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Abstract. In the present paper, the notions of some weaker forms of soft nano continuity namely soft nano semi continuity, soft nano pre continuity, soft nano $\alpha$-continuity and soft nano $\beta$-continuity on soft nano topological spaces are introduced and related properties are studied.

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Keywords: Soft nano topology, Soft nano open set, Soft nano continuity, Soft nano semi continuity, Soft nano $\beta$-continuity.

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## 1. Introduction

In 1999, Molodtsov [19] initiated the theory of soft sets, which is a completely generic mathematical tool for modeling uncertainties. Further, the study on soft set theory was done by Maji et al [16] which includes discussion of the application of soft set theory in decision making problems. Further, Shabir and Naz [22] have introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. S. Hussain and B. Ahmad [13] and Naim Cagman et al [11] have continued the study of properties of soft topological spaces. The study of soft sets and related aspects was also undertaken in [2, 3, 5, 6, 7, 8, 12, 17, 18, 21, 23, 26]. Presently, work on the soft set theory is making progress rapidly.

In 2014, Kandil et al [14] introduced a unification of different kinds of soft continuity in soft topological spaces using the notion of $\gamma$-operation. The notion of Nano topology was introduced by Lellis Thivagar [24]. Based on that, Benchalli et al [9] introduced the notion of soft nano topological spaces using soft set equivalence relation on the universal set. Also, the notion of soft nano continuity and weaker forms of soft nano open sets namely soft nano semi open, soft nano pre open, soft nano $\alpha$-open and soft nano $\beta$-open sets in soft nano topological spaces are introduced and studied in [9] and [10].

In the present paper, the notions of some weaker forms of soft nano continuity namely soft nano semi continuity,soft nano pre continuity, soft nano $\alpha$-continuity and soft nano $\beta$-continuity in soft nano topological spaces are introduced and related properties are studied.

## 2. Preliminaries

**Definition 2.1** ([19])**.** Let $U$ be an initial universe and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F : A \to P(U)$.

In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set $(F, A)$ . Clearly, a soft set need not be a set.

**Definition 2.2** ([3])**.** Let $(F, A)$ and $(G, B)$ be two soft sets over $U$, then the Cartesian product of $(F, A)$ and $(G, B)$ is defined as $(F, A) \times (G, B) = (H, A \times B)$, where $H : A \times B \to P(U \times U)$ and for each $(a, b) \in A \times B$,

$$H(a, b) = \{(h_i, h_j) : h_i \in F(a) \text{ and} h_j \in G(b)\} = F(a) \times F(b).$$

**Theorem 2.3** ([3])**.** *Let $(F, A)$ and $(G, B)$ be two soft sets over a universe $U$. Then a soft set relation from $(F, A)$ to $(G, B)$ is a soft subset of $(F, A) \times (G, B)$.*

*In other words, a soft set relation from $(F, A)$ to $(G, B)$ is of the form $(H_1, S)$, where $S \subseteq A \times B$ and $H_1(a, b) = H(a, b)$, for all $(a, b) \in S$, where $(H, A \times B) = (F, A) \times (G, B)$ as in the above definition.*

*In an equivalent way, w can define the soft set relation $R$ on $(F, A)$ in the parameterized form as follows:*

$$if\ (F, A) = \{F(a), F(b), ...\},\ then\ F(a)RF(b) \Leftrightarrow F(a) \times F(b) \in R.$$

**Definition 2.4** ([3])**.** Let $R$ be a relation on $(F, A)$.

(i) $R$ is reflexive, if $H_1(a, a) \in R, \forall a \in A$.
(ii) $R$ is symmetric, if $H_1(a, b) \in R \Rightarrow H_1(b, a) \in R, \forall (a, b) \in A \times A$.
(iii) $R$ is transitive, if $H_1(a, b) \in R, H_1(b, c) \in R \Rightarrow H_1(a, c) \in R, \forall a, b, c \in A$.

**Definition 2.5** ([3])**.** A soft set relation $R$ on a soft set $(F, A)$ is called an equivalence relation, if it is reflexive, symmetric and transitive.

**Example 2.6** ([3])**.** Consider a soft set $(F, A)$ over $U$, where $U = \{C_1, C_2, C_3, C_4\}$, $A = \{m_1, m_2\}$ and $F(m_1) = \{C_1, C_3\}, F(m_2) = \{C_2, C_4\}$.

Consider a relation $R$ defined on $(F, A)$ as follows:

$$R = \{F(m_1) \times F(m_2), F(m_2) \times F(m_1), F(m_1) \times F(m_1), F(m_2) \times F(m_2)\}.$$

Then $R$ is a soft set equivalence relation.

**Definition 2.7** ([3])**.** Let $(F, A)$ be a soft set. Then equivalence class of $F(a)$ denoted by $[F(a)]$ is defined as follows:

$$[F(a)] = \{F(b) : F(b)RF(a)\}.$$

**Definition 2.8.** [15]: Let $(X, E)$ and $(Y, E')$ be soft classes. Let $u : X \to Y$ and $p : E \to E'$ be mappings. Then a mapping $f : (X, E) \to (Y, E')$ is defined as : for a soft set $(F,A) \in (X,E)$, $(f(F,A),B), B = p(A) \subseteq E'$ is a soft set in $(Y,E')$ given by

$$f(F,A)(\beta) = \begin{cases} u(\bigcup_{\alpha \in p^{-1}(\beta)\cap A})F(\alpha) & if \quad p^{-1}(\beta) \cap A \neq \phi \\ \phi, \quad otherwise, \end{cases}$$

for $\beta \in B \subseteq E'$. $(f(F,A),B)$ is called a soft image of a soft set $(F,A)$. If $B = E'$, then we write $f((F,A),E')$ as $f(F,A)$.

**Definition 2.9.** [15]: Let $f : (X,E) \to (Y,E')$ be a mapping from a soft class $(X,E)$ to another soft class $(Y,E')$ and $(G,C)$ be a soft set in soft class $(Y,E')$ , where $C \subseteq E'$. Let $u : X \to Y$ and $p : E \to E'$ be mappings. Then, inverse image of a soft set denoted as $f^{-1}((G,C),D), D = p^{-1}(C)$ and is a soft set in the soft class $(X,E)$ , defined as:

$$f^{-1}(G,C)(\alpha) = \begin{cases} u(G((p(\alpha))) & if \quad p(\alpha) \in C \\ \phi, otherwise, \end{cases}$$

for $\alpha \in D \subseteq E$. Here $f^{-1}((G,C),D)$ is called a soft inverse image of $(G,C)$.

**Theorem 2.10** ([3])**.** *Let $SS(U)_A$ and $SS(V)_B$ be families of soft sets. For a function $f_pu : SS(U)_A \to SS(V)_B$, the following statements are true:*
(1) $f_{pu}^{-1}((G,B)') = (f_{pu}^{-1}(G,B))'$, *for any soft set $(G,B)$ in $SS(V)_B$,*
(2) $f_{pu}(f_{pu}^{-1}((G,B))) \subseteq (G,B)$, *for any soft set $(G,B)$ in $SS(V)_B$,*
(3) $(F,A) \subseteq f_{pu}^{-1}(f_{pu}(F,A))$, *for any soft set $(F,A)$ in $SS(U)_A$.*

**Definition 2.11** ([9])**.** Let $U$ be a non-empty finite set of objects called the universe and $E$ be a set of parameters. Let $R$ be a soft equivalence relation on $U$. Then the triplet $(U,R,E)$ is said to be the soft approximation space. Let $X \subseteq U$.

(i) The soft lower approximation of $X$ with respect to $R$ and the set of parameters $E$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and it is denoted by $(L_R(X),E)$, equivalently

$$(L_R(X),E) = \cup\{R(x) : R(x) \subseteq X\},$$

where $R(x)$ denotes the equivalence class determined by $x \in U$.

(ii) The soft upper approximation of $X$ with respect to $R$ and the set of parameters $E$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $(U_R(X),E)$, equivalently

$$(U_R(X),E) = \cup\{R(x) : R(x) \cap X \neq \phi\}.$$

(iii) The soft boundary region of $X$ with respect to $R$ and the set of parameters $E$ is the set of all objects , which can be classified neither inside $X$ nor as outside $X$ with respect to $R$ and is denoted by $(B_R(X),E)$, equivalently

$$(B_R(X),E) = (U_R(X),E) \setminus (L_R(X),E).$$

**Definition 2.12** ([9])**.** Let $U$ be a non-empty universal set and $E$ be a set of parameters. Let $R$ be a soft equivalence relation on $U$. Let $X \subseteq U$ and let $\tau_R(X) = \{U, \phi, (L_R(X),E), (U_R(X),E), (B_R(X),E)\}$. Then $\tau_R(X)$ is a soft topology on $(U,E)$.

In this case, $\tau_R(X)$ is called the soft nano topology with respect to $X$. Elements of the soft nano topology are known as the soft nano open sets and $(\tau_R(X),U,E)$

is called a soft nano topological space. The complements of soft nano open sets are called as soft nano closed sets in $(\tau_R(X), U, E)$.

**Definition 2.13** ([9]). Let $(\tau_R(X), U, E)$ and $(\tau_{R'}(Y), V, K)$ be soft nano topological spaces. Then a mapping $f : (\tau_R(X), U, E) \rightarrow (\tau_{R'}(Y), V, K)$ is said to be soft nano continuous over $U$, if the inverse image of each soft nano open set in $V$ is soft nano open in $U$.

**Definition 2.14** ([10]). Let $(\tau_R(X), U, E)$ be a soft nano topological space and $(G, E)$ be any soft set over $U$.Then $(G, E)$ is said to be:
(i) soft nano semi-open if $(G, E) \subseteq NCl(NInt(G, E))$
(ii) soft nano pre-open if $(G, E) \subseteq NInt(NCl(G, E))$
(iii)soft nano $\alpha$-open if $(G, E) \subseteq NInt(NCl(NInt(G, E)))$
(iv) soft nano $\beta$-open if $(G, E) \subseteq NCl(NInt(NCl(G, E)))$

Here $NInt(G, E)$ is the soft nano interior of $(G, E)$, which is the union of all soft nano open sets contained in $(G, E)$ and $NCl(G, E)$ is the soft nano closure of $(G, E)$, which is the intersection of all soft nano closed sets containing $(G, E)$.
Also, here $SNSO(U, E), SNPO(U, E), SN\alpha O(U, E), SN\beta O(U, E)$ denotes the family of all soft nano semi-open, soft nano pre-open, soft nano $\alpha$-open and soft nano $\beta$-open sets over $U$ with respect to an equivalence relation $R$ and parameter set $E$.

**Definition 2.15** ([4]). Let U be the universe, $P = (U, S)$ be a soft approximation space and $\tau_{SR}(X) = \{U, \phi, \underline{RP}(X), \overline{RP}(X), Bnd_p(X)\}$,
where $X \subseteq U$ and $\tau_{SR}(X)$ satisfies the following axioms:
(i) $U$ and $\phi \in \tau SR(X)$,
(ii) the union of the elements of any sub collection of $\tau SR(X)$ is in $\tau SR(X)$,
(iii) the intersection of the elements of any finite sub collection of $\tau SR(X)$ is in $\tau SR(X)$. Here $\tau SR(X)$ forms a topology on $U$ called as the soft rough topology on $U$ with respect to $X$ and $(U, \tau SR(X), E)$ is called a soft rough topological space.

**Definition 2.16** ([4]). Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR'}(Y), E)$ be two soft rough topological spaces. The mapping $f : (U, \tau_{SR}(X), E) \rightarrow (V, \tau_{SR'}(Y), E)$ is called soft rough continuous on $U$, if the inverse image of each soft rough-open set in $V$ is soft rough open in $U$.

**Definition 2.17** ([25]). Let $(U, A)$ be an information system, where $A$ is divided into a set $C$ of condition attributes and a set $D$ of decision attributes. Then a core is a minimal subset of attributes which is such that none of its elements can be removed without affecting the classification powered attributes.

**Definition 2.18** ([14]). Let $(X, \tau, E)$ be a soft topological space. A mapping $\gamma : SS(X)_E \rightarrow SS(X)_E$ is said to be an operation on $SS(X)_E$, if $F_E \subseteq \gamma(F_E), \forall F_E \in OS(X)$. The collection of all $\gamma$-open soft sets is denoted by $OS(\gamma) = \{F_E : F_E \subseteq \gamma(F_E), F_E \in SS(X)_E\}$. The complement of $\gamma$-open soft set is called $\gamma$-open soft closed set.

## 3. Weaker Forms of Soft Nano Continuous Functions

In [4], the deviation between some properties of soft approximation spaces and the same properties of Pawlak approximation spaces[20] are shown. Also, the notions soft rough topology and soft rough continuous functions are defined and related properties are investigated in [4] . In [9, 10], in addition to the notions of soft nano topological spaces and soft nano continuity, the authors have introduced and studied the notions of soft nano open map, soft nano closed map, soft nano homeomorphism and some of the weaker forms of soft nano closed sets in soft nano topological spaces. Further, Abd El-latif [1] introduced the notions of generalized soft rough sets and generated soft ideal rough topological spaces. In this section, the study on soft nano topological spaces is continued and here some weaker forms of soft nano continuous functions are introduced and discussed.

**Definition 3.1.** Let $(\tau_R(X), U, E)$ be a soft nano topological space. Different cases of weaker forms of soft nano open sets can be defined by using $\gamma$-operation defined as $\gamma : SS(X)_E \to SS(X)_E$ as follows:

(i) if $\gamma = NInt(NCl)$, then $\gamma$ is called soft nano pre-open operator and the set of all soft nano pre-open sets is denoted by $SNPO(X, E)$,

(ii) if $\gamma = NInt(NCl(NInt))$, then $\gamma$ is called soft nano $\alpha$-open operator and the set of all soft nano $\alpha$-open sets is denoted by $SN\alpha O(X, E)$,

(iii) if $\gamma = NCl(NInt)$, then $\gamma$ is said to be soft nano semi-open operator and the set of all soft nano semi-open sets is denoted by $SNSO(X, E)$,

(iv) if $\gamma = NCl(NInt(NCl))$, then $\gamma$ is said to be soft nano $\beta$-open operator and the set of all soft nano $\beta$-open sets is denoted by $SN\beta O(X, E)$.

Here $SS(X)_E$ is the set of all soft sets over $X \subseteq U$ and the set of parameters $E$. The complements of soft nano pre open(respectively soft nano semi open , soft nano $\alpha$-open, soft nano $\beta$-open) sets is called soft nano pre closed(respectively soft nano semi closed,soft nano $\alpha$-closed , soft nano $\beta$-closed) set.

**Theorem 3.2.** *In a soft nano topological space $(\tau_R(X), U, E)$ the following statements holds:*

(1) *Every soft nano open (resp. soft nano closed) set is soft nano pre-open (resp. soft nano pre-closed) set,*

(2) *Every soft nano open (resp. soft nano closed) set is soft nano semi-open (resp. soft nano semi-closed) set,*

(3) *Every soft nano open (resp. soft nano closed) set is soft nano $\alpha$-open (resp. soft nano $\alpha$-closed) set,*

(4) *Every soft nano open (resp. soft nano closed) set is soft nano $\beta$-open (resp. soft nano $\beta$-closed) set.*

*Proof.* (1) Let $(G, E) \in SNO(X, E)$. Then $NInt(G, E) = (G, E)$. Since $(G, E) \subseteq NCl(G, E)$, we have $(G, E) \subseteq NInt(NCl(G, E))$. Thus $(G, E) \in SNPO(X, E)$.

(2) Let $(G, E) \in SNO(X, E)$. Then $NInt(G, E) = (G, E)$. Since $(G, E) \subseteq NCl(G, E)$, we have $(G, E) \subseteq NCl(NInt(G, E))$. Thus $(G, E) \in SNSO(X, E)$.

(3) Let $(G, E) \in SNO(X, E)$. Then $NInt(G, E) = (G, E)$. Since $(G, E) \subseteq NCl(G, E)$, we have $(G, E) \subseteq NInt(NCl(NInt(G, E)))$. Thus $(G, E) \in SN\alpha O(X, E)$.

(4) Let $(G, E) \in SNO(X, E)$. Then $NInt(G, E) = (G, E)$. Since $(G, E) \subseteq NCl(G, E)$, we have $(G, E) \subseteq NCl(G, E) \subseteq NCl(NInt(NCl(G, E)))$. Thus $(G, E) \in SN\beta O(X, E)$. □

**Definition 3.3.** Let $(\tau_R(X), U, E)$ and $(\tau_{R'}(Y), V, K)$ be two soft nano topological spaces. Then, a mapping $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ is called:

(i) soft nano semi continuous, if $f^{-1}(G,K) \in SNSO(X,E), \forall (G,K) \in SNO(Y,K)$, equivalently, the inverse image of every soft nano open set over $V$ is soft nano semi open over $U$,

(ii) soft nano pre continuous, if $f^{-1}(G,K) \in SNPO(X,E), \forall (G,K) \in SNO(Y,K)$, equivalently, the inverse image of every soft nano open set over $V$ is soft nano pre open over $U$,

(iii) soft nano $\alpha$-continuous, if $f^{-1}(G,K) \in SN\alpha O(X,E), \forall (G,K) \in SNO(Y,K)$, equivalently, the inverse image of every soft nano open set over $V$ is soft nano $\alpha$-open set over $U$,

(iv) soft nano $\beta$-continuous, if $f^{-1}(G,K) \in SN\beta O(X,E), \forall (G,K) \in SNO(Y,K)$, equivalently, the inverse image of each soft nano open set over $V$ is soft nano $\beta$-open set over $U$.

**Example 3.4.** Let $U = \{a,b,c,d\}, E = \{m_1,m_2,m_3\}$ and let

$$(G,E) = \{(m_1,\{a\}),(m_2,\{c\}),(m_3,\{b,d\})\}$$

be a soft set over $U$. Let $R$ be a soft equivalence relation on $(G,E)$ defined as follows:

$$R = \{F(m_1)\times F(m_2), F(m_2)\times F(m_1), F(m_1)\times F(m_1), F(m_2)\times F(m_2), F(m_3)\times F(m_3)\}.$$

Then the soft equivalence classes are as follows:

$[F(m_1)] = \{F(b) : F(b)RF(a)\} = \{F(m_1), F(m_2)\} = [F(m_2)]$, and $[F(m_3)] = \{F(m_3)\}$.

Now, let $U/R = \{F(m_1), F(m_2), F(m_3)\} = \{\{a\},\{c\},\{b,d\}\}$ . Let $X = \{a,b\} \subseteq U$. Then

$(L_R(X),E) = \{(m_1,\{a\}),(m_2,\{a\}),(m_3,\{a\})\}$,
$(U_R(X),E) = \{(m_1,\{a,b,d\}),(m_2,\{a,b,d\}),(m_3,\{a,b,d\})\}$,
$(B_R(X),E) = \{(m_1,\{b,d\}),(m_2,\{b,d\}),(m_3,\{b,d\})\}$.

Thus $(\tau_R(X),U,E) = \{U,\phi,(L_R(X),E),(U_R(X),E),(B_R(X),E)\}$ is a soft nano topology on $U$. So

soft nano open sets are $U,\phi,(L_R(X),E),(U_R(X),E),(B_R(X),E)$

and

soft nano semi open sets are $U,\phi,(A_1,E),(A_2,E),(A_3,E),(A_4,E)$,

where $(A_1,E) = \{(m_1,\{a\}),(m_2,\{a\}),(m_3,\{a\})\}$,
$(A_2,E) = \{(m_1,\{a,c\}),(m_2,\{a,c\}),(m_3,\{a,c\})\}$,
$(A_3,E) = \{(m_1,\{b,d\}),(m_2,\{b,d\}),(m_3,\{b,d\})\}$,
$(A_4,E) = \{(m_1,\{a,b,d\}),(m_2,\{a,b,d\}),(m_3,\{a,b,d\})\}$,
$(A_5,E) = \{(m_1,\{b,c,d\}),(m_2,\{b,c,d\}),(m_3,\{b,c,d\})\}$.

Furthermore, we can find the followings.

Soft nano pre open sets are $U,\phi$ and

$(B_1,E) = \{(m_1,\{a\}),(m_2,\{a\}),(m_3,\{a\})\}$,
$(B_2,E) = \{(m_1,\{b\}),(m_2,\{b\}),(m_3,\{b\})\}$,
$(B_3,E) = \{(m_1,\{d\}),(m_2,\{d\}),(m_3,\{d\})\}$,
$(B_4,E) = \{(m_1,\{a,b\}),(m_2,\{a,b\}),(m_3,\{a,b\})\}$,
$(B_5,E) = \{(m_1,\{a,d\}),(m_2,\{a,d\}),(m_3,\{a,d\})\}$,
$(B_6,E) = \{(m_1,\{b,d\}),(m_2,\{b,d\}),(m_3,\{b,d\})\}$,
$(B_7,E) = \{(m_1,\{a,b,c\}),(m_2,\{a,b,c\}),(m_3,\{a,b,c\})\}$,
$(B_8,E) = \{(m_1,\{a,b,d\}),(m_2,\{a,b,d\}),(m_3,\{a,b,d\})\}$,

$(B_9, E) = \{(m_1, \{a, c, d\}), (m_2, \{a, c, d\}), (m_3, \{a, c, d\})\}$.

Soft nano $\alpha$-open sets are $U, \phi$ and
$(C_1, E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$,
$(C_2, E) = \{(m_1, \{b, d\}), (m_2, \{b, d\}), (m_3, \{b, d\})\}$,
$(C_3, E) = \{(m_1, \{a, b, d\}), (m_2, \{a, b, d\}), (m_3, \{a, b, d\})\}$.

Soft nano $\beta$-open sets are $U, \phi$ and
$(D_1, E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$,
$(D_2, E) = \{(m_1, \{b\}), (m_2, \{b\}), (m_3, \{b\})\}$,
$(D_3, E) = \{(m_1, \{d\}), (m_2, \{d\}), (m_3, \{d\})\}$,
$(D_4, E) = \{(m_1, \{a, b\}), (m_2, \{a, b\}), (m_3, \{a, b\})\}$,
$(D_5, E) = \{(m_1, \{a, d\}), (m_2, \{a, d\}), (m_3, \{a, d\})\}$,
$(D_6, E) = \{(m_1, \{b, d\}), (m_2, \{b, d\}), (m_3, \{b, d\})\}$,
$(D_7, E) = \{(m_1, \{a, c\}), (m_2, \{a, c\}), (m_3, \{a, c\})\}$,
$(D_8, E) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$,
$(D_9, E) = \{(m_1, \{c, d\}), (m_2, \{c, d\}), (m_3, \{c, d\})\}$,
$(D_{10}, E) = \{(m_1, \{a, b, c\}), (m_2, \{a, b, c\}), (m_3, \{a, b, c\})\}$,
$(D_{11}, E) = \{(m_1, \{b, c, d\}), (m_2, \{b, c, d\}), (m_3, \{b, c, d\})\}$,
$(D_{12}, E) = \{(m_1, \{a, b, d\}), (m_2, \{a, b, d\}), (m_3, \{a, b, d\})\}$,
$(B_{13}, E) = \{(m_1, \{a, c, d\}), (m_2, \{a, c, d\}), (m_3, \{a, c, d\})\}$.

Now let $V = \{x, y, z, w\}, K = \{k_1, k_2, k_3\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{y, w\} \subseteq V$. Then
$(L_{R'}(Y), K) = \{(k_1, \{y, w\}), (k_2, \{y, w\}), (k_3, \{y, w\})\}$,
$(U_{R'}(Y), K) = \{(k_1, \{y, w\}), (k_2, \{y, w\}), (k_3, \{y, w\})\}$,
$(B_{R'}(Y), K) = \phi$.

Thus, soft nano topology is

$$\begin{aligned}(\tau_{R'}(Y), V, K) &= \{V, \phi, (L_{R'}(Y), K), (U_{R'}(Y), K), (B_{R'}(Y), K)\} \\ &= \{V, \phi, \{(k_1, \{y, w\}), (k_2, \{y, w\}), (k_3, \{y, w\})\}\}.\end{aligned}$$

For $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$, let us consider $f : U \to V$ and $p : E \to K$ as $f(a) = x, f(b) = y, f(c) = z, f(d) = w$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}((L_{R'}(Y), k)) = \{(m_1, \{b, d\}), (m_2, \{b, d\}), (m_3, \{b, d\})\}, f^{-1}((V, K)) = (U, E)$ and $f^{-1}(\phi) = \phi$. Thus the inverse image of every soft nano open set over $V$ is soft nano open set over $U$ and also soft nano semi open (respectively soft nano pre open, soft nano $\alpha$-open, soft nano $\beta$-open) over $U$. So $f$ is soft nano continuous and also soft nano semi (respectively soft nano pre, soft nano $\alpha$, soft nano $\beta$) continuous function.

**Example 3.5.** Let $U = \{a, b, c\}, E = \{m_1, m_2, \}$ and let

$$(G, E) = \{(m_1, \{a\}), (m_2, \{b\})\}$$

be a soft set over $U$. Let $R$ be a soft equivalence relation on $(G, E)$ defined as follows:

$$R = \{F(m_1) \times F(m_2), F(m_2) \times F(m_1), F(m_1) \times F(m_1), F(m_2) \times F(m_2)\}.$$

Now let $U/R = \{F(m_1), F(m_2)\} = \{\{a\}, \{b\}\}$ and let $X = \{a\} \subseteq U$. Then

$(L_R(X), E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\} = (U_R(X), E)$ and $(B_R(X), E) = \phi$. Thus $(\tau_R(X), U, E) = \{U, \phi, \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}\}$ is a soft nano topology on $U$. So we can easily obtain the followings:

soft nano open sets are $U, \phi, \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$,

soft nano closed sets are $U, \phi, \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{a\})\}$,
soft nano $\alpha$-open sets are $U, \phi, (H_1, E), (H_2, E), (H_3, E)$,
where $(H_1, E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$,
$(H_2, E) = \{(m_1, \{a, b\}), (m_2, \{a, b\}), (m_3, \{a, b\})\}$,
$(H_3, E) = \{(m_1, \{a, c\}), (m_2, \{a, c\}), (m_3, \{a, c\})\}$.
Now, consider $V = \{x, y, w\}, K = \{k_1, k_2\}$ and let $V/R' = \{\{x\}, \{y, w\}\}$ and $Y = \{y, w\} \subseteq V$. Then soft nano topology is $(\tau_{R'}(Y), V, K) = \{V, \phi, (L, K) = \{(k_1, \{y, w\}), (k_2, \{y, w\})\}\}$.
Let us define $f : U \to V$ and $p : E \to K$ as $f(a) = y, f(b) = w, f(c) = x$ and $p(m_1) = k_1, p(m_2) = k_2$. Then we have $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ and $f^{-1}(L, K) = (H_2, E)$. Thus $f$ is a soft nano $\alpha$-continuous function.

**Remark 3.6.** The converse of the theorem 3.2 need not be true in general.

**Example 3.7.** In example 3.4,
(i) $(A_2, E)$ is a soft nano semi open set but not a soft nano open set,
(ii) $(B_2, E)$ is a soft nano pre open set but not a soft nano open set,
(iii) $(D_2, E)$ is a soft nano $\beta$-open set but not a soft nano open set.
In example 3.5, $(H_2, E)$ is a soft nano $\alpha$-open set but not a soft nano open set.

**Theorem 3.8.** *Let $(\tau_R(X), U, E)$ and $(\tau_{R'}(Y), V, K)$ be two soft nano topological spaces and $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ be a map. Then, for the classes of soft nano semi-continuous (respectively soft nano pre-continuous, soft nano $\alpha$-continuous, soft nano $\beta$-continuous) functions, the following statements are equivalent:*
(1) *$f$ is a soft nano semi-continuous function,*
(2) *$f^{-1}(F, K) \in SNSC(X, E)$, if $(F, K) \in SNC(Y, K)$,*
(3) *$f(NCl(NInt(G, E))) \subseteq NCl(NInt(f(G, E)))$, for every soft set $(G, E)$ over $U$,*
(4) *$NCl(NInt(f^{-1}(G, K))) \subseteq f^{-1}(NCl(NInt((G, K)))$, for every soft set $(G, K)$ over $V$.*

*Proof.* (1) $\implies$ (2): Let $f$ be a soft nano semi continuous. Let $(F, K)$ be a soft nano closed set over $V$. Then $(F, K)'$ is a soft nano open set. Since $f$ is soft nano semi continuous function, we have $f^{-1}((F, K)')$ is soft nano semi open set over $U$. From theorem 2.10 (1), $(f^{-1}(F, K))'$ is soft nano semi open over $U$. Thus $f^{-1}((F, K))$ is soft nano semi closed set over $U$. So inverse image of every soft nano closed set over $V$ is soft nano semi closed over $U$.

(2) $\implies$ (1): Suppose that the inverse image of every soft nano closed set over $V$ is soft nano semi closed over $U$. Let $(G, K)$ be a soft nano open set over $V$. Then $(G, K)' = (F, K)$ is a soft nano closed set over $V$. Thus, by supposition, $f^{-1}(F, K)$ is a soft nano semi closed set over $U$. So, by theorem 2.10 (1), $f^{-1}(G, K)$ is soft nano semi open over $U$. That is, the inverse image of every soft nano open set over $V$ is soft nano semi open over $U$. Hence, by definition, $f$ is soft nano semi continuous.

(1) $\implies$ (3): Let $f$ be a soft nano semi continuous and $(G, E)$ be a soft set over $U$. Then $f((G, E))$ is soft subset over $V$. Now, $NCl(NInt(f(G, E)))$ is a soft nano closed set over $V$. Thus $f^{-1}(NCl(NInt(f(G, E))))$ is soft nano semi closed set over $U$. Since $f((G, E)) \subseteq NCl(NInt(f(G, E)))$, we have $(G, E) \subseteq$

$f^{-1}(NCl(NInt(f(G,E))))$. So $f^{-1}(NCl(NInt(f(G,E))))$ is a soft nano closed set containing $(G,E)$. But $NCl(NInt(G,E))$ is the smallest soft nano closed set containing $(G,E)$. Hence $NCl(NInt(G,E)) \subseteq f^{-1}(NCl(NInt(f(G,E))))$ which implies $f(NCl(NInt(G,E))) \subseteq NCl(NInt(f(G,E)))$.

(3) $\Longrightarrow$ (1): Let $f(NCl(NInt(G,E))) \subseteq NCl(NInt(f(G,E)))$, for every soft set $(G,E)$ over $U$. Let $(H,E)$ be a soft nano closed set over $V$. Since $f^{-1}(H,E)$ is a soft set over $U$, we have

$$f(NCl(NInt(f^{-1}(H,E)))) \subseteq NCl(NInt(f(f^{-1}(H,E)))) \subseteq NCl(NInt(H,E)).$$

Since $(H,E)$ is soft nano closed set,

$$NCl(NInt(f^{-1}(H,E))) \subseteq f^{-1}(NCl(NInt(H,E))) = f^{-1}(H,E).$$

Then we have $NCl(NInt(f^{-1}(H,E))) \subseteq f^{-1}(H,E)$. But

$$f^{-1}(H,E) \subseteq NCl(NInt(f^{-1}(H,E)))$$

is always true. Thus $NCl(NInt(f^{-1}(H,E))) = f^{-1}(H,E)$. So $f^{-1}(H,E)$ is soft nano semi closed over $U$, for every soft nano closed set $(H,E)$ over $V$. Hence $f$ is soft nano semi continuous function.

(1) $\Longrightarrow$ (4): Suppose that $f$ be a soft nano semi continuous function. Let $(G,K)$ be a soft set over $V$. Then $NCl(NInt(G,K))$ is a soft nano closed set over $V$. Since $f$ is a soft nano semi continuous function, $f^{-1}(NCl(NInt(G,K)))$ is a soft nano semi closed set over $U$. Thus we have

$$NCl(NInt(f^{-1}(NCl(NInt)(G,K)))) = f^{-1}(NCl(NInt(G,K))).$$

Since $(G,K) \subseteq NCl(NInt(G,K))$, we have

$$f^{-1}(G,K) \subseteq f^{-1}(NCl(NInt(G,K))).$$

Thus $NCl(NInt(f^{-1}(G,K))) \subseteq NCl(NInt(f^{-1}(NCl(NInt(G,K)))))$
$= f^{-1}(NCl(NInt(G,K)))$.

(4) $\Longrightarrow$ (1) : Suppose that $NCl(NInt(f^{-1}(G,K))) \subseteq f^{-1}(NCl(NInt(G,K)))$, for every soft set $(G,K)$ over $V$. Let $(G,K)$ be a soft nano closed set over $V$. Then $NCl(G,K) = (G,K)$. By assumption,

$$\begin{aligned} NCl(NInt(f^{-1}(G,K))) &\subseteq f^{-1}(NCl(NInt(G,K))) \\ &= f^{-1}(NInt(G,K)) \\ &\subseteq f^{-1}((G,K)). \end{aligned}$$

Thus $NCl(NInt(f^{-1}(G,K))) \subseteq f^{-1}((G,K))$. But

$$f^{-1}(G,K) \subseteq NCl(NInt(f^{-1}(G,K)))$$

is always true. So $NCl(NInt(f^{-1}(G,K))) = f^{-1}(G,K)$. Hence $f^{-1}(G,K)$ is a soft nano semi closed set in $U$, for every soft nano closed set $(G,K)$ over $V$. Therefore $f$ is soft nano semi continuous function. □

**Remark 3.9.** The equality in the theorem 3.8 (3) and (4) need not be true even if $f$ is soft nano semi continuous function. In example 3.4, let us consider a soft set

$$(G,E) = \{(m_1,\{a,c\}),(m_2,\{a,c\}),(m_3,\{a,c\})\}$$

over $U$. Then $f(NCl(NInt(G,E))) = \{(k_1,\{x,z\}),(k_2,\{x,z\}),(k_3,\{x,z\})\}$. But $NCl(NInt(f(G,E))) = \phi$. Thus $f(NCl(NInt(G,E)) \neq NCl(NInt(f(G,E)))$.

Now, let us consider $(H,K) = \{(k_1,\{x,y,z\}),(k_2,\{x,y,z\}),(k_3,\{x,y,z\})\}$ be a soft set over $V$. Then $NCl(NInt(f^{-1}(H,K))) = \{(m_1,\{a,c\}),(m_2,\{a,c\}),(m_3,\{a,c\})\}$. But $f^{-1}(NCl(NInt(H,K))) = \phi$. Thus

$$NCl(NInt(f^{-1}(H,K))) \neq f^{-1}(NCl(NInt(H,K))).$$

**Theorem 3.10.** *In a soft nano topological space $(\tau_R(X),U,E)$, the following statements are true.*
(1) *Each soft nano $\alpha$-open set is soft nano semi open set.*
(2) *Each soft nano semi open set is soft nano $\beta$-open set.*
(3) *Each soft nano pre open set is soft nano $\beta$-open set.*
(4) *Each soft nano $\alpha$-open set is soft nano pre open set.*

*Proof.* (1) Let $(G,E) \in SN\alpha O(X,E)$. Then

$$(G,E) \subseteq NInt(NCl(NInt(G,E))) \subseteq NCl(NInt(G,E)).$$

Thus $(G,E) \in SNSO(X,E)$.
(2) Let $(G,E) \in SNSO(X,E)$. Then $(G,E)NCl(NInt(G,E))$. Since $(G,E) \subseteq NCl(G,E)$,

$$(G,E) \subseteq NCl(NInt(G,E)) \subseteq NCl(NInt(NCl(G,E))).$$

Thus $(G,E) \in SN\beta O(X,E)$.
(3) Let $(G,E) \in SNPO(X,E)$. Then

$$(G,E) \in NInt(NCl(G,E)) \subseteq NCl(NInt(NCl(G,E))).$$

Thus $(G,E) \in SN\beta O(X,E)$.
(4). Let $(G,E) \in SN\alpha O(X,E)$. Since $NINt(G,E) \subseteq (G,E)$, we have

$$NCl(NInt(G,E)) \subseteq NCl(G,E).$$

Then $(G,E) \subseteq NInt(NCl(NInt(G,E))) \subseteq NInt(NCl(G,E))$. Thus $(G,E) \in SNPO(X,E)$. □

**Remark 3.11.** The converse of theorem 3.10 is not true in general.
In example 3.4,
(i) $(A_2,E)$ is soft nano semi open set but not soft nano $\alpha$-open,
(ii) $(D_2,E)$ is soft nano $\beta$-open set but not soft nano semi open,
(iii) $(D_7,E)$ is soft nano $\beta$-open set but not soft nano pre open,
(iv) $(B_2,E)$ is soft nano pre open set but not soft nano $\alpha$-open.

**Theorem 3.12.** *A function $f : (\tau_R(X),U,E) \to (\tau_{R'}(Y),V,K)$ be a mapping.*
(1) *Every soft nano continuous function is soft nano semi continuous function.*
(2) *Every soft nano continuous function is soft nano pre continuous function.*
(3) *Every soft nano continuous function is soft nano $\alpha$- continuous function.*
(4) *Every soft nano continuous function is soft nano $\beta$- continuous function.*

*Proof.* Follows immediately from theorem 3.2. □

**Remark 3.13.** The converse of the theorem 3.12 need not be true in general as seen in the following example.

**Example 3.14.** In the example 3.4, we have

$$(\tau_{R'}(Y), V, K) = \{V, \phi, (H, K) = \{(k_1, \{y, w\}), (k_2, \{y, w\}), (k_3, \{y, w\})\}\}.$$

. Let us define $f : U \to V$ and $p : E \to K$ as follows:

(i) $f(a) = y, f(b) = x, f(c) = w, f(d) = z$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (A_2, E)$, which is a soft nano semi open set over $U$ but not a soft nano open set over $U$. Thus $f$ is a soft nano semi continuous function but not a soft nano continuous function.

(ii) $f(a) = w, f(b) = y, f(c) = x, f(d) = w$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (B_4, E)$, which is a soft nano pre open set over $U$ but not a soft nano open set over $U$. Thus $f$ is a soft nano pre continuous function but not a soft nano continuous function.

(iii) $f(a) = x, f(b) = z, f(c) = y, f(d) = w$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (D_9, E)$, which is a soft nano $\beta$ open set over $U$ but not a soft nano open set over $U$. Thus $f$ is a soft nano $\beta$-continuous function but not a soft nano continuous function.

In example 3.5, we have $(\tau_{R'}(Y), V, K) = \{V, \phi, (F, K) = \{(k_1, \{y, w\}), (k_2, \{y, w\})\}\}$. Let us define $f : U \to V$ and $p : E \to K$ as follows:

$$f(a) = y, f(b) = w, f(c) = x \text{ and } p(m_1) = k_1, p(m_2) = k_2.$$

Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(F, K) = (H_2, E)$, which is a soft nano $\alpha$-open set over $U$ but not a soft nano open set over $U$. Thus $f$ is a soft nano $\alpha$-continuous function but not a soft nano continuous function.

**Theorem 3.15.** *Let $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ be a mapping.*

(1) *Every soft nano $\alpha$-continuous function is soft nano semi continuous function.*
(2) *Every soft nano semi continuous function is soft nano $\beta$-continuous function.*
(3) *Every soft nano pre continuous function is soft nano $\beta$-continuous function.*
(4) *Every soft nano $\alpha$-continuous function is soft nano pre continuous function.*

*Proof.* Follows immediately from theorem 3.10. □

**Remark 3.16.** The converse of the theorem 3.15 need not be true in general as seen in the following example.

**Example 3.17.** In the example 3.4, we have

$$(\tau_{R'}(Y), V, K) = \{V, \phi, (H, K) = \{(k_1, \{y, w\}), (k_2, \{y, w\}), (k_3, \{y, w\})\}\}.$$

Let us define $f : U \to V$ and $p : E \to K$ as follows:

(i) $f(a) = y, f(b) = x, f(c) = w, f(d) = z$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (A_2, E)$, which is a soft nano semi open set over $U$ but not a soft nano $\alpha$-open set over $U$. Thus $f$ is a soft nano semi continuous function but not a soft nano $\alpha$-continuous function.

(ii) $f(a) = x, f(b) = y, f(c) = w, f(d) = z$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (D_8, E)$, which is a soft nano $\beta$-open set over $U$ but not a soft nano semi open set over $U$. Thus $f$ is a soft nano $\beta$-continuous function but not a soft nano semi continuous function.

(iii) $f(a) = y, f(b) = x, f(c) = w, f(d) = z$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (D_7, E)$, which is a soft nano $\beta$-open set over $U$ but not a soft nano pre open set over $U$. Thus $f$ is a soft nano $\beta$-continuous function but not a soft nano pre continuous function.

(iv) $f(a) = y, f(b) = w, f(c) = x, f(d) = z$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi$ but $f^{-1}(H, K) = (B_4, E)$, which is a soft nano pre open set over $U$ but not a soft nano $\alpha$-open set over $U$. Thus $f$ is a soft nano pre continuous function but not a soft nano $\alpha$-continuous function.

**Theorem 3.18.** *A function $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ is soft nano pre continuous if and only if $f^{-1}(NInt(NCl(H, K))) \subseteq NInt(NCl(f^{-1}(H, K)))$, for every soft set $(H, K)$ over $V$.*

*Proof.* Let $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ be soft nano pre continuous function and $(H, K)$ be a soft set over $V$. Then $NInt(NCl(H, K))$ is a soft nano open set over $V$. Since $f$ is soft nano pre continuous function, $f^{-1}(NInt(NCl(H, K)))$ is a soft nano pre open set over $U$. Since each soft nano open set is soft nano pre open, we have $f^{-1}(NInt(NCl(H, K))) = NInt(f^{-1}(NInt(NCl(H, K))))$. Also, $NInt(NCl(H, K)) \subseteq (H, K)$ is always true. Thus

$$f^{-1}(NInt(NCl(H, K))) \subseteq f^{-1}(H, K).$$

So

$$NInt(NCl(f^{-1}(NInt(NCl(H, K))))) \subseteq NInt(NCl(f^{-1}(H, K))),$$

i.e, $f^{-1}(NInt(NCl(H, K))) \subseteq NInt(NCl(f^{-1}(H, K)))$.

Conversely, let $f^{-1}(NInt(NCl(H, K))) \subseteq NInt(NCl(f^{-1}(H, K)))$, for every soft set $(H, K)$ over $V$. Let $(H, K)$ be a soft nano open set over $V$. Then $NInt(NCl(H, K)) = (H, K)$. Also, by assumption, $f^{-1}(NInt(NCl(H, K))) \subseteq NInt(NCl(f^{-1}(H, K)))$, i.e., $f^{-1}(H, K) \subseteq NInt(NCl(f^{-1}(H, K)))$. But

$$NInt(NCl(f^{-1}(H, K))) \subseteq f^{-1}(H, K)$$

is always true. Thus $NInt(NCl(f^{-1}(H, K))) = f^{-1}(H, K)$. So $f^{-1}(H, K)$ is a soft nano pre open set over $U$, for every soft nano open set $(H, K)$ over $V$. Hence $f$ is soft nano pre continuous function. □

**Remark 3.19.** Equality does not hold good in the theorem 3.18 even if $f$ is soft nano pre continuous function.

**Example 3.20.** Let $U = \{a, b, c, d\}, E = \{m_1, m_2, m_3\}$ and let

$$(G, E) = \{(m_1, \{a, d\}), (m_2, \{b\}), (m_3, \{c\})\}$$

be a soft set over $U$. Consider $U/R = \{\{a, d\}, \{b\}, \{c\}\}$ and let $X = \{a, c\} \subseteq U$. Then

$(L_R(X), E) = \{(m_1, \{c\}), (m_2, \{c\}), (m_3, \{c\})\}$,
$(U_R(X), E) = \{(m_1, \{a, c, d\}), (m_2, \{a, c, d\}), (m_3, \{a, c, d\})\}$,
$(B_R(X), E) = \{(m_1, \{a, d\}), (m_2, \{a, d\}), (m_3, \{a, d\})\}$.

Thus $(\tau_R(X), U, E) = \{U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)\}$ is a soft nano topology on $U$.

Let $V = \{x, y, z, w\}, K = \{k_1, k_2, k_3, k_4\}$ with $V/R' = \{\{x\}, \{y\}, \{z\}, \{w\}\}$ and $Y = \{x, w\} \subseteq V$. Then soft nano topology is

$$(\tau_{R'}(Y), V, K) = \{V, \phi, (L_{R'}(Y), K), (U_{R'}(Y), K), (B_{R'}(Y), K)\},$$

where $(L_{R'}(Y), K) = \{(k_1, \{x, w\}), (k_2, \{x, w\}), (k_3, \{x, w\})\}$,
$(U_{R'}(Y), K) = (L_{R'}(Y), K)$ and $(B_{R'}(Y), K) = \phi$.
For $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$, let us consider $u : U \to V$ and $p : E \to K$ as $f(a) = x, f(b) = y, f(c) = z, f(d) = w$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f$ is soft nano continuous function on $U$, because the inverse image of every soft nano open set over $V$ is soft nano open over $U$.

Now, let $(H, K) = \{(k_1, \{z\}), (k_2, \{z\}), (k_3, \{z\}), (k_4, \{z\})\}$ be a soft set over $V$ with respect to $K$. Then we have

$$f^{-1}(NInt(NCl(H, K))) = \{(m_1, \{b, c\}), (m_2, \{b, c\}), (m_3, \{b, c\})\}$$

and

$$NInt(NCl(f^{-1}(H, K))) = \{(m_1, \{c\}), (m_2, \{c\}), (m_3, \{c\})\}.$$

Thus $f^{-1}(NInt(NCl(H, K))) \neq NInt(NCl(f^{-1}(H, K)))$.

**Definition 3.21.** The mapping $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ is called a soft nano semi closed, if image of every soft nano semi closed set over $U$ is soft nano closed over $V$.

**Theorem 3.22.** *A mapping $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ is soft nano semi closed map if and only if $NCl(NInt(f(G, E))) \subseteq f(NCl(NInt(G, E)))$, for every soft set $(G, E)$ over $U$.*

*Proof.* Suppose that $f$ is soft nano semi closed map. Since $NCl(NInt(G, E))$ is a soft nano semi closed set over $U$, $f(NCl(NInt(G, E)))$ is soft nano closed set over $V$. Since $(G, E) \subseteq NCl(NInt(G, E))$, $f((G, E)) \subseteq f(NCl(NInt(G, E)))$. Then $f(NCl(NInt(G, E)))$ is a soft nano closed set containing $f((G, E))$. But, $NCl(NInt(f(G, E)))$ is the smallest nano closed set containing $f((G, E))$. Thus $NCl(NInt(f(G, E))) \subseteq f(NCl(NInt(G, E)))$.

Conversely, suppose that $NCl(NInt(f(G, E))) \subseteq f(NCl(NInt(G, E)))$, for each soft set $(G, E)$ over $U$. Let $(G, E)$ be a soft nano semi closed set over $U$. Then we have $NCl(NInt(G, E)) = (G, E)$. Thus $f((G, E)) \subseteq NCl(NInt(f(G, E))) \subseteq f(NCl(NInt(G, E))) \subseteq f((G, E))$. Therefore, $f((G, E)) = NCl(f(G, E))$. So $f((G, E))$ is soft nano closed over $V$. Hence $f$ is a soft nano semi closed mapping. □

**Definition 3.23.** A mapping $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ is said to be a soft nano semi homeomorphism, if

(i) $f$ is one-one and onto,
(ii) $f$ is soft nano semi continuous,
(iii) $f$ is soft nano semi open.

**Theorem 3.24.** *Suppose $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ be an one-one onto mapping. Then, $f$ is a soft nano semi homeomorphism if and only if $f$ is soft nano semi closed and soft nano semi continuous mapping.*

*Proof.* Suppose that $f$ is a soft nano semi homeomorphism. Then, by definition, $f$ is soft nano semi continuous mapping. Consider $(G, E)$ be an arbitrary soft nano semi closed set over $U$. Then $(G, E)'$ is soft nano semi open set. Since $f$ is soft nano semi open map, $f((G, E)')$ is a soft nano open set over $V$. Thus $(f(G, E))'$ is soft nano open over $V$. So $f(G, E)$ is soft nano closed set over $V$. Hence the image of soft nano semi closed set over $U$ is soft nano closed over $V$. Therefore $f$ is soft nano semi closed mapping.

Conversely, let $f$ be a soft nano semi closed and soft nano semi continuous mapping. Let $(G, E)$ be a soft nano semi open set over $U$. Then $(G, E)'$ is soft nano semi closed set over $U$. Since $f$ is soft nano semi closed map, $f((G, E)') = (f(G, E))'$ is soft nano semi closed over $V$ and thus soft nano closed over $V$. So $f(G, E)$ is soft nano semi open set over $V$. Hence $f$ is soft nano semi open and hence $f$ is a soft nano homeomorphism. □

**Theorem 3.25.** *Let $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ be an one-one and onto map. Then, $f$ is a soft nano semi homeomorphism if and only if $f(NCl(NInt(G, E)))$ $= NCl(NInt(f(G, E)))$, for each soft set $(G, E)$ over $U$.*

*Proof.* Suppose $f$ is a soft nano semi homeomorphism. By theorem 3.24, $f$ is soft nano semi continuous and soft nano semi closed. Let $(G, E)$ be a soft set over $U$. From theorem 3.8, we have

$$f(NCl(NInt(G, E))) \subseteq NCl(NInt(f(G, E))). \quad (3.25.1)$$

Clearly, $NCl(NInt(G, E))$ is a soft nano semi closed set over $U$ and $f$ is soft nano semi closed mapping. Then $f(NCl(NInt(G, E)))$ is a soft nano semi closed set over $V$. Thus $NCl(NInt(f(NCl(NInt(G, E))))) = f(NCl(NInt(G, E)))$. Since $(G, E) \subseteq NCl(NInt(G, E))$, $f(G, E) \subseteq f(NCl(NInt(G, E)$. So

(3.25.2)

$$NCl(NInt(f(G, E))) \subseteq NCl(NInt(f(NCl(NInt(G, E))))) = f(NCl(NInt(G, E))).$$

Hence, from (3.25.1) and (3.25.2), $NCl(NInt(f(G, E))) = f(NCl(NInt(G, E)))$.

Conversely, suppose that $NCl(NInt(f(G, E))) = f(NCl(NInt(G, E)))$. By theorem 3.8, $f$ is soft nano semi continuous. If $(G, E)$ is soft nano semi closed set over $U$, then $NCl(NInt(G, E)) = (G, E)$. Which implies $f(G, E) = f(NCl(NInt(G, E)))$. Thus $f(G, E)$ is soft nano closed set over $V$, for each soft nano semi closed set $(G, E)$ over $V$. So $f$ is soft nano semi closed mapping. Hence $f$ is soft nano semi continuous and semi closed mapping. Therefore $f$ is a soft nano homeomorphism. □

**Definition 3.26.** Let $(\tau_R(X), U, E)$ be a soft nano topological space and $(G, E)$ be a soft set over $U$. Then the intersection of all soft nano $\beta$-closed sets containing $(G, E)$ is called the soft nano $\beta$-closure of $(G, E)$ and is denoted by $N\beta Cl(G, E)$. The union of all soft nano $\beta$-open sets contained in $(G, E)$ is called the soft nano $\beta$-interior of $(G, E)$ and is denoted by $N\beta Int(G, E)$.

**Example 3.27.** Let $U = \{a, b, c, d\}, E = \{m_1, m_2, m_3\}$ and let

$$(G, E) = \{(m_1, \{a\}), (m_2, \{c\}), (m_3, \{b, d\})\}.$$

Consider $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and let $X = \{a, b\} \subseteq U$. Then

$$(\tau_R(X), U, E) = \{U, \phi, (L_R(X), E), (U_R(X), E), (B_R(X), E)\},$$

where $(L_R(X), E) = \{(m_1, \{a\}), (m_2, \{a\}), (m_3, \{a\})\}$,
$(U_R(X), E) = \{(m_1, \{a, b, d\}), (m_2, \{a, b, d\}), (m_3, \{a, b, d\})\}$,
$(B_R(X), E) = \{(m_1, \{b, d\}), (m_2, \{b, d\}), (m_3, \{b, d\})\}$.
ow, let $V = \{x, y, z, w\}, K = \{k_1, k_2, k_3\}$ with $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$ and let $Y = \{x, y\} \subseteq V$. Then soft nano topology is

$$(\tau_{R'}(Y), V, K) = \{V, \phi, (L_{R'}(Y), K), (U_{R'}(Y), K), (B_{R'}(Y), K)\},$$

where $(L_{R'}(Y), K) = \{(k_1, \{x\}), (k_2, \{x\}), (k_3, \{x\})\}$,
$(U_{R'}(Y), K) = \{(k_1, \{x, y, z\}), (k_2, \{x, y, z\}), (k_3, \{x, y, z\})\}$,
$(B_{R'}(Y), K) = \{(k_1, \{y, z\}), (k_2, \{y, z\}), (k_3, \{y, z\})\}$.
For $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$, let us define $u : U \to V$ and $p : E \to K$ as $u(a) = x, u(b) = y, u(c) = w, u(d) = z$ and $p(m_1) = k_1, p(m_2) = k_2, p(m_3) = k_3$. Then $f$ is a soft nano $\beta$-continuous function.

**Theorem 3.28.** *Let $f : (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$ be a soft mapping. Where $(\tau_R(X), U, E)$ and $(\tau_{R'}(Y), V, K)$ are soft nano topological spaces. Then the following statements are equivalent:*
(1) *$f$ is soft nano $\beta$-continuous function,*
(2)*the inverse image of each soft nano closed set $(G, E)$ over $V$ is soft nano $\beta$-closed set over $U$,*
(3) *$f(N\beta Cl(G, E)) \subseteq NCl(f(G, E))$, for each soft set $(G, E)$ over $U$,*
(4) *$N\beta Cl(f^{-1}(H, E)) \subseteq f^{-1}(NCl(H, E))$, for each soft set $(H, E)$ over $V$,*
(5) *$f^{-1}(NInt(H, E)) \subseteq N\beta Int(f^{-1}(H, E))$, for each soft set $(H, E)$ over $V$.*

*Proof.* (1) $\Longrightarrow$ (2): Let $f$ be a soft nano $\beta$-continuous function. Let $(G, E)$ be a soft nano closed set over $V$. Then $(G, E)'$ is soft nano open set over $V$. Since the function $f$ is soft nano $\beta$-continuous, $f^{-1}((G, E)')$ is soft nano $\beta$-open set in $U$. Thus, from theorem 2.10 (1), $f^{-1}(G, E)$ is soft nano $\beta$-closed in $U$.

(2) $\Longrightarrow$ (1): Let $(G, E)$ be a soft nano open set over $V$. Then $(G, E)'$ is a soft nano closed set. From the assumption, $f^{-1}((G, E)')$ soft nano $\beta$-closed set over $U$. Thus $(f^{-1}(G, E))'$ is a soft nano $\beta$-closed set. So $f^{-1}(G, E)$ is a soft nano $\beta$-open set over $U$. Hence $f$ is a soft nano $\beta$-continuous function.

(1) $\Longrightarrow$ (3): Suppose that $f$ be a soft nano $\beta$-continuous function. Let $(G, E)$ be a soft set over $U$. Then clearly, $NCl(f(G, E))$ is a soft nano closed set over $V$. Since $f$ is a soft nano $\beta$-continuous, $f^{-1}(NCl(f(G, E)))$ is soft nano $\beta$-closed over $U$. Since $f(G, E) \subseteq NCl(f(G, E))$, $f^{-1}(f(G, E)) \subseteq f^{-1}(NCl(f(G, E)))$. Since $f^{-1}(NCl(f(G, E)))$ is soft nano $\beta$-closed set,

$$N\beta Cl(G, E) \subseteq N\beta Cl[(f^{-1}(NCl(f(G, E))))] = f^{-1}(NCl(f(G, E))).$$

Thus $N\beta Cl(G, E) \subseteq f^{-1}(NCl(f(G, E)))$. So

$$f(N\beta Cl(G, E)) \subseteq NCl(f(G, E)),$$

for each soft set $(G, E)$ over $U$.

(3) $\Longrightarrow$ (1): Let $f(N\beta Cl(G, E)) \subseteq NCl(f(G, E))$, for each soft set $(G, E)$ over $U$ and let $(H, E)$ be a soft nano closed set over $V$. Then, by assumption, we have

$$f(N\beta Cl(f^{-1}(H, E))) \subseteq NCl(f(f^{-1}(H, E) = NCl(H, E) = (H, E).$$

Thus $f(N\beta Cl(f^{-1}(H,E)) \subseteq (H,E)$. So $N\beta Cl(f^{-1}(H,E) \subseteq f^{-1}(H,E)$. But $f^{-1}(H,E) \subseteq (N\beta Cl(f^{-1}(H,E)))$ is always true. Hence $f^{-1}(H,E)$ is a soft nano $\beta$-closed set over $U$, for each soft nano closed set $(H,E)$ over $V$. Therefore $f$ is soft nano $\beta$-continuous.

(1) $\Longrightarrow$ (4): Let $f$ be a soft nano $\beta$-continuous function and let $(H,E)$ be a soft set over $V$. Then $NCl(H,E)$ is a soft nano closed set over $V$. Since $f$ is a soft nano $\beta$-continuous, $f^{-1}(NCl(H,E))$ is soft nano $\beta$-closed set over $U$. Thus $N\beta Cl[f^{-1}(NCl(H,E))] = f^{-1}(NCL(H,E))$. Since $(H,E) \subseteq N\beta Cl(H,E)$, we have $f^{-1}(H,E) \subseteq f^{-1}(N\beta Cl(H,E))$. So

$$N\beta Cl(f^{-1}(H,E)) \subseteq N\beta Cl[f^{-1}(N\beta Cl(H,E))] = f^{-1}(NCl(H,E)).$$

Hence $N\beta Cl(f^{-1}(H,E)) \subseteq f^{-1}(NCl(H,E))$, for each soft set $(H,E)$ over $V$.

(4) $\Longrightarrow$ (1): Suppose that $N\beta Cl(f^{-1}(H,E)) \subseteq f^{-1}(NCl(H,E))$, for each soft set $(H,E)$ over $V$ and let $(H,E)$ be a soft nano closed set over $V$. Then $NCl(H,E) = (H,E)$. By the assumption, we have

$$N\beta Cl(f^{-1}(H,E)) \subseteq f^{-1}(NCl(H,E)) = f^{-1}(H,E).$$

Thus $N\beta Cl(f^{-1}(H,E)) \subseteq f^{-1}(H,E)$. But $f^{-1}(H,E) \subseteq N\beta Cl(f^{-1}(H,E))$ is always true. So $N\beta Cl(f^{-1}(H,E)) = f^{-1}(H,E)$. Hence $f^{-1}(H,E)$ is a soft nano $\beta$-closed set over $U$, for each soft nano closed set $(H,E)$ over $V$. Therefore $f$ is a soft nano $\beta$-continuous.

(1) $\Longrightarrow$ (5): Suppose $f$ be a soft nano $\beta$-continuous function. Let $(H,E)$ be a soft set over $V$. Then $NInt(H,E)$ is a soft nano open set over $V$ and thus $f^{-1}(NInt(H,E))$ is soft nano $\beta$-open over $U$, by the definition of soft nano $\beta$-continuity. So $N\beta Int[f^{-1}(NInt(H,E))] = f^{-1}(NInt(H,E))$.

On the other hand, $NInt(H,E) \subseteq (H,E)$. Then $f^{-1}(NInt(H,E)) \subseteq f^{-1}(H,E)$. Thus

$$N\beta Int(f^{-1}(NInt(H,E))) \subseteq N\beta Int(f^{-1}(H,E)).$$

So $f^{-1}(NInt(H,E)) \subseteq N\beta Int(f^{-1}(H,E))$.

(5) $\Longrightarrow$ (1): Suppose that $f^{-1}(NInt(H,E)) \subseteq N\beta Int(f^{-1}(H,E))$, for each soft set $(H,E)$ over $V$. If $(H,E)$ is a soft nano open set over $V$, then we have $NInt(H,E) = (H,E)$. By the assumption, we have

$$f^{-1}(NInt(H,E)) \subseteq N\beta Int(f^{-1}(H,E)).$$

Thus $f^{-1}(H,E) \subseteq N\beta Int(f^{-1}(H,E))$. But $N\beta Int(f^{-1}(H,E)) \subseteq f^{-1}(H,E)$ is always true. So $N\beta Int(f^{-1}(H,E)) = f^{-1}(H,E)$. Hence $(H,E)$ is soft nano $\beta$-open set over $U$, for each soft nano open set $(H,E)$ over $V$. Therefore $f$ is a soft nano $\beta$-continuous. □

## 4. Application of soft nano topology

**Example 4.1.** Measles is a viral and infectious disease of the respiratory system. Measles is a contagious disease that can spread rapidly through contact with infected person's mucus and saliva. It can spread into air when an infected person cough or sneeze. The risk of infection increases if one shares drinking glass or eating utensils from an infected person. Measles can now almost be prevented with a proper vaccine. Symptoms of measles generally appear within 14 days of exposure to the virus. Symptoms include: skin rash, red eyes, cough, fatigue, temperature. The following is the information about 8 patients.

Table 1. Data in terms of soft set

| E/U | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ |
|---|---|---|---|---|---|---|---|---|
| $e_1$ | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| $e_2$ | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| $e_3$ | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| $e_4$ | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| $e_5$ | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Measles | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |

Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ be the set of patients and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of attributes (symptoms of the disease Measles),where $e_1, e_2, e_3, e_4, e_5$ stands for skin rash, red eyes, cough, fatigue and temperature respectively.
Let $S = (F, E)$ be a soft set given as in table 1.
For $X = \{p_2, p_3, p_7, p_8\}$ the set of patients having Measles, we have the set of equivalence classes is given by $U/R = \{\{p_1\}, \{p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_7\}\}$.
Then the soft nano topology over $U$ with respect to $X$ and $E$ given by

$$(\tau_R(X), U, E) = \{U, \phi, \{p_2, p_7, p_8\}, \{p_2, p_3, p_6, p_7, p_8\}, \{p_3, p_6\}\}.$$

If the attribute $e_1$ is removed, then we get

$$U/(R - e_1) = \{\{p_1\}, \{p_2, p_8\}, \{p_3, p_4, p_6\}, \{p_5, p_7\}\}.$$

Thus $U/R \neq U/(R - e_1)$ and $(\tau_R(X), U, E) \neq (\tau_{(R-e_1)}(X), U, E)$.
If the attribute $e_2$ is removed, then we get

$$U/(R - e_2) = \{\{p_1\}, \{p_2, p_3, p_6, p_8\}, \{p_4\}, \{p_5\}, \{p_7\}\}.$$

Thus $U/R \neq U/(R - e_2)$ and $(\tau_R(X), U, E) \neq (\tau_{(R-e_2)}(X), U, E)$.
If the attribute $e_3$ is removed, then we get

$$U/(R - e_3) = \{\{p_1\}, \{p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_7\}\}.$$

Thus $U/R = U/(R - e_3)$ and $(\tau_R(X), U, E) = (\tau_{(R-e_3)}(X), U, E)$.
If the attribute $e_4$ is removed, then we get

$$U/(R - e_4) = \{\{p_1\}, \{p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_7\}.$$

Thus $U/R = U/(R - e_4)$ and $(\tau_R(X), U, E) = (\tau_{(R-e_4)}(X), U, E)$.
If the attribute $e_5$ is removed, then we get

$$U/(R - e_5) = \{\{p_1, p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_7\}\}.$$

Thus $U/R \neq U/(R - e_5)$ and $(\tau_R(X), U, E) \neq (\tau_{(R-e_5)}(X), U, E)$.
So $CORE(SR) = \{e_1, e_2, e_5\}$, i.e., skin rash, red eyes and temperature are the key attributes to say that the patient has Measles.

## 5. Conclusion

In the present paper, the notions of some weaker forms of soft nano continuity namely soft nano semi continuity, soft nano pre continuity, soft nano $\alpha$-continuity and soft nano $\beta$-continuity on soft nano topological spaces are introduced and related properties are studied. Also, the problem of Measles is discussed using the notion of soft nano topology. It is shown that the notion of soft nano topology is advantageous to analyze the problems arising from real world situations.

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