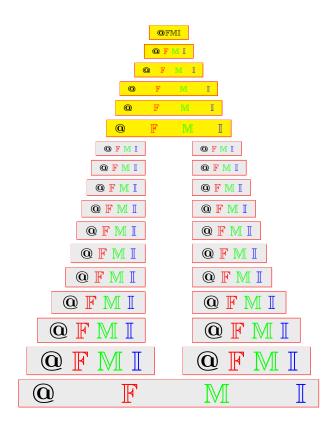
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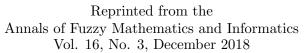


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# Two notes on "On soft Hausdorff spaces"

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# Two notes on "On soft Hausdorff spaces"

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ABSTRACT. One of the well known results in general topology says that every compact subset of a Hausdorff space is closed. This result in soft topology is not true in general as demonstrated throughout this note. We begin this investigation by showing that [Theorem 3.34, p.p.23] which proposed by Varol and Aygün [7] is invalid in general, by giving a counterexample. Then we derive under what condition this result can be generalized in soft topology. Finally, we evidence that [Example 3.22, p.p. 20] which introduced in [7] is false, and we make a correction for this example to satisfy a condition of soft Hausdorffness.

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# 1. INTRODUCTION

To handle problems which contain uncertainties and vagueness, Molodtsov [4] in 1999, proposed a new mathematical tool, namely soft sets. Shabir and Naz [6] in 2011, formulated the concept of soft topological spaces based on the soft sets notion. They initiated some soft topological notions such as some soft operators and soft separation axioms. In [2], the authors introduced a concept of soft compactness and investigated main features. Zorlutuna et al. [8] gave a notion of soft points and then the authors [3, 5] modified this notion in order to investigate some results related to soft limit points and soft metric spaces.

In 2013, Varol and Aygün [7] characterized a soft Hausdorff space and discussed some results related to it. However, they made two errors, as we observe, the first one was in [Theorem 3.34, p.p. 23] which investigated a relationship between soft closed set and soft compact soft Hausdorff space. To show this mistake, we provide a counterexample and then we present the correct form of this result. The second error was in [Example 3.22, p.p. 20]. We explain why this example did not satisfy the second axiom of a soft topology, hence it is not a soft Hausdorff space. To remove an error of this example, we consider the given collection as a soft base. This treatment constructs a soft topology satisfying a soft  $T_2$ -axiom.

## 2. Preliminaries

In what follows, we recall some definitions which will be needed throughout this investigation.

**Definition 2.1** ([4]). A pair (G, A) is said to be a soft set over X, if G is a mapping of A into  $2^X$ .

**Definition 2.2** ([1]). The relative complement of a soft set (G, A), denoted by  $(G^c, A)$ , where  $G^c : A \to 2^X$  is the mapping defined by  $G^c(a) = X \setminus G(a)$ , for each  $a \in A$ .

**Definition 2.3** ([6]). For a soft set (G, A) over X and  $x \in X$ , we say that:

(i)  $x \in (G, A)$ , if  $x \in G(a)$ , for each  $a \in A$ ,

(ii)  $x \notin (G, A)$ , if  $x \notin G(a)$ , for some  $a \in A$ .

**Definition 2.4** ([6]). A collection  $\tau$  of soft sets over a non-empty set X with a fixed set of parameters A is called a soft topology on X, if it satisfies the following three axioms:

(i) the null soft set  $\widetilde{\Phi}$  and the absolute soft set  $\widetilde{X}$  are members of  $\tau$ ,

(ii) the union of an arbitrary number of soft sets in  $\tau$  is also a member of  $\tau$ ,

(iii) the intersection of a finite number of soft sets in  $\tau$  is also a member of  $\tau$ .

The triple  $(X, \tau, A)$  is called a soft topological space. Each soft set in  $\tau$  is called soft open and its relative complement is called soft closed.

**Definition 2.5** ([2]). (i) A family  $\{(G_i, A) : i \in I\}$  of soft open sets is called a soft open cover of  $(X, \tau, A)$ , if  $\widetilde{X} = \bigcup_{i \in I} (G_i, A)$ .

(ii) A soft topological space  $(X, \tau, A)$  is called soft compact (resp. soft Lindelöf), provided that every soft open cover of  $\tilde{X}$  has a finite (resp. countable) subcover.

(iii)]A family  $\mathcal{B}$  of soft open subsets of  $(X, \tau, A)$  is called a soft base of  $\tau$  if every member of  $\tau$  can be expressed as a union of members of  $\mathcal{B}$ .

**Definition 2.6** ([3, 5]). A soft subset (P, A) of  $\widetilde{X}$  is called soft point, if there is  $a \in A$  and there is  $x \in X$  satisfies that  $P(a) = \{x\}$  and  $P(e) = \emptyset$ , for each  $e \in A \setminus \{a\}$ . A soft point will be shortly denoted by  $P_a^x$ .

**Definition 2.7** ([6]). A soft topological space  $(X, \tau, A)$  is said to be a soft  $T_2$ -space (or a soft Hausdorff space), if for every  $x \neq y \in X$ , there are two disjoint soft open sets (G, A) and (F, A) such that  $x \in (G, A)$  and  $y \in (F, A)$ .

# 3. Main Results

In [7], the authors introduced the following result which is numbered as Theorem 3.34 in their paper.

**Theorem 3.1.** Let  $(X, \tau, A)$  be a soft Hausdorff space. If (F, A) is soft compact subset of  $\widetilde{X}$ , then (F, A) is soft closed.

The following example illustrates that the above theorem need not be true in general.

**Example 3.2.** Let  $A = \{a_1, a_2\}$  be a set of parameters and let  $X = \{x, y\}$  be the universe set. Then a collection  $\tau = \{\widetilde{\Phi}, \widetilde{X}, (G, A), (H, A)\}$  is a soft topology on X, where

$$G(a_1) = \{x\}, \ G(a_2) = \{x\}$$

and

$$H(a_1) = \{y\}, \ H(a_2) = \{y\}.$$

Obviously,  $(X, \tau, A)$  is a soft Hausdorff space. On the other hand, a soft set (F, A), which defined as  $F(a_1) = \{x\}$ ,  $F(a_2) = \{y\}$ , is a soft compact subset of  $\widetilde{X}$ . But it is not soft closed.

**Remark 3.3.** We think that an error of the proof of Theorem 3.34 in [7] attributed to that the authors incorrectly expected that: If for each  $x \in (M, A)$  implies that  $x \in (N, A)$ , then  $(M, A) \subseteq (N, A)$ . This result is true via the set theory, but it need not be true via the soft set theory. We can show this by taking the two soft subsets  $(M, A) = \{(a_1, \{x\}), (a_1, X)\}$  and  $(N, A) = \{(a_1, X), (a_1, \{x\})\}$  of  $(X, \tau, A)$ which given in the above example. Now, every  $x \in (M, A)$  implies that  $x \in (N, A)$ and every  $x \in (N, A)$  implies that  $x \in (M, A)$ . However  $(M, A) \not\subseteq (N, A)$  and  $(N, A) \not\subseteq (M, A)$ .

We correctly formulate this result via the soft set theory by utilizing a soft point notion as illustrated in the following result.

**Proposition 3.4.**  $(F, A) \cong (G, A)$  if and only if for each  $P_a^x \in (F, A)$  implies that  $P_a^x \in (G, A)$ .

**Definition 3.5.** A soft set (G, A) over X is said to be stable, if there exists a subset U of X such that G(a) = U, for each  $a \in A$ .

The following result is the correct form of [Theorem 3.34, p.p.23] in [7].

**Theorem 3.6.** Every stable soft compact subset (F, A) of a soft Hausdorff space  $(X, \tau, A)$  is soft closed.

*Proof.* Suppose that (F, A) is a stable soft set. Then  $(F^c, A)$  is stable as well. Let  $P_a^x \in (F^c, A)$ . This means that  $x \in (F^c, A)$ . Similarly, for each  $P_a^{y_i} \in (F, A)$ , we get  $y_i \in (F, A)$ . Thus  $x \neq y_i$ . By hypothesis, there exist two disjoint soft open sets  $(G_i, A)$  and  $(W_i, A)$  such that  $x \in (G_i, A)$  and  $y_i \in (W_i, A)$ . It follows that  $\{(W_i, A) : i \in I\}$  forms a soft open cover of (F, A). Consequently,  $(F, A) \subseteq \widetilde{\bigcup}_{i=1}^{i=n} (W_i, A)$ . Putting  $\widetilde{\bigcap}_{i=1}^{i=n} (G_i, A) = (H, A)$  and  $\widetilde{\bigcup}_{i=1}^{i=n} (W_i, A) = (V, A)$ . Then (H, A) and (V, A) are soft open sets such that  $(H, A) \cap (V, A) = \widetilde{\Phi}$ . Thus  $(H, A) \cap (F, A) = \widetilde{\Phi}$ . So  $(H, A) \subseteq (F^c, A)$ . Since  $P_a^x$  is chosen arbitrary,  $(F^c, A)$  is a soft neighborhood of its soft points. Hence it is soft open. Therefore it is soft closed. □

In the rest of this work, we point out that [Example 3.22, p.p.20] in [7] is incorrect. We firstly mention this example as it originally introduced in [7].

**Example 3.7.** Let  $X = E = \mathcal{R}$ , where  $\mathcal{R}$  is the set of real numbers. Consider  $(F, E)_y = \{(x, (x, y]) : x, y \in E \text{ and } x < y\}$ . Then  $\mathcal{T} = \{\widetilde{\Phi}, \widetilde{\mathcal{R}}, (F, E)_y : y \in E\}$  defined a soft topology on  $\mathcal{R}$ . Furthermore,  $(\mathcal{R}, \mathcal{T}, \mathcal{R})$  is a soft Hausdorff space.

To show an error of this example, let  $B = \{2 - \frac{1}{n} : n = 1, 2, ...\}$ . Then  $\Lambda = \{(F, E)_y : x = 0 \text{ and } y \in B\}$  is a collection of soft open sets. However,  $\bigcup_{y \in B} (0, (0, y]) = (0, (0, 2))$ . This implies that  $\widetilde{\bigcup}_{y \in B} (F, E)_y \notin \mathcal{T}$ . So  $\mathcal{T}$  which given in the above example is not a soft topology.

We observe that  $\widehat{\mathcal{R}}$  can be expressed as a union of members of  $\mathcal{T}$ , and the soft intersection of any two members of  $\mathcal{T}$  can be expressed as a union of members of  $\mathcal{T}$ as well. So we correct the above example by considering  $\mathcal{T}$  as a soft base for a soft topology  $\tau$ . Then  $(\mathcal{R}, \tau, \mathcal{R})$  is a soft Hausdorff space.

**Remark 3.8.** It worthily noted that a soft topology which has  $\mathcal{T}$  as a soft base consider as a version of the upper limit topology via soft topology.

### CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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