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Abstract. In [7], the authors reported that a soft T_i-space need not be a soft T_{i-1}-space, for $i = 3, 4, 5$ [Line 4 and 5 in abstract] and [Theorem 3.21], and the soft T_i-spaces in the sense of [3] and soft T_i-spaces in their work are equivalent, for $i = 0, 1, 2, 3$ [Line 7 and 8 in abstract] and [Line 12 and 13, p.p. 522]. In this note, we correct the errors in these assertions by proving that every soft T_3-space is a soft T_2-space and presenting two counterexamples to show that a soft T_i-space in the sense of [3] is not equivalent to a soft T_i-space in the sense of [7], for $i = 2, 3$.

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Keywords: Soft set, Soft point, Soft T_i-space ($i = 1, 2, 3$).

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1. Introduction

Molodtsov [4] in 1999, initiated the concept of soft sets as a new mathematical tool for dealing with uncertainties. Shabir and Naz [6] in 2011, employed this notion in establishing the concept of soft topological spaces. They introduced soft separation axioms by utilizing ordinary points and investigated its basic properties. The authors of [2] and [5] defined a concept of soft point, which is a special case of the definition of soft point in [8], and verified some results related to soft limit points and soft neighborhood systems. Georgiou et al. [3] in 2013, introduced and studied new soft axioms namely soft T_i-spaces, for $i = 0, 1, 2, 3$, and Tantawy et al. [7] investigated new soft axioms namely soft T_i-spaces, for $i = 0, 1, 2, 3, 4, 5$.

We observe that there are some mistakes in [7]. To correct these mistakes, we prove that every soft T_3-space is a soft T_2-space with respect to [7] and provide two examples to illustrate that soft T_i-spaces in [3] and soft T_i-spaces in [7] are not equivalent, for $i = 2, 3$.

2. Preliminaries

In what follows, we recall some definitions that will be needed in the sequels.

Definition 2.1 ([4]). A pair (G, A) is said to be a soft set over X provided that G is a map of A into the family of all subsets of X. For short, we write (G, A) as ordered pairs $G_A = \{(a, G(a)) : a \in A \text{ and } G(a) \in 2^X\}$.

Definition 2.2 ([1]). The relative complement of a soft set (G, A), denoted by $(G, A)^c$, is given by $(G, A)^c = (G^c, A)$, where a map $G^c : A \to 2^X$ is defined by

$$G^c(a) = X - G(a), \text{ for each } a \in A.$$

Definition 2.3 ([6]). A collection τ of soft sets over X with a fixed set of parameter A is called a soft topology on X, if it satisfies the following three axioms:

(i) the null soft set $\widetilde{\varnothing}$ and the absolute soft set $\widetilde{X}$ are members of τ,

(ii) the soft union of an arbitrary number of soft sets in τ is also a member of τ,

(iii) the soft intersection of a finite number of soft sets in τ is also a member of τ.

The triple (X, τ, A) is called a soft topological space. Each soft set in τ is called soft open and its relative complement is called soft closed.

Definition 2.4 ([2, 5]). A soft subset (P, A) of (X, τ, A) is called soft point, if there is $a \in A$ and $x \in X$ satisfies that $P(a) = \{x\}$ and $P(e) = \varnothing$, for each $e \in A \setminus \{a\}$. A soft point will be shortly denoted by x_a.

Definition 2.5 ([3]). A soft topological space (X, τ, A) is said to be:

(i) soft T_0-space, if for every pair of distinct points $x, y \in X$ and for every $a \in A$, there is a soft open set U_A such that $x \in_a U_A$ and $y \notin_a U_A$ or $y \in_a U_A$ and $x \notin_a U_A$,

(ii) soft T_1-space, if for every pair of distinct points $x, y \in X$ and for every $a \in A$, there are soft open sets U_A and V_A such that $x \in_a U_A, y \notin_a U_A$ and $y \in_a V_A$, $x \notin_a V_A$,

(iii) soft T_2-space, if for every pair of distinct points $x, y \in X$ and for every $a \in A$, there are soft open sets U_A and V_A such that $x \in_a U_A$, $y \in_a V_A$ and $U(a) \bigcap V(a) = \varnothing$,

(iv) soft T_3-space, if for every $x \in X$, for every $a \in A$ and for every soft closed set H_A such that $x \notin_a H_A$, there are soft open sets U_A and V_A such that $x \in V(a)$, $H(a) \subseteq U(a)$ and $U(a) \bigcap V(a) = \varnothing$.

Definition 2.6 ([7]). A soft topological space (X, τ, A) is said to be:

(i) soft T_0-space, if for every pair of distinct soft points $x_a, y_a \in X$, there is a soft open set U_A such that $x_a \in U_A$ and $y_a \notin U_A$ or $y_a \in U_A$ and $x_a \notin U_A$,

(ii) soft T_1-space, if for every pair of distinct soft points $x_a, y_a \in X$, there are soft open sets U_A and V_A such that $x_a \in U_A, y_a \notin U_A$ and $y_a \in V_A$, $x_a \notin V_A$,

(iii) soft T_2-space, if for every pair of distinct soft points $x_a, y_a \in X$, there are disjoint soft open sets U_A and V_A containing x_a and y_a, respectively,

(iv) soft regular, if for every soft closed set H_A and $x_a \in \widetilde{X}$ such that $x_a \notin H_A$, there are disjoint soft open sets U_A and V_A such that $H_A \widetilde{\subseteq} U_A$ and $x_a \in V_A$,

(v) soft T_3-space, if it is both soft regular and soft T_1-space.

3. Main Results

Tantawy et al. [7] claimed that a soft T_3-space is a soft T_2-space provided that x_a is a soft closed set, for each $x \in X$ and $a \in A$ [Line 4 and 5 in abstract] and [Theorem 3.21, p.p. 519]. In the following result, we prove that a soft T_3-space is a soft T_2-space without imposing x_a is a soft closed set.

Theorem 3.1. *Every soft T_3-space (X, τ, A) is a soft T_2-space.*

Proof. Let $x_a \neq y_a \in \widetilde{X}$. Since (X, τ, A) is a soft T_1-space, there exists a soft open sets U_A such that $x_a \in U_A$ and $y_a \notin U_A$. Now, $x_a \notin U_A^c$ and U_A^c is a soft closed set containing y_a. It follows, by soft regularity of (X, τ, A), that there exist disjoint soft open sets W_A and V_A such that $x_a \in W_A$ and $y_a \in U_A^c \widetilde{\subseteq} V_A$. Then the desired result is proved. □

Tantawy et al. [7] claimed that the soft T_i-spaces in the sense of [3] and soft T_i-spaces in their work are equivalent, for $i = 0, 1, 2, 3$ [Line 7 and 8 in abstract] and [Line 12 and 13, p.p. 522]. The following two examples illustrate that this result need not be true in general in case of $i = 2, 3$.

Example 3.2. Let $A = \{a, b, c\}$ be a set of parameters and let the universe set $X = \{x, y\}$. Then a soft collection $\tau = \{\widetilde{\varnothing}, \widetilde{X}, (G_i, A) : i = 1, 2, ..., 8\}$ is a soft topology on X, where

$$(G_1, A) = \{(a, \{x\}), (b, \{y\}), (c, \{x\})\},$$
$$(G_2, A) = \{(a, \{y\}), (b, \{x\}), (c, \{x\})\},$$
$$(G_3, A) = \{(a, X), (b, X), (c, \{x\})\},$$
$$(G_4, A) = \{(a, \varnothing), (b, \varnothing), (c, \{x\})\},$$
$$(G_5, A) = \{(a, \varnothing), (b, \varnothing), (c, \{y\})\},$$
$$(G_6, A) = \{(a, \{x\}), (b, \{y\}), (c, X)\},$$
$$(G_7, A) = \{(a, \{y\}), (b, \{x\}), (c, X)\},$$
$$(G_8, A) = \{(a, \varnothing), (b, \varnothing), (c, X)\}.$$

Obviously (X, τ, A) is a soft T_2-space in the sense of [3]. On the other hand, $x_b \neq y_b$. For any two soft open sets (G_i, A), (G_j, A) such that $x_b \in (G_i, A)$ and $y_b \in (G_j, A)$, we have that $x \in G_i(c) \bigcap G_j(c)$. Hence it is not a soft T_2-space with respect to the definition of [7].

Example 3.3. Let $A = \{a, b\}$ be a set of parameters and let the universe set $X = \{x, y, z\}$. Then a soft collection $\tau = \{\widetilde{\varnothing}, \widetilde{X}, \{(a, \{x, y\}), (b, \{z\})\}, \{(a, \{z\}), (b, \{x, y\})\}\}$ is a soft topology on X. Obviously, (X, τ, A) is a soft T_3-space in the sense of [3]. On the other hand, $x_a \neq y_a$ and any soft open set containing x_a contains y_a as well. Thus it is not a soft T_1-space with respect to the definition of [7]. So it is not a soft T_3-space with respect to [7].

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