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Completely β generalized continuous mappings in intuitionistic fuzzy topological spaces

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Abstract. In this paper we introduce intuitionistic fuzzy completely β generalized continuous mappings. We investigate some of its properties. Also we provide some characterization of intuitionistic fuzzy completely β generalized continuous mappings.

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Corresponding Author: Saranya (saranya20maths@gmail.com)

1. Introduction

Atanassov [1] introduced the idea of intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. M. Saranya and D. Jayanthi [9] introduced intuitionistic fuzzy β generalized continuous mappings. In this paper we introduce the notion of intuitionistic fuzzy completely β generalized continuous mappings and studied some of their properties. We provide some characterizations of intuitionistic fuzzy completely β generalized continuous mappings.

2. Preliminaries

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$)and the degree of non-membership (namely $\nu_A(x)$)of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$

Definition 2.2 ([1])**.** Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$,

(ii) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(v) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 ([3])**.** An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,

(iii) $\bigcup G_i \in \tau$, for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 ([7])**.** An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy β generalized closed set (IFβGCS, for short), if βcl(A) $\subseteq U$, whenever $A \subseteq U$ and U is an IFβOS in (X, τ).

Definition 2.5 ([8])**.** If every IFβGCS in (X, τ) is an IFβCS in (X, τ), then the space can be called as an intuitionistic fuzzy β generalized $T_{1/2}$ space (IF$\beta_g T_{1/2}$, in short).

Definition 2.6 ([9])**.** A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy β generalized continuous (IFβG continuous, for short) mapping, if $f^{-1}(V)$ is an IFβGCS in (X, τ), for every IFCS V of (Y, σ).

Definition 2.7 ([5])**.** Let X and Y be two IFTSs. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle : y \in Y\}$ be IFSs of X and Y, respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by:

$$(A \times B)(x, y) = \langle (x, y), min(\mu_A(x), \mu_B(y)), max(\nu_A(x), \nu_B(y))\rangle.$$

Definition 2.8 ([5])**.** Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by:

$$(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2)), \text{ for every } (x_1, x_2) \in X_1 \times X_2 .$$

Definition 2.9 ([3]). Let X and Y be two non empty sets and $f : X \to Y$ be a function. If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the preimage of B under f is denoted and defined by:

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle : x \in X\},$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$, for every $x \in X$.

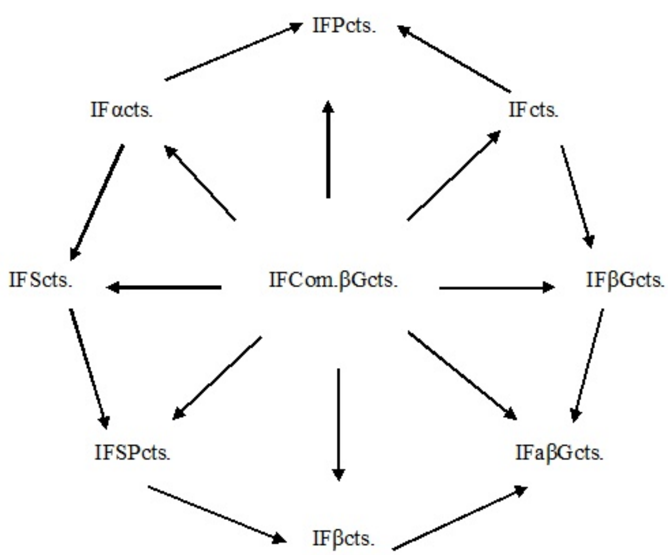
3. Major Section

In this section we introduce intuitionistic fuzzy completely β generalized continuous mappings and study some of their properties.

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an intuitionistic fuzzy completely β generalized continuous (IF completely βG continuous mapping, for short) mapping, if $f^{-1}(V)$ is an IFRCS in X, for every IFβGCS V in Y.

We use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts.' means continuous and IFcom.βGcts. means IF completely βG continuous.



The reverse implications are not true in general in the above diagram. This can be seen from the following examples.

Example 3.2. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are

IFTs on X and Y, respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then,

IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: either $\mu_a < 0.6$ or $\mu_b < 0.8$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: either $\mu_a > 0.4$ or $\mu_b > 0.2$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u < 0.6$ or $\mu_v < 0.8$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u > 0.4$ or $\mu_v > 0.2$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Then f is an IF continuous mapping, IFS continuous mapping, IFP continuous mapping, IFSP continuous mapping, IFβ continuous mapping and IFα continuous mapping but not an IF completely βG continuous mapping, since G_2^c is an IFβGCS in Y, but $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$ is not an IFRCS in X. Since $cl(int(f^{-1}(G_2^c))) = cl(0_\sim) = 0_\sim \neq f^{-1}(G_2^c)$. Thus f is not an IF completely βG continuous mapping.

Example 3.3. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y, respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then

IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u < 0.6$ or $\mu_v < 0.7$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u > 0.4$ or $\mu_v > 0.3$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Now $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. Then

$$f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle.$$

Thus we have $\beta cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c)$. So $f^{-1}(G_2^c) \subseteq G_1$. Hence $\beta cl(f^{-1}(G_2^c)) \subseteq G_1$, where G_1 is an IFβOS in X. This implies that $f^{-1}(G_2^c)$ is an IFβGCS in X. Therefore f is an IFβG continuous mapping.

Since G_2^c is an IFCS in Y, it is an IFβGCS in Y but $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is not an IFRCS in X, since $cl(int(f^{-1}(G_2^c))) = cl(0_\sim) = 0_\sim \neq f^{-1}(G_2^c)$. Then f is not an IF completely βG continuous mapping.

Example 3.4. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.8_a, 0.9_b), (0.2_a, 0.1_b)\rangle$, $G_2 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.3_v), (0.5_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y, respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then

IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: either $\mu_a < 0.6$ or $\mu_b < 0.7$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: either $\mu_a > 0.4$ or $\mu_b > 0.3$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$, IF$\beta$C(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u < 0.5$ or $\mu_v < 0.3$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u > 0.5$ or $\mu_v > 0.7$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Now $G_3^c = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)\rangle$ is an IFRCS in Y, since $cl(int(G_3^c)) = cl(G_3) = G_3^c$. Then we have

$$f^{-1}(G_3^c) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle \subseteq G_1$$

and

$$f^{-1}(G_3^c) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle \subseteq G_2.$$

Thus $\beta cl(f^{-1}(G_3^c)) = f^{-1}(G_3^c)$. So $\beta cl(f^{-1}(G_3^c)) \subseteq G_1$ and $\beta cl(f^{-1}(G_3^c)) \subseteq G_2$. Hence $f^{-1}(G_3^c)$ is an IFβGCS in X and thus f is an IFaβG continuous mapping.

Since G_3^c is an IFCS in Y, it is an IFβGCS in Y. But

$$f^{-1}(G_3^c) = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$$

is not an IFRCS in X, since $cl(int(f^{-1}(G_3^c))) = cl(0_\sim) = 0_\sim \neq f^{-1}(G_3^c)$. Therefore f is not an IF completely βG continuous mapping.

Theorem 3.5. *If $f : (X,\tau) \to (Y,\sigma)$ is an IF completely βG continuous mapping, where X is an IF$\beta_g T_{1/2}$ space, then $\beta cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$, for every IF$\beta OS$ $A \subseteq Y$.*

Proof. Let A be an IFβOS in Y. Then $cl(A)$ is an IFRCS in Y. Thus $cl(A)$ is an IFβGCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFRCS in X. So it is an IFβCS in X. Hence $\beta cl(f^{-1}(A)) \subseteq \beta cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. □

Theorem 3.6. *Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping. Then the following are equivalent:*

(1) *f is an IF completely βG continuous mapping,*

(2) *$f^{-1}(V)$ is an IFROS in X, for every IFβGOS V in Y,*

(3) *for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFβGOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IFROS in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.*

Proof. (1) $\Leftrightarrow$ (2) is obvious.

(2) $\Rightarrow$ (3): Let $p_{(\alpha,\beta)} \in X$ and $B \subseteq Y$ such that $f(p_{(\alpha,\beta)})) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since B is an IF$\beta$GOS in Y, by hypothesis, $f^{-1}(B)$ is an IFROS in X. Let $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)} \subseteq f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B) = A$. Thus $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. So $f(A) \subseteq B$.

(3) $\Rightarrow$ (1): Let $B \subseteq Y$ be an IFβGOS. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. Then by hypothesis, there exists an IFROS C in X such that $p_{(\alpha,\beta)} \in C$ and $f(C) \subseteq B$. Thus $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. So $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(B)$. Hence

$$f^{-1}(B) = \cup_{p_{(\alpha,\beta)} \in f^{-1}(B) p_{(\alpha,\beta)}} \subseteq \cup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C \subseteq f^{-1}(B).$$

This implies $f^{-1}(B) = \cup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C$. Since the union IFROSs is IFRO, $f^{-1}(B)$ is an IFROS in X. Therefore f is an IF completely βG continuous mapping. □

Theorem 3.7. *If a mapping f: $(X, \tau) \to (Y, \sigma)$ is an IF completely βG continuous mapping then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN [11] A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.*

Proof. Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFβGOS, C is an IFβGOS in Y. Thus by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(C) \subseteq f^{-1}(A)$. So $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Then $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$. □

Theorem 3.8. *A mapping f: (X, τ) → (Y, σ) is an IF completely βG continuous mapping then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN A of f($p_{(\alpha,\beta)}$), there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B$ and f(B)⊆ A.*

Proof. Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFβGOS, C is an IFβGOS in Y. Thus by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Then $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. So $f(B) \subseteq A$. □

Theorem 3.9. *A mapping f: (X, τ) → (Y, σ) is an IF completely βG continuous mapping then int(cl(f^{-1}(int(B)))) ⊆ f^{-1}(B), for every IFS B in Y.*

Proof. Let $B \subseteq Y$. Then $int(B)$ is an IFOS in Y. Thus it is an IFβGOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. So $int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$. □

Theorem 3.10. *For any two IF completely βG continuous functions f_1 , f_2 : (X, τ) → (Y, σ), the function (f_1 , f_2) : (X, τ) → (Y × Y, σ × σ) is also an IF completely βG continuous function, where (f_1 , f_2) (x) = (f_1 (x), f_2 (x)), for every $x \in X$.*

Proof. Let $A \times B$ be an IFβGOS in $Y \times Y$. Then

$$\begin{aligned}&(f_1, f_2)^{-1}(A \times B)(x)\\ &= (A \times B)(f_1(x), f_2(x))\\ &= \langle x, min(\mu_A(f_1(x)), \mu_B(f_2(x))), max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle\\ &= \langle x, min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B(x)), max(f_1^{-1}(\nu_A)(x), f_2^{-1}(\nu_B)(x))\rangle\\ &= (_1^{-1}(A) \cap f_1^{-1}A))(x).\end{aligned}$$

Since f_1 and f_2 are IF completely βG continuous functions, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Thus (f_1, f_2) is an IF completely βG continuous mappings.

□

Theorem 3.11. *Let a mapping f: (X, τ) → (Y, σ) be an IF completely βG continuous mapping. Then the following are equivalent:*

(1) *For any IFβGOS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if f($p_{(\alpha,\beta)}$) $_q$ A, then $p_{(\alpha,\beta)}$ $_q$ int(f^{-1}(A)),*

(2) *For any IFβGOS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if f($p_{(\alpha,\beta)}$) $_q$ A, then there exists an IFOS B such that $p_{(\alpha,\beta)}$ $_q$ B and f(B) ⊆ A.*

Proof. (1) ⇒ (2): Let $A \subseteq Y$ be an IFβGOS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})_q A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. (1) implies that $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$, where $int(f^{-1}(A))$ is an IFOS in$X$. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A)$, $B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(2) ⇒ (1): Let $A \subseteq Y$ be an IFβGOS and let $p_{(\alpha,\beta)} \in X$. Suppose $f(p_{(\alpha,\beta)})_q A$. Then by (2), there exists an IFOS B in X such that $p_{(\alpha,\beta)}\ _q B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $B = int(B) \subseteq int(f^{-1}(A))$. Thus $p_{(\alpha,\beta)}\ _q B$ implies $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$. □

Theorem 3.12. *A mapping f: (X, τ) → (Y, σ) be a function and g: X → X × Y the graph of the function f. Then f is an IF completely βG continuous, if g is so.*

Proof. Let B be an IFβGOS in Y. Then $f^{-1}(B) = f^{-1}(1_\sim \times B) = 1_\sim \cap f^{-1}(B) = g^{-1}(1_\sim \times B)$. Since B is an IFβGOS in Y, $1_\sim \times B$ is an IFβGOS in $X \times Y$. Also, since g is an IF completely βG continuous mapping, $g^{-1}(1_\sim \times B)$ is an IFROS in X. Thus $f^{-1}(B)$ is an IFROS in X. So f is an IF completely βG continuous mapping. □

Definition 3.13. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy β generalized irresolute (IFβG irresolute) mapping, if $f^{-1}(V)$ is an IFβGCS in (X, τ), for every IFβGCS V of (Y, σ).

Theorem 3.14. *Let f: (X, τ) → (Y, σ) be an IFβG irresolute mapping. Then f is an IFβG continuous mapping but not conversely.*

Proof. Let f be an IFβG irresolute mapping. Let V be any IFCS in Y. Then V is an IFβGCS and by hypothesis $f^{-1}(V)$ is an IFβGCS in X. Thus f is an IFβG continuous mapping. □

Example 3.15. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.6_a, 0.8_b), (0.2_a, 0.1_b)\rangle$, $G_2 = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.2_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y, respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then

IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: either $\mu_a < 0.3$ or $\mu_b < 0.3$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$: either $\mu_a > 0.2$ or $\mu_b > 0.2$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u < 0.5$ or $\mu_v < 0.6$, $0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$: either $\mu_u > 0.5$ or $\mu_v > 0.4$, $0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1\}$.

Let $A = \langle y, (0.5_u, 0.3_v), (0.2_u, 0.1_v)\rangle$ in Y. Now $A \subseteq 1_\sim$. Then we have $\beta cl(A) = A \subseteq 1_\sim$. Thus A is an IFβGCS in Y. But $f^{-1}(A) = \langle x, (0.5_a, 0.3_b), (0.2_a, 0.1_b)\rangle \subseteq G_1$ and $\beta cl(f^{-1}(A)) = 1_\sim \nsubseteq G_1$. So $f^{-1}(A)$ is not an IFβGCS in X. Hence f is not an IFβG irresolute mapping.

Theorem 3.16. *Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent, if X and Y are IF$\beta_g T_{1/2}$ spaces:*

(1) *f is an IFβG irresolute mapping,*
(2) *$f^{-1}(B)$ is an IFβGOS in X for each IFβGOS in Y,*
(3) *$f^{-1}(\beta int(B)) \subseteq \beta int(f^{-1}(B))$ for each IFS B of Y,*
(4) *$\beta cl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B))$ for each IFS B of Y.*

Proof. (1) ⇒ (2) is obvious.

(2) $\Rightarrow$ (3): Let B be any IFS in Y and $\beta int(B) \subseteq B$. Also $f^{-1}(\beta int(B)) \subseteq f^{-1}(B)$. Since $\beta int(B)$ is an IFβOS in Y, it is an IFβGOS in Y. Then $f^{-1}(\beta int(B))$ is an IFβGOS in X, by hypothesis. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(\beta int(B))$ is an IFβOS in X. Thus $f^{-1}(\beta int(B)) = \beta int(f^{-1}(\beta int(B))) \subseteq \beta int(f^{-1}(B))$.

(3) $\Rightarrow$ (4) is obvious by taking complement in (3).

(4) $\Rightarrow$ (1): Let B be an IFβGCS in Y. Since Y is an IF$\beta_g T_{1/2}$ space, B is an IFβCS in Y and $\beta cl(B) = B$. Then $f^{-1}(B) = f^{-1}(\beta cl(B)) \supseteq \beta cl(f^{-1}(B)) \supseteq f^{-1}(B)$. Thus $\beta cl(f^{-1}(B)) = f^{-1}(B)$. So $f^{-1}(B)$ is an IFβCS. Hence it is an IFβGCS in X. Therefore f is an IFβG irresolute mapping. □

Theorem 3.17. *Let f: (X, τ) → (Y, σ) be an IFβG irresolute mapping. Then* $f^{-1}(B) \subseteq \beta int(f^{-1}(cl(int(cl(B)))))$, *for every IFβGOS B in Y, if X and Y are* IF$\beta_g T_{1/2}$ *spaces.*

Proof. Let B be an IFβGOS in Y. Then by hypothesis, $f^{-1}(B)$ is an IFβGOS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(B)$ is an IFβOS in X. Thus $\beta int(f^{-1}(B)) = f^{-1}(B)$. Since Y is an IF$\beta_g T_{1/2}$ space, B is an IFβOS in Y and $B \subseteq cl(int(cl(B)))$. So $f^{-1}(B) = \beta int(f^{-1}(B))$ implies $f^{-1}(B) \subseteq \beta int(f^{-1}(cl(int(cl(B)))))$. □

Theorem 3.18. *Let f: (X, τ) → (Y, σ) be an IFβG irresolute mapping from an IFTS X into an IFTS Y. Then* $f^{-1}(B) \subseteq \beta int(cl(int(cl(f^{-1}(B)))))$, *if X is an* IF$\beta_g T_{1/2}$ *space.*

Proof. Let B be an IFβGOS in Y. Then by hypothesis, $f^{-1}(B)$ is an IFβGOS in X. Since X is an IF$\beta_g T_{1/2}$ space, $f^{-1}(B)$ is an IFβOS in X. Thus $\beta int(f^{-1}(B)) = f^{-1}(B)$ and $f^{-1}(B) \subseteq cl(int(cl(f^{-1}(B))))$. So $f^{-1}(B) \subseteq \beta int(cl(int(cl(f^{-1}(B)))))$. □

Theorem 3.19. *The composition of any two IF completely βG continuous mapping is an IF completely βG continuous mapping.*

Proof. Let $f : X \to Y$ and $g : Y \to Z$ be any two intuitionistic fuzzy completely β generalized continuous mappings. Let B be an IFβGOS in Z. Since g is an IF completely βG continuous mapping, $g^{-1}(B)$ is an IFROS in Y. Since every IFROS is an IFβGOS, $g^{-1}(B)$ is an IFβGOS in Y. Since f is an IF completely βG continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFROS in X. Then $g \circ f$ is an IF completely βG continuous mapping. □

Theorem 3.20. *Let f: X → Y and g: Y → Z be any two functions. Then the following properties are hold:*

(1) *If f is an IF completely βG continuous mapping and g is an IFβG irresolute mapping, then g ∘ f is an IF completely βG continuous mapping,*

(2) *If f is an IF completely βG continuous mapping and g is an IFβG continuous mapping, then g ∘ f is an IFβG continuous mapping.*

Proof. (1) Let B be an IFβGOS in Z. Since g is an IFβG irresolute mapping, $g^{-1}(B)$ is an IFβGOS in Y. Also, since f is an IF completely βG continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an IF completely βG continuous mapping.

(2)Let B be an IFOS in Z. Since g is an IFβG continuous mapping, $g^{-1}(B)$ is an IFβGOS in Y. Also, since f is an IF completely βG continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Then $f^{-1}(g^{-1}(B))$ is an IFβGOS in X. From the fact that $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, it follows that $g \circ f$ is an IFβG continuous mapping.

□

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M. Saranya (saranya20maths@gmail.com)
Department of Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India

D. Jayanthi (jayanthimathss@gmail.com)
Department of Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India