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Intuitionistic fuzzy topological spaces: Categorical concepts

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Abstract. Coker has introduced the concept of intuitionistic fuzzy topological spaces (X, T) where X is an ordinary set and T is a family of intuitionistic fuzzy sets in X satisfying some axioms. In this paper I introduce the fuzzy version of some universal constructions, namely, fuzzy products, fuzzy equalizers and fuzzy pullbacks for intuitionistic fuzzy topological spaces. Some results concerning all such universal objects are discussed. Finally, functors preserving equalizers and pullbacks are investigated.

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1. Introduction

Zadeh [21] introduced the notion of a fuzzy set as a function from the given set to the unit interval. Chang [6] defined the fuzzy topological spaces. The collection of all fuzzy topological spaces and fuzzy continuous functions form a category. Some applications of category theory in fuzzy topology are presented. Eklund established some important category theoretic properties for categories of fuzzy topological spaces [10]. Alderton and Castellini [2] had used Salbany-type closure operator to characterize the epimorphisms in three categories of separated fuzzy topological spaces. In [17], characterization of the epimorphisms in the category of Hausdorff fuzzy topological spaces was introduced. Behera [5] introduced the concepts of fuzzy equalizers , fuzzy pullbacks and their duals for fuzzy topological spaces in the sense of Chang. It is shown in [9] that various approaches to fuzzy topological spaces can be extended to an abstract category. Geetha S. [11] introduced a new category FTOP, the object is I - fuzzy topological spaces (X, μ, F) where X is an ordinary set, μ is a fuzzy set in X and F is a family of fuzzy sets in X satisfying some axioms. Hamouda E. [12, 13] investigated some categorical concepts for I - fuzzy topological spaces in the sense of Geetha. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [3]. Coker [7] introduced intuitionistic fuzzy topological spaces. Numerous topological concepts in general topology have been generalized to intuitionistic fuzzy settings[4, 8, 15, 16, 18, 19, 20]. In this paper we introduce the universal constructions, namely, fuzzy products, fuzzy equalizers and fuzzy pullbacks for Intuitionistic fuzzy topological spaces. Also we discuss some results concerning all such universal objects. Finally, functors preserving equalizers and pullbacks are investigated.

2. Preliminaries

As usual I denotes the closed unit interval $[0, 1]$. A fuzzy set μ in a set X is a function on X into the closed unit interval I of the real line. If $f : X \to Y$ is a function and μ, γ are fuzzy sets in X, Y respectively, then the fuzzy set $f^{-1}(\gamma)$ in X is defined by $f^{-1}(\gamma) = \gamma \circ f$ and $f(\mu) : Y \to I$ is defined as follows [7]:

$$f(\mu)(y) = \begin{cases} \bigvee\{\mu(x) : x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \phi, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.1 ([3]). Let X be a non-empty fixed set. An intuitionistic fuzzy set (IF-Set for short) A is an object having the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\},$$

where the functions $\mu : X \longrightarrow I$ and $\mu : X \longrightarrow I$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \geq 1$, for each $x \in X$.

In particular, $\mathbf{0}_X$ and $\mathbf{1}_X$ denote the intuitionistic fuzzy empty set and the intuitionistic fuzzy whole set in a set X defined by $\mathbf{0}_X = (0, 1)$ and $\mathbf{1}_X = (1, 0)$ for each $x \in X$ respectively.

Definition 2.2 ([3]). Let X and Y be two nonempty sets and $f : X \longrightarrow Y$ be a function.

(i) If $B = \{< x, \mu_B(x), \nu_B(x) >: x \in X\}$ is an IF-Set in Y ,then the preimage of B under f denoted by $f^{-1}(B)$ is the IF-Set in X defined by

$$f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) >: x \in X\},$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ and $f^{-1}(\nu_B)(x) = \nu_B(f(x))$.

(ii) If $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ is an IF-Set in X ,then the image of A under f ,denoted by $f(A)$ is the IF-Set in Y defined by

$$f(A) = \{< y, f(\mu_A)(y), (1 - f(1 - \nu_A))(y) >: y \in Y\},$$

where

$$f(\mu_A(y) = \begin{cases} \bigvee_{x \in f^{-1}(y)} \quad \nu_A(x), & \text{if } f^{-1}(y) \neq \phi, \\ 0, & \text{otherwise.} \end{cases}$$

$$(1 - f(1 - \nu_A))(y) = \begin{cases} \bigwedge_{x \in f^{-1}(y)} \quad \nu_A(x), & \text{if } f^{-1}(y) \neq \phi, \\ 1, & \text{otherwise.} \end{cases}$$

Definition 2.3 ([3]). An intuitionistic fuzzy topology on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $\mathbf{0}_X$, $\mathbf{1}_X \in T$.

(ii) If $A_i \in T, i \in J$, then $\bigcup A_i \in T$.

(iii) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

The pair (X, T) is called an intuitionistic fuzzy topological space (IFTS for short). The members of T are called intuitionistic fuzzy open sets and their complements are called intuitionistic fuzzy closed sets..

Definition 2.4 ([3]). Let (X_1, T_1) and (X_2, T_2) be two intuitionistic fuzzy topological spaces. A function $f : (X_1, T_1) \longrightarrow (X_2, T_2)$ is called intuitionistic fuzzy continuous (IF- continuous) iff the preimage of each intuitionistic fuzzy set in T_2 is an intuitionistic fuzzy set in T_1.

The notion **IFTOP** will denote the category of intuitionistic fuzzy topological spaces and IF- continuous functions. We shall use the categorical terminology of [1]. For more information about the category **IFTOP**, the reader could consult [11].

3. Main results

In this section we discuss fuzzy products, fuzzy equalizers and fuzzy pullbacks of Intuitionistic fuzzy topological spaces. The word "map" will always mean a continuous function, but the word "function" does not imply continuity.

The concept of intuitionistic fuzzy product has introduced in [8, 20]. The following theorem emphasizes the universal property of intuitionistic fuzzy product in **IFTOP**. Throughout this section, J is referred to as the index set and the word intuitionistic fuzzy spaces means intuitionistic fuzzy topological spaces.

Theorem 3.1. *For given intuitionistic fuzzy spaces $(X_i, T_i), i \in J$, the following hold:*

(1) *There exists an intuitionistic fuzzy space (P, T) and IF-maps $p_i : (P, T) \longrightarrow (X_i, T_i)$ for each $i \in J$.*

(2) *For any intuitionistic fuzzy space (X, H) with IF-maps $\phi_i : (X, H) \longrightarrow (X_i, T_i)$, there is a unique IF- map $\theta : (X, H) \longrightarrow (P, T)$ such that $p_i \circ \theta = \phi_i$ for each $i \in J$.*

Proof. (1) For the given intuitionistic fuzzy spaces (X_i, T_i), we consider their intuitionistic fuzzy product $\prod_{i\in J}(X_i, T_i)$ to be the intuitionistic fuzzy space (P, T), where $P = \prod_{i\in J} X_i$ is the usual set product and T is generated by the subbase $\beta = \{p_i^{-1}(U_i) : U_i \in T_i\}$ for each $i \in J$ [8]. The intuitionistic fuzzy topology on $P = \prod_{i\in J} X_i$ is the weakest intuitionistic fuzzy topology so that the projections $p_i : (P, T) \longrightarrow (X_i, T_i)$ are IF- maps for each $i \in J$ [8].

(2) Define $\theta : (X, H) \longrightarrow (P, T)$ by $\theta(x) = (\phi_i(x))_{i\in J}$ for all $x \in X, i \in J$. With the definition of θ, we have $p_i \circ \theta = \phi_i$. Since ϕ_i is an IF- map for each $i \in J$, for any intuitionistic fuzzy set$V = p_i^{-1}(U_i)$ in T, $\theta^{-1}(V) = \theta^{-1}(p_i^{-1}(U_i)) = \phi_i^{-1}(U_i)$ is an intuitionistic fuzzy set in H . Then θ is IF-continuous. This shows the existence of the universal property. Let $\psi : (X, H) \longrightarrow (P, T)$ be any IF- map with the property

that $p_i \circ \psi = \phi_i$ for each $i \in J$. Then the i-th coordinate of $\psi(x)$ is ϕ_i for each $x \in P$. Thus $\psi = \theta$. This shows the uniqueness of the universal property. □

The pair (P, p_i) in the above theorem is called the intuitionistic fuzzy product of the family $\{(X_i, T_i), i \in J\}$ of IFTSs.

Proposition 3.2. *The intuitionistic fuzzy product of the family $\{(X_i, T_i), i \in J\}$ is unique up to IF- homeomorphism.*

Proof. Let $P = \prod_{i\in J} X_i$ and $Q = \prod_{i\in J} X_i$ be two fuzzy product of spaces (X_i, T_i) with projections $p_i : (P, T) \longrightarrow (X_i, T_i)$ and $q_i : (Q, G) \longrightarrow (X_i, T_i)$ respectively. Then the universal property of the intuitionistic fuzzy product (P, T) implies that there is a unique IF- map $\theta : (Q, G) \longrightarrow (P, T)$ such that $p_i \circ \theta = q_i$ for each $i \in J$. In a similar way, there exists a unique IF- map $\varphi : (P, T) \longrightarrow (Q, G)$ such that $q_i \circ \varphi = p_i$ for each $i \in J$. Thus we have $p_i = q_i \circ \varphi = p_i \circ \theta \circ \varphi = p_i \circ id_P$ for each $i \in J$. From the uniqueness condition of Theorem 3.1, it follows that $\theta \circ \varphi = id_P$. Similarly, we get $\varphi \circ \theta = id_Q$. So, P and Q are IF- homeomorphic . □

From now on, we shall use the symbol (X, T_X) for the intuitionistic fuzzy topological space (X, T).

The following theorem deals with another universal construction in **IFTOP**, namely the intuitionistic fuzzy equalizer.

Theorem 3.3. *Let $f, g : (X, T_X) \longrightarrow (Y, T_Y)$ be IF- maps. Then*

(1) *There exists an intuitionistic fuzzy space (E, T_E) and an IF- map $e : (E, T_E) \longrightarrow (X, T_X)$ such that $f \circ e = g \circ e$.*

(2) *For any intuitionistic fuzzy space (A, T_A) with an IF- map $\phi : (A, T_A) \longrightarrow (X, T_X)$ satisfying $f \circ \phi = g \circ \phi$, there exists a unique IF- map $h : (A, T_A) \longrightarrow (E, T_E)$ such that $\phi = e \circ h$.*

Proof. (1) We consider (E, T_E) as a subspace of (X, T_X), where $E = \{x \in X : f(x) = g(x)\}$ and $T_E = \{V|_E : V \in T_X\}$, where $V|_E$ is the intutionistic fuzzy set $V \in T_X$ restricted to $E \subset X$. Define $e : (E, T_E) \longrightarrow (X, T_X)$ by $e(x) = x$, $x \in E$. Then it is clear that $f \circ e = g \circ e$ and e is an IF- map, since it equals the identity map id_X restricted to E.
(2)We then have to verify the universal property. For any intuitionistic fuzzy space (A, T_A), we define $h : (A, T_A) \longrightarrow (E, T_E)$ by $h(a) = \phi(a), a \in A$. Since $f(\phi(a)) = g(\phi(a))$, we have $\phi(a) \in E$. Thus, we get $\phi = e \circ h$, since $e(h(a)) = h(a) = \phi(a), a \in A$.

Now, we have to show that h is an IF- map. With the definition of T_E, we set $U = V|_E, V \in T_X$. Since ϕ is an IF- map, we get

$$(h^{-1}(U))(a) = U(h(a)) = V(h(a)) = V(\phi(a)) = (\phi^{-1}(V))(a).$$

So $h^{-1}(U) = \phi^{-1}(V) \in T_A$, as desired. The uniqueness of h comes directly from the definition. □

The pair (E, e) in theorem 3.3 is called the intuitionistic fuzzy equalizer of the IF-maps $f, g : (X, T_X) \longrightarrow (Y, T_Y)$.

We now define intutionistic fuzzy pullbacks. Suppose we have IF- maps $f : (X, T_X) \longrightarrow (Z, T_Z)$ and $g : (Y, T_Y) \longrightarrow (Z, T_Z)$. Then the intuitionistic fuzzy pullback of these IF- maps is an IFTS (B, T_B) together with IF- maps $\alpha : (B, T_B) \longrightarrow (X, T_X)$ and $\beta : (B, T_B) \longrightarrow (Y, T_Y)$ such that $f \circ \alpha = g \circ \beta$, and such that the following universal property holds: Suppose that (C, T_C) is an IFTS and that $\alpha' : (C, T_C) \longrightarrow (X, T_X)$ and $\beta' : (C, T_C) \longrightarrow (Y, T_Y)$ are IF- maps with $f \circ \alpha' = g \circ \beta'$. Then there is a unique IF- map $\phi : (C, T_C) \longrightarrow (B, T_B)$ with $\alpha \circ \phi = \alpha'$ and $\beta \circ \phi = \beta'$. We then call (B, α, β) the intuitionistic fuzzy pullback of f and g.

In the following theorem, we show that intuitionistic fuzzy pullbacks exist in the category **IFTOP** by constructing them as intuitionistic fuzzy products.

Theorem 3.4. *Let $f : (X, T_X) \longrightarrow (Z, T_Z)$ and $g : (Y, T_Y) \longrightarrow (Z, T_Z)$ be any IF- maps. Then there exists an intuitionistic fuzzy pullback (B, α, β).*

Proof. We consider $B = \{(x, y) \in X \times Y : f(x) = g(y)\}$ as a subset of $X \times Y$, and T_B is generated by the base $\intercal = \{(U_1 \times U_2)|_B : U_1 \in T_X, U_2 \in T_Y\}$. Define the projection maps $\alpha : (B, T_B) \longrightarrow (X, T_X)$ and $\beta : (B, T_B) \longrightarrow (X, T_X)$ by $\alpha(x, y) = x$ and $\beta(x, y) = y$ for all $(x, y) \in B$, clearly α and β are IF- maps [20]. It is obvious from the definition of B that $f \circ \alpha = g \circ \beta$.

To show that B satisfies the universal property, suppose that (C, T_C) is an intuitionistic fuzzy space with IF- maps $\lambda : (C, T_C) \longrightarrow (X, T_X)$ and $\delta : (C, T_C) \longrightarrow (Y, T_Y)$ with $f \circ \lambda = g \circ \delta$. Define $\phi : (C, T_C) \longrightarrow (B, T_B)$ by $\phi(c) = (\lambda(c), \delta(c))$. Then ϕ maps C into B, since $f(\lambda(c)) = g(\delta(c))$. Also, it is clear that $\alpha \circ \phi = \lambda$ and $\beta \circ \phi = \delta$. By the uniqueness property of the IF- projection maps (Theorem 3.1) , ϕ is unique. Finally, we show that ϕ is intuitionistic fuzzy continuous.

Let U be an intuitionistic fuzzy open set in T_B, that is, $U = (U_1 \times U_2)|_B$ for all $U_1 \in T_X, U_2 \in T_Y$. Since λ and δ are IF- maps,

$$\begin{aligned} \phi^{-1}(U) &= \phi^{-1}(U_1 \times U_2) \\ &= \phi^{-1}(\alpha^{-1}(U_1) \cap \beta^{-1}(U_2)) \\ &= \phi^{-1}(\alpha^{-1}(U_1)) \cap \phi^{-1}(\beta^{-1}(U_2)) \\ &= \lambda^{-1}(U_1) \cap \delta^{-1}(U_2). \end{aligned}$$

Then $\phi^{-1}(U) = \lambda^{-1}(U_1) \cap \delta^{-1}(U_2)$ belongs to T_C. Thus, ϕ is indeed an IF- map. □

Example 3.5. Given the diagram of IF- maps

$$(X, T_X) \longrightarrow (\{\star\}, T_{\{\star\}}) \longleftarrow (Y, T_Y),$$

where $(\{\star\}, T_{\{\star\}})$ is a terminal object in **IFTOP**. Then the intuitionistic fuzzy pullback (B, T_B) is the intuitionistic fuzzy space $(X \times Y, T_{X \times Y})$.

Proposition 3.6. *Given any IF- maps $f : (X, T_X) \longrightarrow (Z, T_Z)$ and $g : (Y, T_Y) \longrightarrow (Z, T_Z)$, the intuitionistic fuzzy pullback (B, α, β) is unique up to IF- homeomorphism.*

Proof. Suppose (W, T_W), with IF- maps $f' : (W, T_W) \longrightarrow (X, T_X)$ and $g' : (W, T_W) \longrightarrow (Y, T_Y)$, is another intuitionistic fuzzy pullback. Take $(C, T_C) = (W, T_W)$. Then we find an IF- map $\phi : (W, T_W) \longrightarrow (B, T_B)$ such that $\alpha \circ \phi = f'$ and $\beta \circ \phi = g'$. By reversing the roles of (W, T_W) and (B, T_B), we find an IF- map $\phi' : (B, T_B) \longrightarrow (W, T_W)$ such that $f' \circ \phi' = \alpha$ and $g' \circ \phi' = \beta$. Thus $\alpha \circ \phi \circ \phi' = \alpha$ and $\beta \circ \phi \circ \phi' = \beta$.

Now take $(C, T_C) = (B, T_B)$,$\alpha = \alpha'$ and $\beta = \beta'$. Then we have two IF- maps, $\phi \circ \phi' : (B, T_B) \longrightarrow (B, T_B)$ and id_B that satisfy the conditions of fuzzy pullback. By the uniqueness, we have $\phi \circ \phi' = id_B$. Similarly, $\phi' \circ \phi = id_W$. So ϕ and ϕ' are inverse IF- homeomorphisms. □

In the following lemma, we investigate the relationship among the universal constructions mentioned above.

Lemma 3.7. . *In* ***IFTOP****, intuitionistic fuzzy pullbacks exist if and only if intuitionistic fuzzy equalizers exist.*

Proof. Suppose that intuitionistic fuzzy pullbacks exist. Given any two IF- maps $f, g : (X, T_X) \longrightarrow (Y, T_Y)$, we write

$$(f, g) : (X, T_X) \longrightarrow (Y \times Y, T_{Y\times Y}),\ x \longmapsto (f(x), g(x))$$

and

$$\triangle : (Y, T_Y) \longrightarrow (Y \times Y, T_{Y\times Y}), y \longmapsto (y, y).$$

Now we show that (f, g) and $\triangle$ are IF- maps. Let U be an intuitionistic fuzzy open set in $T_{Y\times Y}$, that is, $U = U_1 \times U_2$ for all $U_1, U_2 \in T_Y$. Since f, g are IF-maps,

$$(f, g)^{-1}(U) = (f, g)^{-1}(U_1 \times U_2) = [f^{-1}(U_1) \cap g^{-1}(U_2)] \in T_X.$$

Then, (f, g) is an IF- map.

In a similar argument, $\triangle$ is an IF- map. Let (B, T_B) be the intuitiojnistic fuzzy pullback of (f, g) and $\triangle$ so that $\alpha : (B, T_B) \longrightarrow (X, T_X)$ and $\beta : (B, T_B) \longrightarrow (Y, T_Y)$ are IF- maps with $(f, g) \circ \alpha = \triangle \circ \beta$. Then (B, α) is intuitionistic fuzzy equalizer of the IF-maps $f, g : (X, T_X) \longrightarrow (Y, T_Y)$. Indeed, $p_1 \circ (f, g) = f$, $p_2 \circ (f, g) = g$, $p_1 \circ \triangle = id_Y$ and $p_2 \circ \triangle = id_Y$, where $p_i : Y \times Y \longrightarrow Y$ are the projection maps, $i = 1, 2$. Thus $f \circ \alpha = p_1 \circ (f, g) \circ \alpha = p_1 \circ \triangle \circ \beta = id_Y \circ \beta = \beta$ and $g \circ \alpha = p_2 \circ (f, g) \circ \alpha = p_2 \circ \triangle \circ \beta = id_Y \circ \beta = \beta$. So $f \circ \alpha = g \circ \alpha$. For any IFTS (C, T_C) and IF-map $\theta(C, T_C) \longrightarrow (X, T_X)$ with $f \circ \theta = g \circ \theta$,

$$p_1 \circ (f, g) \circ \theta = f \circ \theta = g \circ \theta = id_Y \circ g \circ \theta = p_1 \circ \triangle \circ g \circ \theta,$$

$$p_2 \circ (f, g) \circ \theta = g \circ \theta = id_Y \circ g \circ \theta = p_2 \circ \triangle \circ g \circ \theta.$$

By Theorem 3.1, $(f, g) \circ \theta = \triangle \circ g \circ \theta$. Since (B, T_B) is the intuitionistic fuzzy pullback of (f, g) and $\triangle$, there exists an IF- map $\phi : (C, T_C) \longrightarrow (B, T_B)$ such that $\theta = \alpha \circ \phi$. So the universal property is satisfied.

Conversely, Consider the arbitrary IF- maps $f : (X, T_X) \longrightarrow (Z, T_Z)$ and $g : (Y, T_Y) \longrightarrow (Z, T_Z)$. Let (E, T_E) together with an IF- map $e : (E, T_E) \longrightarrow (X \times Y, T_{X\times Y})$ be the intuitionistic fuzzy equalizer of IF- maps $f \circ p_1, g \circ p_2 : (X \times Y, T_{X\times Y}) \longrightarrow (Z, T_Z)$ such that $f \circ p_1 \circ e = g \circ p_2 \circ e$.

Now we prove that (E, T_E) together with IF- maps $p_1 \circ e : (E, T_E) \longrightarrow (X, T_X)$ and $p_2 \circ e : (E, T_E) \longrightarrow (Y, T_Y)$ is the intuitionistic fuzzy pullback of the IF- maps f and g. Let (C, T_C) be an intuitionistic fuzzy space with IF- maps $\alpha : (C, T_C) \longrightarrow (X, T_X)$ and $\beta(C, T_C) \longrightarrow (Y, T_Y)$ so that $f \circ \alpha = g \circ \beta$. Then by Theorem 3.1, there exists a unique IF- map $\theta : (C, T_C) \longrightarrow (X \times Y, T_{X\times Y})$ such that $p_1 \circ \theta = \alpha, p_2 \circ \theta = \beta$. Since (E, T_E) is the intuitionistic fuzzy equalizer of IF- maps $f \circ p_1, g \circ p_2 : (X \times Y, T_{X\times Y}) \longrightarrow (Z, T_Z)$ and $f \circ p_1 \circ \theta = f \circ \alpha = g \circ \beta = g \circ p_2 \circ \theta)$, Theorem 3.3 implies that there exists a unique IF- map $\phi : (C, T_C) \longrightarrow (E, T_E)$ such that $e \circ \phi = \theta$. Thus $p_1 \circ e \circ \phi = p_1 \circ \theta = \alpha, p_2 \circ e \circ \phi = p_2 \circ \theta = \beta$. So the desired intuitionistic fuzzy pullback is obtained. □

Let **CTOP** be the category of all fuzzy topological spaces in the sense of Chang and fuzzy continuous functions.It is shown that **CTOP** has equalizers and pullbaks [5].

Recall [14] that there are functors $G_1, G_2 : \mathbf{IFTOP} \longrightarrow \mathbf{CTOP}$ defined by

$$G_1(X, T) = (X, G_1(T))\, and\, G_1(f) = f,$$

where $G_1(T) = \{\mu_A : A = (\mu_A, \nu_A) \in T\}$.

$$G_2(X, T) = (X, G_2(T))\, and\, G_2(f) = f,$$

where $G_2(T) = \{1 - \nu_A : A = (\mu_A, \nu_A) \in T\}$.

Proposition 3.8. *The functors G_1, G_2 : **IFTOP** $\longrightarrow$ **CTOP** preserve equalizers and pullbacks*

Proof. Immediate from the definition of G_1, G_2. □

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