Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 14, No. 5, (November 2017), pp. 503–513
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 14, No. 5, November 2017

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Received 26 May 2017; Revised 8 July 2017; Accepted 24 July 2017

Abstract. In this paper, we investigate the relationships between the various forms of soft separation axioms in soft topological spaces.

2010 AMS Classification: 54A05, 54D10

Keywords: Soft topology, Soft points, Soft T_i; $(i = 0, 1, 2, 3)$ spaces, Soft regular spaces.

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1. Introduction

In [11], Shabir and Naz introduced the concept of soft topology as a parametrized form of topological structures using soft set theory. In [7]and [8] Matejdes showed that any soft topological space is homeomorphic to a topological space $(A\times X, \tau_{A\times X})$ so that many soft topological notions and results can be derived from general topology. Since its advent a lot of work has been done in this field and during this period various soft axioms have been introduced and investigated by several authors. In [11], the soft closure of a soft set is defined and has been applied to characterize soft closed sets in soft topological spaces. Further, Georgiou and Megaritis [2] introduced the concept of parametrized soft open neighborhoods of points in soft topological spaces and applied them to characterize soft closure and soft closed sets. Also various forms of soft separation axioms in soft topological spaces are seen in literature. Georgiou et al. [3] have used family of soft open neighborhoods whereas Shabir and Naz [11] have used the family of soft sets of soft topological spaces to define various soft separation axioms. Recently, Hussain and Ahmad [5] and Tantawy et al. [13] have applied the concept of soft points given in [14] to define soft separation axioms in soft topological spaces. Also [1] have investigated various soft separation axioms in terms of α open sets.

In Section 2, we recall the basic concepts related to soft topological structures.

In Section 3, we show the relationships between the various forms of soft-T_i; $(i = 0, 1, 2, 3)$ axioms introduced by several authors in [3], [5], [11] and [13]. We also point out the errors of Remark 2 and Theorem 4 of Section 4 of [5] (See Remark 3.11

and Remark 3.23 below). Further we also show the error made in Theorem 3.33 of [13] (See Remark 3.25 below).

Note: As various forms of soft T_i axioms have been introduced in several papers as mentioned above, so in order to differentiate between these soft separation axioms we use the following notations. Throughout the paper, the soft separation axioms given by Georgiou [3] will be denoted by soft-T_i^G; $(i = 0, 1, 2, 3)$, those given by Hussain [5] will be denoted by soft-T_i^H; $(i = 0, 1, 2, 3)$, those given by Shabir [11] will be denoted by soft-T_i^S; $(i = 0, 1, 2, 3)$ and those given by Tantawy [13] will be denoted by soft-T_i^T; $(i = 0, 1, 2, 3)$. One of the motivation of this paper is to clear the confusion generated due to different concepts being used in the study of soft topological spaces. The relationships between the various soft separation axioms have been clearly shown in this paper.

2. Preliminaries

Throughout this paper, let X be an initial universe set, $\mathbf{P}(X)$ be the power set of X and A be a set of parameters.

Definition 2.1. (i) [10] A pair (F, A) is called a soft set over X, where F is a mapping given by, $F : A \to \mathbf{P}(X)$. $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A). The family of all soft sets (F, A) over X is denoted by $SS(X, A)$.

(ii) [6] A soft set (F, A) over X is said to be a null soft set, denoted by Φ_A , if for all $e \in A$, $F(e) = \phi$.

(iii) [6] A soft set (F, A) over X is said to be an absolute soft set, denoted by X_A, if for all $e \in A$, $F(e) = X$.

(iv) [6] For two soft sets (F, A) and (G, A) over a common universe X, (F, A) is a soft subset of (G, A) if $F(e) \subseteq G(e)$, for all $e \in A$ and is denoted by $(F, A) \mathrel{\widetilde{\subseteq}} (G, A)$.

(v) [14] The soft set $(F, A) \in SS(X, A)$ is called a soft point in X_A, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e^{'}) = \phi$, for all $e^{'} \in A - \{e\}$.

(vi)[14] The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \tilde{\in} (G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.2 ([11])**.** Let τ be the collection of soft sets over X with the fixed set of parameters A. Then τ is said to be a soft topology on X, if

(i) Φ_A, X_A belong to τ,

(ii) the union of any number of soft sets in τ belongs to τ,

(iii) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, A) is called a soft topological space over X. The members of τ are called soft open sets. The soft complement of a soft open set is called a soft closed set in (X, τ, A).

Definition 2.3 ([14])**.** Let X be a soft topological space and let $(F, A) \in SS(X, A)$. Then the soft closure of (F, A), denoted by $\overline{(F, A)}$, is the intersection of all soft closed supersets of (F, A).

Definition 2.4 ([11])**.** Let $(F, A) \in SS(X, A)$ and let $x \in X$.

(i) $x \in (F, A)$ read as x belongs to the soft set (F, A), whenever $x \in F(e)$, for all $e \in A$. For any $x \in X$, $x \notin (F, A)$, if $x \notin F(e)$, for some $e \in A$.

(ii) $(x, A) \in SS(X, A)$ denotes the soft set for which $x(e) = \{x\}$, for all $e \in A$.

Definition 2.5 ([13]). (i) x_e denotes the soft point $(F, A) \in SS(X, A)$ such that for the element $e \in A$, $F(e) = \{x\}$ and $F(e^{'}) = \phi$, for all $e^{'} \in A - \{e\}$.

(ii) The soft point x_e is said to be in the soft set (G, A), denoted by $x_e \widetilde{\in} (G, A)$, if for the element $e \in A$, $F(e) = \{x\} \subseteq G(e)$.

Definition 2.6 ([5]). The soft points $e_G, e_H \in SS(X, A)$ are distinct, denoted as $e_G \neq e_H$, if for their corresponding soft sets (G, A) and (H, A), $(G, A) \widetilde{\cap} (H, A) = \Phi_A$.

Definition 2.7 ([3]). Let $(F, A) \in SS(X, A)$, $e \in A$ and $x \in X$. Then $x \in_e (F, A)$ if and only if $x \in F(e)$ and $x \notin_e (F, A)$ if and only if $x \notin F(e)$.

We now recall some results which are used in further sections to obtain results.

Theorem 2.8 ([11]). *Let (X, τ, A) be a soft topological space over X. Then the collection $t_e = \{F(e) : (F, A) \in \tau\}$ for each $e \in A$, defines a topology on X.*

Theorem 2.9 ([13]). *For $(F, A) \in SS(X, A)$ and $e \in A$, $x_e \widetilde{\in} (F, A)$ if and only if $x \in_e (F, A)$.*

3. Soft Separation Axioms

3.1. **Soft-T_0 spaces.** First of all we give all the definitions of soft-T_0 spaces present in various articles as mentioned in Section 2 above.

Definition 3.1. A soft topological space (X, τ, A) over X is said to be:

(i) soft-T_0^G [3], if for each pair of distinct points $x, y \in X$ and for every $e \in A$, there exists a soft open set (F, A) such that either $x \in_e (F, A)$, $y \notin_e (F, A)$ or $y \in_e (F, A)$, $x \notin_e (F, A)$,

(ii) soft-T_0^H [5], if for each pair of distinct soft points e_F and e_G, there exist soft open sets (F_1, A) and (F_2, A) such that $e_F \widetilde{\in} (F_1, A)$, $e_G \widetilde{\notin} (F_1, A)$ or $e_G \widetilde{\in} (F_2, A)$, $e_F \widetilde{\notin} (F_2, A)$,

(iiiP) soft-T_0^S [11], if for each pair of distinct points $x, y \in X$, there exist soft open sets (F, A) and (G, A) such that $x \in (F, A)$, $y \notin (F, A)$ or $y \in (G, A)$, $x \notin (G, A)$,

(iv) soft-T_0^T [13], if for each pair of soft points x_e and y_e such that $x, y \in X$ and $x \neq y$, there exist soft open sets (F, A) and (G, A) such that $x_e \widetilde{\in} (F, A)$, $y_e \widetilde{\notin} (F, A)$ or $y_e \widetilde{\in} (G, A)$, $x_e \widetilde{\notin} (G, A)$.

Now show the relationships between these various soft-T_0 spaces given in Definition 3.1.

Theorem 3.2. *For a soft topological space (X, τ, A) over X the following hold:*

1. *Every soft-T_0^H space X is soft-T_0^T,*
2. *The space X is soft-T_0^T if and only if it is soft-T_0^G.*

Proof. (1) It follows from the fact that x_e are also soft points of the form e_F.

(2) It follows from Theorem 3.33 of [13]. ∎

The following diagram depicts the relationships of various soft T_0 spaces where $A \not\Leftrightarrow B$ denotes that A and B are independent of each other.

$$\begin{tikzcd}
 & \mathbf{soft-T_0^S} \arrow[dl, Leftrightarrow, "/" marking] \arrow[d, Leftrightarrow, "/" marking] & \\
\mathbf{soft-T_0^H} \arrow[r, Rightarrow] & \mathbf{soft-T_0^T} \arrow[r, Leftrightarrow] & \mathbf{soft-T_0^G}
\end{tikzcd}$$

The following examples justify the relationships shown in the above diagram.

Example 3.3. Let $X = \{x, y\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A)\}$, where

$$F_1(e_1) = \{x\}, F_1(e_2) = \{y\};$$
$$F_2(e_1) = \{y\}, F_2(e_2) = \{x\}.$$

Then (X, τ, A) is a soft-T_0^H space and hence a soft-T_0^T space. But the only soft open set containing the points x and y is X_A. Thus it is not a soft-T_0^S space.

Example 3.4. Let $X = \{x, y\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A)\}$, where

$$F_1(e_1) = \{x\}, F_1(e_2) = X.$$

Then (X, τ, A) is a soft-T_0^S space as for points x and y, $x \in (F_1, A)$ and $y \notin (F_1, A)$. But soft open sets containing the soft point x_{e_2} also contain the soft point y_{e_2}. Thus it is not soft-T_0^T and so not a soft-T_0^H space.

3.2. **Soft-T_1 spaces.** Now we give all the definitions of soft-T_1 spaces present in various articles as mentioned before.

Definition 3.5. A soft topological space (X, τ, A) over X is said to be:

(i) soft-T_1^G [3], if for each pair of distinct points $x, y \in X$ and for every $e \in A$, there exists a soft open set (F, A) such that $x \in_e (F, A)$, $y \notin_e (F, A)$,

(ii) soft-T_1^H [5], if for each pair of distinct soft points e_F and e_G, there exist soft open sets (F_1, A) and (F_2, A) such that $e_F \widetilde{\in} (F_1, A)$, $e_G \widetilde{\notin} (F_1, A)$ and $e_G \widetilde{\in} (F_2, A)$, $e_F \widetilde{\notin} (F_2, A)$,

(iii) soft-T_1^S [11], if for each pair of distinct points $x, y \in X$, there exist soft open sets (F, A) and (G, A) such that $x \in (F, A)$, $y \notin (F, A)$ and $y \in (G, A)$, $x \notin (G, A)$,

(iv) soft-T_1^T [13], if for each pair of soft points x_e and y_e such that $x, y \in X$ and $x \neq y$, there exist soft open sets (F, A) and (G, A) such that $x_e \widetilde{\in} (F, A)$, $y_e \widetilde{\notin} (F, A)$ and $y_e \widetilde{\in} (G, A)$, $x_e \widetilde{\notin} (G, A)$.

Before we show the relationships between the various soft-T_1 spaces given in Definition 3.5, we prove a few results.

The following result is derived for a T_1 space in general topological space.

Lemma 3.6. *A topological space (X, t) is T_1 if and only if for any disjoint non-empty sets F and G in X there exist open sets P and Q in topology t such that $F \subseteq P$, $G \nsubseteq P$ and $G \subseteq Q$, $F \nsubseteq Q$.*

Proof. The proof follows from the fact that every T_1 topological space contains the co-finite topology. $\square$

Theorem 3.7. (1) *If a soft topological space (X, τ, A) is soft-T_1^T then the collection, $t_e = \{F(e) : (F, A) \in \tau\}$ is a T_1 topology on X for each $e \in A$.*

(2) *If a soft topological space (X, τ, A) is soft-T_0^T then the collection, t_e is a T_0 topology on X for each $e \in A$.*

Proof. (1) Let (X, τ, A) be a soft-T_1^T space. Let x and y be any distinct points in X. Let us consider the collection $t_e = \{F(e) : (F, A) \in \tau\}$, for some $e \in A$. Then by Theorem 2.8, t_e is a topology on X. Since (X, τ, A) is soft-T_1^T, for the soft points x_e and y_e,$x \neq y$, there exist soft open sets (F, A) and (G, A) such that $x_e \widetilde{\in} (F, A)$, $y_e \widetilde{\notin} (F, A)$ and $y_e \widetilde{\in} (G, A)$, $x_e \widetilde{\notin} (G, A)$. Thus for the points x and y, $F(e)$ and $G(e)$ are open sets in topology t_e such that $x \in F(e)$, $y \notin F(e)$ and $y \in G(e)$, $x \notin G(e)$. So (X, t_e) is a T_1 topological space.

(2) The proof is similar to that of part (1). □

Theorem 3.8. *For a soft topological space (X, τ, A) over X, the following hold:*

(1) *the space X is soft-T_1^T if and only if soft-T_1^G,*
(2) *the space X is soft-T_1^H if and only if soft-T_1^T.*

Proof. (1) It follows from Theorem 3.33 of [13].

(2) The necessity is obvious. We now prove the sufficient part. Let (X, τ, A) be a soft-T_1^T space and let $e_F = (F, A)$ and $e_G = (G, A)$ be distinct soft points in X for some $e \in A$. By Definition 2.6, we have that $F(e)$ and $G(e)$ are non-empty sets such that $F(e) \cap G(e) = \phi$. Thus by Theorem 2.8, the collection t_e is a T_1 topology on X and by Lemma 3.6, there exist open sets $F_1(e)$ and $F_2(e)$ in t_e corresponding to the soft open sets (F_1, A) and (F_2, A) in soft topology τ such that $F(e) \subseteq F_1(e)$, $G(e) \nsubseteq F_1(e)$ and $G(e) \subseteq F_2(e)$, $F(e) \nsubseteq F_2(e)$. So, by Definition 2.1 (9), $e_F \widetilde{\in} (F_1, A)$, $e_G \widetilde{\notin} (F_1, A)$ and $e_G \widetilde{\in} (F_2, A)$, $e_F \widetilde{\notin} (F_2, A)$. Hence the soft topological space (X, τ, A) is soft-T_1^H. □

The following diagram depicts the relationships of various soft T_1 spaces.

$$\begin{tikzcd}
 & \mathbf{soft-T_1^S} \arrow[d, Leftrightarrow, "/" marking] & \\
\mathbf{soft-T_1^H} \arrow[ur, Leftrightarrow, "/" marking] \arrow[r, Leftrightarrow] & \mathbf{soft-T_1^T} \arrow[r, Leftrightarrow] & \mathbf{soft-T_1^G}
\end{tikzcd}$$

The following examples justify the relationships shown in the above diagram.

Example 3.9. Let $X = \{x, y\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$, where

$$F_1(e_1) = \{x\}, F_1(e_2) = \{y\};$$
$$F_2(e_1) = \{y\}, F_2(e_2) = \{x\};$$
$$F_3(e_1) = \{x\}, F_3(e_2) = \phi;$$
$$F_4(e_1) = X, F_2(e_2) = \{x\}.$$

Then (X, τ, A) is a soft-T_1^T space. For the points $x, y \in X$, there exists a soft open set (F_4, A) such that $x \in (F_4, A)$ and $y \notin (F_4, A)$, but the only soft open set containing y is X_A which also contains x. Thus it is a not a soft-T_1^S space.

Example 3.10. Let $X = \{x, y\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A), (F_3, A)\}$, where

$$F_1(e_1) = \{x\}, F_1(e_2) = X;$$

$$F_2(e_1) = X, F_2(e_2) = \{y\};$$

$$F_3(e_1) = \{x\}, F_3(e_2) = \{y\}.$$

Then (X, τ, A) is a soft-T_1^S space. For the distinct soft points x_{e_1} and y_{e_1}, there exists a soft open set (F_1, A) such that $x_{e_1}\widetilde{\in}(F_1, A)$ and $y_{e_1}\widetilde{\notin}(F_1, A)$, but the soft open sets which contain the soft point y_{e_1} are (F_2, A) and X_A, also contain x_{e_1}. Thus it is a not a soft-T_1^T space.

Remark 3.11. In Remark 2 of [5] it is given that:
In general, if (X, τ, A) is a soft-T_1^H-space (soft-T_0^H-space), then (X, t_e) is not necessarily a T_1-space (T_0-space) for $e \in A$.

But by Theorem 3.2, Theorem 3.7 and Theorem 3.8, we see that the Remark 2 of [5] is incorrect.

3.3. **Soft T_2 spaces.** Now we give the various definitions of soft-T_2 spaces.

Definition 3.12. A soft topological space (X, τ, A) over X is said to be:

(i) soft-T_2^G [3], if for each pair of distinct points $x, y \in X$ and for every $e \in A$, there exist soft open sets (F, A) and (G, A) such that $x \in_e (F, A)$, $y \in_e (G, A)$ and $F(e) \cap G(e) = \phi$,

(ii) soft-T_2^H [5], if for each pair of distinct soft points e_F and e_G, there exist soft open sets (F_1, A) and (F_2, A)such that $e_F\widetilde{\in}(F_1, A)$, $e_G\widetilde{\in}(F_2, A)$ and $(F_1, A)\widetilde{\cap}(F_2, A) = \Phi_A$.

(iii) soft-T_2^S [11], if for each pair of distinct points $x, y \in X$, there exist soft open sets (F, A) and (G, A) such that $x \in (F, A)$, $y \in (G, A)$ and $(F, A)\widetilde{\cap}(G, A) = \Phi_A$,

(iv) soft-T_2^T [13], if for each pair of soft points x_e and y_e such that $x, y \in X$ and $x \neq y$, there exist soft open sets (F, A) and (G, A) such that $x_e\widetilde{\in}(F, A)$, $y_e\widetilde{\in}(G, A)$ and $(F, A)\widetilde{\cap}(G, A) = \Phi_A$.

We now show the relationships between the various soft T_2 spaces of Definition 3.12.

Theorem 3.13. *For a soft topological space (X, τ, A) over X the following hold:*

1. *Every soft-T_2^T space X is soft-T_2^G.*
2. *Every soft-T_2^H space X is soft-T_2^T.*
3. *Every soft-T_2^S space X is soft-T_2^T*

Proof. (1) Let (X, τ, A) be a soft-T_2^T space and let x and y be distinct points in X. For some $e \in A$, x_e and y_e are soft points and since (X, τ, A) is soft-T_2^T, there exist soft open sets (F, A) and (G, A) such that $x_e\widetilde{\in}(F, A)$, $y_e\widetilde{\in}(G, A)$ and $(F, A)\widetilde{\cap}(G, A) = \Phi_A$. Now $(F, A)\widetilde{\cap}(G, A) = \Phi_A$ implies $F(e) \cap G(e) = \phi$. Thus it is soft-T_2^G which follows from Theorem 2.9.

The proofs of the rest of the parts is obvious from definitions. □

The following diagram depicts the relationships of various soft T_2 spaces.

$$\begin{tikzcd} & \mathbf{soft} - \mathbf{T_2^S} \arrow[d, Rightarrow] & \\ \mathbf{soft} - \mathbf{T_2^H} \arrow[ur, Leftrightarrow, "/" marking] \arrow[r, Rightarrow] & \mathbf{soft} - \mathbf{T_2^T} \arrow[r, Rightarrow] & \mathbf{soft} - \mathbf{T_2^G} \end{tikzcd}$$

The following examples justify the relationships shown in the above diagram.

Example 3.14. Let us consider the soft topological space (X, τ, A) of Example 3.3. This space is soft-T_2^H and hence a soft-T_2^T space but is not a soft-T_2^S space as the only soft open set containing the points x and y is X_A.

Example 3.15. Let $X = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_i, A); i = 1, 2, ...20\}$, where

$$\begin{gathered}
F_1(e_1) = \{x\}, F_1(e_2) = \{z\};\\
F_2(e_1) = \{y\}, F_2(e_2) = \{z\};\\
F_3(e_1) = \{z\}, F_3(e_2) = \{x\};\\
F_4(e_1) = \{z\}, F_4(e_2) = \{y\};\\
F_5(e_1) = \{x, y\}, F_5(e_2) = \{z\};\\
F_6(e_1) = \{x, z\}, F_6(e_2) = \{x, z\};\\
F_7(e_1) = \{x, z\}, F_7(e_2) = \{y, z\};\\
F_8(e_1) = \{y, z\}, F_8(e_2) = \{x, z\};\\
F_9(e_1) = \{y, z\}, F_9(e_2) = \{y, z\};\\
F_{10}(e_1) = \{z\}, F_{10}(e_2) = \{x, y\};\\
F_{11}(e_1) = X, F_{11}(e_2) = \{x, z\};\\
F_{12}(e_1) = X, F_{12}(e_2) = \{y, z\};\\
F_{13}(e_1) = \{y, z\}, F_{13}(e_2) = X;\\
F_{14}(e_1) = \{x, z\}, F_{14}(e_2) = X;\\
F_{15}(e_1) = \phi, F_{15}(e_2) = \{z\};\\
F_{16}(e_1) = \{z\}, F_{16}(e_2) = \phi;\\
F_{17}(e_1) = \{x, z\}, F_{17}(e_2) = \{z\};\\
F_{18}(e_1) = \{y, z\}, F_{18}(e_2) = \{z\};\\
F_{19}(e_1) = \{z\}, F_{19}(e_2) = \{x, z\};\\
F_{20}(e_1) = \{z\}, F_{20}(e_2) = \{y, z\}.
\end{gathered}$$

Then (X, τ, A) is a soft-T_2^G space. But for the points x_{e_1} and y_{e_1}, if $x_{e_1} \widetilde{\in} (G, A)$ and $y_{e_1} \widetilde{\in} (H, A)$, for some soft open sets (G, A) and (H, A) in X, then at least $z \in G(e_2)$ and $z \in H(e_2)$. Thus they are not disjoint. So it is not a soft-T_2^T space. (Note that the collection of soft sets, $\mathcal{S} = \{(F_i, A); i = 1, 2, 3, 4\}$ serves as a subbase for the soft topology τ in this Example.)

The following example is obtained from Example 64 of [12] and Example 3.7 (3) of [3].

Example 3.16. Let (X, t) be a topological space, where $X = \mathbf{R}$, the set of real numbers, $K = \{\frac{1}{n} : n = 1, 2, 3....\}$. We define the topology t by letting $O \in t$, if $O = U - B$, where $B \subset K$ and U is the open set in Eucledian topology on X.
Now let $A = \{e_1, e_2, e_3\}$, $\tau = \{(F_U, A) : U \in t\}$, where

$$F_U(e) = U; \text{for every } e \in A.$$

Then soft topological space is (X, τ, A) is a soft-T_2^S space. Thus a soft-T_2^T space as the topological space (X, t) is T_2. Let us consider the distinct soft points $e_{1_G} = \{(e_1, \{0\})\}$ and $e_{1_F} = \{(e_1, K)\}$. But these soft points do not have disjoint soft open sets containing them, as the disjoint sets K and $\{0\}$ do not have disjoint open sets in the topological space (X, t). So it is not a soft-T_2^H space.

3.4. **Soft regular and Soft T_3 spaces.** We now present the various soft regular and soft T_3 spaces.

Definition 3.17. A soft topological space (X, τ, A) over X is said to be:
(i) soft-T_3^G [3], if for every point $x \in X$, every $e \in A$ and each soft closed set (G, A) such that $x \notin_e (G, A)$, there exist soft open sets (F_1, A) and (F_2, A) such that $x \in_e (F_1, A)$, $G(e) \subseteq F_2(e)$ and $F_1(e) \cap F_2(e) = \phi$,
(ii) soft regular^H [5], if for each soft closed set (G, A) and $e_F \in SS(X, A)$ such that $e_F \widetilde{\notin} (G, A)$, there exist soft open sets (F_1, A) and (F_2, A) such that $e_F \widetilde{\in} (F_1, A)$, $(G, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \Phi_A$,
(iii) soft-T_3^H [5], if it is soft regular^H and soft-T_1^H,
(iv) soft regular^S [11], if for each soft closed set (G, A) and $x \in X$ such that $x \notin (G, A)$, there exist soft open sets (F_1, A) and (F_2, A) such that $x \in (F_1, A)$, $(G, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \Phi_A$,
(v) soft-T_3^S [11], if it is soft regular^S and soft-T_1^S,
(vi) soft-regular^T [13], if for each soft closed set (G, A) and $x_e \in SS(X, A)$ such that $x_e \widetilde{\notin} (G, A)$, there exist soft open sets (F_1, A) and (F_2, A) such that $x_e \widetilde{\in} (F_1, A)$, $(G, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \Phi_A$,
(vii) soft-T_3^T [13], if it is soft regular^T and soft-T_1^T.

The relationships between the various soft regular spaces and soft T_3 of Definition 3.17 are as follows:

Theorem 3.18. *For a soft topological space (X, τ, A) over X the following hold:*
(1) *every soft regular^S space X is soft regular^T,*
(2) *every soft regular^H space X is soft regular^T,*
(3) *every soft regular^T space X is soft-T_3^G,*
(4) *every soft-T_3^T space X is soft-T_3^G,*
(5) *every soft-T_3^H space X is soft-T_3^T,*
(6) *every soft-T_3^S space X is soft-T_3^T*

Proof. (1) Let (X, τ, A) be a soft regular^S space and let for some soft point x_e and some soft closed set (G, A) in X, $x_e \widetilde{\notin} (G, A)$. Then $x \notin G(e)$, for $e \in A$ implies $x \notin (G, A)$. Since X is soft regular^S, there exist soft open sets (F_1, A) and (F_2, A) such that $x \in (F_1, A)$, $(G, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \Phi_A$. Since $x \in (F_1, A)$, $x \in F_1(e)$, for all $e \in A$. Thus $x_e \widetilde{\in} (F_1, A)$. So the result follows.

(2) The proof is obvious.

(3) Let (X, τ, A) be a soft regular^T space and let for some point $x \in X$ and some soft closed set (G, A) in X, $x \notin_e (G, A)$, for some $e \in A$. Then by Theorem 2.9, $x_e \widetilde{\notin} (G, A)$. Since X is soft regular^T, there exist soft open sets (F_1, A) and (F_2, A) such that $x_e \widetilde{\in} (F_1, A)$, $(G, A) \widetilde{\subseteq} (F_2, A)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \Phi_A$. Thus $x \in_e (F_1, A)$, $G(e) \subseteq F_2(e)$ and $(F_1, A) \widetilde{\cap} (F_2, A) = \Phi_A$. So $F_1(e) \widetilde{\cap} F_2(e) = \phi$. Hence the result follows.

(4) Since every soft-T_3^T space X is soft regular^T, the proof follows from part (3).

(4) The proof follows from part (2) and Theorem 3.8.

(5) From part (1), it is enough to prove that the space-T_1^T. Since in [9] it is shown that every soft-T_3^S space is soft-T_2^S and in [13] it is shown that every soft-T_2^T space is soft-T_1^T. From Theorem 3.13, we have that every soft-T_3^S space is soft-T_1^T. Then the result follows. □

The following two diagrams depict the relationships of various soft regular and soft T_3 as shown in Theorem 3.18.

$$\begin{tikzcd} & \mathbf{soft\,regular^S} \arrow[d, Rightarrow] \arrow[dl, Leftrightarrow, "/" marking] & \\ \mathbf{soft\,regular^H} \arrow[r, Rightarrow] & \mathbf{soft\,regular^T} \arrow[r, Rightarrow] & \mathbf{soft - T_3^G} \end{tikzcd}$$

$$\begin{tikzcd} & \mathbf{soft - T_3^S} \arrow[d, Rightarrow] \arrow[dl, Leftrightarrow, "/" marking] & \\ \mathbf{soft - T_3^H} \arrow[r, Rightarrow] & \mathbf{soft - T_3^T} \arrow[r, Rightarrow] & \mathbf{soft - T_3^G} \end{tikzcd}$$

The following examples justify the relationships shown in the above diagram.

Example 3.19. Let $X = \{x, y\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A)\}$, where

$$F_1(e_1) = X, F_1(e_2) = \phi;$$
$$F_2(e_1) = \phi, F_2(e_2) = X.$$

Then (X, τ, A) is a soft-T_3^H (soft regular^H) space. Thus X is a soft-T_3^T (soft regular^T) space. The point $x \in X$ does not belong to the soft closed set $(F_1, A)^c$ as $x \notin (F_1(e_1))^c$ and the only soft open set containing x is X_A. So it is not soft regular^S. Hence it not a soft-T_3^S space.

Example 3.20. Let $X = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A)\}$, where

$$F_1(e_1) = \{x\}, F_1(e_2) = \{x, y\};$$
$$F_2(e_1) = \{y, z\}, F_2(e_2) = \{z\}.$$

Then (X, τ, A) is a soft regular^T space. The point $y \in X$ does not belong to the soft closed set $(F_1, A)^c$ as $y \notin (F_1(e_2))^c$ and the only soft open set containing y is X_A. Thus it is not a soft regular^S space. Also the soft point $e_1(G) = (e_1, \{x, y\})$ does not belong to $(F_1, A)^c$ as $\{x, y\} \notin (F_1(e_1))^c$ and the only soft open set containing $e_1(G)$ is X_A. So it is not even a soft regular^H space.

Example 3.21. Let $X = \{x, y\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A)\}$, where

$$F_1(e_1) = \{x\}, F_1(e_2) = \{x\};$$
$$F_2(e_1) = \{y\}, F_2(e_2) = \{y\}.$$

Then (X, τ, A) is a soft-T_3^S (soft regular^S) space and thus a soft-T_3^T (soft regular^T) space. Also the soft point $e_1(G) = (e_1, X)$ does not belong to $(F_1, A)^c$ as $\{x, y\} \notin (F_1(e_1))^c$ and the only soft open set containing $e_1(G)$ is X_A. So a soft-T_3^S (soft regular^S) space need not be soft-T_3^H (soft regular^H).

Example 3.22. Let us consider the soft topological space (X, τ, A) of Example 3.15. Then (X, τ, A) is a soft-T_3^G space. The soft point x_{e_1} does not belong to the soft closed set $(F_6, A)^c$ where $F_6^c(e_1) = \{y\}$, $F_6^c(e_2) = \{y\}$. But if $x_{e_1} \widetilde{\in} (G, A)$ and $(F_6, A)^c \widetilde{\subseteq} (H, A)$, for some soft open sets (G, A) and (H, A) in X, then at least $z \in G(e_2)$ and $z \in H(e_2)$. Thus they are not disjoint. So it is not soft regular^T and hence not a soft-T_3^T space.

Further we show some errors in papers related with the above axioms.

Remark 3.23. In Theorem 4 of Section 4 of [5], the following is proved.

Let (X, τ, A) be a soft topological space over X. Then the following statements are equivalent:

(1) (X, τ, A) is soft regular^H,

(2) for any soft open set (F, A) in (X, τ, A) and $e_G \tilde{\in} (F, A)$, there is a soft open set (G, A) containing e_G such that $e_G \tilde{\in} \overline{(G, A)} \tilde{\subseteq} (F, A)$,

(3) each soft point in (X, τ, A) has a soft neighborhood base consisting of soft closed sets.

The Theorem 4 of Section 4 of [5] is partly incorrect as the parts (2) and (3) are equivalent and part (1) implies (2) and (3). But the converse is not true which is shown in the example below.

Example 3.24. Let $X = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A)\}$, where

$$F_1(e_1) = \{y, z\}, F_1(e_2) = \phi;$$
$$F_2(e_1) = \{x\}, F_2(e_2) = X.$$

The soft topological (X, τ, A) satisfies parts (2) and (3) of the Theorem mentioned in Remark 3.23 above. The soft point, $e_{1_G} = (e_1, \{x, y\})$ is such that it does not belong to the soft closed set $(F_2, A)^c$, where $F_2^c(e_1) = \{y, z\}$, $F_2^c(e_2) = \phi$. But they do not have disjoint soft open sets containing them as the only soft open set containing the soft point e_{1_G} is X_A. Then the space X is not soft regular^H.

Remark 3.25. In Theorem 3.33 of [13] it is proved that

A soft topological space (X, τ, A) is a soft-T_i^T $(i = 0, 1, 2, 3)$ if and only if it is soft-T_i^G $(i = 0, 1, 2, 3)$.

In view of Theorem 3.2, Theorem 3.8, Example 3.15 and Example 3.22, the Theorem 3.33 of [13] is correct only for soft-T_i $(i = 0, 1)$ spaces but is incorrect for soft-T_i $(i = 2, 3)$ spaces.

Acknowledgements. The authors are grateful to the editor and the referees for their valuable suggestions which has improved the quality of the paper.

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