Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 14, No. 5, (November 2017), pp. 463–474
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 14, No. 5, November 2017

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Decompositions of soft continuity and soft AB-continuity

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Received 7 May 2016; Revised 28 May 2016; Accepted 31 March 2017

Abstract. In this paper, we introduce soft semi-regular sets, soft AB-sets and soft αAB-sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We discuss their relationships with other weaker forms of soft open sets like soft semi-open sets, soft A-sets and soft B-sets. We also introduce the concepts of soft AB-continuous function and soft αAB-continuous function. The new decompositions of soft continuity are provided.

2010 AMS Classification: 06D72, 54A40

Keywords: Soft set, Soft topological space, Soft semi-regular set, Soft AB-set, Soft αAB-set, Soft AB-continuity, Soft αAB-continuity.

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1. Introduction

Molodtsov [15] initiated a novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Later Maji et al. [13] presented some new definitions on soft sets such as a subset, the complement of a soft set. Research works on soft sets are progressing rapidly in recent years.

Shabir and Naz [16] defined the soft topological spaces which are defined over an initial universe with a fixed set of parameters and studied some basic notions of soft topological spaces such as soft open and closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft separation axioms. Later Aygünoğlu and Aygün [3], Min [14], Zorlutuna et al. [20] and Hussain and Ahmad [7] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces.

Recently, weak and strong forms of soft open sets and soft closed sets were studied by many authors. First, Chen [4] investigated the concept of soft semi-open sets in soft topological spaces and studied some properties of them. Later Arockiarani and Arokialancy [2] defined soft β-open sets and some other weak forms of soft open sets in soft topological spaces. Kandil et al. [8] studied the notions of γ-operation, pre-open soft sets, α-open soft sets, semi-open soft sets and β-open soft sets to soft topological spaces. Akdağ and Özkan [1] studied soft α-open sets and soft α-continuous functions in soft topological spaces. Also, Kannan [9] defined soft generalized closed and open sets in soft topological spaces and he showed that every soft closed set is soft generalized closed. Yuksel et al. [18] continued investigating the properties of soft generalized closed and open sets. After then Yuksel et al. [19] defined soft regular generalized closed and open sets in soft topological spaces. Also, they showed that every soft generalized closed set is soft regular generalized closed. Güzel Ergül et al. [6] defined soft generalized preregular closed sets which are weaker form of the above mentioned generalizations such as soft generalized closed set and soft regular generalized closed set and study the properties of them in soft topological spaces. Then, Mahmood [12] studied soft regular generalized b-closed sets and their properties in soft topological spaces. Also, the concepts of soft A-set, soft B-set, soft A-continuous and soft B-continuous functions in soft topological spaces were introduced by Tozlu and Yuksel [17].

The aim of this paper is to study soft AB-sets very closely related to soft A-sets and soft B-sets, in fact properly placed between them. The concept of soft αAB-set is also introduced. The new decompositions of soft continuity and a decomposition of soft AB-continuity are produced at the end of the paper.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies.

Definition 2.1 ([15])**.** Let X be an initial universe set and E be the set of all possible parameters with respect to X. Let $P(X)$ denote the power set of X. A pair (F, A) is called a soft set over X, where $A \subseteq E$ and $F : A \to P(X)$ is a set valued mapping.

In other words, a soft set over X is a parameterized family of subsets of the universe X.

The set of all soft sets over X is denoted by $SS(X)_E$.

Definition 2.2 ([13])**.** A soft set (F, A) over X is said to be a null soft set, denoted by Φ, if for all $e \in A$, $F(e) = \varnothing$.

A soft set (F, A) over X is said to be an absolute soft set, denoted by $\widetilde{A}$, if for all $e \in A$, $F(e) = X$.

Definition 2.3 ([16])**.** Let Y be a nonempty subset of X. Then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.

Definition 2.4 ([13])**.** For two soft sets (F, A) and (G, B) over X,

(i) (F, A) is said to be a soft subset of (G, B), denoted by $(F, A) \sqsubseteq (G, B)$, if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$,

(ii) (F, A) is said to be a soft super set of (G, B), denoted by $(G, B) \sqsubseteq (F, A)$, if (G, B) is a soft subset of (F, A),

(iii) (F, A) and (G, B) are said to be soft equal, denoted by $(F, A) = (G, B)$, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.5 ([13]). The union of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$, if $e \in A \backslash B$, $H(e) = G(e)$, if $e \in B \backslash A$, $H(e) = F(e) \cup G(e)$, if $e \in A \cap B$. We write $(F, A) \sqcup (G, B) = (H, C)$.

Definition 2.6 ([5]). The intersection (H, C) of two soft sets (F, A) and (G, B) over X, denoted by $(F, A) \sqcap (G, B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, for all $e \in C$.

Definition 2.7 ([16]). The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \backslash (G, E)$, is defined as $H(e) = F(e) \backslash G(e)$, for all $e \in E$.

Definition 2.8 ([16]). The relative complement of a soft set (F, E) is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$, where $F^c : E \longrightarrow P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$, for all $e \in E$.

Definition 2.9 ([20]). Let (F, E) be a soft set over X. (F, E) is called a soft point in X, denoted by e_F, if for the element $e \in E$, $F(e) \neq \varnothing$ and $F(e') = \varnothing$, for all $e' \in E \backslash \{e\}$. The soft point e_F is said to be in the soft set (G, E), denoted by $e_F \in (G, E)$, if for the element $e \in E$ and $F(e) \subseteq G(e)$.

Proposition 2.10 ([20]). *Let $e_F \in \widetilde{X}$ and $(G, E) \sqsubseteq \widetilde{X}$. If $e_F \in (G, E)$, then $e_F \notin (G, E)^c$.*

Definition 2.11 ([16]). Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X, if

(i) Φ, $\widetilde{X} \in \tau$,
(ii) if (F, E), $(G, E) \in \tau$, then $(F, E) \sqcap (G, E) \in \tau$,
(iii) if $\{(F_i, E)\}_{i \in I} \in \tau$, $\forall i \in I$, then $\sqcup_{i \in I}(F_i, E) \in \tau$.

The triplet (X, τ, E) is called a soft topological space over X. Every member of τ is called a soft open set in X. A soft set (F, E) over X is called a soft closed set in X, if its relative complement $(F, E)^c$ belongs to τ. We will denote the family of all soft open sets (resp., soft closed sets) of a soft topological space (X, τ, E) by SOS(X, τ, E) (resp., SCS(X, τ, E)).

Definition 2.12. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X.

(i) [16] The soft closure of (F, E) is the soft set in X defined as:

$$cl(F, E) = \sqcap \{(G, E) : \ (G, E) \text{ is soft closed and } (F, E) \sqsubseteq (G, E)\}.$$

(ii) [20] The soft interior of (F, E) is the soft set in X defined as:

$$int(F, E) = \sqcup \{(H, E) : (H, E) \text{ is soft open and } (H, E) \sqsubseteq (F, E)\}.$$

Clearly, $cl(F, E)$ is the smallest soft closed set over X which contains (F, E) and $int(F, E)$ is the largest soft open set over X which is contained in (F, E).

Throughout the paper, the spaces X and Y (or (X, τ, E) and (Y, ν, K)) stand for soft topological spaces assumed unless stated otherwise.

Definition 2.13. A soft set (F,E) is called:

(i) soft semi-open [4] in a soft topological space X, if $(F,E) \sqsubseteq cl(int(F,E))$.

(ii) soft pre-open [2] in a soft topological space X, if $(F,E) \sqsubseteq int(cl(F,E))$.

(iii) soft α-open [1] in a soft topological space X, if $(F,E) \sqsubseteq int(cl(int(F,E)))$.

The relative complement of a soft semi-open (resp., soft pre-open, soft α-open) set is called a soft semi-closed (resp., soft pre-closed, soft α-closed) set.

We will denote the family of all soft semi-open (resp., soft pre-open and soft α-open) sets of a soft topological space (X,τ,E) by SSOS(X,τ,E) (resp., SPOS(X,τ,E) and SαOS(X,τ,E)).

Remark 2.14 ([4]). A soft set (F,E) in a soft topological space (X,τ,E) will be termed soft semi-open if and only if there exists a soft open set (G,E) such that

$$(G,E) \sqsubseteq (F,E) \sqsubseteq cl(G,E).$$

Definition 2.15 ([4]). Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X.

(i) The soft semi-closure of (F,E) is the soft set in X defined as:

$$cl_s(F,E) = \sqcap\{(H,E) : \ (H,E) \text{ is soft semi-closed and } (F,E) \sqsubseteq (H,E)\}.$$

(ii) The soft semi-interior of (F,E) is the soft set in X defined as:

$$int_s(F,E) = \sqcup\{(G,E) : (G,E) \text{ is soft semi-open and } (G,E) \sqsubseteq (F,E)\}.$$

$int_s(F,E)$ is soft semi-open and $cl_s(F,E)$ is soft semi-closed [4].

Theorem 2.16. [4] *Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. We have*

$$int(F,E) \sqsubseteq int_s(F,E) \sqsubseteq (F,E) \sqsubseteq cl_s(F,E) \sqsubseteq cl(F,E).$$

Definition 2.17 ([19]). Let (X,τ,E) be a soft topological space. A soft set (F,E) is called a soft regular open (resp. soft regular closed) set in X, if $(F,E) = int(cl(F,E))$ (resp. $(F,E) = cl(int(F,E))$).

We will denote the family of all soft regular open sets (resp. soft regular closed sets) of a soft topological space (X,τ,E) by $SROS(X,\tau,E)$ (resp. $SRCS(X,\tau,E)$).

Definition 2.18 ([17]). Let (X,τ,E) be a soft topological space. A soft set (F,E) is called a soft A-set in X, if $(F,E) = (G,E)\backslash(H,E)$, where (G,E) is a soft open set and (H,E) is a soft regular open set in X.

It is obvious that a soft set (F,E) is a soft A-set if and only if $(F,E) = (G,E) \sqcap (K,E)$, where (G,E) is a soft open set and (K,E) is a soft regular closed set.

Definition 2.19 ([17]). Let (X,τ,E) be a soft topological space. A soft set (F,E) is called a soft t-set in X, if $int(cl(F,E)) = int(F,E)$.

Definition 2.20 ([17]). Let (X,τ,E) be a soft topological space. A soft set (F,E) is called a soft B-set in X ,if $(F,E) = (G,E) \sqcap (H,E)$, where (G,E) is a soft open set and (H,E) is a soft t-set in X.

Remark 2.21. In a soft topological space (X, τ, E),
(1) every soft open set is soft α-open [1],
(2) every soft α-open set is soft pre-open and soft semi-open [1],
(3) every soft regular open (closed) set is soft open (closed) [19],
(4) every soft open set is a soft A-set [17],
(5) every soft A-set is soft semi-open [17],
(6) every soft open set is a soft B-set [17],
(7) every soft A-set is a soft B-set [17].

Definition 2.22 ([10])**.** Let $SS(X)_E$ and $SS(Y)_K$ be families of soft sets, $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings. Then the mapping $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ is defined as: for each $(F, E) \in SS(X)_E$ and each $(G, K) \in SS(Y)_K$,

(i) the image of (F, E) under f_{pu}, written as $f_{pu}(F, E) = (f_{pu}(F), p(E))$, is a soft set in $SS(Y)_K$ such that

$$f_{pu}(F)(y) = \begin{cases} \cup_{x \in p^{-1}(y) \cap A} u(F(x)), & p^{-1}(y) \cap A \neq \varnothing \\ \varnothing, & otherwise, \end{cases}$$

for all $y \in K$,

(ii) the inverse image of (G, K) under f_{pu}, written as $f_{pu}^{-1}(G, K) = (f_{pu}^{-1}(G), p^{-1}(K))$, is a soft set in $SS(X)_E$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))), & p(x) \in K \\ \varnothing, & otherwise, \end{cases}$$

for all $x \in E$.

Definition 2.23 ([20])**.** Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is called a soft continuous function, if for each $(G, K) \in \upsilon$, we have $f_{pu}^{-1}(G, K) \in \tau$.

Definition 2.24. Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is called:

(i) a soft semi-continuous function [11], if for each $(G, K) \in SOS(Y)$, we have $f_{pu}^{-1}(G, K) \in SSOS(X)$,

(ii) a soft α-continuous function [1], if for each $(G, K) \in SOS(Y)$, we have $f_{pu}^{-1}(G, K) \in S\alpha OS(X)$,

(iii) a soft pre-continuous function [1], if for each $(G, K) \in SOS(Y)$, we have $f_{pu}^{-1}(G, K) \in SPOS(X)$,

(iv) a soft A-continuous function [17], if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft A-set in X.

(v) a soft B-continuous function [17], if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft B-set in X.

Remark 2.25. Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then,
(1)[1] every soft continuous function is soft α-continuous,
(2)[1] every soft α-continuous function is soft semi-continuous and soft pre-continuous,
(3)[17] every soft continuous function is soft A-continuous,
(4)[17] every soft A-continuous function is soft semi-continuous,
(5)[17] every soft continuous function is soft B-continuous,

(6)[17] every soft A-continuous function is soft B-continuous.

Corollary 2.26 ([17])**.** *Let* (X, τ, E) *and* (Y, υ, K) *be soft topological spaces and* $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ *be a function. Then* f_{pu} *is soft continuous if and only if it is both soft pre-continuous and soft A-continuous.*

3. Soft AB-sets and Soft αAB-sets

Definition 3.1. A soft set (F, E) is called a soft semi-regular set in a soft topological space X, if it is both soft semi-open and soft semi-closed.

The family of all soft semi-regular sets of a soft topological space (X, τ, E) is denoted by $SSRS(X, \tau, E)$.

Theorem 3.2 ([8])**.** *Let* (X, τ, E) *be a soft topological space and* (F, E) *be a soft set over* X*. Then* (F, E) *is soft semi-closed if and only if* $(F, E) = (F, E) \sqcup int(cl(F, E))$*. That is, for any soft set* (F, E) *over* X*,* $cl_s(F, E) = (F, E) \sqcup int(cl(F, E))$*.*

Remark 3.3. Let (X, τ, E) be a soft topological space. If (F, E) is a soft set over X, then $int(cl(F, E)) \sqsubseteq cl_s(F, E)$.

Proof. The proof is obvious from Theorem 3.2. □

Proposition 3.4. *Let* (X, τ, E) *be a soft topological space. For a soft set* (F, E) *over* X*, the following are equivalent:*
(1) (F, E) *is soft semi-regular in* X*,*
(2) $(F, E) = int_s(cl_s(F, E))$*,*
(3) *there exists a soft regular open set* (G, E) *in* X *such that* $(G, E) \sqsubseteq (F, E) \sqsubseteq cl(G, E)$*.*

Proof. (1)$\Longrightarrow$ (2): Suppose (F, E) is a soft semi-regular set. Then $int_s(cl_s(F, E)) = int_s(F, E) = (F, E)$.
(2)$\Longrightarrow$ (3): Suppose $(F, E) = int_s(cl_s(F, E))$. Since $int(cl(G, E)) \sqsubseteq cl_s(G, E)$, for each soft set (G, E) over X by Remark 3.3, $int(cl(F, E)) \sqsubseteq int_s(cl_s(F, E)) = (F, E)$. Since (F, E) is soft semi-open, we have $(F, E) \sqsubseteq cl(int(F, E))$. Then we obtain

$$int(cl(F, E)) \sqsubseteq (F, E) \sqsubseteq cl(int(F, E)) \sqsubseteq cl(int(cl(F, E))),$$

where $int(cl(F, E))$ is soft regular open, since $int(cl(int(cl(F, E)))) = int(cl(F, E))$.
(3)$\Longrightarrow$ (1): It is clear that (F, E) is soft semi-open in X. Then

$$int(cl(F, E)) \sqsubseteq int(cl(G, E)) = (G, E) \sqsubseteq (F, E).$$

Thus (F, E) is soft semi-closed. So (F, E) is soft semi-regular in X. □

Proposition 3.5. *Let* (X, τ, E) *be a soft topological space and* (F, E) *a soft set over* X*. If* (F, E) *is soft semi-open, then* $cl_s(F, E)$ *is a soft semi-regular set in* X*.*

Proof. Since $cl_s(F, E)$ is soft semi-closed, we show that $cl_s(F, E)$ is soft semi-open in X. Since (F, E) is soft semi-open, $(G, E) \sqsubseteq (F, E) \sqsubseteq cl(G, E)$, for soft open set (G, E) in X. Then, we obtain $(G, E) \sqsubseteq cl_s(G, E) \sqsubseteq cl_s(F, E) \sqsubseteq cl(G, E)$. Thus $cl_s(F, E)$ is a soft semi-regular set in X. □

Definition 3.6. A soft set (F, E) is called a soft AB-set in a soft topological space X, if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is soft open and (H, E) is soft semi-regular.

The family of all soft AB-sets of a soft topological space (X, τ, E) is denoted by $S\mathrm{AB}S(X, \tau, E)$.

Example 3.7. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X, defined as follows:

$$\begin{aligned}(F_1, E) &= \{(e_1, \{x_2\}),\ (e_2, \{x_2\})\},\\ (F_2, E) &= \{(e_1, \{x_3\}),\ (e_2, \{x_3\})\},\\ (F_3, E) &= \{(e_1, \{x_2, x_3\}),\ (e_2, \{x_2, x_3\})\}.\end{aligned}$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Let (H, E) be a soft set over X such that $(H, E) = \{(e_1, \{x_2\}),\ (e_2, \{x_1, x_2\})\}$. Then clearly, (H, E) is a soft AB-set in X but not soft open.

Example 3.8. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), ..., (F_{11}, E)\}$, where (F_1, E), (F_2, E), ..., (F_{11}, E) are soft sets over X, defined as follows:

$$\begin{aligned}(F_1, E) &= \{(e_1, \{x_1\}),\ (e_2, \{x_1\})\},\\ (F_2, E) &= \{(e_1, \{x_2\}),\ (e_2, \{x_2\})\},\\ (F_3, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1, x_2\})\},\\ (F_4, E) &= \{(e_1, \{x_1, x_2, x_3\}),\ (e_2, \{x_1, x_3\})\},\\ (F_5, E) &= \{(e_1, \{x_1, x_2, x_4\}),\ (e_2, \{x_1, x_2, x_3\})\},\\ (F_6, E) &= \{(e_1, \{x_2\}),\ (e_2, \varnothing)\},\\ (F_7, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1\})\},\\ (F_8, E) &= \{(e_1, \{x_1, x_2, x_3\}),\ (e_2, \{x_1, x_2, x_3\})\},\\ (F_9, E) &= \{(e_1, X),\ (e_2, \{x_1, x_2, x_3\})\},\\ (F_{10}, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1, x_2, x_3\})\},\\ (F_{11}, E) &= \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1, x_3\})\}.\end{aligned}$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Let (G, E) be a soft set over X such that $(G, E) = \{(e_1, \{x_2, x_3\}),\ (e_2, \{x_3\})\}$. Then clearly, (G, E) is a soft AB-set in X but not soft semi-regular.

Proposition 3.9. *Every soft regular closed set in a soft topological space (X, τ, E) is soft semi-regular.*

Proof. Let (F, E) be a soft regular closed set. From Definition 2.17, $(F, E) \sqsubseteq cl(int(F, E))$ is obvious and (F, E) is soft semi-open. Since soft regular closed set is soft closed, $(F, E) = cl(F, E)$. Then, $int(cl(F, E)) \sqsubseteq int(F, E) \sqsubseteq (F, E)$ and $int(cl(F, E)) \sqsubseteq (F, E)$. Thus (F, E) is soft semi-closed. So (F, E) is soft semi-regular. $\square$

Remark 3.10. Since soft regular closed sets are soft semi-regular and since soft semi-regular sets are soft semi-closed, then the following implications are obvious:

$$\text{Soft A-set} \Longrightarrow \text{Soft AB-set} \Longrightarrow \text{Soft B-set}$$

The examples given below show that the converses of these implications are not true.

Example 3.11. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X and the soft set $(H, E) = \{(e_1, \{x_2\}),\ (e_2, \{x_1, x_2\})\}$ in Example 3.7. Then clearly, (H, E) is a soft AB-set in X but not a soft A-set.

Example 3.12. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X in Example 3.7. Let (G, E) be a soft set over X such that $(G, E) = \{(e_1, \{x_1\}),\ (e_2, \{x_1\})\}$. Then clearly, (G, E) is a soft B-set in X but not a soft AB-set.

Remark 3.13. Since the intersection of a soft open set and a soft semi-regular set is always soft semi-open in a soft topological space X, then the following implication is clear:

$$\text{Soft AB-set} \Longrightarrow \text{Soft semi-open set}$$

Example 3.14. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X, defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{x_4\}),\ (e_2, \{x_2\})\}, \\ (F_2, E) &= \{(e_1, \{x_2, x_4\}),\ (e_2, \{x_2, x_3, x_4\})\}, \\ (F_3, E) &= \{(e_1, \{x_1, x_2, x_4\}),\ (e_2, X)\}. \end{aligned}$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Let (G, E) be a soft set over X such that

$$(G, E) = \{(e_1, \{x_2, x_3, x_4\}),\ (e_2, \{x_2, x_3, x_4\})\}.$$

Then clearly, (G, E) is a soft semi-open set in X but not a soft AB-set.

Theorem 3.15. *Let (X, τ, E) be a soft topological space. A soft set (F, E) over X is a soft AB-set if and only if it is both a soft semi-open set and a soft B-set.*

Proof. Necessity is obvious from Remarks 3.10 and 3.13, we prove the sufficiency. Since (F, E) is a soft B-set, we have $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and $int(cl(H, E)) = int(H, E)$. Since (F, E) is soft semi-open, we have

$$\begin{aligned} (F, E) &\sqsubseteq cl(int(F, E)) = cl(int((G, E) \sqcap (H, E))) \\ &= cl(int(G, E) \sqcap int(H, E)) \sqsubseteq cl(int(G, E)) \sqcap cl(int(H, E)) \\ &= cl(G, E) \sqcap cl(int(H, E)). \end{aligned}$$

Then

$$\begin{aligned} (F, E) &= (G, E) \sqcap (H, E) = ((G, E) \sqcap (H, E)) \sqcap (G, E) \\ &\sqsubseteq (cl(G, E) \sqcap cl(int(H, E))) \sqcap (G, E) \\ &= (cl(G, E) \sqcap (G, E)) \sqcap cl(int(H, E)) \\ &= (G, E) \sqcap cl(int(H, E)). \end{aligned}$$

Since $(H, E) \sqsubseteq cl(int(H, E))$, we have (H, E) is soft semi-open. Also, since

$$int(cl(H, E)) \sqsubseteq int(H, E) \sqsubseteq (H, E),$$

(H, E) is soft semi-closed. So it is a soft semi-regular set. Hence we obtain (F, E) is a soft AB-set. ☐

Theorem 3.16. *Let (X, τ, E) be a soft topological space. A soft set (F, E) over X is a soft open set if and only if it is both a soft pre-open set and a soft AB-set.*

Proof. Necessity is trivial, we prove the sufficiency. Since every soft AB-set is a soft B-set, we have $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and $int(cl(H, E)) = int(H, E)$. Since (F, E) is soft pre-open, we have

$$\begin{aligned}(F, E) &\sqsubseteq int(cl(F, E)) = int(cl((G, E) \sqcap (H, E)))\\ &\sqsubseteq int(cl(G, E) \sqcap cl(H, E))\\ &= int(cl(G, E)) \sqcap int(cl(H, E))\\ &= int(cl(G, E)) \sqcap int(H, E).\end{aligned}$$

Then

$$\begin{aligned}(F, E) &= (G, E) \sqcap (H, E) = ((G, E) \sqcap (H, E)) \sqcap (G, E)\\ &\sqsubseteq (int(cl(G, E)) \sqcap int(H, E)) \sqcap (G, E)\\ &= (int(cl(G, E)) \sqcap (G, E)) \sqcap int(H, E)\\ &= (G, E) \sqcap int(H, E).\end{aligned}$$

Notice $(F, E) = (G, E) \sqcap (H, E) \sqsupseteq (G, E) \sqcap int(H, E)$, we have $(F, E) = (G, E) \sqcap int(H, E)$. Thus we obtain (F, E) is soft open. □

Definition 3.17. A soft set (F, E) is called a soft αAB-set in a soft topological space X, if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft α-open set and (H, E) is a soft semi-regular set in X.

The family of all soft αAB-sets of a soft topological space (X, τ, E) is denoted by $S\alpha\mathrm{AB}S(X, \tau, E)$.

Example 3.18. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X in Example 3.7 and (G, E) be a soft set over X such that $(G, E) = \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_1, x_2\})\}$. Then clearly, (G, E) is a soft αAB-set but not soft α-open.

Example 3.19. Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X and the soft set $(G, E) = \{(e_1, \{x_2, x_3, x_4\}),\ (e_2, \{x_2, x_3, x_4\})\}$ in Example 3.14. Clearly, (G, E) is a soft αAB-set but not a soft semi-regular set.

Theorem 3.20. *In a soft topological space (X, τ, E), every soft AB-set is a soft αAB-set.*

Proof. The proof is obvious, since every soft open set is a soft α-open set. □

Example 3.21. Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X and the soft set $(G, E) = \{(e_1, \{x_2, x_3, x_4\}),\ (e_2, \{x_2, x_3, x_4\})\}$ in Example 3.14. Then clearly, (G, E) is a soft αAB-set but not a soft AB-set.

The examples below show that a soft B-set need not be a soft αAB-set and a soft αAB-set need not be a soft B-set.

Example 3.22. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X in Example 3.7 and (H, E) be a soft set over X such that $(H, E) = \{(e_1, \{x_1\}),\ (e_2, \{x_1\})\}$. Then clearly, (H, E) is a soft B-set but not a soft αAB-set.

Example 3.23. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F, E)\}$, where (F, E) is a soft set over X, defined as follows:

$$(F, E) = \{(e_1, \{x_1\}),\ (e_2, \{x_2\})\}.$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Let (G, E) be a soft set over X such that $(G, E) = \{(e_1, \{x_1, x_2\}),\ (e_2, \{x_2\})\}$. Then clearly, (G, E) is a soft αAB-set in X but not a soft B-set.

Corollary 3.24. *Let (X, τ, E) be a soft topological space. A soft set (F, E) over X is a soft open set if and only if it is both a soft pre-open set and a soft αAB-set.*

Proof. It is obvious from Theorem 3.16. □

We have the following implications for a soft topological space (X, τ, E). These implications are not reversible.

$$\begin{tikzcd}
& \text{soft open set} \arrow[rrr] \arrow[d] & & & \text{soft } \alpha\text{-open set} \arrow[dl] \arrow[dr] & \\
& \text{soft A-set} \arrow[d] \arrow[rr] & & \text{soft semi-open set} & & \text{soft pre-open set} \\
& \text{soft AB-set} \arrow[urr] \arrow[dl] \arrow[dr] & & & & \\
\text{soft } \alpha\text{AB-set} & & \text{soft B-set} & & &
\end{tikzcd}$$

4. Soft AB-continuous functions and soft αAB-continuous functions

Definition 4.1. Let (X, τ, E) and (Y, υ, K) be soft topological spaces. Let $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings. Let $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then the function f_{pu} is called a soft AB-continuous function, if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft AB-set in X.

Definition 4.2. Let (X, τ, E) and (Y, υ, K) be soft topological spaces. Let $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings. Let $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then the function f_{pu} is called a soft αAB-continuous function, if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft αAB-set in X.

We have immediate results by Theorem 3.15, Theorem 3.16, Corollary 3.24. Then their proofs are omitted. Theorem 4.3 gives the relations between soft AB-continuous functions and other weaker forms of soft continuity. Theorem 4.4 gives the relation between soft AB-continuity and soft αAB-continuity. Theorem 4.5 gives a decomposition of soft AB-continuity. Also, Theorem 4.6 and Corollary 4.7 give decompositions of soft continuity.

Theorem 4.3. *Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then*

(1) *every soft A-continuous function is soft AB-continuous,*
(2) *every soft AB-continuous function is soft B-continuous,*
(3) *every soft AB-continuous function is soft semi-continuous.*

Theorem 4.4. *Let (X, τ, E) and (Y, υ, K) be soft topological spaces and f_{pu} : $SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then, every soft AB-continuous function is soft αAB-continuous.*

Theorem 4.5. *Let (X, τ, E) and (Y, υ, K) be soft topological spaces and f_{pu} : $SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is a soft AB-continuous function if and only if it is both soft semi-continuous and soft B-continuous.*

Theorem 4.6. *Let (X, τ, E) and (Y, υ, K) be soft topological spaces and f_{pu} : $SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is a soft continuous function if and only if it is both soft pre-continuous and soft AB-continuous.*

Corollary 4.7. *Let (X, τ, E) and (Y, υ, K) be soft topological spaces and f_{pu} : $SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is a soft continuous function if and only if it is both soft pre-continuous and soft αAB-continuous.*

5. Conclusions

In the present study, we have introduced soft semi-regular sets, soft AB-sets and soft αAB-sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We have presented their basic properties with the help of some counterexamples. Also, we have introduced soft AB-continuous and soft αAB-continuous functions and we have obtained the new decompositions of soft continuity.

In our future work, we will go on studying the different soft sets such as soft αB-sets, soft αC-sets and the relationships between them and soft AB-sets, soft αAB-sets.

We expect that results in this paper will be helpfull for further studies in soft topological spaces.

Acknowledgements. This work was supported by Ahi Evran University Scientific Research Projects Coordination Unit. Project Number: FEF. A3.16.020.

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