the soft function f, denoted by $f^{-1}[(G, A)]$, is defined by $f^{-1}[(G, A)] = SS\{f^{-1}(SE(G, A))\}$.

Definition 2.46 ([7])**.** Let $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$. Then f is said to be
(i) injective, if $\tilde{x} \neq \tilde{y}$ implies $f(\tilde{x}) \neq f(\tilde{y})$,
(ii) surjective, if $f(\tilde{X}, A) = (\tilde{Y}, A)$,
(iii) bijective, if both injective and surjective.

Definition 2.47 ([7])**.** Let $(\tilde{X}, \tau, A)$ and $(\tilde{Y}, \nu, A)$ be two soft topological spaces and $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$. Then we denote this soft function as $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$.

(i) f is said to be soft continuous at $\tilde{x}_0 \tilde{\in} (\tilde{X}, A)$, if for every $(V, A) \in \nu$ such that $f(\tilde{x}_0) \tilde{\in} (V, A)$, there exists $(U, A) \in \tau$ such that $\tilde{x}_0 \tilde{\in} (U, A)$ and $f(U, A) \tilde{\subseteq} (V, A)$.

(ii) f is said to be soft continuous in $(\tilde{X}, \tau, A)$, if it is soft continuous at each soft element $\tilde{x}_0 \tilde{\in} (\tilde{X}, A)$.

Proposition 2.48 ([7])**.** *Let $(\tilde{X}, \tau, A)$ and $(\tilde{Y}, \nu, A)$ be two soft topological spaces and $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ be a soft function. Then the following relation holds:*

$$(1) \Leftrightarrow (2) \Leftrightarrow (3) \text{ and } (2) \Rightarrow (4).$$

(1) *f is soft continuous.*
(2) *For all $(V, A) \in \nu$, $f^{-1}(V, A) \in \tau$.*
(3) *There exists a subbase $\wp$ for ν such that $f^{-1}(V, A) \in \tau$ for all $(V, A) \in \wp$.*
(4) *for any closed soft set (F, A) in $(\tilde{Y}, \nu, A)$, $f^{-1}(F, A)$ is soft closed in $(\tilde{X}, \tau, A)$.*

Definition 2.49 ([7])**.** A soft function $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ is said to be soft open if for all $(V, A) \in \tau$, $f(V, A) \in \nu$.

Definition 2.50 ([7])**.** A soft function $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ is said to be soft homeomorphism if it satisfioes the following conditions:
(i) f is bijective,
(ii) f is soft continuous,
(iii) f^{-1} is soft continuous.

Proposition 2.51 ([7])**.** *Let $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ be a soft function. Then the followings are equivalent:*
(1) *f is a soft homeomorphism,*
(2) *f is bijective, soft open and soft continuous,*
(3) *f^{-1} is a soft homeomorphism.*

Proposition 2.52. *Let $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \nu, A)$ be a soft homeomorphism. Then for any $(F, A) \in S(\tilde{X})$, $f[(F, A)^\circ] = [f(F, A)]^\circ$.*

Proof. Let $(F, A) \in S(\tilde{X})$. Then, by the hypothesis, f is soft open and soft continuous. Thus

$$f[(F, A)^\circ] \tilde{\subseteq} [f(F, A)]^\circ \text{ and } f^{-1}\left([f(F, A)]^\circ\right) \tilde{\subseteq} \left[f^{-1}f(F, A)\right]^\circ.$$

So, $f[(F, A)^\circ] \tilde{\subseteq} [f(F, A)]^\circ$ and $[f(F, A)]^\circ \tilde{\subseteq} f[(F, A)^\circ]$. Hence $f[(F, A)^\circ] = [f(F, A)]^\circ$. □

Definition 2.53 ([26]). Let (F, A) and (G, A) be two soft sets over X. Then the product of (F, A) and (G, A) is defined as $(F, A)\tilde{\times}(G, A) = (F\tilde{\times}G, A)$, where $[F\tilde{\times}G](\lambda) = F(\lambda) \times G(\lambda)$, for each $\lambda \in A$.

Definition 2.54. Let $(F, A), (G, A) \in S(\tilde{X})$ and $\mathscr{B} = \{(\tilde{x}, \tilde{y}) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\}$. Now to each $\lambda \in A$, consider the set $\{(\tilde{x}(\lambda), \tilde{y}(\lambda)) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\}$ and define a soft set (H, A) such that $H(\lambda) = \{(\tilde{x}(\lambda), \tilde{y}(\lambda)) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\}$. Then $(H, A) = SS(\mathscr{B})$.

Proposition 2.55. *If $(F, A), (G, A) \in S(\tilde{X})$ then $(F\tilde{\times}G, A) = SS(\mathscr{B})$, where $\mathscr{B} = \{(\tilde{x}, \tilde{y}) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\}$.*

Proof. Let $\mathscr{B} = \{(\tilde{x}, \tilde{y}) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\}$. If any one of (F, A) and (G, A) is null soft set, then $\mathscr{B}$ is empty and obviously $(F\tilde{\times}G, A) = (\tilde{\Phi}, A)$.

Let (F, A) and $(G, A)(\neq (\tilde{\Phi}, A))$ be two members of $S(\tilde{X})$. Then for each $\lambda \in A$,

$$\begin{aligned} SS(\mathscr{B})(\lambda) &= \{(\tilde{x}(\lambda), \tilde{y}(\lambda)) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\} \\ &= \{(\tilde{x}(\lambda), \tilde{y}(\lambda)) : \tilde{x}(\lambda)\tilde{\in}F(\lambda), \tilde{y}(\lambda)\tilde{\in}G(\lambda),\ for\ all\ \lambda \in A\} \\ &= F(\lambda) \times G(\lambda) \\ &= [F\tilde{\times}G](\lambda). \end{aligned}$$

Thus, $(F\tilde{\times}G, A) = SS(\mathscr{B})$. □

Remark 2.56. Obviously $\mathscr{B} = \{(\tilde{x}, \tilde{y}) : \tilde{x}\tilde{\in}(F, A), \tilde{y}\tilde{\in}(G, A)\} = SE(F\tilde{\times}G, A)$.

Proposition 2.57. *Let $\{(V_i, A) : i \in \triangle\}$ be any collection of soft sets over a linear space X over the field K and A be the parameter set. Then for any $\hat{\alpha}\tilde{\in}(\widetilde{K}, A)$,*

(1) $\hat{\alpha} \underset{i\in\triangle}{\Cup} (V_i, A) = \underset{i\in\triangle}{\Cup} \hat{\alpha}(V_i, A)$,

(2) $\hat{\alpha} \underset{i\in\triangle}{\Cap} (V_i, A) = \underset{i\in\triangle}{\Cap} \hat{\alpha}(V_i, A)$, *if* $\underset{i\in\triangle}{\Cap} (V_i, A) \neq (\tilde{\Phi}, A)$.

Proof. (1) Let $\lambda \in A$. Then

$$\begin{aligned} \left(\hat{\alpha} \underset{i\in\triangle}{\Cup} (V_i, A)\right)(\lambda) &= \hat{\alpha}(\lambda) \left[\underset{i\in\triangle}{\Cup} (V_i, A)\right](\lambda) \\ &= \hat{\alpha}(\lambda) \left[\underset{i\in\triangle}{\tilde{\cup}} (V_i, A)\right](\lambda) \\ &= \hat{\alpha}(\lambda) \underset{i\in\triangle}{\cup} V_i(\lambda) \\ &= \underset{i\in\triangle}{\cup} \hat{\alpha}(\lambda)V_i(\lambda) \\ &= \left[\underset{i\in\triangle}{\tilde{\cup}} \hat{\alpha}(V_i, A)\right](\lambda) \\ &= \left[\underset{i\in\triangle}{\Cup} \hat{\alpha}(V_i, A)\right](\lambda). \end{aligned}$$

Thus, $\hat{\alpha} \underset{i\in\triangle}{\Cup} (V_i, A) = \underset{i\in\triangle}{\Cup} \hat{\alpha}(V_i, A)$.

(2) If $\underset{i\in\triangle}{\Cap} (V_i, A) \neq (\tilde{\Phi}, A)$, then $\underset{i\in\triangle}{\Cap} (V_i, A) = \underset{i\in\triangle}{\tilde{\cap}} (V_i, A)$. Thus, by similar arguments as above, we have $\hat{\alpha} \underset{i\in\triangle}{\Cap} (V_i, A) = \underset{i\in\triangle}{\Cap} \hat{\alpha}(V_i, A)$. □

3. Balanced, convex and absorbing soft sets

Definition 3.1. Let $(\tilde{X}, A)$ be an absolute soft vector space. A soft set $(F, A) \in S(\tilde{X})$ is said to be

(i) balanced, if for all $\tilde{x}\tilde{\in}(F,A)$ and $\hat{\alpha}\tilde{\in}(\widetilde{K},A)$ with $\mid \hat{\alpha} \mid \tilde{\leq}\bar{1}$, $\hat{\alpha}(\lambda)\neq 0,\ for\,all\,\lambda\in A$ or $\mid\hat{\alpha}\mid=\bar{0}$, $\hat{\alpha}\tilde{x}\tilde{\in}(F,A)$,

(ii) absorbing, if for all $\tilde{x}\tilde{\in}(F,A)$, there exists a soft real number $\hat{t}$ with $\hat{t}(\lambda)>0$, for each $\lambda\in A$ such that $\hat{t}^{-1}\tilde{x}\tilde{\in}(F,A)$,

(iii) symmetrical, if $(-\bar{1})(F,A)=(F,A)$.

Proposition 3.2. *Let $(\tilde{X},A)$ be an absolute soft vector space. A soft set $(F,A)\in S(\tilde{X})$ is*

(1) *a convex soft set iff for each $\lambda\in A$, $F(\lambda)$ is a convex set of X,*

(2) *a balanced soft set iff for each $\lambda\in A$, $F(\lambda)$ is a balanced set of X,*

(3) *an absorbing soft set iff for each $\lambda\in A$, $F(\lambda)$ is an absorbing set of X.*

Proof. (1) Let (F,A) be a convex soft set and choose $\lambda\in A$. Let $\xi,\eta\in F(\lambda)$ and $r\in[0,1]$. Then there exist $\tilde{x},\tilde{y}\tilde{\in}(F,A)$ such that $\tilde{x}(\lambda)=\xi,\tilde{y}(\lambda)=\eta$. Define a soft scalar $\bar{r}$ such that $\bar{r}(\lambda)=r$, for each $\lambda\in A$. Since (F,A) is a convex soft set, $\bar{r}\tilde{x}+(\bar{1}-\bar{r})\tilde{y}\tilde{\in}(F,A)$, i.e., $\bar{r}(\lambda)\tilde{x}(\lambda)+(1-\bar{r}(\lambda))\tilde{y}(\lambda)\tilde{\in}F(\lambda)$, for all $\lambda\in A$. In particular, $\xi+(1-r)\eta\in F(\lambda)$. Thus $F(\lambda)$ is convex set of X. Since $\lambda\in A$ is arbitrary, for each $\lambda\in A$, $F(\lambda)$ is convex set of X.

Conversely, let $(F,A)\in S(\tilde{X})$ and for each $\lambda\in A$, $F(\lambda)$ be convex set of X. Let $\tilde{x},\tilde{y}\tilde{\in}(F,A)$ and $\hat{k}\tilde{\in}(\widetilde{K},A)$ with $\hat{k}(\lambda)\in[0,1]$, for each $\lambda\in A$. Then $\tilde{x}(\lambda),\tilde{y}(\lambda)\in F(\lambda)$, for each $\lambda\in A$. Since $F(\lambda)$ is convex set of X for each $\lambda\in A$,

$$[\hat{k}\tilde{x}+(\bar{1}-\hat{k})\tilde{y}](\lambda)=\hat{k}(\lambda)\tilde{x}(\lambda)+(1-\hat{k}(\lambda))\tilde{y}(\lambda)\tilde{\in}F(\lambda),$$

for each $\lambda\in A$. Thus $\hat{k}\tilde{x}+(\bar{1}-\hat{k})\tilde{y}\tilde{\in}(F,A)$. So (F,A) is a convex soft set of $(\tilde{X},A)$.

(2) Let (F,A) be a balanced soft set and choose $\lambda\in A$. Let $\xi\in F(\lambda)$ and $\mid k\mid\leq 1$. Then there exists $\tilde{x}\tilde{\in}(F,A)$ such that $\tilde{x}(\lambda)=\xi$. Define a soft scalar $\bar{k}$ such that $\bar{k}(\lambda)=k$, for all $\lambda\in A$. Then either $\mid\bar{k}\mid\tilde{\leq}\bar{1}$ with $\bar{k}(\lambda)\neq 0$, for each $\lambda\in A$ or $\mid\bar{k}\mid=\bar{0}$, Then since (F,A) is a balanced soft set, $\bar{k}\tilde{x}\tilde{\in}(F,A)$, i.e., $\bar{k}(\lambda)\tilde{x}(\lambda)\in F(\lambda)$, for all $\lambda\in A$. In particular, $k\xi\in F(\lambda)$. Thus $F(\lambda)$ is a balanced set of X. Since $\lambda\in A$ is arbitrary, for each $\lambda\in A$, $F(\lambda)$ is balanced set of X.

Conversely, let $(F,A)\in S(\tilde{X})$ and for each $\lambda\in A$, $F(\lambda)$ be a balanced set of X. Let $\tilde{x}\tilde{\in}(F,A)$ and $\hat{k}\tilde{\in}(\widetilde{K},A)$ with $\mid\hat{k}\mid\tilde{\leq}\bar{1},\hat{k}(\lambda)\neq 0$, for each $\lambda\in A$ or $\mid\hat{k}\mid=\bar{0}$. Then $\tilde{x}(\lambda)\in F(\lambda)$, for all $\lambda\in A$. Since $F(\lambda)$for each $\lambda\in A$, is a balanced set of X

$$\text{for } \mid\hat{k}\mid\tilde{\leq}\bar{1},\hat{k}(\lambda)\neq 0, \text{ for each } \lambda\in A \text{ or } \mid\hat{k}\mid=\bar{0},\ [\hat{k}\tilde{x}](\lambda)=\hat{k}(\lambda)\tilde{x}(\lambda)\tilde{\in}F(\lambda),$$

for each $\lambda\in A$. Thus (F,A) is a balanced soft set of $(\tilde{X},A)$.

Proof of (3) is similar. □

Example 3.3. Consider the vector spce $\mathbb{R}^3$ over $\mathbb{R}$ and let $\mathbb{N}$, the set of natural numbers, be the parameter set. Then $(\tilde{\mathbb{R}^3},\mathbb{N})$ is an absolute soft vector space over $\mathbb{R}$. Define $(F,\mathbb{N})\in S(\tilde{\mathbb{R}^3})$ such that for each $n\in\mathbb{N}$, $F(n)$ is a closed ball of radius $1+\frac{1}{n}$ with center at $\theta(=(0,0,0))$ in $\mathbb{R}^3$. Then $F(n)$ is balanced and absorbing for each $n\in\mathbb{N}$ and hence, by Proposition 3.2, $(F,\mathbb{N})$ is a balanced and absorbing soft set in $(\tilde{\mathbb{R}^3},\mathbb{N})$.

Remark 3.4. If a soft set $(F,A)\in S(\tilde{X})$ is absorbing, then $\Theta\tilde{\in}(F,A)$.

Proposition 3.5. *Let $(\tilde{X}, A)$ be an absolute soft vector space and $(F, A) \in S(\tilde{X})$. Then*

(1) (F, A) is a balanced soft set iff $\hat{\alpha}(F, A)\tilde{\subseteq}(F, A)$, for all $\hat{\alpha}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{\alpha} \mid \tilde{\leq}\bar{1}, \hat{\alpha}(\lambda) \neq 0$, for each $\lambda \in A$ or $\mid \hat{\alpha} \mid= \bar{0}$,

(2) If (F, A) is a balanced soft set and $\hat{\alpha}, \hat{\beta}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{\alpha} \mid \tilde{\leq} \mid \hat{\beta} \mid$ and either $\hat{\alpha}(\lambda), \hat{\beta}(\lambda) \neq 0$, for each $\lambda \in A$ or $\mid \hat{\alpha} \mid, \mid \hat{\beta} \mid= \bar{0}$, then $\mid \hat{\alpha} \mid (F, A)\tilde{\subseteq} \mid \hat{\beta} \mid (F, A)$,

(3) If (F, A) is a balanced soft set and $\hat{\alpha}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{\alpha} \mid= \bar{1}$, then $\mid \hat{\alpha} \mid (F, A) = (F, A)$. Hence every balanced soft set is symmetrical,

(4) If $\{(F_i, A) : i \in \triangle\}$ be a collection of balanced soft sets in $(\tilde{X}, A)$, then $\underset{i\in\triangle}{\Cap}(F_i, A)$ is also a balanced soft set in $(\tilde{X}, A)$,

(5) If $\{(F_i, A) : i \in \triangle\}$ be a collection of balanced soft sets in $(\tilde{X}, A)$, then $\underset{i\in\triangle}{\Cup}(F_i, A)$ is also a balanced soft set in $(\tilde{X}, A)$.

Proof. (1) Suppose (F, A) is a balanced soft set. Let $\hat{\alpha}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{\alpha} \mid \tilde{\leq}\bar{1}, \hat{\alpha}(\lambda) \neq 0$, for each $\lambda \in A$ or $\mid \hat{\alpha} \mid= \bar{0}$. Then, by the hypothesis, $F(\lambda)$ is a balanced set, for each $\lambda \in A$. Thus $\hat{\alpha}(\lambda)F(\lambda) \subseteq F(\lambda)$, for each $\lambda \in A$. So $\hat{\alpha}(F, A)\tilde{\subseteq}(F, A)$.

Conversely, suppose the necessary condition holds and choose $\lambda \in A$ and let $\alpha \in K$ with $\mid \alpha \mid\leq 1$. Let $\bar{\alpha}(\lambda) = \alpha$, for each $\lambda \in A$. Then $\bar{\alpha}\tilde{\in}(\widetilde{K}, A)$ with $\mid \bar{\alpha} \mid \tilde{\leq}\bar{1}, \bar{\alpha}(\lambda) \neq 0$, for each $\lambda \in A$ or $\mid \bar{\alpha} \mid= \bar{0}$. By given condition, $\bar{\alpha}(F, A)\tilde{\subseteq}(F, A)$, i.e., $\bar{\alpha}(\lambda)F(\lambda) \subseteq F(\lambda)$, for each $\lambda \in A$. Thus $F(\lambda)$ is a balanced set. This is true for all $\lambda \in A$, i.e., $F(\lambda)$ is a balanced set, for each $\lambda \in A$. So, by Proposition 3.2, (F, A) is a balanced soft set.

Proof of (2) and (3) are similar as of (1).

(4) Let $(F, A) = \underset{i\in\triangle}{\Cap}(F_i, A)$ and let $\tilde{x}\tilde{\in}(F, A)$ and $\hat{\alpha}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{\alpha} \mid \tilde{\leq}\bar{1}, \hat{\alpha}(\lambda) \neq 0$, for each $\lambda \in A$ or $\mid \hat{\alpha} \mid= \bar{0}$. Then $\tilde{x}\tilde{\in}(F_i, A)$, for all $i \in \triangle$. Since each (F_i, A) is balanced soft set, $\hat{\alpha}\tilde{x}\tilde{\in}(F_i, A)$, for all $i \in \triangle$. Thus $\hat{\alpha}\tilde{x}\tilde{\in}(F, A)$. So (F, A) is a balanced soft set.

(5) Let $(F, A) = \underset{i\in\triangle}{\Cup}(F_i, A) = \underset{i\in\triangle}{\tilde{\cup}}(F_i, A)$. Since (F_i, A) is balanced soft set for each $i \in \triangle$, for each $\lambda \in A$, $F_i(\lambda)$ is balanced for each $i \in \triangle$. Thus $F(\lambda) = \underset{i\in\triangle}{\cup} F_i(\lambda)$ is balanced, for each $\lambda \in A$. So, by Proposition 3.2, (F, A) is balanced soft set. □

4. Soft topological vector space

Definition 4.1. Let K be the field of real or complex numbers and d be the usual metric on K. Let $(\widetilde{K}, d, A)$ be the soft metric space generated by the crisp metric space (K, d) as in Remark 2.17, where A be the set of parameter. Let ν be the soft metric topology on $(\widetilde{K}, A)$ as in Definition 2.23. Then ν is a soft topology as in Definition 2.38 and it is called usual soft topology on $(\widetilde{K}, A)$.
Thus $d(\tilde{x}, \tilde{y}) =\mid \tilde{x}-\tilde{y} \mid$, $\tilde{x}, \tilde{y}\tilde{\in}(\widetilde{K}, A)$, is the soft metric and $B(\tilde{x}, \hat{r}) = \{\tilde{y} : \mid \tilde{x}-\tilde{y} \mid \tilde{<}\hat{r}\}$ are open balls and $SS\{B(\tilde{x}, \hat{r})\}$ are soft open balls on $(\widetilde{K}, A)$. Also we see that $SS\{B(\tilde{x}, \hat{r})\}$ are soft basic open set in $(\widetilde{K}, \tau, A)$ as per Definition 2.39.

Definition 4.2. Let X be a vector space over the field of real or complex numbers and $(\tilde{X}, A)$ be the absolute soft vector space of X, i.e., $\tilde{X}(\lambda) = X$, for all $\lambda \in A$. Let τ be a soft topology on $(\tilde{X}, A)$ as in Definition 2.38 and ν be the usual soft topology on $(\widetilde{K}, A)$. Then $(\tilde{X}, \tau, A)$ is called a soft topological vector space, if the mappings

$$\begin{aligned} f : SE(\tilde{X}) \times SE(\tilde{X}) &\rightarrow SE(\tilde{X})\, defined\, by \\ f(\tilde{x}, \tilde{y}) &= \tilde{x} + \tilde{y} \end{aligned}$$

and

$$\begin{aligned} g : SE(\widetilde{K}) \times SE(\tilde{X}) &\rightarrow SE(\tilde{X})\, defined\, by \\ g(\hat{\alpha}, \tilde{x}) &= \hat{\alpha}\tilde{x} \end{aligned}$$

are continuous in the sense that

for any soft nbd (W, A) of $\tilde{x} + \tilde{y}$, there exist soft nbds $(V_1, A), (V_2, A)$ of $\tilde{x}$ and $\tilde{y}$ respectively such that $(V_1, A) + (V_2, A) \tilde{\subseteq} (W, A)$

and

for any soft nbd (U, A) of $\hat{\alpha}\tilde{x}$, there exist soft nbds (U_1, A) of $\hat{\alpha}$ in $(\widetilde{K}, \nu, A)$ and (U_2, A) of $\tilde{x}$ in $(\tilde{X}, \tau, A)$ such that $(U_1, A) \cdot (U_2, A) \tilde{\subseteq} (U, A)$.

By definition of ν, there exists $\hat{r} \tilde{>} \overline{0}$ such that $\hat{\alpha}$ where $B(\hat{\alpha}, \hat{r})$ is the open ball centred at $\hat{\alpha}$.

Let $SS\{B(\hat{\alpha}, \hat{r})\} = (G_1, A)$. Then $(G_1, A) \cdot (U_2, A) \tilde{\subseteq} (U, A)$. Thus, g is continuous iff there exists $\hat{r} \tilde{>} \overline{0}$ such that $\hat{\beta} \cdot \tilde{y} \tilde{\in} (U, A)$, for all $\hat{\beta} \tilde{\in} (\widetilde{K}, A)$ with $\mid \hat{\beta} - \hat{\alpha} \mid \tilde{<} \hat{r}$ and for all $\tilde{y} \tilde{\in} (U_2, A)$.

Definition 4.3. Let $(\tilde{X}, A)$ be the absolute soft vector space and $(\widetilde{K}, A)$ be the absolute soft set over K, the field of real or complex numbers. For a fixed $\tilde{a} \tilde{\in} (\tilde{X}, A)$ and $\hat{k} \tilde{\in} (\widetilde{K}, A)$, define operators $T_{\tilde{a}}$ and $M_{\hat{k}}$ from $SE(\tilde{X})$ to $SE(\tilde{X})$ by $T_{\tilde{a}}(\tilde{x}) = \tilde{a} + \tilde{x}$ and $M_{\hat{k}}(\tilde{x}) = \hat{k} \cdot \tilde{x}$, for all $\tilde{x} \tilde{\in} (\tilde{X}, A)$.

Operators $T_{\tilde{a}}$ and $M_{\hat{k}}$ are called soft translation operator and soft multiplication operator, respectively.

Remark 4.4. The operators $T_{\tilde{a}}$ and $M_{\hat{k}}$ in Definition 4.3 can be interpreted as soft functions as per Definition 2.44 associated with the families of functions $\{(T_{\tilde{a}})_\lambda : \lambda \in A\}$ and $\left\{\left(M_{\hat{k}}\right)_\lambda : \lambda \in A\right\}$, where $(T_{\tilde{a}})_\lambda : X \rightarrow X$ and $\left(M_{\hat{k}}\right)_\lambda : X \rightarrow X$ are defined by

$$(T_{\tilde{a}})_\lambda(\xi) = \tilde{a}(\lambda) + \xi \text{ and } \left(M_{\hat{k}}\right)_\lambda(\xi) = \hat{k}(\lambda) \cdot \xi, \text{ for all } \lambda \in A\, and\, \xi \in X.$$

Proposition 4.5. *Let $(\tilde{X}, \tau, A)$ be a soft topological vector space. For $\tilde{a} \tilde{\in} (\tilde{X}, A)$ and $\hat{k} \tilde{\in} (\widetilde{K}, A)$ with $\hat{k}(\lambda) \neq 0$, for each $\lambda \in A$ then soft translation operator $T_{\tilde{a}}$ and soft multiplication operator $M_{\hat{k}}$ are soft homeomorphism from $(\tilde{X}, \tau, A)$ to $(\tilde{X}, \tau, A)$.*

Proof. Clearly $T_{\tilde{a}}$ and $M_{\hat{k}}$ are bijective mappings on $SE(\tilde{X})$ and thus inverses exist. In fact $(T_{\tilde{a}})^{-1} = T_{-\tilde{a}}$ and $(M_{\hat{k}})^{-1} = M_{(\hat{k})^{-1}}$.

Consider the mapping $T_{\tilde{a}}$ and take a soft nbd (W, A) of $T_{\tilde{a}}(\tilde{x}) (= \tilde{a} + \tilde{x})$. Then, by continuity of the function f at $(\tilde{x}, \tilde{a})$, there exist soft nbds (U, A) and (V, A) of $\tilde{x}$ and $\tilde{a}$ respectively such that $(U, A) + (V, A) \tilde{\subseteq} (W, A)$. Thus, in particular, $(U, A) + \tilde{a} \tilde{\subseteq} (W, A)$, i.e., $T_{\tilde{a}}(U, A) \tilde{\subseteq} (W, A)$. So, $T_{\tilde{a}}$ is soft continuous. Similarly, $(T_{\tilde{a}})^{-1}$ is soft continuous. Hence $T_{\tilde{a}}$ is a soft homeomorphism.

By the continuity of the function g at $(\hat{k}, \tilde{x})$, for any soft nbd (W, A) of $\hat{k} \cdot \tilde{x}$, there exist $\hat{r}\tilde{>}\bar{0}$ and a soft nbd (U, A) of $\tilde{x}$ such that $\hat{s}\tilde{y}\tilde{\in}(W, A)$, for all $\hat{s}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{s} - \hat{k} \mid \tilde{<}\hat{r}$ and for all $\tilde{y}\tilde{\in}(U, A)$. Then, in particular, $\hat{k} \cdot (U, A)\tilde{\subseteq}(W, A)$, i.e., $M_{\hat{k}}(U, A)\tilde{\subseteq}(W, A)$. Thus, $M_{\hat{k}}$ is soft continuous. Since $(M_{\hat{k}})^{-1} = M_{(\hat{k})^{-1}}$, it follows that $M_{(\hat{k})^{-1}}$ is also soft continuous. So $M_{\hat{k}}$ is a soft homeomorphism. □

Proposition 4.6. *Let $(\tilde{X}, \tau, A)$ be a soft topological vector space. Then*
(1) *If (C, A) is convex soft set, then $(C, A)^{\circ}$ is also convex soft set,*
(2) *If (B, A) is balanced soft set and $\Theta\tilde{\in}(B, A)^{\circ}$, then $(B, A)^{\circ}$ is also balanced soft set.*

Proof. (1) Let $\hat{t}\tilde{\in}\mathbb{R}(A)$ with $\hat{t}(\lambda) \in [0, 1]$, for each $\lambda \in A$. Since (C, A) is convex soft set and $(C, A)^{\circ}\tilde{\subset}(C, A)$, we have

$$\hat{t}(\lambda)C^{\circ}(\lambda) + (1 - \hat{t}(\lambda))C^{\circ}(\lambda) \subset \hat{t}(\lambda)C(\lambda) + (1 - \hat{t}(\lambda))C(\lambda) \subset C(\lambda),$$

i.e., $\hat{t}(C, A)^{\circ} + (\bar{1} - \hat{t})(C, A)^{\circ}\tilde{\subset}(C, A)$. Since $(C, A)^{\circ}$ is soft open, by Proposition 2.51, $\hat{t}(C, A)^{\circ} + (\bar{1} - \hat{t})(C, A)^{\circ}$ is a soft open subset of (C, A). Since $(C, A)^{\circ}$ is the largest soft open subset of (C, A), $\hat{t}(C, A)^{\circ} + (\bar{1} - \hat{t})(C, A)^{\circ}\tilde{\subset}(C, A)^{\circ}$. Thus $(C, A)^{\circ}$ is convex soft set.

(2) Since soft multiplication $M_{\hat{k}}$ is a soft homeomorphism, by Proposition 2.52, $M_{\hat{k}}[(B, A)^{\circ}] = \left(M_{\hat{k}}[(B, A)]\right)^{\circ}$, for all $\hat{k}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{k} \mid \tilde{\leq}\bar{1}$, $\hat{k}(\lambda) \neq 0$, for each $\lambda \in$A. Since (B, A) is a balanced soft set, $\hat{k}(B, A)^{\circ} = \left(\hat{k}(B, A)\right)^{\circ}\tilde{\subseteq}(B, A)^{\circ}$.

Now, if $\Theta\tilde{\in}(B, A)^{\circ}$, then $\bar{0}.(B, A)^{\circ} = \Theta\tilde{\subseteq}(B, A)^{\circ}$. Thus, $\hat{k}(B, A)^{\circ}\tilde{\subseteq}(B, A)^{\circ}$, for all $\hat{k}\tilde{\in}(\widetilde{K}, A)$ with $\mid \hat{k} \mid \tilde{\leq}\bar{1}$, $\hat{k}(\lambda) \neq 0$, for each $\lambda \in$A and $\hat{k} = \bar{0}$. So $(B, A)^{\circ}$ is a balanced soft set. □

Proposition 4.7. *Let $(\tilde{X}, \tau, A)$ be a soft topological vector space. Then*
(1) *Every soft nbd of Θ contains a balanced soft nbd of Θ,*
(2) *Every convex soft nbd of Θ contains a balanced convex soft nbd of Θ.*

Proof. (1) Let (V, A) be any soft nbd of Θ. By continuity of the mapping $g(\hat{\alpha}, \tilde{x}) = \hat{\alpha}\tilde{x}$ at $(\bar{0}, \Theta)$, for any soft nbd (V, A) of Θ, there exists a soft nbd (W, A) of Θ and $\hat{\delta}\tilde{>}\bar{0}$ such that $\hat{\alpha}\tilde{y}\tilde{\in}(V, A)$ for all soft scalar $\mid \hat{\alpha} \mid \tilde{<}\hat{\delta}$ and for all $\tilde{y}\tilde{\in}(W, A)$. Then $\hat{\alpha}(W, A)\tilde{\subseteq}(V, A)$, for all $\mid \hat{\alpha} \mid \tilde{<}\hat{\delta}$.

Let $(W', A) = \Cup_{|\hat{\alpha}|\tilde{<}\hat{\delta}} \hat{\alpha}(W, A)$. Then (W', A) is a soft nbd of Θ. Let $\hat{\beta}$ be any soft scalar with $\mid \hat{\beta} \mid \tilde{\leq}\bar{1}, \hat{\beta}(\lambda) \neq 0$, for each $\lambda \in A$ or $\hat{\beta} = \bar{0}$. Then $\mid \hat{\alpha}\hat{\beta} \mid \tilde{<}\hat{\delta}$. Thus $\hat{\beta}(W', A) = \Cup_{|\hat{\alpha}|\tilde{<}\hat{\delta}} \hat{\alpha}\hat{\beta}(W, A)\tilde{\subseteq}(W', A)$. So, (W', A) is a balanced soft nbd of Θ such that $(W', A)\tilde{\subseteq}(V, A)$.

(2) Let (U, A) be any convex soft nbd of Θ and let $(V, A) = \Cap_{|\hat{\delta}|=\bar{1}} \hat{\delta}(U, A)$. We shall show that (V, A) is a convex soft set.

Let $\tilde{x}, \tilde{y} \tilde{\in} \hat{\delta}(U, A)$. Then there exist $\tilde{x}', \tilde{y}' \tilde{\in} (U, A)$ such that $\tilde{x} = \hat{\delta}\tilde{x}', \tilde{y} = \hat{\delta}\tilde{y}'$. Now for any soft scalar $\hat{\alpha}$ with $\hat{\alpha}(\lambda) \in [0, 1]$,

$$\hat{\alpha}\tilde{x} + (\bar{1} - \hat{\alpha})\tilde{y} = \hat{\alpha}\hat{\delta}\tilde{x}' + (\bar{1} - \hat{\alpha})\hat{\delta}\tilde{y}' = \hat{\delta}(\hat{\alpha}\tilde{x}' + (\bar{1} - \hat{\alpha})\tilde{y}') \tilde{\in} \hat{\delta}(U, A).$$

Thus $\hat{\delta}(U, A)$ is convex soft set. Since arbitary elementary intersection of convex soft sets is also convex, it follows that (V, A) is convex.

We now show that (V, A) is a soft nbd of Θ. By (1), there exists a balanced soft nbd of Θ such that $(W, A) \tilde{\subseteq} (U, A)$. Since (W, A) is balanced, $\hat{\delta}(W, A) = (W, A)$ for all soft scalar $\hat{\delta}$ with $|\hat{\delta}| = \bar{1}$. Then, $(W, A) = \hat{\delta}(W, A) \tilde{\subseteq} \hat{\delta}(U, A)$ for all soft scalar $\hat{\delta}$ with $|\hat{\delta}| = \bar{1}$. Thus, $(W, A) \tilde{\subseteq} \Cap_{|\hat{\delta}|=\bar{1}} \hat{\delta}(U, A) = (V, A)$. So (V, A) is a soft nbd of Θ and $(V, A) \tilde{\subseteq} (U, A)$.

Let $|\hat{\beta}| \tilde{\leq} \bar{1}$, with $\hat{\beta}(\lambda) \neq 0$, for each $\lambda \in A$ or $\hat{\beta} = \bar{0}$. If $\hat{\beta} = \bar{0}$, $\hat{\beta}(V, A) = \Theta \tilde{\subset} (V, A)$. For $|\hat{\beta}| \tilde{\leq} \bar{1}$ such that $\hat{\beta}(\lambda) \neq 0$, for each $\lambda \in A$, $\hat{\beta}$ can be expressed as $\hat{\beta} = \hat{r}\hat{\gamma}$ where $\bar{0} \tilde{<} \hat{r} \tilde{\leq} \bar{1}$, $|\hat{\gamma}| = \bar{1}$. Since $|\hat{\delta}\tilde{\gamma}| = \bar{1}$,

$$\hat{r}\hat{\gamma}(V, A) = \Cap_{|\hat{\delta}|=\bar{1}} \hat{\delta}\hat{r}\hat{\gamma}(U, A) = \Cap_{|\hat{\delta}|=\bar{1}} \hat{r}\hat{\delta}(U, A).$$

Now, let $\tilde{y} \tilde{\in} \hat{r}\hat{\delta}(U, A)$. Since $\hat{\delta}(U, A)$ is convex soft set containing Θ, there exists $\tilde{x} \tilde{\in} \hat{\delta}(U, A)$ such that

$$\tilde{y} = \hat{r}\tilde{x} = \hat{r}\tilde{x} + (\bar{1} - \hat{r})\Theta \tilde{\in} \hat{\delta}(U, A).$$

Then $\hat{r}\hat{\delta}(U, A) \tilde{\subseteq} \hat{\delta}(U, A)$. Therefore $\Cap_{|\hat{\delta}|=\bar{1}} \hat{r}\hat{\delta}(U, A) \tilde{\subseteq} \Cap_{|\hat{\delta}|=\bar{1}} \hat{\delta}(U, A) = (V, A)$. So (V, A) is a balanced soft set. Hence (U, A) contains a convex balanced soft nbd (V, A) of Θ. □

Definition 4.8. Let $(\tilde{X}, \tau, A)$ be a soft topological vector space. A soft set $(F, A) \in S(\tilde{X})$ is said to be bounded soft set, if for any soft nbd (V, A) of Θ, there exists $\hat{t} > \bar{0}$ such that $(F, A) \tilde{\subseteq} \hat{s}(V, A)$, for every $\hat{s} \tilde{\geq} \hat{t}$.

Definition 4.9. A soft nbd base at Θ in a soft topological vector space $(\tilde{X}, \tau, A)$ is a collection $\mathcal{F}$ of soft nbds of Θ such that every soft nbd of Θ contains a member of $\mathcal{F}$.

Proposition 4.10. *Let (V, A) be any bounded soft nbd of Θ and $\{\hat{\delta}_n(\tilde{>}\bar{0})\}$ be a sequence of soft scalars tending to $\bar{0}$ as $n \to \infty$. Then for any soft nbd (U, A) of Θ, there exists a member $\hat{\delta}_{n_0}(V, A)$, of $\{\hat{\delta}_n(V, A)\}$ such that $\hat{\delta}_{n_0}(V, A) \tilde{\subseteq} (U, A)$ i.e., $\{\hat{\delta}_n(V, A)\}$ forms a soft nbd base at Θ.*

Proof. Let (U, A) be a soft nbd of Θ. Since (V, A) is bounded, there exists $\hat{\alpha} \tilde{>} \bar{0}$ such that $(V, A) \tilde{\subseteq} \hat{\beta}(U, A)$ for each $\hat{\beta} \tilde{>} \hat{\alpha}$. Since $\hat{\delta}_n \to \bar{0}$, there exists $n_0 \in \mathbb{N}$ such that $\frac{\bar{1}}{\hat{\delta}_n} \tilde{>} \hat{\alpha}$, for all $n \geq n_0$. Then, $(V, A) \tilde{\subseteq} \frac{\bar{1}}{\hat{\delta}_n}(U, A)$, for all $n \geq n_0$. Thus $\hat{\delta}_{n_0}(V, A) \tilde{\subseteq} (U, A)$. So $\{\hat{\delta}_n(V, A)\}$ forms a soft nbd base at Θ. □

5. Soft seminorm and soft normability

Definition 5.1. Let $(\tilde{X}, A)$ be an absolute soft vector space of X over K. A mapping $p : SE(\tilde{X}) \to \mathbb{R}(A)$ is said to be a soft seminorm on $(\tilde{X}, A)$, if it satisfies the following axioms:

$(SN1)$ $p(\tilde{x} + \tilde{y}) \tilde{\leq} p(\tilde{x}) + p(\tilde{y})$,

$(SN2)$ $p(\hat{\alpha}\tilde{x}) = \mid \hat{\alpha} \mid p(\tilde{x})$, for all $\tilde{x}, \tilde{y} \tilde{\in} (\tilde{X}, A)$, for each $\hat{\alpha} \tilde{\in} (\widetilde{K}, A)$.

Proposition 5.2. *Let p be a soft seminorm on the absolute soft vector space $(\tilde{X}, A)$. Then*

(1) *If $\tilde{x} = \Theta$, then $p(\tilde{x}) = \bar{0}$,*

(2) $p(\tilde{x} - \tilde{y}) \tilde{\geq} \mid p(\tilde{x}) - p(\tilde{y}) \mid$,

(3) $p(\tilde{x}) \tilde{\geq} \bar{0}$,

(4) *If $p(\tilde{x}) = \bar{0} \Rightarrow \tilde{x} = \Theta$, then p is a soft norm on $(\tilde{X}, A)$.*

Proof. (1) Follows from definition, taking $\hat{\alpha} = \bar{0}$.

(2) Clearly, $p(\tilde{x}) = p(\tilde{x} - \tilde{y} + \tilde{y}) \tilde{\leq} p(\tilde{x} - \tilde{y}) + p(\tilde{y})$ Then, $p(\tilde{x}) - p(\tilde{y}) \tilde{\leq} p(\tilde{x} - \tilde{y})$. On one hand,

$$\begin{aligned} p(\tilde{y}) - p(\tilde{x}) &\tilde{\leq} p(\tilde{y} - \tilde{x}) \\ &= p((-\bar{1})(\tilde{x} - \tilde{y})) \\ &= \mid -\bar{1} \mid p(\tilde{x} - \tilde{y}) \\ &= p(\tilde{x} - \tilde{y}). \end{aligned}$$

Thus, $p(\tilde{x} - \tilde{y}) \tilde{\geq} \mid p(\tilde{x}) - p(\tilde{y}) \mid$.

(3) By (1), $p(\tilde{x}) \tilde{\geq} \mid p(\tilde{x}) - p(\Theta) \mid = \mid p(\tilde{x}) \mid \tilde{\geq} \bar{0}$.

(4) It follows from (1) and (3). □

Theorem 5.3. *(Decomposition Theorem) Every soft seminorm on a soft vector space satisfies the following condition*

$(\mathcal{I})$ *For $\xi \in X$ and $\lambda \in A$, $\{[p(\tilde{x})](\lambda) : \tilde{x}(\lambda) = \xi\}$ is a singleton set.*

And hence each soft seminorm p can be decomposed into a family of crisp seminorms $\{p_\lambda : \lambda \in A\}$, where $p_\lambda : X \to \mathbb{R}^+$ is defined by the following:
for each $\xi \in X$, $p_\lambda(\xi) = [p(\tilde{x})](\lambda)$ with $\tilde{x} \tilde{\in} (\tilde{X}, A)$ such that $\tilde{x}(\lambda) = \xi$.

Proof. Let $\mu \in A$ and $\tilde{x} \tilde{\in} (\tilde{X}, A)$ be such that $\tilde{x}(\mu) = \theta$. Let us consider a soft scalar $\hat{\alpha}$ such that $\hat{\alpha}(\mu) = 1$ and $\hat{\alpha}(\lambda) = 0$, for each $\lambda \in A \setminus \{\mu\}$. Then $\hat{\alpha} \cdot \tilde{x} = \Theta$. By definition, $p(\hat{\alpha}\tilde{x}) = \mid \hat{\alpha} \mid p(\tilde{x})$. Thus $p(\Theta) = \mid \hat{\alpha} \mid p(\tilde{x})$. Hence $\bar{0} = \mid \hat{\alpha} \mid p(\tilde{x})$. In particular, $0 = \mid \hat{\alpha}(\mu) \mid [p(\tilde{x})](\mu)$, i.e., $0 = 1 \cdot [p(\tilde{x})](\mu)$. So, $0 = [p(\tilde{x})](\mu)$. Hence we see that for any soft element $\tilde{x} \tilde{\in} (\tilde{X}, A)$, $\tilde{x}(\mu) = \theta$ implies that $[p(\tilde{x})](\mu) = 0$.

Next let $\xi \in X$ and $\mu \in A$ be arbitrary. Let $\tilde{x}, \tilde{y} \tilde{\in} (\tilde{X}, A)$ be any two soft elements such that $\tilde{x}(\mu) = \tilde{y}(\mu) = \xi$. Then $(\tilde{x} - \tilde{y})(\mu) = \theta$. Thus, by the above argument, we get $[p(\tilde{x} - \tilde{y})](\mu) = 0$.

On one hand, by Proposition 5.2, $\mid p(\tilde{x}) - p(\tilde{y}) \mid \tilde{\leq} p(\tilde{x} - \tilde{y})$. In particular,
$\mid [p(\tilde{x})](\mu) - [p(\tilde{y})](\mu) \mid \tilde{\leq} [p(\tilde{x} - \tilde{y})](\mu)$. Since $[p(\tilde{x} - \tilde{y})](\mu) = 0$, we have
$\mid [p(\tilde{x})](\mu) - [p(\tilde{y})](\mu) \mid = 0$. Thus $[p(\tilde{x})](\mu) = [p(\tilde{y})](\mu)$.
So, $\{[p(\tilde{x})](\lambda) : \tilde{x}(\lambda) = \xi\}$ is a singleton set. Hence the condition $(\mathcal{I})$ is satisfied.

Now we prove the second part of the theorem.
Clearly, $p_\lambda : X \to \mathbb{R}^+$ is a rule that assign a vector of X to a non-negative real number, for all $\lambda \in A$. Now for each $\lambda \in A$, the well defined property of p_λ follows

from condition ($\mathcal{I}$) and the soft seminorm axioms gives the seminorm conditions of p_λ. Thus every soft seminorm gives a parametrized family of crisp seminorms. With this point of view, it also follows that, a soft seminorm, is a 'soft function' as per Definition 2.44. □

Proposition 5.4. *Every parametrized family of crisp seminorm $\{p_\lambda : \lambda \in A\}$ on a crisp linear space X can be considered as a soft seminorm on the soft vector space $(\tilde{X}, A)$.*

Proof. Let $(\tilde{X}, A)$ be the absolute soft vector space of X over a field K, where A be a non-empty set of parameters and $\{p_\lambda : \lambda \in A\}$ be a family of crisp seminorms on the vector space X. If $\tilde{x}\tilde{\in}\left(\tilde{X}, A\right)$, then $\tilde{x}(\lambda) \in X$, for each $\lambda \in A$. We define a soft mapping $p : SE(\tilde{X}) \to \mathbb{R}(A)^*$ by

$$p(\tilde{x})(\lambda) = p_\lambda(\tilde{x}(\lambda)), \text{ for each } \lambda \in A \text{ and } \tilde{x}\tilde{\in}\left(\tilde{X}, A\right).$$

Let us show that p satisfies the conditions $(SN1), (SN2)$ for soft seminorm.

$(SN1)$ Let $\tilde{x}, \tilde{y}\tilde{\in}\left(\tilde{X}, A\right)$. Then for each $\lambda \in A$,

$$\begin{aligned} &[p(\tilde{x}+\tilde{y})](\lambda) \\ &= p_\lambda(\tilde{x}(\lambda)+\tilde{y}(\lambda)) \\ &\leq p_\lambda(\tilde{x}(\lambda)) + p_\lambda(\tilde{y}(\lambda)) \text{ [Since } p_\lambda \text{ is a seminorm, for each } \lambda \in A] \\ &= [p(\tilde{x})](\lambda) + [p(\tilde{y})](\lambda). \end{aligned}$$

Thus, $p(\tilde{x}+\tilde{y})\tilde{\leq}p(\tilde{x}) + p(\tilde{y})$.

$(SN2)$. Let $\tilde{x}, \tilde{y}\tilde{\in}(\tilde{X}, A), \hat{\alpha}\tilde{\in}(\widetilde{K}, A)$. Then for each $\lambda \in A$,

$$\begin{aligned} &[p(\hat{\alpha}\tilde{x})](\lambda) \\ &= p_\lambda((\hat{\alpha}\tilde{x})(\lambda)) \\ &= p_\lambda(\hat{\alpha}(\lambda)\tilde{x}(\lambda)) \\ &=\mid \hat{\alpha}(\lambda) \mid [p(\tilde{x})](\lambda) \text{ [Since } p_\lambda \text{ is a seminorm, for each } \lambda \in A] \\ &= [\mid \hat{\alpha} \mid p(\tilde{x})](\lambda). \end{aligned}$$

Thus $p(\hat{\alpha}\tilde{x}) =\mid \hat{\alpha} \mid p(\tilde{x})$.

So p is a soft seminorm on$(\tilde{X}, A)$. □

Proposition 5.5. *Let $p : SE(\tilde{X}) \to \mathbb{R}(A)$ be a soft seminorm on $(\tilde{X}, A)$. Then*

(1) *$SS(\mathscr{B})$ is a soft subspace of the soft vector space $(\tilde{X}, A)$, where $\mathscr{B} = \{\tilde{x} : p(\tilde{x}) = \bar{0}\}$,*

(2) *if $\mathscr{B} = \{\tilde{x} : p(\tilde{x}) \tilde{<} \bar{1}\}$, then $SS(\mathscr{B})$ is a convex, balanced and absorbing soft set.*

Proof. (1) Let $\tilde{x}, \tilde{y}\tilde{\in}SS(\mathscr{B})$ be any two soft elements and $\hat{\alpha}, \hat{\beta}$ be any two soft scalars. Let $\lambda \in A$. Since p_λ is a seminorm,

$$\begin{aligned} [p(\hat{\alpha}\tilde{x}+\hat{\beta}\tilde{y})](\lambda) &= p_\lambda(\hat{\alpha}(\lambda)\tilde{x}(\lambda) + \hat{\beta}(\lambda)\tilde{y}(\lambda)) \\ &\leq \hat{\alpha}(\lambda)p_\lambda(\tilde{x}(\lambda)) + \hat{\beta}(\lambda)p_\lambda(\tilde{y}(\lambda)) \\ &= 0. \end{aligned}$$

Then $p(\hat{\alpha}\tilde{x}+\hat{\beta}\tilde{y}) = \bar{0}$. Thus $\hat{\alpha}\tilde{x}+\hat{\beta}\tilde{y}\tilde{\in}SS(\mathscr{B})$. So $SS(\mathscr{B})$ is a soft subspace of the soft vector space $(\tilde{X}, A)$.

(2) Let $\tilde{x}, \tilde{y} \tilde{\in} SS(\mathscr{B})$ be any two soft elements and $\hat{t}$ be any soft real number with $\hat{t}(\lambda) \in [0, 1]$, for all $\lambda \in A$. Let $\lambda \in A$. Since p_λ is a seminorm,

$$\begin{aligned}[p(\hat{t}\tilde{x} + (\bar{1} - \hat{t})\tilde{y})](\lambda) &= p_\lambda(\hat{t}(\lambda)\tilde{x}(\lambda) + (\bar{1} - \hat{t})(\lambda)\tilde{y}(\lambda)) \\ &\leq \hat{t}(\lambda)p_\lambda(\tilde{x}(\lambda)) + (1 - \hat{t}(\lambda))p_\lambda(\tilde{y}(\lambda) \\ &< \hat{t}(\lambda) + (1 - \hat{t}(\lambda)) \\ &= 1.\end{aligned}$$

Then $\hat{t}\tilde{x} + (\bar{1} - \hat{t})\tilde{y} \tilde{\in} SS(\mathscr{B})$. Thus $SS(\mathscr{B})$ is a convex soft set.

Similarly, it can be shown that $SS(\mathscr{B})$ is balanced and absorbing soft set. □

Definition 5.6. Let $(\tilde{X}, A)$ be an absolute soft vector space and $(F, A) \in S(\tilde{X})$ be a convex absorbing soft set. Then a mapping $\mu_{(F,A)} : SE(\tilde{X}) \to \mathbb{R}(A)^*$, where $\mathbb{R}(A)^*$ is the set of all non-negative soft real numbers, defined by

$$[\mu_{(F,A)}(\tilde{x})](\lambda) = inf\{t > 0 : t^{-1}(\tilde{x}(\lambda)) \in F(\lambda)\}, \text{ for each } \lambda \in A,$$

is called soft Minkowski functional associated with (F, A).

Proposition 5.7. *Let $(\tilde{X}, A)$ be an absolute soft vector space and p be a soft seminorm on $(\tilde{X}, A)$. Then*

(1) *For any convex absorbing soft set $(F, A) \in S(\tilde{X})$ and any soft real number $\hat{t} \tilde{>} \bar{0}$, $\mu_{(F,A)}(\hat{t}^{-1}\tilde{x}) = \hat{t}^{-1}\mu_{(F,A)}(\tilde{x})$,*

(2) *Let $(F, A) = SS(\mathscr{B})$, where $\mathscr{B} = \{\tilde{x} : \bar{1} \tilde{>} p(\tilde{x})\}$. Then $\mu_{(F,A)}(\tilde{x}) = p(\tilde{x})$, for all $\tilde{x} \tilde{\in} (\tilde{X}, A)$.*

Proof. (1) For each $\lambda \in A$ and soft real number $\hat{t} \tilde{>} \bar{0}$,

$$\begin{aligned}\left[\mu_{(F,A)}(\hat{t}^{-1}\tilde{x})\right](\lambda) &= inf\{s > 0 : s^{-1}\hat{t}^{-1}(\lambda)\tilde{x}(\lambda) \in F(\lambda)\} \\ &= inf\{\hat{t}^{-1}(\lambda)\hat{t}(\lambda)s > 0 : s^{-1}\hat{t}^{-1}(\lambda)\tilde{x}(\lambda) \in F(\lambda)\} \\ &= \hat{t}^{-1}(\lambda) inf\{\hat{t}(\lambda)s > 0 : \left(\hat{t}(\lambda)s\right)^{-1}\tilde{x}(\lambda) \in F(\lambda)\} \\ &= \hat{t}^{-1}(\lambda) inf\{r > 0 : r^{-1}\tilde{x}(\lambda) \in F(\lambda)\} \\ &= \hat{t}^{-1}(\lambda)\left[\mu_{(F,A)}(\tilde{x})\right](\lambda). \text{ Then, } \mu_{(F,A)}(\hat{t}^{-1}\tilde{x}) = \hat{t}^{-1}\mu_{(F,A)}(\tilde{x}).\end{aligned}$$

(2) Choose $\lambda \in A$ and $\tilde{x} \tilde{\in} (\tilde{X}, A)$. Then

$$\begin{aligned}\left[\mu_{(F,A)}(\tilde{x})\right](\lambda) &= inf\{t > 0 : t^{-1}(\tilde{x}(\lambda)) \in F(\lambda)\} \\ &= inf\{t > 0 : p_\lambda(t^{-1}\tilde{x}(\lambda)) < 1\} \\ &= inf\{t > 0 : \left[p(\bar{t}^{-1}\tilde{x})\right](\lambda) < 1\} \\ &= inf\{t > 0 : \bar{t}^{-1}(\lambda)\left[p(\tilde{x})\right](\lambda) < 1\} \\ &= inf\{t > 0 : \left[p(\tilde{x})\right](\lambda) < t\} \\ &= \left[p(\tilde{x})\right](\lambda).\end{aligned}$$

Then $\mu_{(F,A)}(\tilde{x}) = p(\tilde{x})$, for all $\tilde{x} \tilde{\in} (\tilde{X}, A)$. □

Proposition 5.8. *Let $(\tilde{X}, A)$ be an absolute soft vector space and $(F, A) \in S(\tilde{X})$ be a convex absorbing soft set. Then for all $\tilde{x}, \tilde{y} \tilde{\in} (\tilde{X}, A)$,*

(1) $\mu_{(F,A)}(\tilde{x} + \tilde{y}) \tilde{\leq} \mu_{(F,A)}(\tilde{x}) + \mu_{(F,A)}(\tilde{y})$,

(2) $\mu_{(F,A)}(\hat{t}\tilde{x}) = \hat{t}\mu_{(F,A)}(\tilde{x}), \hat{t} \tilde{\geq} \bar{0}$,

(3) $\mu_{(F,A)}$ *is a soft seminorm, if (F, A) is a balanced soft set,*

(4) *If $B_\lambda = \{\xi = \tilde{x}(\lambda) \in X : [\mu_{(F,A)}(\tilde{x})](\lambda) < 1\}$ and $C_\lambda = \{\eta = \tilde{x}(\lambda) \in X : [\mu_{(F,A)}(\tilde{x})](\lambda) \leq 1\}$, then $B_\lambda \subseteq F(\lambda) \subseteq C_\lambda$, for each $\lambda \in A$.*

If we define soft sets (B, A) and (C, A) such that $B(\lambda) = B_\lambda$ and $C(\lambda) = C_\lambda$, for each $\lambda \in A$, then $(B, A)\tilde{\subseteq}(F, A)\tilde{\subseteq}(C, A)$ and for any $\tilde{x}\tilde{\in}(B, A), \mu_{(B,A)}(\tilde{x})\tilde{<}\bar{1}$ and for any $\tilde{x}\tilde{\in}(C, A), \mu_{(F,A)}(\tilde{x})\tilde{\leq}\bar{1}$.

Proof. Let $\left[H_{(F,A)}(\tilde{x})\right](\lambda) = \{t > 0 : t^{-1}\tilde{x}(\lambda) \in F(\lambda)\}$, for each $\lambda \in A$.

(1) Choose $\lambda \in A$ and $t > [\mu_{(F,A)}(\tilde{x})](\lambda)$, $s > [\mu_{(F,A)}(\tilde{y})](\lambda)$. Then

$$t > [\mu_{(F,A)}(\tilde{x})](\lambda) = inf\{u > 0 : u^{-1}\tilde{x}(\lambda) \in F(\lambda)\}.$$

Thus, there exists $u^\circ > 0$ such that $t > u^\circ$ and $u^{\circ -1}\tilde{x}(\lambda) \in F(\lambda)$. Since (F, A) is convex and absorbing soft set,

$$t^{-1}\tilde{x}(\lambda) = \left(t^{-1}u^\circ\right) u^{\circ -1}\tilde{x}(\lambda) + \left(1 - \left(t^{-1}u^\circ\right)\right)\theta \in F(\lambda).$$

Similarly, $s^{-1}\tilde{y}(\lambda) \in F(\lambda)$.

On one hand,

$$\begin{aligned}
&(t+s)^{-1}[\tilde{x}(\lambda) + \tilde{y}(\lambda)]\\
&= \left[(t+s)^{-1}\tilde{x}(\lambda)\right] + \left[(t+s)^{-1}\tilde{y}(\lambda)\right]\\
&= \left[(t+s)^{-1}tt^{-1}\tilde{x}(\lambda)\right] + \left[(t+s)^{-1}ss^{-1}\tilde{y}(\lambda)\right]\\
&= \tfrac{t}{(t+s)}t^{-1}\tilde{x}(\lambda) + \tfrac{s}{(t+s)}s^{-1}\tilde{y}(\lambda)\\
&= \tfrac{t}{(t+s)}t^{-1}\tilde{x}(\lambda) + \left[1 - \tfrac{t}{(t+s)}\right]s^{-1}\tilde{y}(\lambda)\\
&\in F(\lambda) \text{ [Since } t^{-1}\tilde{x}(\lambda),\ s^{-1}\tilde{y}(\lambda) \in F(\lambda) \text{ and } (F, A) \text{ is convex soft set].}
\end{aligned}$$

So, $\left[\mu_{(F,A)}(\tilde{x} + \tilde{y})\right](\lambda) \leq t + s$. Since $t > [\mu_{(F,A)}(\tilde{x})](\lambda)$ and $s > [\mu_{(F,A)}(\tilde{y})](\lambda)$ are arbitrary, we have

$$\left[\mu_{(F,A)}(\tilde{x} + \tilde{y})\right](\lambda) \leq \left[\mu_{(F,A)}(\tilde{x})\right](\lambda) + \left[\mu_{(F,A)}(\tilde{y})\right](\lambda).$$

Since $\lambda \in A$ is arbitrary, we have,

$$\mu_{(F,A)}(\tilde{x} + \tilde{y})\tilde{\leq}\mu_{(F,A)}(\tilde{x}) + \mu_{(F,A)}(\tilde{y}).$$

(2) Let $\hat{t}\tilde{\geq}\bar{0}$. Then $\hat{t}(\lambda)\geq 0$, for all $\lambda \in A$.

If $\lambda \in A$ such that $\hat{t}(\lambda)>0$, then for $s > 0$,

$$\begin{aligned}
&s \in \left[H_{(F,A)}(\hat{t}\tilde{x})\right](\lambda)\\
&\Leftrightarrow s^{-1}(\hat{t}\tilde{x})(\lambda) \in F(\lambda)\\
&\Leftrightarrow \left(s\hat{t}^{-1}(\lambda)\right)^{-1}\tilde{x}(\lambda) \in F(\lambda)\\
&\Leftrightarrow s\hat{t}^{-1}(\lambda) \in \left[H_{(F,A)}(\tilde{x})\right](\lambda)\\
&\Leftrightarrow s \in \hat{t}(\lambda)\left[H_{(F,A)}(\tilde{x})\right](\lambda).
\end{aligned}$$

Thus, $\left[H_{(F,A)}(\hat{t}\tilde{x})\right](\lambda) = \hat{t}(\lambda)\left[H_{(F,A)}(\tilde{x})\right](\lambda)$. So

$$\begin{aligned}
\left[\mu_{(F,A)}(\hat{t}\tilde{x})\right](\lambda) &= inf\left\{\left[H_{(F,A)}(\hat{t}\tilde{x})\right](\lambda)\right\}\\
&= inf\left\{\hat{t}(\lambda)\left[H_{(F,A)}(\tilde{x})\right](\lambda)\right\}\\
&= \hat{t}(\lambda)\left[\mu_{(F,A)}(\tilde{x})\right](\lambda).
\end{aligned}$$

Also, if $\hat{t}(\lambda) = 0$, then

$$\begin{aligned}
\left[\mu_{(F,A)}(\hat{t}\tilde{x})\right](\lambda) &= inf\left\{s > 0 : s^{-1}(\hat{t}\tilde{x})(\lambda) \in F(\lambda)\right\}\\
&= inf\left\{s > 0 : s^{-1}\theta \in F(\lambda)\right\}\\
&= 0 \text{ [Since } (F, A) \text{ is absorbing soft set and } \Theta\tilde{\in}(F, A)]\\
&= \hat{t}(\lambda)\left[\mu_{(F,A)}(\tilde{x})\right](\lambda).
\end{aligned}$$

So $\left[\mu_{(F,A)}(\hat{t}\tilde{x})\right](\lambda) = \hat{t}(\lambda)\left[\mu_{(F,A)}(\tilde{x})\right](\lambda)$, for $\hat{t}(\lambda)\geq 0$.

Since $\lambda \in A$ is arbitrary, we have,

$$\left[\mu_{(F,A)}(\hat{t}\tilde{x})\right](\lambda) = \hat{t}(\lambda)\left[\mu_{(F,A)}(\tilde{x})\right](\lambda),$$

for all $\lambda \in A$. Hence (2) is proved.

(3) Let $\hat{\alpha}$ be any scalar and $\lambda \in A$. Then either $\hat{\alpha}(\lambda) \neq 0$ or $\hat{\alpha}(\lambda) = 0$.

If $\hat{\alpha}(\lambda) = 0$, then

$$\begin{aligned}\left[\mu_{(F,A)}(\hat{\alpha}\tilde{x})\right](\lambda) &= inf\left\{s > 0 : s^{-1}(\hat{\alpha}\tilde{x})(\lambda) \in F(\lambda)\right\}\\ &= inf\left\{s > 0 : s^{-1}\theta \in F(\lambda)\right\}\\ &= 0 \text{ [Since } (F,A) \text{ is absorbing soft set and } \Theta\tilde{\in}(F,A)]\\ &= \hat{\alpha}(\lambda).\left[\mu_{(F,A)}(\tilde{x})\right](\lambda).\end{aligned}$$

If $\hat{\alpha}(\lambda) \neq 0$, then

$$\begin{aligned}\left[\mu_{(F,A)}(\hat{\alpha}\tilde{x})\right](\lambda) &= inf\left\{s > 0 : s^{-1}(\hat{\alpha}\tilde{x})(\lambda) \in F(\lambda)\right\}\\ &= inf\left\{s > 0 : s^{-1} \mid \hat{\alpha}(\lambda) \mid \frac{\hat{\alpha}(\lambda)}{|\hat{\alpha}(\lambda)|}\tilde{x}(\lambda) \in F(\lambda)\right\}\\ &= inf\left\{s > 0 : s^{-1} \mid \hat{\alpha}(\lambda) \mid \tilde{x}(\lambda) \in F(\lambda)\right\} \text{ [Since } \left|\frac{\hat{\alpha}(\lambda)}{|\hat{\alpha}(\lambda)|}\right| = 1\\ &\quad \text{and } (F,A) \text{ is balanced implies that } \frac{\hat{\alpha}(\lambda)}{|\hat{\alpha}(\lambda)|}F(\lambda) = F(\lambda)\text{]}\\ &= inf\left\{s > 0 : \left(s \mid \hat{\alpha}(\lambda) \mid^{-1}\right)^{-1}\tilde{x}(\lambda) \in F(\lambda)\right\}\\ &= \mid \hat{\alpha}(\lambda) \mid inf\left\{s > 0 : s^{-1}\tilde{x}(\lambda) \in F(\lambda)\right\}\\ &= \mid \hat{\alpha}(\lambda) \mid \left[\mu_{(F,A)}(\tilde{x})\right](\lambda).\end{aligned}$$

Thus $\left[\mu_{(F,A)}(\hat{\alpha}\tilde{x})\right](\lambda) = \hat{\alpha}(\lambda).\left[\mu_{(F,A)}(\tilde{x})\right](\lambda)$. Since $\lambda \in A$ is arbitrary, we have, $\mu_{(F,A)}(\hat{\alpha}\tilde{x}) = \mid \hat{\alpha} \mid \mu_{(F,A)}(\tilde{x})$. So (3) is proved.

(4) Let $\xi \in B_\lambda$. Then there exists $\tilde{x}\tilde{\in}(\tilde{X}, A)$ such that $\tilde{x}(\lambda) = \xi$ and $\left[\mu_{(F,A)}(\tilde{x})\right](\lambda) < 1$. Thus, there exists $t > 0$ such that $\left[\mu_{(F,A)}(\tilde{x})\right](\lambda) < t < 1$. So approaching as in (1), $t^{-1}\tilde{x}(\lambda) \in F(\lambda)$. Since $1 > t$ and $F(\lambda)$ is absorbing, $1.\tilde{x}(\lambda) \in F(\lambda)$. Hence $\tilde{x}(\lambda) = \xi \in F(\lambda)$. Therefore $B_\lambda \subset F(\lambda)$.

Next let $\eta \in F(\lambda)$. Then there exists $\tilde{y}\tilde{\in}(\tilde{X}, A)$ such that $\tilde{y}(\lambda) = \eta$ and $\tilde{y}(\lambda) \in F(\lambda)$. Thus, $1.\tilde{y}(\lambda) \in F(\lambda)$ and this imlpies that $\left[\mu_{(F,A)}(\tilde{y})\right](\lambda) \leq 1$. So $\tilde{y}(\lambda) \in C_\lambda$. Hence $F(\lambda) \subset C_\lambda$. Since $\lambda \in A$ is arbitrary, $B_\lambda \subset F(\lambda) \subset C_\lambda$, for each $\lambda \in A$. Therefore if we define soft sets (B, A) and (C, A) such that $(B, A)(\lambda) = B_\lambda$ and $(C, A)(\lambda) = C_\lambda$, for each $\lambda \in A$, then $(B, A)\tilde{\subseteq}(F, A)\tilde{\subseteq}(C, A)$.

Again, $\tilde{x}\tilde{\in}(B, A)$ implies that for all $\lambda \in A$, $\tilde{x}(\lambda) \in B_\lambda = B(\lambda)$. Then, by definition, $\mu_{(F,A)}(\tilde{x})\tilde{<}\bar{1}$.

Similarly, for any $\tilde{x}\tilde{\in}(C, A), \mu_{(F,A)}(\tilde{x})\tilde{\leq}\bar{1}$. □

Proposition 5.9. *Let $(\tilde{X}, A)$ be an absolute soft vector space, $(F, A) \in S(\tilde{X})$ be a convex absorbing soft set and $\mu_{(F,A)}$ be the soft Minkowski functional associated with (F, A). Then for any $\hat{r}\tilde{>}\bar{0}$, $(W, A) = \hat{r} \cdot (V, A)$, where $(W, A) = SS\{\tilde{x} : \mu_{(F,A)}(\tilde{x})\tilde{<}\hat{r}\}$ and $(V, A) = SS\{\tilde{x} : \mu_{(F,A)}(\tilde{x})\tilde{<}\bar{1}\}$.*

Proof. Choose $\lambda \in A$ and $\xi \in W(\lambda)$. Then there exists $\tilde{x}\tilde{\in}(W, A)$ such that $\mu_{(F,A)}(\tilde{x})\tilde{<}\hat{r}$ and $\tilde{x}(\lambda) = \xi$. Thus $\mu_{(F,A)}(\frac{\hat{r}\cdot\tilde{x}}{\hat{r}})\tilde{<}\hat{r}$. By Proposition 5.8 (2), $\hat{r}\mu_{(F,A)}(\frac{\tilde{x}}{\hat{r}})\tilde{<}\hat{r}$. So $\mu_{(F,A)}(\frac{\tilde{x}}{\hat{r}})\tilde{<}\frac{\hat{r}}{\hat{r}} = \bar{1}$. Hence $\frac{\tilde{x}}{\hat{r}}\tilde{\in}(V, A)$, i.e., $\tilde{x}\tilde{\in}\hat{r} \cdot (V, A)$. Therefore $\tilde{x}(\lambda) \in \hat{r}(\lambda)V(\lambda)$, for all $\lambda \in A$. In particular, $\xi \in \hat{r}(\lambda)V(\lambda)$ and thus $W(\lambda) \subseteq \hat{r}(\lambda)V(\lambda)$.
Since $\lambda \in A$ is arbitrary, $(W, A)\tilde{\subseteq}\hat{r} \cdot (V, A)$.

Reverse inclusion is similar. Hence, $(W, A) = \hat{r} \cdot (V, A)$. □

Definition 5.10. Let $(\tilde{X}, \tau, A)$ be a soft topological vector space. Then $(\tilde{X}, \tau, A)$ is said to be soft normable, if there exists a soft norm on $(\tilde{X}, A)$ such that the soft topology on $(\tilde{X}, A)$ is the soft metric topology induced from the soft norm. In that case τ is said to be the soft topology induced by the soft norm.

Definition 5.11. A soft topological vector space $(\tilde{X}, \tau, A)$ is said to satisfy the property $(*)$, if for any $\hat{r}(\tilde{>}\bar{0})\tilde{\in}(\tilde{K}, A)$ and for any bounded soft nbd (V, A) of Θ, $\Cap \hat{r}(V, A) = \Theta$.

Proposition 5.12. *A soft topological vector space $(\tilde{X}, \tau, A)$ with the property $(*)$ as in Definition 5.11, is soft normable iff there is a bounded convex soft nbd of Θ.*

Proof. Let $(\tilde{X}, \tau, A)$ be a soft topological vector space which satisfies the property $(*)$ and normable. Then there exists a soft norm $\| \cdot \|$ on $(\tilde{X}, A)$ such that the induced soft topology from $\| \cdot \|$ coincides with the topology of the soft topological vector space $(\tilde{X}, \tau, A)$. Then the soft open ball $SS(B(\Theta, \bar{1}))$ will be a bounded soft open set. By Proposition 2.36, $SS(B(\Theta, \bar{1}))$ convex soft set containing Θ. Since $SS(B(\Theta, \bar{1}))$ is soft open, it is a nbd of Θ.

Conversely, suppose that $(\tilde{X}, \tau, A)$ is a soft topological vector space with the property that there is a bounded convex soft nbd (U, A) of Θ. By Proposition 4.7, (U, A) contains a balanced convex soft nbd (V, A) of Θ. Clearly (V, A) is bounded as (U, A) is so. Then the soft Minkowski functional $\mu_{(V,A)}$ associated with (V, A) is a soft seminorm by Proposition 5.8. Since $\Cap \hat{r}(V, A) = \Theta$, if $\tilde{x} \neq \Theta$, then there exists $\hat{r}_0 \tilde{>} \bar{0}$ such that $\tilde{x} \tilde{\notin} \hat{r}_0(V, A)$, i.e., $\hat{r}_0^{-1}\tilde{x} \tilde{\notin} (V, A)$.

Now $\mu_{(V,A)}(\hat{r}_0^{-1}\tilde{x}) \tilde{<} \bar{1}$ implies $\hat{r}_0^{-1}\tilde{x} \tilde{\in} (V, A)$. Then there exists $\lambda \in A$ such that $\left[\mu_{(V,A)}(\hat{r}_0^{-1}\tilde{x})\right](\lambda) \geq 1$. Thus, $\hat{r}_0^{-1}(\lambda)\left[\mu_{(V,A)}(\tilde{x})\right](\lambda) \geq 1$. So $\left[\mu_{(V,A)}(\tilde{x})\right](\lambda) \geq \hat{r}_0(\lambda) > 0$. Hence $\tilde{x} \neq \Theta$ implies that $\mu_{(V,A)}(\tilde{x}) \neq \bar{0}$. This shows that $\mu_{(V,A)}$ is a soft norm on $(\tilde{X}, A)$.

Let $\mu_{(V,A)}(\tilde{x}) = \| \tilde{x} \|$ and let $(W, A) = SS\{\tilde{x} : \mu_{(V,A)}(\tilde{x}) \tilde{<} \bar{1}\}$. Taking $\tilde{x}$ be such that $\mu_{(V,A)}(\tilde{x}) \tilde{<} \bar{1}$. Then $\tilde{x} \tilde{\in} (V, A)$. Thus $\{\tilde{x} : \mu_{(V,A)}(\tilde{x}) \tilde{<} \bar{1}\} \subseteq SE(V, A)$. So $SS\{\tilde{x} : \mu_{(V,A)}(\tilde{x}) \tilde{<} \bar{1}\} \tilde{\subseteq} SS(SE(V, A))$. Hence $(W, A) \tilde{\subseteq} (V, A)$.

Conversely, suppose $\tilde{x} \tilde{\in} (V, A)$. By continuity of scalar multiplication, we can say that there exists $\hat{t} \tilde{>} \bar{0}$ such that $\bar{1} \tilde{>} \hat{t}$ and $\hat{t}^{-1}\tilde{x} \tilde{\in} (V, A)$. Then, by Proposition 5.8 (4), $\mu_{(V,A)}(\hat{t}^{-1}\tilde{x}) \tilde{\leq} \bar{1}$. Thus, $\hat{t}^{-1}\mu_{(V,A)}(\tilde{x}) \tilde{\leq} \bar{1}$. So $\mu_{(V,A)}(\tilde{x}) \tilde{\leq} \hat{t} \tilde{<} \bar{1}$. Hence $\tilde{x} \tilde{\in} (W, A)$. Therefore $(V, A) = (W, A)$.

Further, since (V, A) is bounded, $\{\hat{r}(V, A) : \hat{r} \tilde{>} \bar{0}\}$ will form a soft nbd base of Θ. Then, by Proposition 5.9,

$$\hat{r}(V, A) = \hat{r}(W, A) = SS\{\tilde{x} : \mu_{(V,A)}(\tilde{x}) \tilde{<} \hat{r}\} = SS\{\tilde{x} : \| \tilde{x} \| \tilde{<} \hat{r}\}.$$

Thus the soft topology induced by $\| \, . \, \|$ coincides with the soft topology of the soft topological vector space $(\tilde{X}, \tau, A)$. □

6. Conclusion

In this paper, we introduce soft topological vector space, soft semi-norm, soft Minkowski functional and study some basic properties of those concepts. Also, we study the problem of soft normability of soft topological vector spaces. There is an ample scope for further research on many problems such as the problems of finite dimensionality, metrizability, open mapping theorem, closed graph theorem etc. in soft topological vector spaces.

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