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Abstract. A novel idea namely Pseudo Fuzzy (PF) Set has been presented here to handle imprecise or vague uncertainties. Pseudo fuzzy set is an ordered triplet of elements where last two elements designate two membership grades. Both the membership grades may or may not be dependent. The hypothesis is that every positive sense has some negative sense in general and both the senses try to balance the system. So, one membership grade has been considered as positive and another as negative. Considering this concept, here the notion of Pseudo Fuzzy (PS) set and its property along with PS numbers have been introduced.

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1. Introduction

Uncertainty plays an important role in real world problems. As such, every system contains uncertainty which may not be avoided. Traditionally these uncertainties have been handled by using probabilistic approach. Due to the involvement of linguistic variables and imprecise parameters in the system, it may be very difficult to get large number of experimental or observed data. In this context, a great philosopher Zadeh [17] introduced the concept of fuzzy which may help to manage uncertainty and reduce the difficulty of availability of large number of data. Further, the fuzzy set theory has been extensively used by various authors viz. (Chakraverty and Nayak [4], Dubois and Prade [6], Hanss [9], Zimmermann [18]). Zadeh [17] introduced fuzzy sets and generalized the concepts of union, intersection, complement, relation and convexity etc. form crisp sets to fuzzy sets. Some logical connective properties are proposed by Alsina et al. [1]. Dubois and Prade [7] gave a thorough survey on fuzzy set-theoretic operations. The relevance of functional in the axiomatically construction and the derivation of functional representations are emphasized. Diamond and Kloeden [5] addressed two classes of metrics for spaces of fuzzy sets and equivalence relation as well as some basic properties are established. Further, Yager [16] discussed the union and intersection of fuzzy sets along with the selection of appropriate operators to perform these operations. Then, Mendel [10] presented a new theory of fuzzy sets which is based on the collection of data from people that reflect intra- and inter-levels of uncertainties. Trillas [14] studied the fuzzy sets theories to extend it into applications in the area of both language and reasoning. Montero et al. [11] mentioned the importance of a specific family of fuzzy sets in which each element is the result of a classification problem. Whereas, Pedrycz [13] operated a category of logic operators to view the fuzzy sets which are inherently associated with membership grades. Then, Garcia-Honrado and Trillas [8] investigated the membership function of a fuzzy set which exhibits some intrinsic property.

Atanassov and Gargov [3] introduced intuitionistic fuzzy set which is based on interval valued fuzzy sets and generalized interval valued intuitionistic fuzzy set. Atanassov [2] proposed modal operator necessity and possibility on intuitionistic fuzzy sets. In this paper, the topological operator-interior and closure properties are also discussed. In this context, Xu and Yager [15] defined the notions of uncertain intuitionistic fuzzy variable. Further, Zhou and Wu [19] introduced a general framework to understand the relation-based intuitionistic fuzzy rough approximation. Recently, Nayak and Chakraverty [12] have developed a transformation and modified the fuzzy number into a crisp form. This transformation has been used in various problems and the uncertain solution has been investigated.

From the above literature it has been revealed that only one membership function has been used for each element which is not sufficient to describe the cause and effect of that element in general. The Newtonian philosophy viz. "Every action has equal and opposite reaction" may give us some idea about the involvement of some other membership function in addition with the known membership function used in usual fuzzy set. The law of nature tells us that if there is positive then we must have its default negative. In view of this we have introduced here another membership function and the modified fuzzy number has been named as Pseudo Fuzzy set or PF set.

The above concept of PF set has been systematically presented in the following sections. In the Section 2, the development of Pseudo fuzzy and various properties has been discussed. Further, in Section 3, Pseudo Fuzzy (PF) number have been introduced and conclusion is in the Section 4.

2. Pseudo fuzzy (PF) set and its property

Almost every system possesses uncertainties which may be vague or imprecise. The uncertainties may be categorized into probabilistic and non-probabilistic (i.e. imprecise or vague). Here, the imprecise or vague uncertainties are considered as fuzzy, where a fuzzy set is a pair of element and its membership value. The usual fuzzy set $\tilde{X}$ may be defined as $\tilde{X} = \{(x, \mu_X)|x \in \Re, \mu_X(x) \in [0,1]\}$. But, we may introduce here another type of membership value $\lambda_X : X \to [-1, 0]$. Below we include few examples of independent and dependent fuzzy set.

Example 2.1. Let us consider a fuzzy set $\tilde{X}$ = real numbers larger than 25 (Zimmermann [18]) as $\tilde{A} = \{(x, \mu_{\tilde{A}})|x \in X\}$,
where

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq 25 \\ \left(1 + \dfrac{1}{(x-25)^2}\right)^{-1}, & x > 25. \end{cases}$$

Example 2.2. Consider the set of tall men. We can say that the people having equal to 6 feet or taller than this, are tall. So the membership function can be constructed as the 'tall' set, and a person is either in the set of tall or are not in it. This implies that the membership functions work good for binary operations. But, it does not work as good as, when we deal in the real world. There will be no difference of membership function between two persons having heights 6'2" and 7', they are both counted as tall. In general, there is a significant difference between the heights of two persons. On the other side if we consider two person having heights 5'11" and 6', there is only a difference of one inch and this tells one is tall and the other is not tall. Whereas, the fuzzy set theory explains a much better representation of the tallness of a person. Hence, the membership function defines the fuzzy set for the possible values and a scale of 0 to 1, provides the membership value of the height in the fuzzy set. That is, the person has a membership of 0.3 is not very tall and the person has a membership of 0.95 is definitely tall.

The membership values may be divided into negative and positive to differentiate the nature and sense of the membership values. As we know that nothing is perfect, so we introduce the hypothesis of cause and effect into uncertainty. As such, if we consider the effect as positive then the cause may be assumed as negative. For positiveness, there should be some negativeness which may balance or unbalance the system that may not be avoided. Let us consider that the positive effect has membership value $\mu_X : X \to [0, 1]$ then negative effect (cause) has membership value $\lambda_X : X \to [-1, 0]$.

If we consider only the positive part, then we may not sometime define the system in global sense as followed by nature or practical problems. As such, we must study the negative impact too.

In view of the above we may now define a system as a triplet (x, μ_x, λ_x), where

$$\tilde{\tilde{X}} = \{(x, \mu_x, \lambda_x)|x \in \Re, \mu_x \in [0, 1], \lambda_x \in [-1, 0]\}. \tag{2.1}$$

In this regard, corresponding properties may be written as:
Properties:

(i) description

$$\mu_x \in [0, 1], \lambda_x \in [-1, 0], \tag{2.2}$$

(ii)

$$|\mu_x| \leq 1, |\lambda_x| \leq 1, \tag{2.3}$$

(iii)

$$0 \leq |\mu_x| + |\lambda_x| \leq 2. \tag{2.4}$$

Depending on the nature of membership values, system becomes balanced or unbalanced. So, the membership values may be independent or dependent with respect to the system (problem). As such, we may define now the following three special cases:

(i)

$$0 \leq |\mu_x| + |\lambda_x| \leq 1, \tag{2.5}$$

(ii)

$$|\mu_x| + |\lambda_x| = 1, \tag{2.6}$$

(iii)

$$1 \leq |\mu_x| + |\lambda_x| \leq 2. \tag{2.7}$$

Below we include few examples of independent and dependent PF set.

Example 2.3. (Independent pseudo fuzzy set) To know the hypothesis, we may see the following example of cold fever. When a person is affected by cold fever, then we may see that temperature of the body rises as well as body feels cold inside. Further it may be seen from the case study that when the body temperature rises i.e. for high fever we feel colder for this type of cold fever. So from these we may see that the hotness and coldness occurs simultaneously and so membership functions should have been defined for hotness and coldness accordingly. Here, μ correspond the hotness of the body and λ correspond the coldness felt by the body, where $\mu \in [0, 1]$ and $\lambda \in [-1, 0]$.

Example 2.4. (Dependent pseudo fuzzy set) We may also understand the hypothesis by considering the expansion of the solid rod. We see, the iron rod expands when it is heated. As such, we may consider two membership values viz. rise of temperature and expansion of the solid rod. If μ corresponds the rise of temperature and then λ is expansion of the solid rod where, $\mu \in [0, 1]$ and $\lambda \in [-1, 0]$.

Example 2.5. (Dependent pseudo fuzzy set) Let us consider an example of fuzzy relation between symptoms and diseases of the physician's medical knowledge in the following. Let $S = \{S_1, S_2, ..., S_n\}$, $D = \{D_1, D_2, ..., D_n\}$ and $P = \{P_1, P_2, ..., P_n\}$ denote the sets of symptoms, diseases and patients respectively. Two fuzzy relations A and B are defined as:

$$A = \{((p, s), \mu_A(p, s), \lambda_A(p, s))|(p, s) \in P \times S\},$$
$$B = \{((s, d), \mu_B(s, d), \lambda_B(s, d))|(s, d) \in S \times D\},$$

where,

$\mu_A(p, s)$ represents the degree to which symptom s appears in patient p,

$\lambda_A(p, s)$ represents the degree to which the symptom s does not appear in patient p,

$\mu_B(s, d)$ represents the degree to which symptom s confirms the presence of disease d,

$\lambda_B(s, d)$ represents the degree to which the symptom s does not confirm the presence of disease d.

Considering the above idea, we are now going to define Pseudo Fuzzy or PF number.

3. Pseudo fuzzy (PF) number

Depending upon the nature of the uncertainty, Pseudo Fuzzy Set may be classified as follows.

(i) Dependent pseudo fuzzy set:
In this case, both the membership functions are complement to each other. Let us consider $x \in X$ be an arbitrary element of PF set X and μ and λ are the membership functions then $|\mu_x| + |\lambda_x| = 1$.
The dependent Pseudo Triangular Fuzzy Number (TFN) (Figure 1) may be represented as follows:

$$\tilde{\tilde{X}} = \{(x, \mu_x, \lambda_x) | x \in \Re, \mu_x \in [0,1], \lambda_x \in [-1,0]\},$$

where

$$\mu_x = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x \ge c, \end{cases}$$

$$\lambda_x = \begin{cases} -1, & x \le a \\ \dfrac{x-b}{b-a}, & a \le x \le b \\ \dfrac{b-x}{c-b}, & b \le x \le c \\ -1, & x \ge c. \end{cases}$$

Similarly, dependent Pseudo Trapezoidal Fuzzy Number (TRFN) (Figure 2) may be represented as follows:

$$\tilde{\tilde{Y}} = \{(y, \mu_y, \lambda_y) | y \in \Re, \mu_y \in [0,1], \lambda_y \in [-1,0]\},$$

where

$$\mu_y = \begin{cases} 0, & y \le a \\ \dfrac{y-a}{b-a}, & a \le y \le b \\ \dfrac{d-y}{d-c}, & c \le y \le d \\ 0, & y \ge d, \end{cases}$$

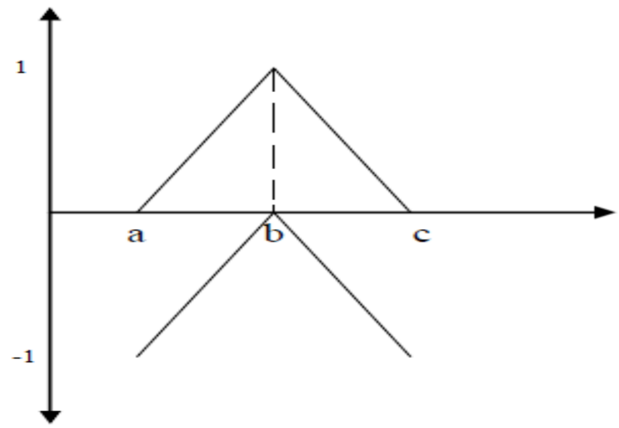

Figure 1. Pseudo TFN (Dependent)

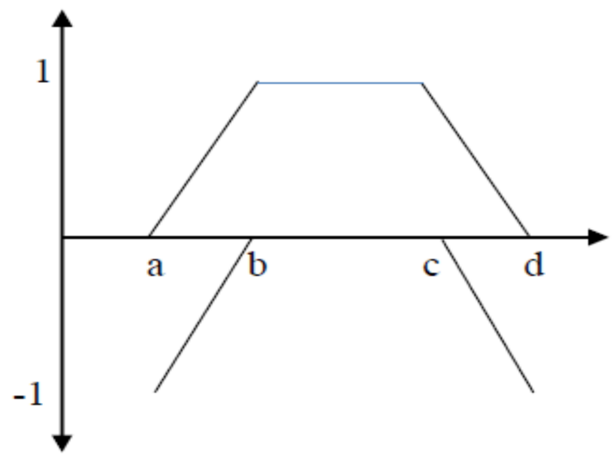

Figure 2. Pseudo TRFN (Dependent)

$$\lambda_y = \begin{cases} -1, & y \leq a \\ \dfrac{y-b}{b-a}, & a \leq y \leq b \\ \dfrac{c-y}{d-c}, & c \leq y \leq d \\ -1, & y \geq d. \end{cases}$$

(ii) Independent pseudo fuzzy set:

Here, both the membership functions are complement to each other. Let us consider $x \in X$ be an arbitrary element of PF set X and μ and λ are the membership functions then $0 \leq |\mu_x| + |\lambda_x| \leq 1$ or $0 \leq |\mu_x| + |\lambda_x| \leq 2$.

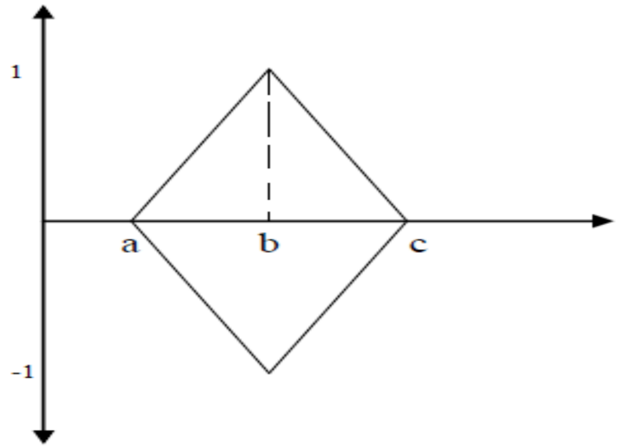

Figure 3. Pseudo TFN (Independent)

The independent Pseudo Triangular Fuzzy Number (TFN) (Figure 3) may be represented as follows:

$$\tilde{\tilde{X}} = \{(x, \mu_x, \lambda_x)|x \in \Re, \mu_x \in [0,1], \lambda_x \in [-1,0]\},$$

where

$$\mu_x = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x \ge c, \end{cases}$$

$$\lambda_x = \begin{cases} 0, & x \le a \\ \dfrac{a-x}{b-a}, & a \le x \le b \\ \dfrac{x-c}{c-b}, & b \le x \le c \\ 0, & x \ge c. \end{cases}$$

Similarly, dependent Pseudo Trapezoidal Fuzzy Number (TRFN) (Figure 4) may be represented as follows:

$$\tilde{\tilde{Y}} = \{(y, \mu_y, \lambda_y)|y \in \Re, \mu_y \in [0,1], \lambda_y \in [-1,0]\},$$

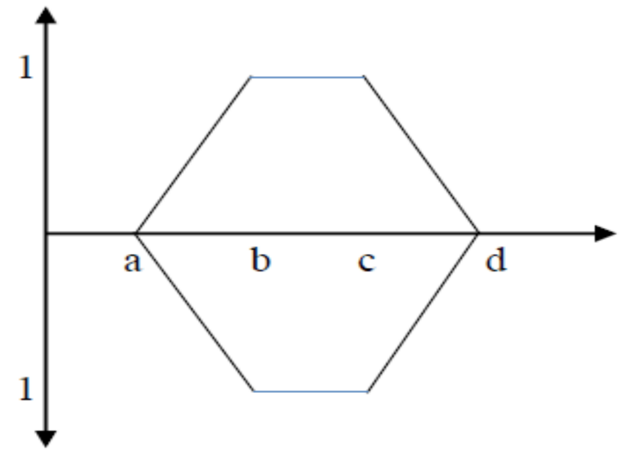

FIGURE 4. Pseudo TRFN (Independent)

where

$$\mu_y = \begin{cases} 0, & y \le a \\ \dfrac{y-a}{b-a}, & a \le y \le b \\ \dfrac{d-y}{d-c}, & c \le y \le d \\ 0, & y \ge d, \end{cases}$$

$$\lambda_y = \begin{cases} 0, & y \le a \\ \dfrac{a-y}{b-a}, & a \le y \le b \\ \dfrac{y-d}{d-c}, & c \le y \le d \\ 0, & y \ge d. \end{cases}$$

If we consider two Pseudo Fuzzy Sets (PFS) $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, then we have the following operations:

(i) $\tilde{\tilde{A}} \subset \tilde{\tilde{B}}$ iff $\mu_A(x) \le \mu_B(x)$ and $\lambda_A(x) \le \lambda_B(x)$, $\forall x \in \Re$,

(ii) $\tilde{\tilde{A}} = \tilde{\tilde{B}}$ iff $\tilde{\tilde{A}} \subset \tilde{\tilde{B}}$ and $\tilde{\tilde{A}} \supset \tilde{\tilde{B}}$,

(iii) $\tilde{\tilde{A}} \cup \tilde{\tilde{B}} = \{(x, \max[\mu_A(x), \mu_B(x)], \min[\lambda_A(x), \lambda_B(x)])|x \in \Re\}$,

(iv) $\tilde{\tilde{A}} \cap \tilde{\tilde{B}} = \{(x, \min[\mu_A(x), \mu_B(x)], \max[\lambda_A(x), \lambda_B(x)])|x \in \Re\}$,

(v) $\tilde{\tilde{A}} \bigoplus \tilde{\tilde{B}} = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x),$
$\lambda_A(x) + \lambda_B(x) + \lambda_A(x).\lambda_B(x))|x \in \Re\}$,

(vi) $\tilde{\tilde{A}} \bigotimes \tilde{\tilde{B}} = \{(x, \mu_A(x).\mu_B(x), -[\lambda_A(x).\lambda_B(x)])|x \in \Re\}$.

It is easy to verify the above defined operations as well. For better visualization of the above operations we have presented the union and intersection operations for two pseudo fuzzy sets (Figure 5) in Figures 6 and 7 respectively.

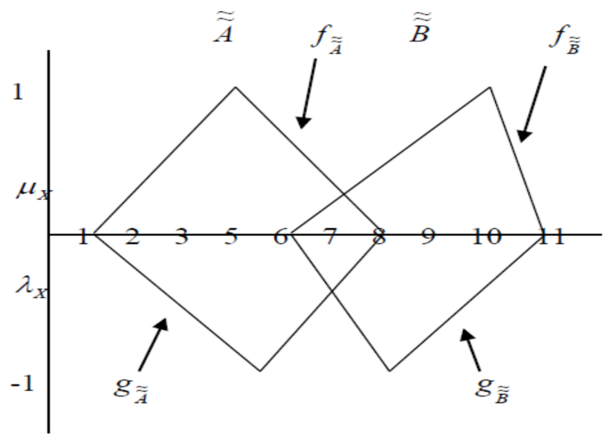

FIGURE 5. Pseudo fuzzy sets (PFS) $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$

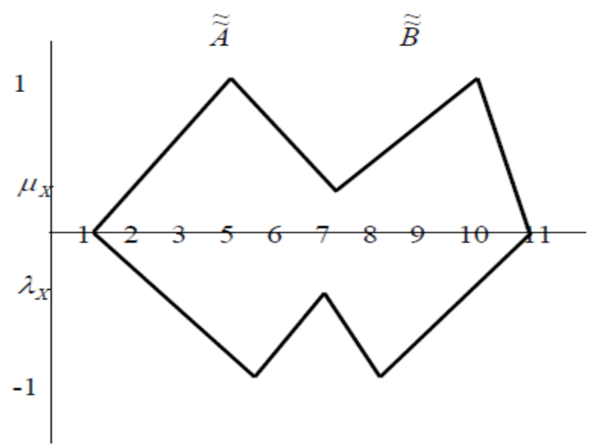

FIGURE 6. Union of pseudo fuzzy sets (PFS) $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, i.e., $(\tilde{\tilde{A}} \cup \tilde{\tilde{B}})$

With the above defined union and intersection operations along with complementation, we may extend most of the basic identities for pseudo fuzzy set too such as those of distributive and De Morgan's laws.

Theorem 3.1. *The operations addition* ($\bigoplus$) *and multiplication* ($\bigotimes$) *are commutative, associative, and satisfy De Morgan law:*

$$\left(\tilde{\tilde{A}} \cup \tilde{\tilde{B}}\right)' = \left(\tilde{\tilde{A}}\right)' \cap \left(\tilde{\tilde{B}}\right)', \tag{3.1}$$

$$\left(\tilde{\tilde{A}} \cap \tilde{\tilde{B}}\right)' = \left(\tilde{\tilde{A}}\right)' \cup \left(\tilde{\tilde{B}}\right)'. \tag{3.2}$$

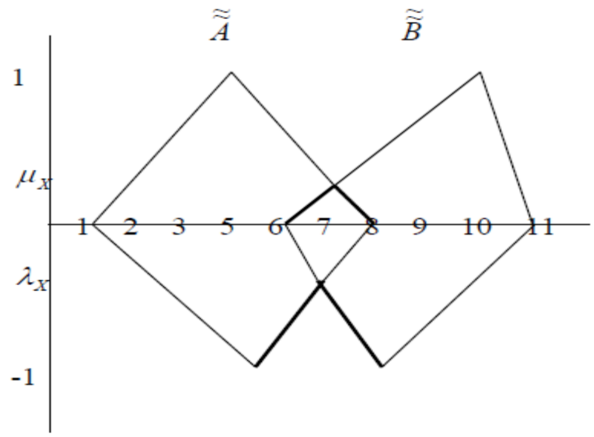

FIGURE 7. Intersection of pseudo fuzzy sets (PFS) $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, i.e., $(\tilde{\tilde{A}} \cap \tilde{\tilde{B}})$

Proof. The above equalities may easily be established by showing that the corresponding relations for membership functions of $\tilde{\tilde{A}}$ and $\tilde{\tilde{A}}$ are nothing but the identities. For Eqs. 3.1 and 3.2, we have

$$1 - \max[f_{\tilde{\tilde{A}}}, f_{\tilde{\tilde{B}}}] = \min[1 - f_{\tilde{\tilde{A}}}, 1 - f_{\tilde{\tilde{B}}}] \tag{3.3}$$

and

$$-1 - \max[g_{\tilde{\tilde{A}}}, g_{\tilde{\tilde{B}}}] = \min[-1 - g_{\tilde{\tilde{A}}}, -1 - g_{\tilde{\tilde{B}}}] \tag{3.4}$$

which can be verified by testing two cases $f_{\tilde{\tilde{A}}}(x) > f_{\tilde{\tilde{B}}}(x)$ and $f_{\tilde{\tilde{A}}}(x) < f_{\tilde{\tilde{B}}}(x)$, [17]. In the same fashion Eq. 3.4 can be verified. □

These operations may be useful in the conventional parallel and series network and circuit theory.

In Table 3, we have presented the comparison among fuzzy set, intuitionistic fuzzy set and pseudo fuzzy set.

From the above comparisons (Table 3), we may say that the intuitionistic fuzzy set is a special case of pseudo fuzzy set, when the pseudo fuzzy set is dependent.

4. CONCLUSION

In this paper, the concept of fuzzy set has been generalized and notion of a new set viz. Pseudo Fuzzy (PF) set has been introduced. This set is an ordered triplet of elements and its two membership grades are based on the hypothesis of every positive sense and the default negative sense. Hence, membership grades have been considered as positive and negative. Various set properties, examples and operations are verified by using pseudo fuzzy set notions. Further, it is believed that pseudo fuzzy set theory may play a crucial role to study the uncertainties and its quantification in various fields of science and engineering problems.

Table 1. Comparison among fuzzy set, intuitionistic fuzzy set and pseudo fuzzy set

Fuzzy set (FS)	Intuitionistic fuzzy set (IFS)	Pseudo fuzzy set (PFS)
If X is a collection of objects denoted by x, then a fuzzy set $\tilde{A}$ in X is a set of order pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\|x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function which maps X to the membership space.	Let us consider universal set E and A be a subset of E. Then the set A^*, $A^* = \{(x, \mu_A(x), \nu_A(x))\|x \in E\}$ is called the intuitionistic fuzzy set.	If X is a collection of objects denoted by x, then a pseudo fuzzy set $\tilde{\tilde{A}}$ in X is a set of triplets $\tilde{\tilde{A}} = \{(x, \mu_{\tilde{A}}(x), \lambda_{\tilde{A}}(x))\|x \in X\}$, where $\mu_{\tilde{A}}(x)$ and $\lambda_{\tilde{A}}(x)$ are called the cause and effect membership functions which maps X to the membership spaces.
A fuzzy set $\tilde{X}$ may be defined as $\tilde{X} = \{(x, \mu_x)\|x \in \Re, \mu_x(x) \in [0, 1]\}$.	An intuitionistic fuzzy set X^* may be defined as $X^* = \{(x, \mu_x, \nu_x)\|x \in \Re, \nu_x = 1 - \mu_x(x), \mu_x \in [0, 1]\}$.	A pseudo fuzzy set $\tilde{\tilde{X}}$ may be defined as $\tilde{\tilde{A}} = \{(x, \mu_x, \lambda_x)\|x \in \Re, \mu_x \in [0, 1], \lambda_x \in [-1, 0]\}$.
The membership value is defined as $\mu_x : X \to [0, 1]$.	The membership value is defined as $\mu_x : X \to [0, 1]$.	here the membership values are defined as $\mu_x : X \to [0, 1]$ and $\lambda_x : X \to [-1, 0]$.
Membership values satisfy, $0 \leq \mu_x \leq 1$.	Membership values satisfy, $0 \leq \mu_x + \nu_x \leq 1$.	Membership values satisfy, $0 \leq \|\mu_x\| + \|\lambda_x\| \leq 2$.
It follows De Morgan's law.	It follows De Morgan's law.	It follows De Morgan's law.

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