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Abstract. In this paper we introduce the concept of secure domination and secure total domination in fuzzy graphs and intuitionistic fuzzy graphs. Secure domination number $\gamma_s(G)$ and secure total domination number $\gamma_{st}(G)$ for several classes of fuzzy graphs and intuitionistic fuzzy graphs have been determined. The definition of 2-total dominating set, 2-secure dominating set, 2-secure total dominating set and its domination number in fuzzy graphs and intuitionistic fuzzy graphs are defined and some properties are analysed with suitable examples.

2010 AMS Classification: 05C72, 03E72, 03F55

Keywords: Fuzzy secure dominating set, Fuzzy secure domination number, Intuitionistic fuzzy secure dominating set, Intuitionistic fuzzy secure domination number.

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1. Introduction

One of the most interesting problem and fast growing area in graph theory is the study of domination. The study of dominating sets in graph was introduced by Ore[15] and Berge[6]. Merouane and Chellali[10] introduced the concept of secure domination set and 2-dominating set. A.Somasundaram and S.Somasundaram[20] introduced the concept of domination in fuzzy graphs and obtain several bounds for the domination number. Parvathi and Thamizhendhi [18] introduced dominating set, domination number, independent set, total dominating and total domination number in intuitionistic fuzzy graphs. Motivated by the notion of dominating number and their applicability [1, 2, 3, 4, 5, 7, 9, 16, 19, 20, 21], we focused on introducing secure domination and secure total domination in fuzzy graphs and intuitionistic fuzzy graphs.

This paper is organized as follows. Section 2 contains preliminaries and in section 3, a secure domination number, secure total domination number and 2-secure domination number of a fuzzy graph is defined and their relationship has been formulated. Section 4 deals with a secure domination number, secure total domination number and 2-secure domination number of an intuitionistic fuzzy graph and their properties are given as theorems and lemmas.

2. Preliminaries

Definition 2.1 ([11]). Let V be a non empty set. A fuzzy graph is a pair of functions $G = (\sigma, \mu)$, where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ, i.e., $\sigma : V \longrightarrow [0, 1]$ and $\mu : V \times V \longrightarrow [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, for all u, v in V.

Definition 2.2 ([11]). A fuzzy graph G is said to be complete if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$.

Definition 2.3 ([20]). Let $G = (\sigma, \mu)$ be a fuzzy graph on V. Let S of V and the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Definition 2.4 ([20]). The domination number of a fuzzy graph G is the minimum cardinality taken over all dominating sets in G and is denoted by $\gamma(G)$ or simply γ.

Definition 2.5 ([20]). A arc (u, v) of a fuzzy graph is called strong arc if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$.

Definition 2.6 ([16]). An intuitionistic fuzzy graph (IFG) is of the form $G = \langle V, E \rangle$ said to be a minmax IFG, if

(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\nu_1 : V \to [0, 1]$, denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V, (i = 1, 2, ..., n)$,

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\nu_2 : V \times V \to [0, 1]$, are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)] \text{ and } \nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)],$$

denotes the degree of membership and non-membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

For each intuitionistic fuzzy graph G,

the degree of hesitance(hesitation degree) of the vertex $v_i \in V$ in G is

$$\pi_1(v_i) = 1 - \mu_1(v_i) - \nu_1(v_i)$$

and

the degree of hesitance(hesitation degree) of an edge $e_{ij} = (v_i, v_j) \in E$ in G is

$$\pi_2(e_{ij}) = 1 - \mu_2(e_{ij}) - \nu_2(e_{ij}).$$

Definition 2.7 ([17]). An IFG, $G = (V, E)$ is said to be complete IFG, if

$$\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)) \text{ and } \nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)),$$

for every $v_i, v_j \in V$.

Definition 2.8 ([17]). An IFG, $G = (V, E)$ is said to be strong IFG, if

$$\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)) \text{ and } \nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)),$$

for every $(v_i, v_j) \in E$.

Definition 2.9 ([17])**.** The complement of an IFG, $G = (V, E)$ is an IFG, $\overline{G} = (\overline{V}, \overline{E})$, where

(i) $\overline{V} = V$,

(ii) $\overline{\mu_1(v_i)} = \mu_1(v_i)$ and $\overline{\nu_1(v_i)} = \nu_1(v_i)$, for all $i = 1, 2, \ldots, n$,

(iii) $\overline{\mu_2(v_i, v_j)} = \min(\mu_1(v_i), \mu_1(v_j)) - \mu_2(v_i, v_j)$ and $\overline{\nu_2(v_i, v_j)} = \max(\nu_1(v_i), \nu_1(v_j)) - \nu_2(v_i, v_j)$, for all $i, j = 1, 2, \ldots, n$.

Definition 2.10 ([13])**.** The neighbourhood degree of a vertex is defined as

$$d_N(v) = (d_{N\mu}(v), d_{N\nu}(v)),$$

where

$$d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w) \text{ and } d_{N\nu}(v) = \sum_{w \in N(v)} \nu_1(w).$$

Definition 2.11 ([12])**.** Let $G = (V, E)$ be an IFG. Then the degree of a vertex v_i is defined by $d_G(v_i) = (d_\mu(v_i), d_\nu(v_i)) = (K_1, K_2)$
where $K_1 = d_\mu(v_i) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $K_2 = d_\nu(v_i) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$.

Definition 2.12 ([8])**.** An intuitionistic fuzzy graph $G = (V, E)$ is said to be a (K_1, K_2)-regular, if $d_G(v_i) = (K_1, K_2)$, for all $v_i \in V$ and also G is said to be a regular intuitionistic fuzzy graph of degree (K_1, K_2).

Definition 2.13 ([18])**.** An intuitionistic fuzzy graph $G = (V, E)$ is said to be a bipartite, if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that

(i) $\mu_2(v_i, v_j) = 0$ and $\nu_2(v_i, v_j) = 0$ if $(v_i, v_j) \in V_1$ or $(v_i, v_j) \in V_2$.

(ii) $\mu_2(v_i, v_j) > 0$, $\nu_2(v_i, v_j) < 0$ if $v_i \in V_1$ or $v_j \in V_2$, for some i and j, (or)
$\mu_2(v_i, v_j) = 0$, $\nu_2(v_i, v_j) < 0$ if $v_i \in V_1$ or $v_j \in V_2$, for some i and j, (or)
$\mu_2(v_i, v_j) > 0$, $\nu_2(v_i, v_j) = 0$ if $v_i \in V_1$ or $v_j \in V_2$, for some i and j.

Definition 2.14 ([18])**.** A bipartite intuitionistic fuzzy graph $G = (V, E)$ is said to be complete, if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$, for all $v_i \in V_1$ and $v_j \in V_2$ and is denoted by K_{V_1, V_2}.

Definition 2.15. ([18]) If $v_i, v_j \in V \subseteq G$, then
the μ-strength of connectedness between v_i and v_j is

$$\mu_2^\infty(v_i, v_j) = sup\{\mu_2^k(v_i, v_j) | k = 1, 2, ...n\}$$

and
ν-strength of connectedness between v_i and v_j is

$$\nu_2^\infty(v_i, v_j) = inf\{\nu_2^k(v_i, v_j) | k = 1, 2, ...n\}.$$

.

If u, v are connected by means of paths of length k, then
$\mu_2^k(u, v)$ is defined as:

$$sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \mu_2(v_2, v_3) ... \wedge \mu_2(v_{k-1}, v) | (u, v_1, v_2 ... v_{k-1}, v \in V)\}$$

and

$\nu_2^k(u,v)$ is defined as:

$inf\{\nu_2(u,v_1) \vee \nu_2(v_1,v_2) \vee \nu_2(v_2,v_3)... \vee \nu_2(v_{k-1},v) | (u,v_1,v_2...v_{k-1},v \in V)\}$.

Definition 2.16 ([18]). Let $G = (V, E)$ be an IFG on V. Let $u, v \in V$, we say that u dominates v in G, if there exists a strong edge between them.

Definition 2.17 ([18]). A subset S of V is called dominating set in G, if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

Definition 2.18 ([18]). A dominating set S of an IFG is said to be minimal dominating set, if no proper subset of S is a dominating set.

Definition 2.19 ([18]). Minimum cardinality among all minimal dominating set is called vertex domination number of G and is denoted by $\gamma(G)$.

Definition 2.20 ([18]). Let $G = (V, E)$ be an IFG, then the vertex cardinality of V is defined by $|V| = \sum_{v_i \in V} \left(\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right)$, for all $v_i \in V$.

Definition 2.21 ([18]). Let $G = (V, E)$ be an IFG, then the edge cardinality of E is defined by $|E| = \sum_{v_i, v_j \in E} \left(\frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right)$, for all $(v_i, v_j) \in E$.

Theorem 2.22 ([20]). *Every arc in a complete fuzzy graph is a strong arc.*

3. Secure domination in fuzzy graphs

The concept of secure dominaton in graphs was introduced by Cockayne, Favaron and Mynhardt in 2003[10] and further secure domination number and 2-dominating set in graphs are studied by Merouane and Chellali[7]. We refer to [20] for the terminology of secure domination in graphs.

Definition 3.1 ([20]). Let $G = (V, E)$ be a fuzzy graph. Let $u, v \in V$ and we say that u dominates v in G, if $\mu(u, v) = \min(\sigma(u), \sigma(v))$. A subset S of V is called dominating set in G, if for every $v \in V - S$, there exists $u \in S$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$.

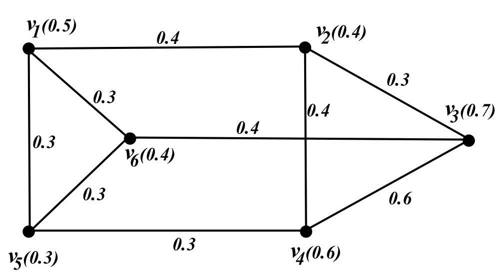

Figure 1. Domination of a fuzzy graph

Here $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$, $\{v_1, v_2, v_3, v_5\}$, $\{v_1, v_3, v_4\}$, $\{v_1, v_4, v_6\}$, $\{v_1, v_4, v_5\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_2, v_4, v_6\}$, $\{v_2, v_5, v_6\}$, $\{v_3, v_4, v_5\}$, $\{v_1, v_2, v_6\}$, $\{v_1, v_5, v_6\}$, $\{v_4, v_5\}$ are some dominating sets of G and $\gamma(G) = 0.9$.

Definition 3.2 ([20])**.** Let $G = (V, E)$ be a fuzzy graph without isolated vertices. A dominating set S of G is called a total dominating set, if the subgraph $< S >$ induced by S has no isolated vertices. The minimum fuzzy cardinality taken over all total dominating sets of G is called the total domination number of G and is denoted by $\gamma_t(G)$.

From figure 1, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$, $\{v_1, v_4, v_5\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_2, v_4, v_6\}$, $\{v_2, v_5, v_6\}$, $\{v_1, v_5, v_6\}$, $\{v_4, v_5\}$ are total dominating sets of G and $\gamma_t(G) = 0.9$.

Definition 3.3. Let $G = (V, E)$ be a fuzzy graph. A dominating set S of V is a secure dominating set, if for each vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is dominating set. The secure domination number of G is minimum fuzzy cardinality taken over all secure dominating sets of G and is denoted by $\gamma_s(G)$.

From figure 1, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$, $\{v_3, v_4, v_5\}$, $\{v_1, v_2, v_6\}$, $\{v_2, v_4, v_6\}$, $\{v_1, v_3, v_5\}$, $\{v_2, v_5, v_6\}$, $\{v_1, v_3, v_4\}$ are secure dominating sets of G and $\gamma_s(G) = 1.1$.

Definition 3.4. Let $G = (V, E)$ be a fuzzy graph without isolated vertices. A secure dominating set S of G is called a total secure dominating set, if the subgraph $< S >$ induced by S has no isolated vertices. The total secure domination number of G is minimum fuzzy cardinality taken over all secure total dominating sets of G and is denoted by $\gamma_{st}(G)$.

From figure 1, $\{v_1, v_4, v_5\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_6\}$, $\{v_1, v_2, v_4\}$, $\{v_3, v_4, v_5\}$ are secure total dominating sets of G and $\gamma_{st}(G) = 1.2$.

Definition 3.5 ([14])**.** A subset S of V is a 2-dominating set in G, if every vertex of $V - S$ has atleast two neighbour in S. The 2-domination number of G is minimum fuzzy cardinality taken over all 2-dominating sets of G and is denoted by $\gamma_2(G)$.

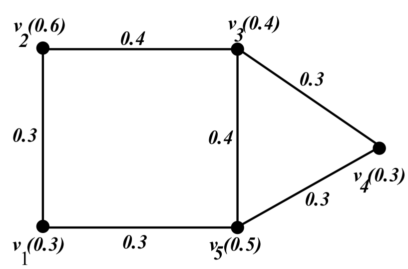

FIGURE 2. 2-Domination of a fuzzy graph

Here $\{v_1, v_3, v_5\}$,$\{v_1, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_1, v_2, v_4\}$,$\{v_2, v_3, v_5\}$ are 2-dominating sets of G and $\gamma_2(G) = 1.0$.

Definition 3.6. A subset S of V is a 2-total dominating set in G, if S is 2-dominating set and the subgraph induced by S has no isolated vertices. The 2-total domination number of G is minimum fuzzy cardinality taken over all 2-total dominating sets of G and is denoted by $\gamma_{2t}(G)$.

From figure 2, $\{v_1, v_3, v_5\}$,$\{v_2, v_3, v_5\}$ are 2-total dominating sets of G and $\gamma_{2t}(G) = 1.2$.

Definition 3.7. Let $G = (V, E)$ be a fuzzy graph. A 2-dominating set S of V is a secure 2-dominating set, if for every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is 2-dominating set. The 2-secure domination number of G is minimum fuzzy cardinality taken over all 2-secure dominating sets of G and is denoted by $\gamma_{2s}(G)$.

From figure 2, $\{v_1, v_3, v_5\}$,$\{v_1, v_3, v_4\}$ are 2-secure dominating sets of G and $\gamma_{2s}(G) = 1.0$.

Definition 3.8. Let $G = (V, E)$ be a fuzzy graph without isolated vertices. A 2-secure dominating set S of G is called a 2- secure total dominating set, if the subgraph $< S >$ induced by S has no isolated vertices. The 2- secure total domination number of G is minimum fuzzy cardinality taken over all 2-secure total dominating sets of G and is denoted by $\gamma_{2st}(G)$.

From figure 2, $\{v_1, v_3, v_5\}$ is 2-secure total dominating sets of G and $\gamma_{2st}(G) = 1.2$.

Theorem 3.9. *If S is a minimal dominating set in a complete fuzzy graph G, then*
(1) *S is a secure dominating set,*
(2) *S is not a secure total dominating set.*

Proof. Given that S is a minimal dominating set of a complete fuzzy graph G. By Theorem 2.22, every arc in a complete fuzzy graph is a strong arc. Then minimal dominating set S contains a only one vertex v, i.e., $S = \{v\}$. Now any vertex $v_i \in V - S$ and v_i is adjacent to v. thus $(S - \{v\})U\{v_i\} = \{v_i\}$ is a dominating set. So S is secure dominating set. Since any secure dominating set of a complete fuzzy graph contains a vertex v_i, by definition of total dominating, S is not secure total dominating set. □

Theorem 3.10. *If S is a minimal dominating set in a complete fuzzy graph G, then*
(1) *S is not a 2-dominating set,*
(2) *S is not a 2-total dominating set.*

Proof. If S is a minimal dominating set in a complete fuzzy graph G, then S contains a vertex of minimum cardinality but 2-dominating set should contain atleast two vertices. Thus S is not a 2-dominating set. Similarly, S is not a 2-total dominating set. □

Theorem 3.11. *For a complete fuzzy graph, $\gamma_s(G) = \gamma(G)$.*

Proof. Let $G = (V, E)$ be a complete fuzzy graph G. Let S be a minimal dominating set of G. Then S contains a vertex $\{v\}$, i.e., $S = \{v\}$. The minimum cardinality of S is denoted by $\gamma(G)$. By Theorem 3.9, S is also secure dominating set and the minimum cardinality of secure dominating set is denoted by $\gamma_s(G)$. Thus $\gamma_s(G) = \gamma(G)$. □

Theorem 3.12. *Every 2-secure dominating set of a fuzzy graph G is a secure dominating set of G.*

Proof. Let S be a 2-secure dominating set of a fuzzy graph G. Then every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is 2-dominating set. Since G is 2-secure dominating set, by definition, G is 2-dominating set and every 2-dominating set is a dominating set. Thus every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is dominating set. So S is a secure dominating set of G. □

Theorem 3.13. *If G is a fuzzy graph then $\gamma_{2s}(G) \geq \gamma_2(G)$.*

Proof. By Theorem 3.12, every 2-secure dominating set of a fuzzy graph G is a secure dominating set of G. Then every minimum 2-secure dominating set of G is also a secure dominating set of G. Thus $\gamma_{2s}(G) \geq \gamma_2(G)$. □

Theorem 3.14. *If S is a 2-dominating set of a path of a fuzzy graph G, then S is not 2-secure dominating set.*

Proof. Let P_n be a path of a fuzzy graph G and let S be a 2-dominating set of a path P_n of a fuzzy graph G. Then S contain two pendent vertices v_i and v_j. Now for some $u \in V - S$ and u is adjacent to v_i. Thus $(S - \{v_i\})U\{u\}$ is not 2-dominating set. So S is not 2-secure dominating set. □

Theorem 3.15. *If S is a dominating set of a complete bipartite fuzzy graph then S is not a secure dominating set.*

Proof. Given that S is a dominating set of a complete bipartite fuzzy graph say, $k_{m,n}$. Then S should contain a vertex in V_1 say u and a vertex in V_2 say v. Now for some $v_i \in V - S$ and v_i is adjacent to $u \in V_1$. Thus $(S - \{u\})U\{v_i\}$ is not dominating set. So S is not a secure dominating set. □

Theorem 3.16. *Let G be a fuzzy graph with only strong edges and without isolated vertices and let S be a minimal secure dominating set. Then $V - S$ is a secure dominating set of G.*

Proof. Given that S is a minimal secure dominating set. Then by definition every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is dominating set. We have to prove that $V - S$ is a secure dominating set of G. Suppose $V - S$ is not secure dominating set. Then there exist vertex $w \in S$ is adjacent to a vertex $x \in V - S$ such that $(S - \{x\})U\{w\}$ is not dominating set. Thus x is not dominated by any vertex in S which is contradiction to our assumption that S is minimal secure dominating set and G has no isolated vertices and has only strong edges. So $V - S$ is a secure dominating set of G. □

Theorem 3.17. *Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^* = (V, E)$. If σ is a constant function and γ_s is a secure domination number, then $\gamma_s(k_n) = \sigma$*

Proof. The proof is obvious. □

4. Secure domination in intuitionistic fuzzy graphs

Definition 4.1 ([18]). Let $G = (V, E)$ be an intuitionistic fuzzy graph. A subset S of V is called dominating set in G, if for every $v \in V - S$, there exists $u \in S$ such that u dominates v. The domination number of G is minimum cardinality taken over all dominating sets of G and is denoted by $\gamma^*(G)$.

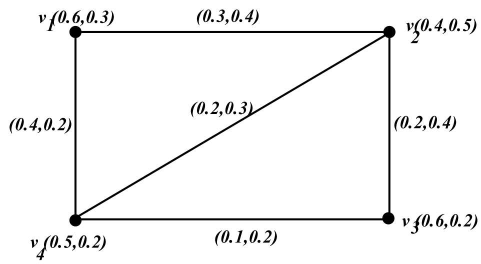

Figure 3. Domination of an intuitionistic fuzzy graph

Here $\{v_1, v_3, v_4\}$,$\{v_1, v_2\}$, $\{v_3, v_4\}$ are some dominating sets of G and $\gamma^*(G) = 1.1$.

Definition 4.2 ([18]). Let $G = (V, E)$ be an intuitionistic fuzzy graph without isolated vertices. A dominating set S of G is called a total dominating set, if the subgraph $< S >$ induced by S has no isolated vertices. The total domination number of G is minimum cardinality taken over all total dominating sets of G and is denoted by $\gamma_t^*(G)$.

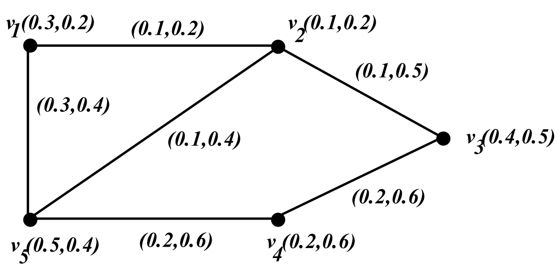

Figure 4. Total domination of an intuitionistic fuzzy graph

From above figure, $\{v_2, v_3\}$,$\{v_2, v_5\}$, $\{v_4, v_5\}$ are total dominating sets of G and $\gamma_t^*(G) = 0.85$.

Definition 4.3. Let $G = (V, E)$ be an intuitionistic fuzzy graph. A dominating set S of V is a secure dominating set, if for every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is dominating set. The secure domination number of G is minimum cardinality taken over all secure dominating sets of G and is denoted by $\gamma_s^*(G)$.

From figure 4, $\{v_1, v_3\}$,$\{v_1, v_4\}$, $\{v_2, v_4\}$, $\{v_2, v_3\}$, $\{v_3, v_5\}$,$\{v_4, v_5\}$ are secure dominating sets of G and $\gamma_s^*(G) = 0.75$.

Definition 4.4. Let $G = (V, E)$ be an intuitionistic fuzzy graph without isolated vertices. A secure dominating set S of G is called a total secure dominating set, if the subgraph $< S >$ induced by S has no isolated vertices. The total secure domination number of G is minimum fuzzy cardinality taken over all secure total dominating sets of G and is denoted by $\gamma^*_{st}(G)$.

From figure 4, $\{v_2, v_3\}$, $\{v_4, v_5\}$ are secure total dominating sets of G and $\gamma^*_{st}(G) = 0.85$.

Definition 4.5. A subset S of V is a 2-dominating set in G if every vertex of $V - S$ has atleast two neighbour in S. The 2-domination number of G is minimum cardinality taken over all 2-dominating sets of G and is denoted by $\gamma^*_2(G)$.

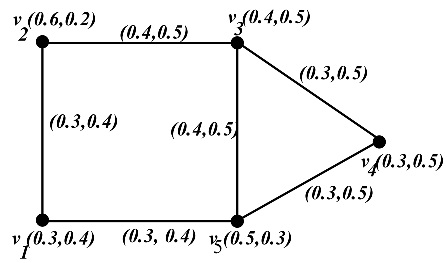

FIGURE 5. 2-Domination of an intuitionistic fuzzy graph

Here $\{v_1, v_3, v_5\}$,$\{v_1, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_1, v_2, v_4\}$,$\{v_2, v_3, v_5\}$ are secure dominating sets of G and $\gamma^*_2(G) = 1.3$.

Definition 4.6. A subset S of V is a 2-total dominating set in G, if S is 2-dominating set and the subgraph induced by S has no isolated vertices. The 2-total domination number of G is minimum cardinality taken over all 2-total dominating sets of G and is denoted by $\gamma^*_{2t}(G)$. From figure 5, $\{v_1, v_3, v_5\}$,$\{v_2, v_3, v_5\}$ are secure dominating sets of G and $\gamma^*_{2t}(G) = 1.4$.

Definition 4.7. Let $G = (V, E)$ be an intuitionistic fuzzy graph. A 2-dominating set S of V is a secure 2-dominating set, if for every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is 2-dominating set. The 2-secure domination number of G is minimum cardinality taken over all 2-secure dominating sets of G and is denoted by $\gamma^*_{2s}(G)$. Here $\{v_1, v_3, v_5\}$,$\{v_1, v_3, v_4\}$ are 2-secure dominating sets of G and $\gamma^*_{2s}(G) = 1.2$.

Definition 4.8. Let $G = (V, E)$ be an intuitionistic fuzzy graph without isolated vertices. A 2-secure dominating set S of G is called a 2- secure total dominating set, if the subgraph $< S >$ induced by S has no isolated vertices. The 2- secure total domination number of G is minimum cardinality taken over all 2-secure total dominating sets of G and is denoted by $\gamma^*_{2st}(G)$.

From figure 6, $\{v_1, v_3, v_5\}$ is 2-secure dominating sets of G and $\gamma^*_{2st}(G) = 1.35$.

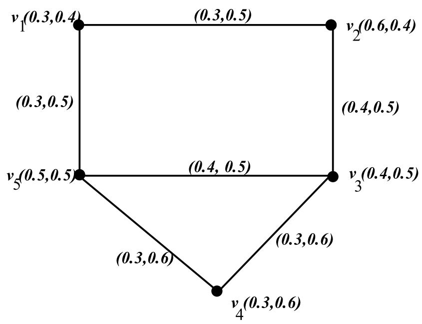

FIGURE 6. 2- Secure domination of an intuitionistic fuzzy graph

Theorem 4.9. *If S is a minimal dominating set in complete intuitionistic fuzzy graph G, then*
(1) *S is a secure dominating set,*
(2) *S is not a secure total dominating set.*

Proof. Given that S is a minimal dominating set of a complete intuitionistic fuzzy graph G. By Theorem 2.22, every arc in a complete intuitionistic fuzzy graph is a strong arc then minimal dominating set S contains a only one vertex v, i.e., $S = \{v\}$. Now any vertex $v_i \in V - S$ and v_i is adjacent to v. Then $(S - \{v\})U\{v_i\} = \{v_i\}$ is a dominating set. Thus S is secure dominating set. Since any secure dominating set of a complete intuitionistic fuzzy graph contains a vertex v_i, by the definition of total dominating, S is not secure total dominating set. □

Theorem 4.10. *If S is a minimal dominating set in complete intuitionistic fuzzy graph G, then*
(1) *S is not a 2-dominating set,*
(2) *S is not a 2-total dominating set.*

Proof. If S is a minimal dominating set in a complete intuitionistic fuzzy graph G, then S contains a vertex of minimum cardinality but 2-dominating set should contain atleast two vertices. Thus S is not a 2-dominating set. Similarly, S is not a 2-total dominating set. □

Theorem 4.11. *For a complete intuitionistic fuzzy graph, $\gamma_s^*(G) = \gamma^*(G)$.*

Proof. Let $G = (V, E)$ be a complete intuitionistic fuzzy graph G. Let S be a minimal dominating set of G. Then S contains a vertex $\{v\}$, i.e., $S = \{v\}$. The minimum cardinality of S is denoted by $\gamma^*(G)$. By Theorem 4.9, S is also secure dominating set and the minimum cardinality of secure dominating set is denoted by $\gamma_s^*(G)$. Thus $\gamma_s^*(G) = \gamma^*(G)$. □

Theorem 4.12. *Every 2-secure dominating set of an intuitionistic fuzzy graph G is a secure dominating set of G.*

Proof. Let S be a 2-secure dominating set of an intuitionistic fuzzy graph G. Then every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is 2-dominating set. Since G is 2-secure dominating set, by definition, G is 2-dominating set and every 2-dominating set is a dominating set. Thus every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is dominating set. So S is a secure dominating set of G. □

Theorem 4.13. *If G is an intuitionistic fuzzy graph, then $\gamma^*_{2s}(G) \geq \gamma^*_2(G)$.*

Proof. By Theorem 4.12, every 2-secure dominating set of an intuitionistic fuzzy graph G is a secure dominating set of G. Thus every minimum 2-secure dominating set of G is also a secure dominating set of G. So $\gamma^*_{2s}(G) \geq \gamma^*_2(G)$. □

Theorem 4.14. *If S is a 2-dominating set of a path of an intuitionistic fuzzy graph G, then S is not 2-secure dominating set.*

Proof. Let P_n be a path of G and S is a 2-dominating set of a path P_n of an intuitionistic fuzzy graph G. Then S contain two pendent vertices v_i and v_j. Now for some $u \in V - S$ and u is adjacent to v_i. Thus $(S - \{v_i\})U\{u\}$ is not 2-dominating set. So S is not 2-secure dominating set. □

Theorem 4.15. *If S is a dominating set of a complete bipartite intuitionistic fuzzy graph, then S is not a secure dominating set.*

Proof. Given that S is a dominating set of a complete bipartite intuitionistic fuzzy graph say, $k_{m,n}$. Then S should contain a vertex in V_1 say u and a vertex in V_2 say v. Now for some $v_i \in V - S$ and v_i is adjacent to $u \in V_1$. Thus $(S - \{u\})U\{v_i\}$ is not dominating set. So S is not a secure dominating set. □

Theorem 4.16. *Let G be an intuitionistic fuzzy graph with only strong edges and without isolated vertices and S is a minimal secure dominating set. Then $V - S$ is a secure dominating set of G.*

Proof. Given that S is a minimal secure dominating set. Then by definition, every vertex $u \in V - S$ is adjacent to a vertex $v \in S$ such that $(S - \{v\})U\{u\}$ is dominating set. We have to prove that $V - S$ is a secure dominating set of G. Suppose $V - S$ is not secure dominating set. Then there exist vertex $w \in S$ is adjacent to a vertex $x \in V - S$ such that $(S - \{x\})U\{w\}$ is not dominating set. Thus x is not dominated by any vertex in S which is contradiction to our assumption that S is minimal secure dominating set and G has no isolated vertices and has only strong edges. So $V - S$ is a secure dominating set of G.

□

5. Conclusion

In this paper, the secure domination set and secure total domination set in fuzzy graphs and intuitionistic fuzzy graphs have been investigated. Secure domination number $\gamma_s(G)$ and secure total domination number $\gamma_{st}(G)$ for several classes of fuzzy graphs and intuitionistic fuzzy graphs have been determined. The concepts of fuzzy graphs and intuitionistic fuzzy graphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks, and medical diagnosis. We plan to extend our research of fuzzification to secure connectivity of an IFG and its applicability.

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