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Abstract. The main aim of this paper, is to introduce and study some new weak soft separation axioms in soft spaces by using the notions of soft α-open sets and soft α-closure. We investigate some of their properties. Some nice results and relations are obtained with some necessary examples.

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1. Introduction and preliminaries

The concept of a soft set was first introduced by Molodtsov [24] in 1999, as a general mathematical tool for dealing with uncertain objects. In 2011, Shabir-Naz [27] introduced the concept of soft topological spaces and studied some related notions such as open(closed)soft sets, soft subspaces, soft closure and soft separation axioms. In the recent years in development in the fields of soft sets have been done. Of late many authors [7, 8, 13, 16, 23, 25, 27, 30, 33] have studied various properties of soft topological spaces. The soft separation axioms studied by many authors (see, for example, [3, 11, 12, 14, 15, 25, 26, 27, 29, 31]). Some of these separation axioms have been found to be useful in computer science and digital topology. Weak forms of soft open sets were introduced and studied by many authors (see for example, [4, 10, 17, 18, 21, 28, 32]). Recently, soft separation axioms via semi-open soft sets, α-open soft sets, pre-open soft sets and β-open soft sets was studied by some authors (see, for example, [2, 5, 6, 10, 19, 20, 21]). In this paper, we offer some new lower soft separations axioms such as soft α-R_0, soft α-symmetric and soft α-R_1 axioms by utilizing soft α-open sets, soft points and soft α-closure operator. We characterize their basic properties, we also, investigate some relations between them. Some nice results and properties are obtained. Moreover, some necessary examples are given.

Throughout this paper, X refers to an initial universe set, E be the set of all parameters for X and $P(X)$ is the power set of X.

Definition 1.1 ([24])**.** A soft set (F, E) over X, denoted by F_E and defined by the set of ordered pairs $F_E = \{(e, F(e)) : e \in E \; and \; F(e) \in P(X)\}$. The family of all soft sets over X denoted by $SS(X_E)$.

The relative complement of F_E, denoted by F^c_E, where $F^c : E \longrightarrow P(X)$ is mapping given by $(F(e))^c = X - F(e)$ for all $e \in E$. Clearly, $(F^c_E)^c = F_E$ (see [1]).

Definition 1.2 ([1, 22, 33])**.** Let F_E be a soft set over X. Then F_E is called:

(i) a null soft set, denoted by $\varnothing_E$, if $F(e) = \varnothing$, for all $e \in E$,

(ii) an absolute soft set, denoted by X_E, if $F(e) = X$, for all $e \in E$.

(iii) a soft point in X_E, denoted by x_e, if there are $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e^{'}) = \varnothing$, for all $e^{'} \in E - \{e\}$.

The soft point x_e is said to belong to F_E, denoted by $x_e \widetilde{\in} F_E$, if for the element $e \in E,\ x \in F(e)$.

The set of all soft points in X_E, denoted by $SP(X_E)$. The soft pints x_e, y_e in X_E are called distinct, if $x \neq y$.

Definition 1.3 ([1, 33])**.** Let F_E and G_E be two soft sets over X. Then

(i) we say that F_E is a soft superset of G_E, denoted by $G_E \widetilde{\subseteq} F_E$, if $G(e) \subseteq F(e)$, for all $e \in E$,

(ii) we say that F_E is equal to G_E, denoted by $F_E = G_E$, if $G_E \widetilde{\subseteq} F_E$ and $F_E \widetilde{\subseteq} G_E$,

(iii) the soft union of F_E and G_E, denoted by $F_E \widetilde{\cup} G_E$, is the soft set defined as follows:

$$(F_E \widetilde{\cup} G_E)(e) = F(e) \cup G(e), \text{ for all } e \in E,$$

(iv) the soft intersection of F_E and G_E, denoted by $F_E \widetilde{\cap} G_E$, is the soft set defined as follows:

$$(F_E \widetilde{\cap} G_E)(e) = F(e) \cap G(e), \quad \text{for all } e \in E.$$

Definition 1.4 ([27, 33])**.** Let F_E be a soft set in X, $\varnothing \neq Y \subseteq X$ and $x \in X$. Then:

(i) $x \widetilde{\in} F_E$, if $x \in F(e)$ for all $e \in E$, and $x \widetilde{\notin} F_E$, if $x \notin F(e)$ for some $e \in E$.

(ii) If $F(e) = \{x\}$ for all $e \in E$, then F_E is called a singleton soft point, denoted by x_E. And we have, $x_E \widetilde{\subseteq} F_E \Longleftrightarrow x \widetilde{\in} F_E \Longleftrightarrow x_e \widetilde{\in} F_E$ for all $e \in E$.

(iii) $Y_E = (Y, E)$ denotes the soft set over X for which, $Y(e) = Y$ for all $e \in E$.

Definition 1.5 ([27])**.** A collection τ of soft sets over X with a fixed set of parameters E is called a soft topology on X if it satisfies the following conditions:

(i) τ contains the null and absolute soft sets,

(ii) τ is closed under arbitrary soft union and finite soft intersection.

In this case, the triple (X, τ, E) is called a soft topological space. Any member of τ is called a soft open set and its relative complement is called a soft closed set.

Definition 1.6 ([27, 33])**.** Let (X, τ, E) be a soft topological space and F_E be a soft set in X. Then the soft closure of F_E, denoted by $cl(F_E)$, is the intersection of all soft closed super sets of F_E. The soft interior of F_E, denoted by $int(F_E)$, is the union of all soft open sets which are contained in F_E.

Definition 1.7 ([4, 17]). Let (X, τ, E) be a soft topological space and U_E be a soft set over X, then U_E is called a soft α-open set, if $U_E \widetilde{\subseteq} int(cl\,(int\,(U_E)))$. The set of all soft α-open sets, denoted by $S\alpha O(X_E)$. The relative complement of any soft α-open set is called soft α-closed set. The set of all soft α-closed sets is denoted by $S\alpha C(X_E)$.

Clearly, every soft open set is a soft α-open set. The converse may not be true.

Definition 1.8 ([4, 18]). Let (X, τ, E) be a soft topological space and U_E be a soft set over X. Then the soft α-closure of U_E, denoted by $cl_\alpha(U_E)$, is the soft intersection of all soft α-closed super sets of U_E. The soft interior of U_E, denoted by $int_\alpha(U_E)$, is the soft union of all soft α-open sets which are contained in U_E.

Result 1.9 ([17, 18]). Let (X, τ, E) be a soft topological space over X and U_E, V_E be two soft sets over X. Then

(1) U_E is a soft α-closed set $\Longleftrightarrow$ if $cl_\alpha(U_E) = U_E$,
(2) $U_E \widetilde{\subseteq} V_E \Longrightarrow cl_\alpha(U_E) \widetilde{\subseteq} cl_\alpha(V_E)$,
(3) $x_e \widetilde{\in}\ cl_\alpha(V_E) \Longleftrightarrow U_E \widetilde{\cap} V_E \neq \varnothing_E$ for all soft α-open set U_E containing x_e.

Definition 1.10 ([27]). Let (X, τ, E) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau_Y = \left\{ Y_E \widetilde{\cap} F_E : F_E \in \tau \right\}$ is called a soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Result 1.11 (Theorem 23, [20]). Let (Y, τ_Y, E) be a soft subspace of (X, τ, E) and F_E be a soft set over X. Then we have:

(1) if F_E is a soft α-open set in Y and $Y_E \in \tau$, then $F_E \in \tau$,
(2) F_E is a soft α-open set in Y if and only if $F_E = Y_E \widetilde{\cap} G_E$, for some $G_E \in \tau$.

Definition 1.12 ([13, 27]). Let (X, τ, E) be a soft topological space, then the collection $\tau_e = \{F(e) : F_E \in \tau\}$ for every $e \in E$, defines a topology on X.
On other hand, if (X, σ) is a topological space, then the family $\tau_\sigma = \{F_E \in SS(X_E) : F(e) = A$ for all $e \in E$ and for all $A \in \tau\}$, defines a soft topology on X.

Definition 1.13 ([14, 20]). A soft topological space (X, τ, E) is called a soft single point space, if $\tau = \{U_E : U(e) = U$ for all $e \in E$ and for all $U \subset X\}$.

In this case,
(1) every singleton soft point x_E is a soft α-open set, for all $x \in X$,
(2) every soft element of (X, τ, E) is both soft α-open and soft α-closed set,
(3) (X, τ_e) is a discrete space, for all $e \in E$.

Remark 1. If (X, σ) is a discrete topological space, then the soft topology τ_σ, is a soft single point topology on X.

Definition 1.14 ([9]). A topological space (X, τ) is said to be:

(i) $\alpha - R_0$, if every α-open set contains the α-closure of all its singletons,
(ii) $\alpha - R_1$ if for every pair of distinct points $x, y \in X$ with $cl_\alpha\{x\} \neq cl_\alpha\{y\}$, there are disjoint α-open sets U, V such that $x \in U$, $y \in V$.

Definition 1.15 ([25]). A soft topological space (X, τ, E) is said to be:

(i) soft R_0 (SR_0, for short), if every soft α-open set contains the soft α-closure of all its soft points,

(ii) soft R_1 (SR_1, for short), if for every pair of distinct soft points x_e, y_e with $cl(x_e)\neq cl(y_e)$, there are disjoint soft open sets F_E, G_E such that $cl(x_e)\tilde{\subseteq}F_E$ and $cl(y_e)\tilde{\subseteq}G_E$.

2. The properties of some soft α-separation axioms

In this section, we introduce some new weak separation axioms such as soft α-R_0, soft α-symmetry, and α-R_1 and investigate some characterizations for them.

Definition 2.1. A soft topological space (X,τ,E) is soft α-R_0 (briefly, $S\alpha$-R_0), if for every soft α-open set F_E and for every $x_e\tilde{\in}F_E$, $cl_\alpha(x_e)\tilde{\subseteq}F_E$.

Example 2.2. Let $X=\{x,y\}$ and $E=\{e_1,e_2\}$. Then the family $\tau=\{\varnothing_E, X_E, F_{1E}, F_{2E}\}$ is a soft topology on X, where $F_{1E}=\{(e_1,\{x\}),(e_2,\{y\})\}$, $F_{2E}=\{(e_1,\{y\}),(e_2,\{x\})\}$. Now, the soft α-open set F_{1E} contains the soft α-closure of all it's soft points, that is, $cl_\alpha(x_{e_1})\tilde{\subseteq}F_{1E}$ and $cl_\alpha(y_{e_2})\tilde{\subseteq}F_{1E}$. Also, $cl_\alpha(y_{e_1})\tilde{\subseteq}F_{2E}$ and $cl_\alpha(x_{e_2})\tilde{\subseteq}F_{2E}$. Thus (X,τ,E) is $S\alpha$-R_0.

Remark 2. The following example shows generally that $S\alpha$-R_0 need not be SR_0.

Example 2.3. Let X be an infinite universe set, $E=\{e\}$ and let y_e be a fixed soft point in (X,τ,E) with τ as a soft cofinite topology over X, i.e.,

$$\tau=\{\varnothing_E\}\cup\{F_E\ \tilde{\subset}X_E:(F(e))^c \text{ is a finite subset of } X \text{ and } y_e\tilde{\notin}F_E\ \}.$$

Then we can verify that (X,τ,E) is $S\alpha$-R_0. But it is not SR_0. Indeed, if F_E is a soft open set and $x_e\tilde{\in}F_E$, then $cl(x_e)=X_E\tilde{\not\subseteq}F_E$.

Theorem 2.4. *Let (X,τ,E) be a soft topological space. Then the following properties are equivalent:*

(1) *(X,τ,E) is $S\alpha$-R_0 ,*

(2) *for any pair of distinct soft points x_e, y_e and $cl_\alpha(x_e)\neq cl_\alpha(y_e)$, $cl_\alpha(x_e)\tilde{\cap}$ $cl_\alpha(y_e)=\varnothing_E$.*

Proof. (1) $\Longrightarrow$ (2): Suppose that (X,τ,E) is $S\alpha$-R_0 and x_e, y_e are two distinct soft points with $cl_\alpha(x_e)\neq cl_\alpha(y_e)$. Then there is z_e in X_E such that $z_e\tilde{\in}cl_\alpha(x_e)$ and $z_e\tilde{\notin}cl_\alpha(y_e)$. If $x_e\tilde{\in}cl_\alpha(y_e)$, then $cl_\alpha(x_e)\tilde{\subseteq}cl_\alpha(y_e)$. Thus $z_e\tilde{\in}cl_\alpha(y_e)$ but this is a contradiction. So $x_e\tilde{\notin}cl_\alpha(y_e)$. Hence $x_e\tilde{\in}(cl_\alpha(y_e))^c=U_E\in S\alpha O(X_E)$. Since (X,τ,E) is $S\alpha$-R_0, $cl_\alpha(x_e)\tilde{\subseteq}U_E$. Therefore the result holds.

(2)$\Longrightarrow$ (1): Let $F_E\in S\alpha O(X_E)$ and $x_e\tilde{\in}F_E$. We will show that $cl_\alpha(x_e)\tilde{\subseteq}F_E$. Let $y_e\tilde{\notin}F_E$, that is, $y_e\tilde{\in}F^c_E$. Then $x_e\neq y_e$ and $x_e\tilde{\notin}cl_\alpha(y_e)$. This show that $cl_\alpha(x_e)\neq cl_\alpha(y_e)$. By assumption, $cl_\alpha(x_e)\tilde{\cap}cl_\alpha(y_e)=\varnothing_E$. Thus $y_e\tilde{\notin}cl_\alpha(x_e)$. So $cl_\alpha(x_e)\tilde{\subseteq}F_E$. □

Theorem 2.5. *For a soft topological space (X,τ,E), the following properties are equivalent:*

(1) *(X,τ,E) is $S\alpha$-R_0,*

(2) *$x_e\tilde{\in}cl_\alpha(y_e)$ if and only if $y_e\tilde{\in}cl_\alpha(x_e)$, for all $x_e, y_e\in SP(X_E)$.*

Proof. (1) $\Longrightarrow$ (2): Let (X,τ,E) be $S\alpha$-R_0 and $x_e\tilde{\notin}cl_\alpha(y_e)$. Then there is a soft α-open set F_E that contains x_e such that $y_e\tilde{\cap}F_E=\varnothing_E$, that is, $y_e\tilde{\notin}F_E$. Since $x_e\tilde{\in}F_E$

and (X, τ, E) is $S\alpha$-R_0, $cl_\alpha(x_e) \widetilde{\subseteq} F_E$ which implies, $y_e \widetilde{\notin} cl_\alpha(x_e)$. Similarity for the converse.

(2) $\Longrightarrow$ (1): Let $U_E \in S\alpha O(X_E)$ and $x_e \widetilde{\in} U_E$. We will show that $cl_\alpha(x_e) \widetilde{\subseteq} U_E$. Let $y_e \widetilde{\notin} U_E$. Then $x_e \widetilde{\notin} cl_\alpha(y_e)$. Thus by (2), we get $y_e \widetilde{\notin} cl_\alpha(x_e)$. So $cl_\alpha(x_e) \widetilde{\subseteq} U_E$. So the result holds. □

Corollary 2.6. *A soft topological space (X, τ, E) is $S\alpha$-R_0 if and only if for any $G_E \in S\alpha C(X)_E$ with $x_e \widetilde{\notin} G_E$, $cl_\alpha(x_e) \widetilde{\cap} G_E = \varnothing_E$.*

Proof. It follows directly from the above theorems. □

Proposition 2.7. *If (X, τ, E) is $S\alpha$-R_0 and $\tau \leq \tau^*$, then (X, τ^*, E) need not be $S\alpha$-R_0.*

Proof. From the Example 2.2, we showed that, the soft topology

$$\tau = \{\varnothing_E,\ X_E,\ F_{1E} = \{(e_1, \{x\}), (e_2, \{y\})\},\ F_{2E} = \{(e_1, \{y\}), (e_2, \{x\})\}\}$$

is $S\alpha$-R_0. Let us consider a soft topology on X,

$$\tau^* = \{\varnothing_E, X_E, F_{1E} = \{(e_1, \{x\}), (e_2, \{y\})\}, F_{2E} = \{(e_1, \{y\}), (e_2, \{x\})\},$$
$$F_{3E} = \{(e_1, \{x\})\}, F_{4E} = \{(e_1, X), (e_2, \{x\})\}\}$$

such that $\tau \leq \tau^*$. But (X, τ^*, E) is not $S\alpha$-R_0. Indeed, for a soft α-open set F_{4E} in (X, τ^*, E), we have $x_{e_1} \widetilde{\in} F_{4E}$ but $cl_\alpha(x_{e_1}) = \{(e_1, \{x\}), (e_2, \{y\})\} \widetilde{\nsubseteq} F_{4E}$. □

Definition 2.8. Let (X, τ, E) be a soft topological space and $F_E \in SS(X_E)$. Then A soft α-kernel of F_E, denoted by $SK_\alpha(F_E)$, is the soft set defined by:

$$SK_\alpha(F_E) = \widetilde{\cap}\{G_E \in S\alpha O(X_E) : F_E \widetilde{\subseteq} G_E\}.$$

In particular, the soft α-kernel of $x_e \in SP(X_E)$, is the soft set given by:

$$SK_\alpha(x_e) = \widetilde{\cap}\{G_E \in S\alpha O(X_E) : x_e \widetilde{\in} G_E\}.$$

To present more properties of soft α-R_0 we need the following lemmas whose proofs are similar to that of (Lemma 3.6 and Lemma 3.7 and Lemma 3.12, in [25]).

Lemma 2.9. *Let (X, τ, E) be a soft topological space and $F_E \in SS(X_E)$. Then*

$$SK_\alpha(F_E) = \widetilde{\cup}\left\{x_e \in SP(X_E) : cl_\alpha(x_e) \widetilde{\cap} F_E \neq \varnothing_E\right\}.$$

Lemma 2.10. *Let (X, τ, E) be a soft topological space and $x_e \widetilde{\in} X_E$. Then $y_e \widetilde{\in} SK_\alpha(x_e)$ if and only if $x_e \widetilde{\in} cl_\alpha(y_e)$.*

Lemma 2.11. *Let (X, τ, E) be a soft topological space and $x_e, y_e \in SP(X_E)$. Then $SK_\alpha(x_e) \neq SK_\alpha(y_e)$ if and only if $cl_\alpha(x_e) \neq cl_\alpha(y_e)$.*

Theorem 2.12. *For a soft topological space (X, τ, E), the following properties are equivalence:*

(1) *(X, τ, E) is $S\alpha$-R_0,*

(2) *for every pair of distinct soft points x_e, y_e with $SK_\alpha(x_e) \neq SK_\alpha(y_e)$, $SK_\alpha(x_e) \widetilde{\cap} SK_\alpha(y_e) = \varnothing_E$.*

Proof. (1) $\Longrightarrow$ (2): Let (X, τ, E) be $S\alpha$-R_0 and x_e, y_e be two distinct soft points with $SK_\alpha(x_e) \neq SK_\alpha(y_e)$. Then by Lemma 2.11, $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Suppose that $SK_\alpha(x_e)\widetilde{\cap}SK_\alpha(y_e) \neq \varnothing_E$, Then there is a soft point $z_e\widetilde{\in}SK_\alpha(x_e)\widetilde{\cap}SK_\alpha(y_e)$. Now, if $z_e\widetilde{\in}SK_\alpha(x_e)$, then by Lemma 2.10, $x_e\widetilde{\in}cl_\alpha(z_e)$. Thus $cl_\alpha(x_e)\widetilde{\subset}cl_\alpha(z_e)$. Since $x_e\widetilde{\in}cl_\alpha(x_e)$, by Theorem 2.4, $cl_\alpha(x_e) = cl_\alpha(z_e)$. Similarity, if $z_e\widetilde{\in}SK_\alpha(y_e)$, we have $cl_\alpha(y_e) = cl_\alpha(z_e) = cl_\alpha(x_e)$. This is a contradiction. So $SK_\alpha(x_e)\widetilde{\cap}SK_\alpha(y_e) = \varnothing_E$.

(2) $\Longrightarrow$ (1): Let x_e, y_e be two distinct soft points with $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Then by Lemma 2.11, $SK_\alpha(x_e) \neq SK_\alpha(y_e)$. Thus by hypothesis, $SK_\alpha(x_e)\widetilde{\cap}SK_\alpha(y_e) = \varnothing_E$. Suppose that $cl_\alpha(x_e)\widetilde{\cap}cl_\alpha(y_e) \neq \varnothing_E$. Then there is a soft point z_e such that $z_e\widetilde{\in}cl_\alpha(x_e)$, $z_e\widetilde{\in}cl_\alpha(y_e)$. Then by Lemma 2.10, $x_e\widetilde{\in}SK(z_e)$ and $y_e\widetilde{\in}SK(z_e)$. Thus by Lemma 2.9, we obtain $SK_\alpha(x_e)\widetilde{\cap}SK_\alpha(z_e) \neq \varnothing_E$ and $SK_\alpha(y_e)\widetilde{\cap}SK_\alpha(z_e) \neq \varnothing_E$. By the hypothesis, $SK_\alpha(x_e) = SK_\alpha(z_e)$ and $SK_\alpha(y_e) = SK_\alpha(z_e) = SK_\alpha(x_e)$. So $SK_\alpha(x_e)\widetilde{\cap}SK_\alpha(y_e) \neq \varnothing_E$. This is a contradiction. Hence $cl_\alpha(x_e)\widetilde{\cap}cl_\alpha(y_e) = \varnothing_E$. Therefore by Theorem 2.4, the result holds. □

Theorem 2.13. *Let (X, τ, E) be a soft topological space. Then the following properties are equivalent:*

(1) *(X, τ, E) is $S\alpha$-R_0,*
(2) *$H_E = SK_\alpha(H_E)$, whenever $H_E \in S\alpha C(X_E)$,*
(3) *if $H_E \in S\alpha C(X_E)$ and $x_e\widetilde{\in}H_E$, then $SK_\alpha(x_e)\widetilde{\subseteq}H_E$,*
(4) *$SK_\alpha(x_e)\widetilde{\subseteq}\, cl_\alpha(x_e)$, for a soft point x_e in X_E.*

Proof. (1) $\Longrightarrow$(2): Let $H_E \in S\alpha C(X_E)$ and $x_e\widetilde{\notin}H_E$. Then $x_e\widetilde{\in}H_E^c$ which is a soft α-open set containing x_e. Since (X, τ, E) is $S\alpha$-R_0, $cl_\alpha(x_e)\widetilde{\subseteq}H_E^c$ implies $cl_\alpha(x_e)\widetilde{\cap}H_E = \varnothing_E$. Thus by Lemma 2.9, $x_e\widetilde{\notin}SK_\alpha(H_E)$. So $H_E = SK(H_E)$.

(2) $\Longrightarrow$(3): It follows from the fact, $F_E\widetilde{\subseteq}G_E$ implies $SK_\alpha(F_E)\widetilde{\subseteq}SK_\alpha(G_E)$.

(3) $\Longrightarrow$ (4): It is obvious.

(4) $\Longrightarrow$ (1): Let x_e, y_e be two distinct soft points and $x_e\widetilde{\in}cl_\alpha(y_e)$. Then by Lemma 2.10, we get $y_e\widetilde{\in}SK_\alpha(x_e)$. Since $x_e\widetilde{\in}cl_\alpha(x_e)$ which is soft α-closed set, by (4), we obtain $y_e\widetilde{\in}SK_\alpha(x_e)\widetilde{\subseteq}cl_\alpha(x_e)$, that is, $y_e\widetilde{\in}cl_\alpha(x_e)$. Similarity of the converse. Thus by Theorem 2.5, the result holds. □

Definition 2.14. A soft topological space (X, τ, E) is soft α-symmetric, if $x_e\widetilde{\in}cl_\alpha(y_e)$ implies $y_e\widetilde{\in}cl_\alpha(x_e)$, for all $x_e, y_e \in SP(X_E)$.

Definition 2.15. A soft set F_E in a soft topological space (X, τ, E) is called a soft α-generalized closed set (briefly, $S\alpha$-g-closed), if $cl_\alpha(F_E)\widetilde{\subseteq}U_E$, whenever $F_E\widetilde{\subseteq}U_E$, $U_E \in S\alpha O(X_E)$.

Remark 3. Clearly, every soft α-closed set is a $S\alpha$-g-closed set. But by using the Example 2.2, we can show that the converse of this fact is not necessary true. Indeed, for the soft topology $\tau = \{\varnothing_E, X_E,\ F_{1E} = \{(e_1, \{x\}), (e_2, \{y\})\},\ F_{2E} = \{(e_1, \{y\}), (e_2, \{x\})\}\}$. One can check that, x_{e_1} is a $S\alpha$-g-closed set but it is clear that x_{e_1} is not a soft α-closed set.

Theorem 2.16. *A soft topological space (X, τ, E) is soft α-symmetric if and only if x_e is a $S\alpha$-g-*closed *set, for all $x_e \in SP(X_E)$.*

Proof. Necessity: Let (X, τ, E) be soft α-symmetric. Suppose that $x_e \widetilde{\in} U_E$, $U_E \in S\alpha O(X_E)$ and $cl_\alpha(x_e) \widetilde{\nsubseteq} U_E$. Then there is $y_e \widetilde{\in} X_E$ such that $y_e \widetilde{\in} cl_\alpha(x_e) \widetilde{\cap} U_E^c$. Thus $y_e \widetilde{\in} cl_\alpha(x_e)$, $y_e \widetilde{\in} U_E^c$, that is, $cl_\alpha(y_e) \widetilde{\subseteq} cl_\alpha(U_E^c) = U_E^c$. Since (X, τ, E) is soft α-symmetric and $y_e \widetilde{\in} cl_\alpha(x_e)$, $x_e \widetilde{\in} cl_\alpha(y_e) \widetilde{\subseteq} U_E^c$. This contradiction with $x_e \widetilde{\in} U_E$. So $cl_\alpha(x_e) \widetilde{\subseteq} U_E$.

Conversely, let $x_e \in SP(X_E)$ be a $S\alpha$-g-closed set. Suppose that $x_e \widetilde{\in} cl_\alpha(y_e)$ and $y_e \widetilde{\notin} cl_\alpha(x_e)$, that is, $y_e \widetilde{\in} (cl_\alpha(x_e))^c \in S\alpha O(X_E)$. Since y_e is $S\alpha$-g-closed,

$$cl_\alpha(y_e) \widetilde{\subseteq} (cl_\alpha(x_e))^c.$$

Then $y_e \widetilde{\in} (cl_\alpha(x_e))^c \widetilde{\subseteq} (x_e)^c$. This is a contradiction. Thus the result holds. □

Remark 4. From Definition 2.14 and Theorem 2.5. The notions of soft α-symmetric and soft α-R_0 are equivalence.

Definition 2.17. A soft topological space (X, τ, E) is said to be:
(i) soft weakly R_0(briefly, $Sw - R_0$), if $\widetilde{\cap} \{cl(x_e) : x_e \in SP(X_E)\} = \varnothing_E$,
(ii) soft weakly α-R_0(briefly, $Sw\alpha - R_0$), if $\widetilde{\cap} \{cl_\alpha(x_e) : x_e \in SP(X_E)\} = \varnothing_E$.

Theorem 2.18. *A soft topological space (X, τ, E) is $Sw\alpha$-R_0 if and only if $SK_\alpha(x_e) \neq X_E$, for every x_e in X_E.*

Proof. Necessity: Let (X, τ, E) is $Sw\alpha$-R_0. Suppose that there is y_e in X_E such that $SK_\alpha(y_e) = X_E$. Then $y_e \widetilde{\notin} G_E$, for some $G_E \in S\alpha O(X_E)$. Thus

$$y_e \widetilde{\in} \widetilde{\cap} \{cl_\alpha(x_e) : x_e \in SP(X_E)\}.$$

But this is a contradiction.

Conversely, let $SK_\alpha(x_e) \neq X_E$, for any $x_e \in X_E$. If there is y_e in X_E such that $y_e \widetilde{\in} \widetilde{\cap} \{cl_\alpha(x_e) : x_e \in SP(X_E)\}$, then any soft α-open set containing y_e must contain any soft point in X_E. This mean that X_E is the unique soft α-open set containing y_e. Thus $SK_\alpha(x_e) = X_E$. This is a contradiction. So (X, τ, E) is $Sw\alpha$-R_0. □

Definition 2.19. A soft topological space (X, τ, E) is soft α-R_1 (briefly, $S\alpha$-R_1), if for every pair of distinct soft points x_e, y_e with $cl_\alpha(x_e) \neq cl_\alpha(y_e)$, there are two disjoint α-open sets U_E and V_E such that $cl_\alpha(x_e) \widetilde{\subseteq} U_E$ and $cl_\alpha(y_e) \widetilde{\subseteq} V_E$.

Example 2.20. Let $X = \{x, y\}$ and $E = \{e_1, e_2\}$. The family $\tau = \{\varnothing_E,\ X_E,\ F_{1E},\ F_{2E}\}$ is a soft topology on X, where $F_{1E} = \{(e_1, \{x\})\}$, $F_{2E} = \{(e_1, \{y\}), (e_2, X)\}$. Now, for distinct soft points x_{e_1}, y_{e_1} with $cl_\alpha(x_{e_1}) \neq cl_\alpha(y_{e_1})$, there are disjoint soft α-open set F_{1E}, F_{2E} such that $cl_\alpha(x_{e_1}) \widetilde{\subseteq} F_{1E}$ and $cl_\alpha(y_{e_1}) \widetilde{\subseteq} F_{2E}$. Also, for distinct soft points x_{e_2}, y_{e_2}, we have $cl_\alpha(x_{e_2}) = cl_\alpha(y_{e_2})$. Then (X, τ, E) is $S\alpha$-R_1.

Theorem 2.21. *A soft topological space (X, τ, E) is $S\alpha$-R_1 if and only if for every distinct soft points x_e, y_e with $SK(x_e) \neq SK(y_e)$, there exist disjoint soft disjoint α-open sets F_E, G_E such that $cl_\alpha(x_e) \widetilde{\subseteq} F_E, cl_\alpha(y_e) \widetilde{\subseteq} G_E$.*

Proof. It follows from Lemma 2.10. □

Proposition 2.22. *A soft topological space (X, τ, E) is $S\alpha$-R_1 if and only if for every distinct soft points x_e, y_e with $x_e \widetilde{\notin} cl_\alpha(y_e)$, there are disjoint soft α-open sets F_E, G_E such that $x_e \widetilde{\in} F_E$ and $y_e \widetilde{\in} G_E$.*

Proof. It follows from Definition 2.19 , Lemma 2.11 and Theorem 2.21. □

Theorem 2.23. *Let (X, τ, E) be a soft single point space. Then (X, τ, E) is $S\alpha$-R_1 and $S\alpha$-R_0.*

Proof. Let (X, τ, E) be a soft singlet point space and let x_e, y_e be two distinct soft points with $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Then there are disjoint soft α-open sets x_E , y_E such that $x_e \widetilde{\in} x_E$, $y_e \widetilde{\in} y_E$. Thus (X, τ, E) is $S\alpha$-R_1. Similarity of the other case. □

3. Some basic relations.

First, we recall the definition of soft α-T_0 and soft α-T_1 as in [20].

Definition 3.1. A soft topological space (X, τ, E) is said to be:

(i) soft α-T_0, if for every two distinct soft points x_e, y_e, there is $F_E \in S\alpha O(X_E)$ such that $x_e \widetilde{\in} F_E, y_e \widetilde{\notin} F_E$ or there is $G_E \in S\alpha O(X_E)$ such that $y_e \widetilde{\in} G_E, x_e \widetilde{\notin} G_E$,

(ii) soft α-T_1, if for every two distinct soft points x_e, y_e, there are $G_E, H_E \in S\alpha O(X_E)$ such that $x_e \widetilde{\in} G_E, y_e \widetilde{\notin} G_E$ and $y_e \widetilde{\in} H_E, x_e \widetilde{\notin} H_E$.

Remark 5. Clearly, every $S\alpha$-T_1 is $S\alpha$-T_0. But the converse is not necessary true.

Example 3.2. Let $X = \{x, y\}$ and $E = \{e_1, e_2\}$. The family $\tau = \{\varnothing_E, X_E, F_E\}$ is a soft topology on X, where $F_E = \{(e_1, \{x\}), (e_2, \{y\})\}$. One can shows that (X, τ, E) is soft α-T_0. But is not soft α-T_1. Indeed, for distinct soft points x_{e_1}, y_{e_1}, there are only two soft α-open sets F_E, X_E such that $x_{e_1} \widetilde{\in} F_E$ and $y_{e_1} \widetilde{\notin} F_E$ but X_E contains both x_{e_1}, y_{e_1}.

Theorem 3.3. *If (X, τ, E) is $S\alpha$-R_1, then is $S\alpha$-R_0 (soft α-symmetric).*

Proof. Let x_e, y_e be two distinct soft points and $x_e \widetilde{\notin} cl_\alpha(y_e)$. Then $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Since (X, τ, E) is $S\alpha$-R_1, there is $U_E \in S\alpha O(X_E)$ such that $y_e \widetilde{\in} U_E$ and $x_e \widetilde{\notin} U_E$. Thus $y_e \widetilde{\notin} cl_\alpha(x_e)$. Similarity for the converse. So the result hold. □

The converse of the above theorem is not true, the Example 3.17 in [25] shows it.

Theorem 3.4. *If (X, τ, E) is soft α-T_1, then is soft α-symmetric ($S\alpha$-R_0).*

Proof. Let x_e, y_e be two distinct soft points and $x_e \widetilde{\notin} cl_\alpha(y_e)$. Since (X, τ, E) is $S\alpha$-T_1, there is $F_E \in S\alpha O(X_E)$ such that $y_e \widetilde{\in} F_E$ and $x_e \widetilde{\notin} F_E$. This means that $y_e \widetilde{\notin} cl_\alpha(x_e)$. Then (X, τ, E) is soft α-symmetric. □

The following example shows that the converse of the above theorem is not true.

Example 3.5. Let $X = \{x, y\}$ and $E = \{e_1, e_2\}$. Then the family, $\tau = \{\varnothing_E, X_E, U_E, V_E\}$ is a soft topology on X, where $U_E = \{(e_1, X)\}$, $V_E = \{(e_2, X)\}$. Now we can verify that (X, τ, E) is soft α-symmetric. But it is not soft $\alpha - T_1$ because, for two distinct soft points x_{e_1}, y_{e_1}, the soft α-open sets which are containing x_{e_1} are X_E and U_E but also, they are containing y_{e_1}.

Theorem 3.6. *A soft topological space (X, τ, E) is soft α-T_1 if and only if is both soft α-symmetric and soft α-T_0.*

Proof. Necessity: It follows from Theorem 3.4 and Remark 5.

Conversely, Let x_e , y_e are distinct soft points in X_E. Since (X, τ, E) is $S\alpha$-T_0, we can assume that $x_e \widetilde{\in} U_E \widetilde{\subseteq} (y_e)^c$, for some $U_E \in S\alpha O(X_E)$. Then $x_e \widetilde{\notin} cl_\alpha(y_e)$. Thus $y_e \widetilde{\notin} cl_\alpha(x_e)$. So there is $V_E \in S\alpha O(X_E)$ such that $y_e \widetilde{\in} V_E \widetilde{\subseteq} (x_e)^c$. Hence (X, τ, E) is a soft α-T_1 space. □

Definition 3.7. A soft topological space (X, τ, E) is soft α-$T_{\frac{1}{2}}$, if every soft α-g-closed set is soft α-closed.

Theorem 3.8. *For a soft α-symmetric space (X, τ, E), the following properties are equivalence:*

(1) *(X, τ, E) is soft $\alpha - T_0$,*
(2) *(X, τ, E) is soft $\alpha - T_{\frac{1}{2}}$,*
(3) *(X, τ, E) is soft $\alpha - T_1$,*

Proof. (1)$\Longrightarrow$(2): Let (X, τ, E) be $S\alpha$-T_0 and F_E be a $S\alpha$-g-closed set. Suppose that $F_E \neq cl_\alpha(F_E)$. Then there is x_e in X_E such that $x_e \widetilde{\in} F_E$ and $x_e \widetilde{\notin} cl_\alpha(F_E)$, that is, $x_e \widetilde{\in} (cl_\alpha(F_E))^c \in S\alpha O(X_E)$. Since (X, τ, E) is $S\alpha$-symmetric and $S\alpha$-T_0,

$$cl_\alpha(x_e) \widetilde{\subseteq} (cl_\alpha(F_E))^c \widetilde{\subseteq} F_E^c,$$

that is, $x_e \widetilde{\in} F_E^c$. This is a contradiction. Thus $F_E = cl_\alpha(F_E)$, that is, F_E is a $S\alpha$-closed set. So the result holds.

(2)$\Longrightarrow$(3): Let (X, τ, E) be $S\alpha$-symmetric and $S\alpha$-$T_{\frac{1}{2}}$. By Theorem 2.16, every x_e in X_E is a $S\alpha$-g-closed set, and by(2), x_e is a $S\alpha$-closed set. This mean that every soft pint x_e in X_E is a $S\alpha$-closed set. Thus (X, τ, E) is $S\alpha$-T_1.

(3)$\Longrightarrow$ (1): It follows directly from Theorem 3.6. □

Theorem 3.9. *For a soft topological space (X, τ, E), we have:*

(1) *if (X, τ, E) is $S\alpha$-R_1 , then (X, τ_e) is α-R_1 for all $e \in E$,*
(2) *if (X, τ, E) is $S\alpha$-R_0 , then (X, τ_e) is α-R_0 for all $e \in E$.*

Proof. (1) Let $x, y \in X$ and $x \neq y$ with $cl_\alpha\{x\} \neq cl_\alpha\{y\}$. Then either $x \notin cl_\alpha\{y\}$ *or* $y \notin cl_\alpha\{x\}$. Thus $x_e \widetilde{\notin} cl_\alpha(y_e)$ *or* $y_e \widetilde{\notin} cl_\alpha(x_e)$. So $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Since (X, τ, E) is $S\alpha$-R_1, there are $F_E, G_E \in S\alpha O(X_E)$ such that $x_e \widetilde{\in} F_E, y_e \widetilde{\in} G_E$ and $F_E \widetilde{\cap} G_E = \varnothing_E$. Hence there are disjoint α-open sets $F(e), G(e)$ such that $x \in F(e)$ and $y \in G(e)$. Therefore (X, τ_e) is α-R_1. Similarity for the case (2). □

The following example shows that the converse of the above theorem is not true.

Example 3.10. Let $X = \{x, y\}$ and $E = \{e_1, e_2\}$. Then the family $\tau = \{\varnothing_E, X_E, F_{1E}, F_{2E}, F_{3E}, F_{4E}\}$, where $F_{1E} = \{(e_1, \{x\})\}, F_{2E} = \{(e_1, \{x\}), (e_2, \{x\})\}$, $F_{3E} = \{(e_1, \{x\}), (e_2, \{y\})\}$ and $F_{4E} = \{(e_1, \{x\}), (e_2, X)\}$ is a soft topology on X and the family $\tau_{e_2} = \{\varnothing, X, \{x\}, \{y\}\}$ is a discrete topology on X which is α-R_1 and α-R_0. Bet (X, τ, E) is not $S\alpha$-R_0. Indeed, for distinct soft points x_{e_1}, y_{e_1} we have, $X_E = cl_\alpha(x_{e_1}) \neq cl_\alpha(y_{e_1}) = y_{e_1}$ but $cl_\alpha(x_{e_1}) \widetilde{\cap} cl_\alpha(y_{e_1}) \neq \varnothing_E$.

Proposition 3.11. *Let (X, τ, E) be a soft single point space, then we have:*

(1) *(X, τ, E) is $S\alpha$-R_1 if and only if (X, τ_e) is α-R_1 , $\forall\ e \in E$,*
(2) *(X, τ, E) is $S\alpha$-R_0 if and only if (X, τ_e) is α-R_0 , $\forall\ e \in E$,*

Proof. Necessity: It follows from Theorem 3.9, and also, from (3) of Definition 1.13. Conversely, It follows from Theorem 2.23. □

Theorem 3.12. *For a topological space (X, σ), then we have:*

(1) *(X, σ) is α-R_1 if and only if (X, τ_σ, E) is $S\alpha$-R_1,*
(2) *(X, σ) is α-R_0 if and only if (X, τ_σ, E) is $S\alpha$-R_0,*

Proof. (1) Necessity: Let x_e, y_e be two distinct soft points and $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Then $x \neq y$ and $cl_\alpha\{x\} \neq cl_\alpha(\{y\}$. Since (X, σ) is α-R_1, there are disjoint α-open subsets A, B of X such that $x \in A$, $y \in B$. Thus there are $U_E, V_E \in S\alpha O(X_E)$ such that $A = U(e)$ and $B = V(e)$ for all $e \in E$ with $x_e \widetilde{\in} U_E, y_e \widetilde{\in} V_E$ and $U_E \widetilde{\cap} V_E = \varnothing_E$. So the result holds.

Conversely, let $x, y \in X$ and $x \neq y$ with $cl_\alpha\{x\} \neq cl_\alpha\{y\}$. Then either $x \notin cl_\alpha\{y\}$ *or* $y \notin cl_\alpha\{x\}$. Thus $x_e \notin cl_\alpha(y_e)$ *or* $y_e \notin cl_\alpha(x_e)$, then $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Since (X, τ_σ, E) is $S\alpha$-R_1, there are disjoint soft α-open sets U_E , V_E such that $x_e \widetilde{\in} U_E$, $y_e \widetilde{\in} V_E$. So there are disjoint α-open sets F, G in (X, σ) such that $x \in U(e) = F$ *and* $y \in V(e) = G$, for all $e \in E$. Hence (X, σ) is α-R_1.

(2) The proof of the case (2) is similar. □

Remark 6. If (X, σ) is a discrete topology on X, then we obtain the same results of Proposition 3.11.

The next proposition shows that the $S\alpha$-R_0 and $S\alpha$-R_1 are hereditary.

Proposition 3.13. *Let (X, τ, E) be a soft topological space, we have:*

(1) *if (X, τ, E) is $S\alpha$-R_1, then every soft subspace (Y, τ_Y, E) is $S\alpha$-R_1,*
(2) *if (X, τ, E) is $S\alpha$-R_0, then every soft subspace (Y, τ_Y, E) is $S\alpha$-R_0.*

Proof. (1). Let x_e, y_e are distinct soft points in Y_E with $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Then x_e , y_e are distinct soft points in X_E with $cl_\alpha(x_e) \neq cl_\alpha(y_e)$. Since (X, τ, E) is $S\alpha$-R_1, there are F_E , $G_E \in S\alpha O(X_E)$ such that $x_e \widetilde{\in} F_E$, $y_e \widetilde{\in} G_E$ with $F_E \widetilde{\cap} G_E = \varnothing_E$. Thus there are soft α-open sets $U_E^Y = Y_E \widetilde{\cap} F_E$ and $V_E^Y = Y_E \widetilde{\cap} G_E$ in (Y, τ_Y, E) which are contains x_e, y_e respectively, and $U_E^Y \widetilde{\cap} V_E^Y = \varnothing_E$. So (Y, τ_Y, E) is $S\alpha$-R_1. Similarity of the case (2). □

4. Conclusion.

In this paper, we defined and investigated some new lower separation axioms in soft topological spaces via soft α-open sets. We characterize their basic properties. Some nice results and relations for them are studied. In the next work, we study some properties of soft weakly R_0 and soft weakly α-R_0 spaces.

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