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Abstract. In this paper, the concepts of fuzzy cs dense sets and somewhere fuzzy continuous and somewhere fuzzy open functions between fuzzy topological spaces, are introduced and studied. Several characterizations of fuzzy somewhere dense sets and fuzzy cs sets are established.

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Keywords: Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy somewhere dense set, Fuzzy simply* open set, Fuzzy hyper connected space, Fuzzy P-space, Fuzzy submaximal space, Fuzzy Baire space.

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1. Introduction

In 1965 , L. A. Zadeh [17] introduced the concept of fuzzy sets as a new approach for modeling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by C. L. Chang [5] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

Continuity is one of the most important and fundamental properties that have been widely used in Mathematical Analysis. In the recent years, a considerable amount of research has been done on many types of continuity in general topology. T. M. Al-Shami [1] introduced a new class of sets, namely somewhere dense sets, in topological spaces. T. K. Das [6] introduced and studied density preserving mappings by means of somewhere dense sets in classical topology. The concept of fuzzy somewhere dense sets in fuzzy topological spaces was introduced by G. Thangaraj in [10]. In this paper, the notions of fuzzy cs dense sets in fuzzy topological spaces and somewhere fuzzy continuous functions, somewhere fuzzy open functions between fuzzy topological spaces, are introduced and studied. Several characterizations of fuzzy somewhere dense sets and fuzzy cs dense sets are established. In this paper, the conditions for fuzzy topological spaces to become fuzzy Baire spaces are obtained by means of somewhere fuzzy continuous functions and somewhere fuzzy open functions between topological spaces. Several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 ([5]). Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior and the closure of λ are defined respectively as follows :

(i) $Int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,

(ii). $Cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$.

Lemma 2.2 ([2]). *For a fuzzy set λ of a fuzzy topological space X,*

(1) $1 - Int(\lambda) = Cl(1-\lambda)$,

(2) $1 - Cl(\lambda) = Int(1-\lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called:

(i) fuzzy pre-open, if $\lambda \leq intcl(\lambda)$ and fuzzy pre-closed, if $clint(\lambda) \leq \lambda$ [4],

(ii) fuzzy regular-open, if $\lambda = intcl(\lambda)$ and fuzzy regular-closed, if $\lambda = clint(\lambda)$ [2],

(iii)] fuzzy β-open, if $\lambda \leq clintcl(\lambda)$ and fuzzy β-closed, if $intclint(\lambda) \leq \lambda$ [7].

Theorem 2.4 ([2]). *In a fuzzy topological space,*

(1) *the closure of a fuzzy open set is a fuzzy regular closed set,*

(2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

Definition 2.5 ([9]). A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense, if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X, T).

Definition 2.6 ([9]). A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense, if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $intcl(\lambda) = 0$, in (X, T).

Definition 2.7 ([9]). A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set, if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.8 ([11]). Let λ be a fuzzy first category set in a fuzzy topological space (X, T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).

Definition 2.9 ([11]). A fuzzy topological space (X, T) is called a fuzzy Baire space, if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T).

Definition 2.10 ([3]). A fuzzy topological space (X, T) is called a fuzzy submaximal space, if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1,\ \lambda \in T$.

Definition 2.11 ([8]). A fuzzy topological space (X, T) is said to be fuzzy hyperconnected, if every non-null fuzzy open subset of (X, T) is fuzzy dense in (X, T).

Definition 2.12 ([16]). A fuzzy topological space (X, T) is called a fuzzy globally disconnected space, if each fuzzy semi-open set in (X, T) is fuzzy open.

Definition 2.13 ([12]). A fuzzy topological space (X, T) is called a fuzzy P-space, if every non-zero fuzzy G_δ-set in (X, T) is fuzzy open.

Definition 2.14 ([14]). Let (X, T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy Baire set, if $\lambda = (\mu \wedge \delta)$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X, T).

Definition 2.15 ([15]). A fuzzy topological space (X, T) is called a fuzzy perfectly disconnected space, if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq (1 - \mu)$, $cl(\lambda) \leq 1 - cl(\mu)$, in (X, T).

Theorem 2.16 ([11]). *If $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, where (λ_i)'s are fuzzy dense and open sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy Baire space.*

Definition 2.17 ([9]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a somewhat fuzzy continuous function, if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ in (X, T) such that $\mu \leq f^{-1}(\lambda)$. That is, $int[f^{-1}(\lambda)] \neq 0$, in (X, T).

Definition 2.18 ([9]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a somewhat fuzzy open function, if $\lambda \in T$ and $f(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ in (Y, S) such that $\mu \leq f(\lambda)$. That is, $int[f(\lambda)] \neq 0$, in (Y, S).

Definition 2.19 ([7]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a fuzzy β-continuous function, if for each fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is a fuzzy β-open set in (X, T).

Definition 2.20 ([13]). A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy simply* open set, if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X, T) and $1 - \lambda$ is called a fuzzy simply* closed set in (X, T).

Definition 2.21 ([13]). Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a fuzzy simply* continuous function, if for each fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is a fuzzy simply* open set in (X, T).

Theorem 2.22 ([14]). *If λ is a fuzzy Baire set in a fuzzy globally disconnected and fuzzy P-space (X, T), then λ is a fuzzy open set in (X, T).*

Theorem 2.23 ([14]). *If λ is a fuzzy Baire set in a fuzzy Baire space (X, T), then λ is not a fuzzy dense set in (X, T).*

3. Fuzzy somewhere dense sets

Definition 3.1 ([10])**.** Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy somewhere dense set, if $intcl(\lambda) \neq 0$ in (X,T). That is, λ is a fuzzy somewhere dense set in (X,T) if there exists a non-zero fuzzy open set μ in (X,T) such that $\mu \leq cl(\lambda)$.

Definition 3.2. If λ is a fuzzy somewhere dense set in a fuzzy topological space (X,T), then $1-\lambda$ is called a fuzzy complement of fuzzy somewhere dense set in (X,T). It is to be denoted as fuzzy cs dense set in (X,T).

Example 3.3. Let $X = \{a,b,c\}$. The fuzzy sets α,β,γ,δ and η are defined on X as follows :

$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.4$; $\alpha(c) = 0.6$,
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.5$; $\beta(c) = 0.7$,
$\gamma : X \to [0,1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.3$,
$\delta : X \to [0,1]$ is defined as $\delta(a) = 0.3$; $\delta(b) = 0.1$; $\delta(c) = 0.7$,
$\eta : X \to [0,1]$ is defined as $\eta(a) = 0.6$; $\eta(b) = 0.6$; $\eta(c) = 0.7$.

Then $T = \{0,\alpha,\beta,1\}$ is a fuzzy topology on X. On computation one can see that $intcl(\alpha) \neq 0$; $intcl(\beta) \neq 0$; $intcl(1-\gamma) \neq 0$; $intcl(\delta) \neq 0$; $intcl(1-\delta) \neq 0$; $intcl(\eta) \neq 0$ and also $intcl(\gamma) = 0$; $intcl(1-\alpha) = 0$; $intcl(1-\beta) = 0$; $intcl(1-\eta) = 0$. Hence the fuzzy somewhere dense sets in (X,T) are α, β, $1-\gamma$, δ, $1-\delta$, η and $1-\alpha$, $1-\beta$, γ, $1-\delta$, δ, $1-\eta$ are fuzzy cs dense sets in (X,T).

Remark 3.4. A fuzzy somewhere dense set in a fuzzy topological space need not be a fuzzy open set. For, in Example 3.3, η is a fuzzy somewhere dense set in (X,T), but η is not a fuzzy open set in (X,T).

Proposition 3.5. *If λ is a fuzzy somewhere dense set in a fuzzy topological space (X,T), then there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq cl(\lambda)$.*

Proof. Let λ be a fuzzy somewhere dense set in (X,T). Then, there exists a non-zero fuzzy open set μ in (X,T) such that $\mu \leq cl(\lambda)$. Now $cl(\mu) \leq cl[cl(\lambda)]$. Since μ is a fuzzy open set in (X,T), by Theorem 2.4, the closure of μ is a fuzzy regular closed set in (X,T). Let $cl(\mu) = \eta$. Then, for the fuzzy somewhere dense set λ in (X,T), there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq cl(\lambda)$. □

Proposition 3.6. *If λ is a fuzzy cs dense set in a fuzzy topological space (X,T), then*

(1) *$int(\lambda)$ is not a fuzzy dense set in (X,T),*
(2) *There exists a fuzzy regular open set η in (X,T) such that $int(\lambda) < \eta$.*

Proof. (1) Let λ be a fuzzy cs dense set in (X,T). Then $1-\lambda$ is a fuzzy somewhere dense set in (X,T). Thus $intcl(1-\lambda) \neq 0$ in (X,T). This implies that $1-clint(\lambda) \neq 0$. So $clint(\lambda) \neq 1$. Hence $int(\lambda)$ is not a fuzzy dense set in (X,T).

(2) By (1), $int(\lambda)$ is not a fuzzy dense set in (X,T). Then there exists a fuzzy closed set μ in (X,T) such that $int(\lambda) < \mu < 1$. Thus $int[int(\lambda)] < int(\mu)$. That is, $int(\lambda) < int(\mu)$ in (X,T). Since μ is a fuzzy closed set in (X,T), by Theorem 2.4, $int(\mu)$ is a fuzzy regular open set in (X,T). Let $\eta = int(\mu)$. Then there exists a fuzzy regular open set η in (X,T) such that $int(\lambda) < \eta$. □

Proposition 3.7. *If λ is a non-zero fuzzy β-open set in a fuzzy topological space (X,T), then λ is a fuzzy somewhere dense set in (X,T).*

Remark 3.8. The following example shows that the converse of the above proposition need not be true. That is, a fuzzy somewhere dense set in a fuzzy topological space need not be a β-open set.

Example 3.9. Let $X = \{a, b, c\}$. The fuzzy sets α, β, γ and λ are defined on X as follows :

$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.7$; $\alpha(c) = 0.3$,
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.5$; $\beta(b) = 0.3$; $\beta(c) = 0.6$,
$\gamma : X \to [0,1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.5$; $\gamma(c) = 0.3$,
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.4$; $\lambda(b) = 0.3$; $\lambda(c) = 0.6$.

Then $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \beta \vee \gamma, \alpha \wedge \beta, \beta \wedge \gamma, \gamma \vee (\alpha \wedge \beta), 1\}$ is a fuzzy topology on X. On computation, one can see that $int(\lambda) = \beta \wedge \gamma$; $cl(\lambda) = 1 - \alpha; int(1 - \lambda) = \alpha$ and $cl(1 - \lambda) = 1 - (\beta \wedge \gamma)$ in (X,T). Now $intcl(\lambda) = int(1 - \alpha) = \beta \wedge \gamma \neq 0$, shows that λ is a fuzzy somewhere dense set in (X,T). But λ is not a fuzzy β-open set in (X,T), since $clintcl(\lambda) = 1 - (\alpha \vee \beta)$ and $\lambda \geq 1 - (\alpha \vee \beta)$.

Proposition 3.10. *If λ and μ are fuzzy somewhere dense sets in a fuzzy topological space (X,T), then $\lambda \vee \mu$ is a fuzzy somewhere dense set in (X,T).*

Proposition 3.11. *If λ is fuzzy closed set in a fuzzy topological space (X,T) with $int(\lambda) \neq 0$, then λ is a fuzzy somewhere dense set in (X,T).*

Proposition 3.12. *If λ is a fuzzy somewhere dense set in (X,T), then $cl(\lambda)$ is a fuzzy somewhere dense set in (X,T).*

Proposition 3.13. *If $\lambda \leq \mu$ and λ is a fuzzy somewhere dense set in a fuzzy topological space (X,T), then μ is a fuzzy somewhere dense set in (X,T).*

Proposition 3.14. *If λ and μ are fuzzy cs dense sets in a fuzzy topological space (X,T), then $\lambda \wedge \mu$ is a fuzzy cs dense set in (X,T).*

Proof. Let λ and μ be fuzzy cs dense sets in (X,T). Then $1 - \lambda$ and $1 - \mu$ are fuzzy somewhere dense sets in (X,T). That is, $int[cl(1 - \lambda)] \neq 0$ and $int[cl(1 - \mu)] \neq 0$, in (X,T). On the other hand,

$$\begin{aligned} intcl[1 - (\lambda \wedge \mu)] &= intcl[(1 - \lambda) \vee (1 - \mu)] \\ &= int[cl(1 - \lambda) \vee cl(1 - \mu)] \\ &\geq int[cl(1 - \lambda)] \vee int[cl(1 - \mu)]. \end{aligned}$$

Thus $intcl[1 - (\lambda \wedge \mu)] \neq 0$. So $1 - (\lambda \wedge \mu)$ is a fuzzy somewhere dense set in (X,T). Hence $\lambda \wedge \mu$ is a fuzzy cs dense set in (X,T). □

Proposition 3.15. *If λ is a fuzzy set in a fuzzy topological space (X,T) and μ is a fuzzy somewhere dense set in (X,T), then $\lambda \vee \mu$ is a fuzzy somewhere dense set in (X,T).*

Proposition 3.16. *If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then λ is a fuzzy somewhere dense set in (X,T).*

Proof. Let λ be a fuzzy simply* open set in (X,T). Then $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Now $intcl(\lambda) =$

$intcl(\mu \vee \delta) = int[cl(\mu) \vee cl(\delta) \geq intcl(\mu) \vee intcl(\delta)$. Since δ is a fuzzy nowhere dense set in (X, T), $intcl(\delta) = 0$. Thus $intcl(\lambda) \geq intcl(\mu) \vee 0 = intcl(\mu) \geq int(\mu) = \mu \neq 0$. So $intcl(\lambda) \neq 0$. Hence λ is a fuzzy somewhere dense set in (X, T). □

Remark 3.17. The following example shows that the converse of the above proposition need not be true. That is, a fuzzy somewhere dense set in a fuzzy topological space need not be a simply* open set.

Example 3.18. Let λ, μ, γ and δ be fuzzy sets defined on $I = [0, 1]$ as follows :

$$\lambda(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{2} \\ 2 - 2x, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2x - 1, & \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\gamma(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2 - 2x, & \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\delta(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then $T = \{0, \lambda, \delta, \lambda \vee \delta, 1\}$ is a fuzzy topology on I. The fuzzy somewhere dense set in (I, T) are $\lambda, \delta, \lambda \vee \delta, 1 - \lambda, 1 - \delta, \mu, 1 - \gamma$. The fuzzy nowhere dense sets in (I, T) are $1 - \lambda \vee \delta$, $1 - \mu$ and γ. Now μ is a fuzzy somewhere dense set in (I, T) but not a fuzzy simply* open set (I, T).

Proposition 3.19. *Let X and Y be fuzzy topological spaces such that X is product related to Y. If λ is a fuzzy somewhere dense set in X and μ is a fuzzy somewhere dense set in Y, then the product $\lambda \times \mu$ is a fuzzy somewhere dense set in the product space $X \times Y$.*

Proof. Let λ be a fuzzy somewhere dense set in X and μ be a fuzzy somewhere dense set in Y. Then $intcl(\lambda) \neq 0$ in (X, T) and $intcl(\mu) \neq 0$ in (Y, S). Since X is product related to Y, $intcl(\lambda \times \mu) = int[cl(\lambda) \times cl(\mu)] = intcl(\lambda) \times intcl(\lambda) \neq 0 \times 0 \neq 0$. Thus the product $\lambda \times \mu$ is a fuzzy somewhere dense set in the product space $X \times Y$. □

Theorem 3.20 ([14])**.** *If λ is a fuzzy Baire set in a fuzzy topological space (X, T), then $1 - \lambda = \alpha \vee \beta$, where α is a fuzzy closed set in (X, T) and β is a fuzzy first category set in (X, T).*

Proposition 3.21. *If λ is a fuzzy Baire set in a fuzzy topological space (X, T) in which fuzzy first category sets are fuzzy dense sets, then λ is a fuzzy cs dense set in (X, T).*

Proof. Let λ be a fuzzy Baire set in (X, T). Then, by Theorem 3.20, $1 - \lambda = \alpha \vee \beta$, where α is a fuzzy closed set in (X, T) and β is a fuzzy first category set in (X, T). Now $int[cl(1 - \lambda)] = int[cl(\alpha \vee \beta)] = int[cl(\alpha) \vee cl(\beta)] = int[cl(\alpha) \vee 1] = int(\alpha \vee 1) =$

$int(1) = 1 \neq 0$. That is, $intcl(1-\lambda) \neq 0$. Thus $1-\lambda$ is a fuzzy somewhere dense set in (X, T). So λ is a fuzzy cs dense set in (X, T). □

Proposition 3.22. *If λ is a fuzzy residual set in a fuzzy topological space (X, T) in which fuzzy first category sets are not fuzzy dense sets, then λ is a fuzzy somewhere dense set in (X, T).*

Theorem 3.23 ([12])**.** *If the fuzzy topological space (X, T) is a fuzzy P-space and if λ is a fuzzy first category set in (X, T), then λ is not a fuzzy dense set in (X, T).*

Proposition 3.24. *If λ is a fuzzy residual set in a fuzzy P-space (X, T), then λ is a fuzzy somewhere dense set in (X, T).*

Proposition 3.25. *If λ is a fuzzy somewhere dense set in a fuzzy perfectly disconnected space (X, T), then $cl(\lambda)$ is a fuzzy pre-closed set in (X, T).*

Proof. Let λ be a fuzzy somewhere dense set in (X, T). Then, $int\ cl(\lambda) \neq 0$ in (X, T). Now $int\ cl(\lambda) \leq cl(\lambda)$ implies that $int\ cl(\lambda) \leq 1-[1-cl(\lambda)]$ in (X, T). Since the fuzzy topological space (X, T) is fuzzy perfectly disconnected, $cl[int\ cl(\lambda)] \leq 1-cl[1-cl(\lambda)]$ and then $cl[int\ cl(\lambda)] \leq 1-[1-int\ cl(\lambda)]$. This implies that

$$cl[int\ cl(\lambda)] \leq int\ cl(\lambda). \tag{3.25.1}$$

But,

$$int\ cl(\lambda) \leq cl[int\ cl(\lambda)]. \tag{3.25.2}$$

From (3.25.1) and (3.25.2), $cl[int\ cl(\lambda)] = int\ cl(\lambda)$ in (X, T). Thus, $cl\ int[cl(\lambda)] \leq cl(\lambda)$, implies that $cl(\lambda)$ is a fuzzy pre-closed set in (X, T). □

Proposition 3.26. *If λ is a fuzzy cs dense set in a fuzzy topological space (X, T), then $clint(\lambda) \neq 1$ in (X, T).*

Proposition 3.27. *A fuzzy set λ in a fuzzy topological space (X, T) is a fuzzy cs dense set if and only if there exists a fuzzy closed set μ in (X, T) such that $int(\lambda) \leq \mu$.*

Proof. Let λ be a fuzzy cs dense set in (X, T). Then, by Proposition 3.26, $clint(\lambda) \neq 1$ in (X, T). Thus $int(\lambda)$ is not a fuzzy dense set in (X, T). So there exists a fuzzy closed set μ in (X, T) such that $int(\lambda) \leq \mu < 1$.

Conversely, suppose that α is a fuzzy set defined on X such that $int(\alpha) \leq \delta$, where $1-\delta \in T$ and $\delta \neq 1$. Then $1 - int(\alpha) \geq 1-\delta$. This implies that $cl(1-\alpha) \geq 1-\delta$. thus $intcl(1-\alpha) \geq int(1-\delta) = 1-\delta \neq 0$. So $1-\alpha$ is a fuzzy somewhere dense set in (X, T). Hence α is a fuzzy cs dense set in (X, T). □

Proposition 3.28. *A fuzzy set λ defined on X in a fuzzy topological space (X, T) is either a fuzzy nowhere dense set in (X, T) or a fuzzy somewhere dense set in (X, T).*

Proposition 3.29. *In a fuzzy topological space (X, T),*

(1) *if λ is a fuzzy nowhere dense set in (X, T), then $clint(1-\lambda) = 1$,*
(2) *if λ is a fuzzy somewhere dense set in (X, T), then $clint(1-\lambda) \neq 1$.*

4. Somewhere fuzzy continuous functions

Definition 4.1. A function $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) is called a somewhere fuzzy continuous function, if whenever $int(\lambda) \neq 0$, for a fuzzy set λ defined on Y, then $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X,T). That is, $f : (X,T) \to (Y,S)$ is a somewhere fuzzy continuous function, if $intcl[f^{-1}(\lambda)] \neq 0$ in (X,T), whenever $int(\lambda) \neq 0$, for a fuzzy set λ defined on Y.

Example 4.2. Let $X = \{a,b,c\}$. The fuzzy sets λ, μ, α and β are defined on X as follows :

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.7$; $\lambda(b) = 0.8$; $\lambda(c) = 0.6$,
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.9$; $\mu(b) = 0.6$; $\mu(c) = 0.5$,
$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.8$; $\alpha(b) = 0.7$; $\alpha(c) = 0.6$,
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.9$; $\beta(c) = 0.5$.

Then $T = \{0, \lambda, \mu, \lambda\vee\mu, \lambda\wedge\mu, 1\}$ and $S = \{0, \alpha, \beta, \alpha\vee\beta, \alpha\wedge\beta, 1\}$ are fuzzy topologies on X. On computation, one can see that, for the fuzzy sets $\lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1 - \lambda, 1 - \mu, 1 - (\lambda \vee \mu), 1 - (\lambda \wedge \mu), 1 - \alpha, 1 - \beta, 1 - (\alpha \vee \beta), 1 - (\alpha \wedge \beta)$ defined on X, $int(\lambda) = \alpha \wedge \beta \neq 0$; $int(\mu) = 0$; $int(\lambda \vee \mu) = \alpha$; $int(\lambda \wedge \mu) = 0$; $int(1 - \lambda) = 0$; $int(1-\mu) = 0$; $int[1-(\lambda\vee\mu)] = 0$; $int[1-(\lambda\wedge\mu)] = 0$; $int(1-\alpha) = 0$; $int(1-\beta) = 0$; $int[1 - (\alpha \vee \beta)] = 0$, $int[1 - (\alpha \wedge \beta)] = 0$ in (X,S). Thus $\alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, \lambda$ and $\lambda \vee \mu$ are the fuzzy sets defined on X with non-zero interior in (X,S). Define a function $f : (X,T) \to (X,S)$ by $f(a) = b$; $f(b) = a$; $f(c) = c$. On computation, $f^{-1}(\alpha) = \lambda \neq 0$; $f^{-1}(\beta) = \mu \neq 0$; $f^{-1}(\alpha \vee \beta) = \lambda \vee \mu \neq 0$; $f^{-1}(\alpha \wedge \beta) = \lambda \wedge \mu \neq 0$; $f^{-1}(\lambda) = \alpha \neq 0$ and $f^{-1}(\lambda\vee\mu) \neq 0$, in (X,T), $int\,cl[f^{-1}(\alpha)] \neq 0$; $int\,cl[f^{-1}(\beta)] \neq 0$; $int\,cl[f^{-1}(\alpha\vee\beta)] \neq 0$; $int\,cl[f^{-1}(\alpha\wedge\beta)] \neq 0$; $int\,cl[f^{-1}(\lambda)] \neq 0$; $int\,cl[f^{-1}(\lambda\vee\mu)] \neq 0$, in (X,T). Thus, for each fuzzy set $\delta(= \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, \lambda, \lambda \vee \mu)$ with $int(\delta) \neq 0$, $int\,cl[f^{-1}(\delta)] \neq 0$ in (X,T), implies that f is a somewhere fuzzy continuous function from (X,T) into (Y,S).

Example 4.3. Let μ_1, μ_2 and μ_3 be fuzzy sets defined on $I = [0,1]$ as follows :

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3}(4x - 1), & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then, $T_1 = \{0, \mu_2, \mu_3, \mu_2 \vee \mu_3, \mu_2 \wedge \mu_3, 1\}$ and $T_2 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ are fuzzy topologies on I. On computation, the fuzzy sets with non-zero interior in (I,T_1) are $\mu_2, \mu_3, 1 - \mu_1, 1 - \mu_2, 1 - \mu_3, \mu_1 \vee \mu_2, \mu_2 \vee \mu_3, \mu_1 \vee \mu_3, 1 - (\mu_1 \vee \mu_2), 1 - (\mu_2 \vee \mu_3), 1 - (\mu_1 \vee \mu_3), \mu_2 \wedge \mu_3, 1 - (\mu_1 \wedge \mu_2), 1 - (\mu_2 \wedge \mu_3)$ and $1 - (\mu_1 \wedge \mu_3)$. Define a function $g : (I,T_2) \to (I,T_1)$ by $g(x) = x$, for each $x \in I$. Now $int\,cl\{g^{-1}(1 - (\mu_2 \wedge \mu_3))\} = int\,cl(1 - (\mu_2 \wedge \mu_3)) = int(1 - (\mu_1 \vee \mu_2)) = 0$, in (I,T_2). Thus, for a fuzzy set

$1-(\mu_2 \wedge \mu_3)$ with $int(1-(\mu_2 \wedge \mu_3)) \neq 0$, $int\ cl\{g^{-1}(1-(\mu_2 \wedge \mu_3))\} = 0$ in (I, T_1), shows that $1-(\mu_2 \wedge \mu_3)$ is not a fuzzy somewhere dense set in (I, T_2). So g is not a somewhere fuzzy continuous function from (I, T_2) into (I, T_1).

Proposition 4.4. *If $f : (X, T) \to (Y, S)$ is a fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), then f is a somewhere fuzzy continuous function.*

Proof. Let λ be a non-zero fuzzy set defined on Y with $int(\lambda) \neq 0$ in (Y, S). Since the function f is a fuzzy continuous function, $f^{-1}[int(\lambda)]$ is a fuzzy open set in (X, T). That is, $int(f^{-1}[int(\lambda)]) = f^{-1}[int(\lambda)] \neq 0$, [For, $f^{-1}[int(\lambda)] = 0$, will imply that $int(\lambda) = 0$, a contradiction to $int(\lambda) \neq 0$]. But $f^{-1}[int(\lambda)] \leq f^{-1}(\lambda)$ in (X, T) and then $f^{-1}[int(\lambda)] \leq f^{-1}(\lambda) \leq cl[f^{-1}(\lambda)]$, implies that $int(f^{-1}[int(\lambda)]) \leq intcl[f^{-1}(\lambda)]$ and $int(f^{-1}[int(\lambda)]) \neq 0$, implies that $intcl[f^{-1}(\lambda)] \neq 0$ in (X, T). Thus f is a somewhere fuzzy continuous function. □

Proposition 4.5. *If $f : (X, T) \to (Y, S)$ is a somewhat fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), then f is a somewhere fuzzy continuous function.*

Proof. Let λ be a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S). Now $int(\lambda)$ is a non-zero fuzzy open set in (Y, S). Since f is a somewhat fuzzy continuous function, $int(f^{-1}[int(\lambda)]) \neq 0$, in (X, T). Now $int(f^{-1}[int(\lambda)]) \leq int(f^{-1}(\lambda)) \leq intcl[f^{-1}(\lambda)]$, implies that $intcl[f^{-1}(\lambda)] \neq 0$, in (X, T). Thus, for the fuzzy set λ with $int(\lambda) \neq 0$, $intcl[f^{-1}(\lambda)] \neq 0$ in (X, T), implies that f is a somewhere fuzzy continuous function. □

Remark 4.6. The converse of the above proposition need not be true. That is, a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), need not be a somewhat fuzzy continuous function. For, consider the following example :

Example 4.7. Let μ_1, μ_2 and μ_3 be fuzzy sets defined on $I = [0, 1]$ as follows :

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x-1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x+2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3}(4x-1), & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then, $T_1 = \{0, \mu_2, \mu_3, \mu_2 \vee \mu_3, \mu_2 \wedge \mu_3, 1\}$ and $T_2 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ are fuzzy topologies on I. On computation, $int(1-\mu_1) = \mu_2; int(1-\mu_2) = \mu_1; int(1-\mu_3) = \mu_2; int(\mu_3) = \mu_1; int[1-(\mu_1 \vee \mu_2)] = 0; int[1-(\mu_2 \vee \mu_3)] = 0; int(\mu_2 \vee \mu_3) = (\mu_1 \vee \mu_2)$, in (I, T_2). Define a function $f : (I, T_1) \to (I, T_2)$ by $f(x) = x$ for each $x \in I$. On computation for each fuzzy set $\delta(= \mu_1, \mu_2, \mu_3, \mu_1 \vee \mu_2, 1-\mu_1, 1-\mu_2, 1-\mu_3, \mu_2 \vee \mu_3)$ with $int(\delta) \neq 0, intcl[f^{-1}(\delta)] \neq 0$ in (I, T_1), implies that f is a somewhere fuzzy continuous function from (I, T_1) into (I, T_2). Now for the non-zero fuzzy open set μ_1 in (I, T_2), $int[f^{-1}(\mu_1)] = int(\mu_1) = 0$, shows that f is not a somewhat fuzzy continuous function from (I, T_1) into (I, T_2). Also, since $f^{-1}(\mu_1) = \mu_1$ and μ_1 is not a fuzzy open set in (I, T_1) shows that f is not a fuzzy continuous function from (I, T_1) into (I, T_2).

Proposition 4.8. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) and if λ is a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S), then there exists a fuzzy regular closed set η in (X, T) such that $\eta \leq cl[f^{-1}(\lambda)]$.*

Proposition 4.9. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) and if λ is a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S), then*

(1) *$clint[f^{-1}(1 - \lambda)] \neq 1$, in (X, T),*
(2) *There exists a fuzzy closed set μ in (X, T) such that $int[f^{-1}(1 - \lambda)] < \mu$*
(3) *There exists a fuzzy regular open set η in (X, T) such that $int[f^{-1}(1-\lambda)] < \eta$.*

Proposition 4.10. *If $f : (X, T) \to (Y, S)$ is a fuzzy β-continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), then f is a somewhere fuzzy continuous function.*

Remark 4.11. The converse of the above proposition need not be true. That is, a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), need not be a fuzzy β-continuous function. For, consider the following example :

Example 4.12. Let λ, μ, γ and δ be fuzzy sets defined on $I = [0, 1]$ as follows :

$$\lambda(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{2} \\ 2 - 2x, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2x - 1, & \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\gamma(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2 - 2x, & \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\delta(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then $T_1 = \{0, \lambda, 1 - \lambda, \mu, \gamma, 1\}$ and $T_2 = \{0, \lambda, \delta, \lambda \vee \delta, 1\}$ are fuzzy topologies on I. Define a function $h : (I, T_1) \to (I, T_2)$ by $h(x) = x$ for each $x \in I$. On computation, for each fuzzy set $\tau(= \lambda, \delta, \lambda \vee \delta, 1-\lambda, 1-\delta, \mu, 1-\gamma)$ with $int(\tau) \neq 0, intcl[f^{-1}(\tau)] \neq 0$ in (I, T_1), implies that h is a somewhere fuzzy continuous function. Now for the non-zero fuzzy open set $\lambda \vee \delta$ in (I, T_2), $h^{-1}(\lambda \vee \delta) = \lambda \vee \delta$ and $clintcl(\lambda \vee \delta) = 1 - \lambda$.

Since $\lambda \vee \delta > clintcl(\lambda \vee \delta)$ in (I, T_1), $\lambda \vee \delta$ is not a fuzzy β-open set in (I, T_1). Thus h is not a fuzzy β-continuous function from (I, T_1) into (I, T_2).

Proposition 4.13. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) and if λ is a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S), then $cl[f^{-1}(\lambda)]$ is a somewhere fuzzy dense set in (X, T).*

Proposition 4.14. *If $f : (X, T) \to (Y, S)$ is a fuzzy simply* continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), then f is a somewhere fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let λ be a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S). Now $int(\lambda)$ is a non-zero fuzzy open set in (Y, S). Since f is a fuzzy simply* continuous function, $f^{-1}[int(\lambda)]$ is a fuzzy simply* open set in (X, T). Then, by Proposition 3.16, $f^{-1}[int(\lambda)]$ is a fuzzy somewhere dense set in (X, T). thus $intcl[f^{-1}\{int(\lambda)\}] \neq 0$, in (X, T). Now $intcl[f^{-1}\{int(\lambda)\}] \leq intcl[f^{-1}(\lambda)]$, implies that $intcl[f^{-1}(\lambda)] \neq 0$, in (X, T). So f is a somewhere fuzzy continuous function. □

Remark 4.15. The converse of the above proposition need not be true. That is, a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), need not be a fuzzy simply* continuous function. For, consider the following example :

Example 4.16. Let λ, μ, γ and δ be fuzzy sets defined on $I = [0, 1]$ as follows :

$$\lambda(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{2} \\ 2 - 2x, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2x - 1, & \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\gamma(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 2 - 2x, & \frac{3}{4} \leq x \leq 1, \end{cases}$$

$$\delta(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then $T_1 = \{0, \lambda, 1 - \lambda, \mu, \gamma, 1\}$ and $T_2 = \{0, \lambda, \delta, \lambda \vee \delta, 1\}$ are fuzzy topologies on I. Define a function $i : (I, T_2) \to (I, T_1)$ by $i(x) = x$ for each $x \in I$. On computation, for fuzzy set $\tau (= \lambda, 1 - \lambda, \mu, 1 - \mu, \gamma, \lambda \vee \delta, 1 - \delta, 1 - \gamma, 1 - (\lambda \vee \delta))$ with $int(\tau) \neq 0$ in (I, T_1), $intcl[f^{-1}(\tau)] \neq 0$, in (I, T_2), implies that i is a somewhere fuzzy continuous function. The fuzzy nowhere dense sets in (I, T_2) are γ and $1 - [\lambda \vee \delta]$ and the fuzzy simply* open sets in (I, T_2) are $\{(1 - [\lambda \vee \delta]) \vee \lambda\}, \{(1 - [\lambda \vee \delta]) \vee \delta\}, \{(1 - [\lambda \vee \delta]) \vee [\lambda \vee \delta]\}, \lambda, \lambda \vee \delta$ and $\gamma \vee \delta$. Now for the non-zero fuzzy open sets μ ,γ in (I, T_1), $i^{-1}(\mu) = \mu$ and $i^{-1}(\gamma) = \gamma$ and μ, γ are not fuzzy simply* open sets in (I, T_2). Thus i is not a fuzzy simply* continuous function from (I, T_2) into (I, T_1).

Proposition 4.17. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from a fuzzy topological space (X, T) onto another fuzzy topological space (Y, S) and if λ is a fuzzy open and fuzzy dense set in (X, T), then $f(\lambda)$ is a fuzzy dense set in (Y, S).*

Proof. Let λ be a fuzzy open and fuzzy dense set in (X, T). It is to be proved that $f(\lambda)$ is a fuzzy dense set in (Y, S). Assume the contrary. Suppose that $f(\lambda)$ is not a fuzzy dense set in (Y, S). That is, $cl[f(\lambda)] \neq 1$, in (Y, S) and then $1 - cl[f(\lambda)] \neq 0$. This implies that $int(1 - [f(\lambda)]) \neq 0$. Since f is one-to-one and onto, $f(1 - \lambda) = 1 - f(\lambda)$ and then $int[f(1 - \lambda)] \neq 0$ in (Y, S). Since the function f is a somewhere fuzzy continuous function from (X, T) into (Y, S), $f^{-1}[f(1 - \lambda)]$ is a fuzzy somewhere dense set in (X, T). Then, $intclf^{-1}[f(1 - \lambda)] \neq 0$. Since f is one-to-one, $f^{-1}[f(1 - \lambda)] = 1 - \lambda$. Thus $intclf^{-1}[f(1 - \lambda)] = intcl(1 - \lambda)$ and this will imply that $intcl(1 - \lambda) \neq 0$. So, $1 - clint(\lambda) \neq 0$, in (X, T). This will imply that $clint(\lambda) \neq 1$ and then $cl(\lambda) \neq 1$, [since λ is a fuzzy open set in (X, T), $int(\lambda) = \lambda$], a contradiction to λ being a fuzzy dense set in (X, T). Hence the assumption that $f(\lambda)$ is not a fuzzy dense set in (Y, S), does not hold and therefore $f(\lambda)$ is a fuzzy dense set in (Y, S). □

Proposition 4.18. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from a fuzzy hyperconnected space (X, T) onto another fuzzy topological space (Y, S) and if λ is a fuzzy open in (X, T), then $f(\lambda)$ is a fuzzy dense set in (Y, S).*

Proof. Let $f : (X, T) \to (Y, S)$ be a somewhere fuzzy continuous and one-to-one function from a fuzzy hyperconnected space (X, T) onto another fuzzy topological space (Y, S) and λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy hyperconnected space, the fuzzy open set λ is a fuzzy dense set in (X, T). Then λ is a fuzzy open and fuzzy dense set in (X, T). Thus, by Proposition 4.17, $f(\lambda)$ is a fuzzy dense set in (Y, S). □

Proposition 4.19. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from a fuzzy submaximal space (X, T) onto another fuzzy topological space (Y, S) and if λ is a fuzzy dense set in (X, T), then $f(\lambda)$ is a fuzzy dense set in (Y, S).*

Proposition 4.20. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from a fuzzy hyperconnected space (X, T) onto a fuzzy submaximal space (Y, S) then f is a fuzzy open function from (X, T) onto (Y, S).*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy hyperconnected space, the fuzzy open set λ is a fuzzy dense set in (X, T). Then λ is a fuzzy open and fuzzy dense set in (X, T). Since $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from (X, T) onto (Y, S), by Proposition 4.17, $f(\lambda)$ is a fuzzy dense set in (Y, S). Since (Y, S) is a fuzzy submaximal space, the fuzzy dense set $f(\lambda)$ is a fuzzy open set in (Y, S). Thus f is a fuzzy open function from (X, T) onto (Y, S). □

Proposition 4.21. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous, one-to-one and fuzzy open function from a fuzzy topological space (X, T) onto another* *fuzzy topological space (Y, S) and if $cl[\wedge_{i=1}^{\infty} f(\lambda_i)] = 1$, where (λ_i)'s are fuzzy dense and fuzzy open sets in (X, T), then (Y, S) is a fuzzy Baire space.*

Proof. Let (λ_i)'s ($i = 1$ to ∞) be fuzzy dense and fuzzy open sets in (X, T). Since $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from the topological space (X, T) onto the fuzzy topological space (Y, S) and by Proposition 4.17, $[f(\lambda_i)]$'s are fuzzy dense sets in (Y, S). Since f is a fuzzy open function from (X, T) into (Y, S), $[f(\lambda_i)]$'s are fuzzy open sets in (Y, S). By hypothesis, $cl[\wedge_{i=1}^{\infty} f(\lambda_i)] = 1$, in (Y, S). Then $cl[\wedge_{i=1}^{\infty} f(\lambda_i)] = 1$, where $[f(\lambda_i)]$'s are fuzzy dense and fuzzy open sets in (Y, S), implies, by Theorem 2.16, that (Y, S) is a fuzzy Baire space. □

Proposition 4.22. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous and one-to-one function from a fuzzy hyperconnected space (X, T) onto another fuzzy topological space (Y, S) and if $cl[\wedge_{i=1}^{\infty} f(\lambda_i)] = 1$, where (λ_i)'s are fuzzy open sets in (X, T), then (Y, S) is a fuzzy Baire space.*

Proof. Let (λ_i)'s ($i = 1$ to ∞) be fuzzy open sets in (X, T). Since (X, T) is a fuzzy hyperconnected space, (λ_i)'s are fuzzy dense sets in (X, T). Then (λ_i)'s are fuzzy dense and fuzzy open sets in (X, T). Since f is a somewhere fuzzy continuous and one-to-one function from the fuzzy hyperconnected space (X, T) onto the fuzzy topological space (Y, S), by Proposition 4.18, $[f(\lambda_i)]$'s are fuzzy dense sets in (Y, S). Since f is a fuzzy open function from (X, T) into (Y, S), $[f(\lambda_i)]$'s are fuzzy open sets in (Y, S). Thus $cl[\wedge_{i=1}^{\infty} f(\lambda_i)] = 1$, where $[f(\lambda_i)]$'s are fuzzy dense and fuzzy open sets in (Y, S), implies by Theorem 2.16 that (Y, S) is a fuzzy Baire space. □

Proposition 4.23. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy hyperconnected space (X, T) into another fuzzy topological space (Y, S) and if λ is a non-zero fuzzy open set in (Y, S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy non-zero fuzzy open set in (Y, S). Then $int(\lambda) = \lambda \neq 0$, in (Y, S). Since $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from (X, T) into (Y, S) and λ is a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S), $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X, T). That is, $intcl[f^{-1}(\lambda)] \neq 0$, in (X, T) and then there exists a fuzzy open set μ in (X, T) such that $\mu \leq cl[f^{-1}(\lambda)]$. This implies that $cl(\mu) \leq cl[cl[f^{-1}(\lambda)]]$. Thus $cl(\mu) \leq cl[f^{-1}(\lambda)]$ in (X, T). Since μ is a fuzzy open set in the fuzzy hyperconnected space (X, T), $cl(\mu) = 1$. So $1 \leq cl[f^{-1}(\lambda)]$. That is, $cl[f^{-1}(\lambda)] = 1$, in (X, T). Hence $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T). □

Proposition 4.24. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy hyperconnected space (X, T) into a fuzzy submaximal space (Y, S) and if λ is a fuzzy dense set in (Y, S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T).*

The following proposition gives a condition for a fuzzy function defined between any two fuzzy topological spaces to become a somewhere fuzzy continuous function.

Proposition 4.25. *If $f : (X, T) \to (Y, S)$ is a fuzzy function from a fuzzy P-space (X, T) into a fuzzy topological space (Y, S) such that $f^{-1}(\lambda)$ is a fuzzy residual set in (X, T) for a fuzzy set defined on Y with $int(\lambda) \neq 0$, in (Y, S), then f is a somewhere fuzzy continuous function from (X, T) into (Y, S).*

Proposition 4.26. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into a fuzzy perfectly disconnected space (Y, S) and if λ is a non-zero fuzzy set defined on Y with $int(\lambda) \neq 0$ in (Y, S), then $clintcl[f^{-1}(\lambda)] \leq cl[f^{-1}(\lambda)]$, in (X, T).*

Proof. Let λ be a non-zero fuzzy set defined on Y with $int(\lambda) \neq 0$ in (Y, S). Since $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from (X, T) into (Y, S) , $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in the fuzzy perfectly disconnected space (X, T). Then, by Proposition 3.25, $cl[f^{-1}(\lambda)]$ is a fuzzy preclosed set in (X, T). Thus $clintcl[f^{-1}(\lambda)] \leq cl[f^{-1}(\lambda)]$, in (X, T). □

Proposition 4.27. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into a fuzzy globally disconnected and fuzzy P-space (Y, S) and if λ is a fuzzy Baire set in (Y, S), then $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X, T).*

Proposition 4.28. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into a fuzzy Baire space (Y, S) and if λ is a fuzzy Baire set in (Y, S), then $clint[f^{-1}(\lambda)] \neq 1$ in (X, T).*

Proof. Let λ be a fuzzy Baire set in (Y, S). Since (Y, S) is a fuzzy Baire space, by Theorem 2.23, λ is not a fuzzy dense set in (Y, S). Then, $cl(\lambda) \neq 1$, in (Y, S). This implies that $int(1 - \lambda) \neq 0$, in (Y, S). Since f is a somewhere fuzzy continuous function from (X, T) into (Y, S), $f^{-1}(1-\lambda)$ is a fuzzy somewhere dense set in (X, T). Thus, $intcl[f^{-1}(1 - \lambda)] \neq 0$, in (X, T). This implies that $intcl[(1 - f^{-1}(\lambda)] \neq 0$ and then $1 - clint(f^{-1}(\lambda)) \neq 0$ in (X, T). So $clint[f^{-1}(\lambda)] \neq 1$ in (X, T). □

Proposition 4.29. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy continuous function from a fuzzy topological space (X, T) into a fuzzy topological space (Y, S) and $g : (Y, S) \to (Z, W)$ is a fuzzy continuous function from (Y, S) into a fuzzy topological space (Z, W), then $g \circ f$ is a somewhere fuzzy continuous function from (X, T) into (Z, W).*

5. Somewhere fuzzy open functions

Definition 5.1. A function $f : (X, T) \to (Y, S)$ from a fuzzy topological space (X, T) into a fuzzy topological space (Y, S) is called a somewhere fuzzy open function, if whenever $int(\lambda) \neq 0$, for a fuzzy set λ defined on X, then $f(\lambda)$ is a fuzzy somewhere dense set in (Y, S). That is, $f : (X, T) \to (Y, S)$ is a somewhere fuzzy open function, if $intcl[f(\lambda)] \neq 0$ in (Y, S), whenever $int(\lambda) \neq 0$, for a fuzzy set λ defined on X.

Proposition 5.2. *If λ ($\neq 1$) is a fuzzy closed set in a fuzzy topological space (X, T) and f is a fuzzy somewhere fuzzy open and one-to-one function from (X, T) into a fuzzy topological space (Y, S), then $int[f(\lambda)]$ is not a fuzzy dense set in (Y, S).*

Proposition 5.3. *If $f : (X, T) \to (Y, S)$ is a fuzzy open function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), then f is a somewhere fuzzy open function.*

Proof. Let λ be a non- zero fuzzy set defined on X with $int(\lambda) \neq 0$, in (X, T). Since the function f is a fuzzy open function, $f[int(\lambda)]$ is a fuzzy open set in (Y, S). That is, $intf[int(\lambda)] = f[int(\lambda)] \neq 0$. But $f[int(\lambda)] \leq f(\lambda)$ in (Y, S). Then $f[int(\lambda)] \leq f(\lambda) \leq cl[f(\lambda)]$ implies that $f[int(\lambda)] \leq cl[f(\lambda)]$. Thus $int(f[int(\lambda)]) \leq intcl[f(\lambda)]$. Since $int[f(int(\lambda))] \neq 0$, in (Y, S), $intcl[f(\lambda)] \neq 0$ in (Y, S). So f is a somewhere fuzzy open function from (X, T) into (Y, S). □

Remark 5.4. The converse of the above proposition need not be true. That is, a somewhere fuzzy open function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S), need not be a fuzzy open function. For, consider the following example :

Example 5.5. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ, α and β are defined on X as follows :

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.4$; $\lambda(b) = 0.6$; $\lambda(c) = 0.4$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.5$; $\mu(c) = 0.4$,
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.5$; $\alpha(c) = 0.8$,
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.4$; $\beta(b) = 0.9$; $\beta(c) = 0.7$.

Then $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ and $S = \{0, \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1\}$ are fuzzy topologies on X. On computation, one can see that $int(\lambda) = \lambda, int(\mu) = \mu, int(\lambda \vee \mu) = \lambda \vee \mu, int(\lambda \wedge \mu) = \lambda \wedge \mu, int(\alpha) = \mu, int(\beta) = \lambda, int(1 - \mu) = \lambda \wedge \mu, int(1 - [\lambda \wedge \mu]) = \mu, int(\alpha \vee \beta) = \lambda \vee \mu, int(\alpha \wedge \beta) = \lambda \wedge \mu$, in (X, T) and $\lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1 - \lambda, 1 - \mu, 1 - [\lambda \wedge \mu], 1 - [\lambda \vee \mu]$ are fuzzy dense sets in (X, S). Define a function $f : (X, T) \to (X, S)$ by $f(a) = a; f(b) = b; f(c) = c$. On computation one can see that $intcl[f(\lambda)] \neq 0$; $intcl[f(\mu)] \neq 0$; $intcl[f(\lambda \vee \mu)] \neq 0$; $intcl[f(\lambda \wedge \mu)] \neq 0$, $intcl[f(\alpha)] \neq 0$; $intcl[f(\beta)] \neq 0$; $intcl[f(\alpha \vee \beta)] \neq 0$; $intcl[f(\alpha \wedge \beta)] \neq 0$; $intcl[f(1 - \mu)] \neq 0$; $intcl[f(1 - \lambda)] \neq 0$, $intcl\{f(1 - [\lambda \vee \mu])\} \neq 0$; $intcl\{f(1 - [\lambda \wedge \mu])\} \neq 0$, in (X, S). Then f is a somewhere fuzzy open function from (X, T) into (Y, S). Since $f(\lambda) = \lambda \in S$, f is not a fuzzy open function from (X, T) into (Y, S).

Proposition 5.6. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy open function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) and if λ is a fuzzy set defined on X with $int(\lambda) \neq 0$ in (X, T), then $cl[f(\lambda)]$ is a fuzzy somewhere dense set in (Y, S).*

The following proposition gives a condition for a fuzzy function defined between fuzzy topological spaces to become a somewhere fuzzy open function.

Proposition 5.7. *If $f : (X, T) \to (Y, S)$ is a fuzzy function from a fuzzy topological space (X, T) into a fuzzy P-space (Y, S) such that $f(\lambda)$ is a fuzzy residual set in (Y, S) for a fuzzy set defined on X with $int(\lambda) \neq 0$, in (X, T), then f is a somewhere fuzzy open function from (X, T) into (Y, S).*

Proposition 5.8. *If $f : (X, T) \to (Y, S)$ is a somewhere fuzzy open function from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) and if λ is a fuzzy open and fuzzy dense set in (Y, S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy open and fuzzy dense set in (Y, S) . It is to be proved that $f^{-1}(\lambda)$ is a fuzzy dense set in (X, T). Assume the contrary. Suppose that $f^{-1}(\lambda)$ is not a fuzzy dense set in (X, T) . That is, $cl[f^{-1}(\lambda)] \neq 1$, in (X, T) and then $1 - cl[f^{-1}(\lambda)] \neq 0$. This implies that $int[1 - f^{-1}(\lambda)] \neq 0$. Since $f^{-1}(1 - \lambda) =$

$1 - f^{-1}(\lambda)$, $int[f^{-1}(1-\lambda)] \neq 0$ in (X,T). Since the function f is a somewhere fuzzy open function from (X,T) into (Y,S), $f[f^{-1}(1-\lambda)]$ is a fuzzy somewhere dense set in (Y,S). Then, $intcl\{f[f^{-1}(1-\lambda)]\} \neq 0$, in (Y,S). Since $f[f^{-1}(1-\lambda)] \leq 1-\lambda$, $intcl\{f[f^{-1}(1-\lambda)]\} \leq intcl(1-\lambda)$ and this will imply that $intcl(1-\lambda) \neq 0$. Thus, $clint(\lambda) \neq 1$, in (Y,S). This will imply that $clint(\lambda) \neq 1$. So $cl(\lambda) \neq 1$, [since λ is a fuzzy open set in (Y,S), $int(\lambda) = \lambda$] a contradiction to λ being a fuzzy dense set in (Y,S). Hence the assumption that $f^{-1}(\lambda)$ is not a fuzzy dense set in (X,T), does not hold. Therefore $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T). □

Proposition 5.9. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open function from a fuzzy globally disconnected and fuzzy P-space (X,T) into a fuzzy topological space (Y,S) and if λ is a fuzzy Baire set in (X,T), then $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S).*

Proposition 5.10. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open and fuzzy continuous function from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) and if $cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 1$, where (λ_i)'s are fuzzy dense and fuzzy open sets in (Y,S), then (X,T) is a fuzzy Baire space.*

Proof. Let (λ_i)'s be fuzzy dense and fuzzy open sets in (Y,S). Since $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open function from (X,T) into (Y,S), by Proposition 5.8, $[f^{-1}(\lambda_i)]$'s are fuzzy dense sets in (X,T). Since f is a fuzzy continuous function from (X,T) into (Y,S), $[f^{-1}(\lambda_i)]$'s are fuzzy open sets in (X,T). By hypothesis, $cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 1$, in (X,T). Then $cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 1$, where $[f^{-1}(\lambda_i)]$'s are fuzzy open and dense sets in (X,T), implies by Theorem 2.22, that (X,T) is a fuzzy Baire space. □

Proposition 5.11. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open and fuzzy continuous function from a fuzzy topological space (X,T) into a fuzzy hyperconnected space (Y,S) and if $cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 1$, where (λ_i)'s are fuzzy open sets in (Y,S), then (X,T) is a fuzzy Baire space.*

Proof. Let (λ_i)'s be fuzzy open sets in (Y,S). Since (Y,S) is a fuzzy hyperconnected space, the fuzzy open sets (λ_i)'s are fuzzy dense sets in (Y,S). Since $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open and fuzzy continuous function from (X,T) into (Y,S) and $cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 1$, where (λ_i)'s are fuzzy dense and fuzzy open sets in (Y,S), by Proposition 5.10, (X,T) is a fuzzy Baire space. □

Proposition 5.12. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open function from a fuzzy topological space (X,T) into a fuzzy hyperconnected space (Y,S) and if λ is a non-zero fuzzy open set in (X,T), then $f(\lambda)$ is a fuzzy dense set in (Y,S).*

Proof. Let be a fuzzy non-zero fuzzy open set in (X,T). Then $int(\lambda) = \lambda \neq 0$, in (X,T). Since f is a somewhere fuzzy open function from (X,T) into (Y,S) and λ is a fuzzy set defined on X with $int(\lambda) \neq 0$, in (X,T), $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S). That is, $intcl[f(\lambda)] \neq 0$, in (Y,S). Then there exists a fuzzy open set μ in (Y,S) such that $\mu \leq cl[f(\lambda)]$. This implies that $cl(\mu) \leq cl[cl[f(\lambda)]]$. Thus, $cl(\mu) \leq cl[f(\lambda)]$ in (Y,S). Since μ is a fuzzy open set in the fuzzy hyperconnected space (Y,S), $cl(\mu) = 1$, in (Y,S). Then $1 \leq cl[f(\lambda)]$. That is, $cl[f(\lambda)] = 1$, in (Y,S). So $f(\lambda)$ is a fuzzy dense set in (Y,S). □

Proposition 5.13. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open function from a fuzzy topological space (X,T) into a fuzzy submaximal space (Y,S) and if λ is a fuzzy dense set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy dense set in (X,T).*

Proposition 5.14. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open function from a fuzzy submaximal space (X,T) into a fuzzy hyperconnected space (Y,S) and if λ is a fuzzy dense set in (X,T), then $f(\lambda)$ is a fuzzy dense set in (Y,S).*

Proposition 5.15. *If $f : (X,T) \to (Y,S)$ is a somewhere fuzzy open function from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) and if λ is a fuzzy set defined on X with $int(\lambda) \neq 0$, in (X,T), then there exist a fuzzy regular closed set η in (Y,S) such that $\eta \leq cl[f(\lambda)]$.*

Proof. Let λ be a fuzzy set defined on X with $int(\lambda) \neq 0$, in (X,T). Since f is a somewhere fuzzy open function from (X,T) into (Y,S), $f(\lambda)$ is a fuzzy somewhere dense set in (Y,S). Then, by Proposition 3.5, there exist a fuzzy regular closed set η in (Y,S) such that $\eta \leq cl[f(\lambda)]$. □

Proposition 5.16. *If $f : (X,T) \to (Y,S)$ is a somewhat fuzzy open function from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S), then f is a somewhere fuzzy open function from (X,T) into (Y,S).*

Proof. Let λ be a fuzzy set defined on X with $int(\lambda) \neq 0$, in (X,T). Now $int(\lambda)$ is a non-zero fuzzy open set in (X,T). Since f is a somewhat fuzzy open function from (X,T) into (Y,S), $int[f\{int(\lambda)\}] \neq 0$, in (Y,S). Now $int[f\{int(\lambda)\}] \leq int[f(\lambda)] \leq intcl[f(\lambda)]$, implies that $intcl[f(\lambda)] \neq 0$, in (Y,S). Thus, for the fuzzy set λ defined on X with $int(\lambda) \neq 0$, in (X,T), $intcl[f(\lambda)] \neq 0$, implies that f is a somewhere fuzzy open function from (X,T) into (Y,S) . □

Proposition 5.17. *If $f : (X,T) \to (Y,S)$ is a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S) and $g : (Y,S) \to (Z,W)$ is a somewhere fuzzy open function from (Y,S) into a fuzzy topological space (Z,W), then $g \circ f$ is a somewhere fuzzy open function from (X,T) into (Z,W).*

Proposition 5.18. *If $f : (X,T) \to (Y,S)$ is a fuzzy simply* open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S), then f is a somewhere fuzzy open function from (X,T) into (Y,S).*

Proof. Let λ be a fuzzy set defined on X with $int(\lambda) \neq 0$, in (X,T). Then $int(\lambda)$ is a fuzzy open set in (X,T). Since f is a fuzzy simply* open function, $f[int(\lambda)]$ is a fuzzy simply* open set in (Y,S). Thus, by Proposition 3.16, $f[int(\lambda)]$ is a fuzzy somewhere dense set in (Y,S) and hence $intcl(f[int(\lambda)]) \neq 0$, in (Y,S). But $intcl(f[int(\lambda)]) \leq intcl(f[\lambda])$, implies that $intcl(f[\lambda]) \neq 0$ in (Y,S). So f is a somewhere fuzzy open function from (X,T) into (Y,S). □

6. Conclusions

The notions of fuzzy cs dense sets in fuzzy topological spaces and somewhere fuzzy continuous functions, somewhere fuzzy open functions between fuzzy topological spaces are introduced and studied. Several characterizations of fuzzy somewhere dense sets and fuzzy cs dense sets are established. The existence of fuzzy regular closed sets in a fuzzy topological space, is established by means of fuzzy somewhere dense sets. The inter-relations between fuzzy continuous functions, somewhat fuzzy continuous functions and somewhere fuzzy continuous functions are established in this paper. The conditions under which somewhere fuzzy continuous functions between fuzzy topological spaces become fuzzy open functions are obtained. The conditions for fuzzy topological spaces to become fuzzy Baire spaces are obtained by means of somewhere fuzzy continuous functions and somewhere fuzzy open functions.

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